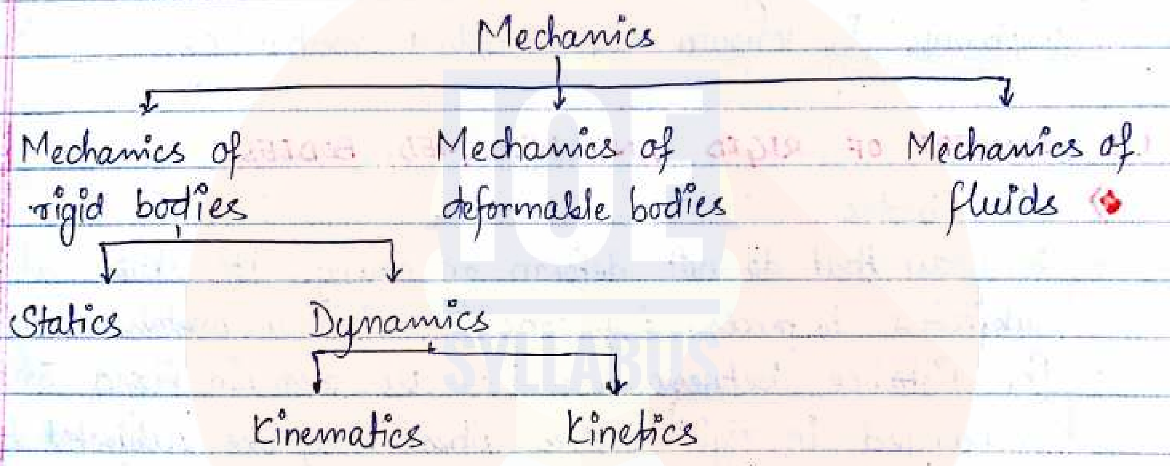
1. Introduction

1.1 DEFENITIONS AND SCOPE OF APPLIED MECHANICS

- Mechanics can be described as that physical science which describes and predicts the conditions of rest or motion of bodies under the action of forces.
- It is divided into three parts: mechanics of rigid bodies, mechanics of deformable bodies and mechanics of fluids.
- The mechanics of rigid bodies is subdivided into statics and dynamics.



Statics
- The branch of mechanics which deals with the forces and theirs effects, while acting upon the bodies at rest.

Dynamics
- The branch of mechanics which deals with the forces and their effects, while acting upon the bodies in motion.

Kinematics
- The branch of dynamics which deals with the bodies in motion without any reference to the forces which are responsible

Applied mechanics acts as bridge between principal theory and its application to technology.

for the motion.

~~Kinema~~

- Kinetics

- The branch of dynamics which deals with the bodies in motion due to the application of forces.

A systematic study of different laws and principles of mechanics along with their applications to engineering problems is known as applied mechanics.

1.2 CONCEPT OF RIGID AND DEFORMED BODIES

- Rigid Bodies
  - A body that do not deform or change its shape when subjected to forces. E.g. concrete, timber, metals, etc.
  - The distance between two particles remain fixed and unchanged in rigid bodies when they are subjected to forces.
  - All the structures and machines are considered rigid from theoritical point of view.

- Deformable Bodies (Elastic Bodies).

  A body that deforms or change its shape and size when subjected to forces. E.g. rubber, plastic, etc.

1.3 FUNDAMENTAL CONCEPTS AND PRINCIPLES OF MECHANICS :

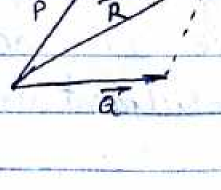
NEWTONIAN MECHANICS

- In Newtonian mechanics, space, time and mass are absolute concepts, independent ~~of~~ each other.(It is not true in relativistic mechanics) and the concept of force is not independent to the other three.
- These concepts cannot be truely defined; they should be accepted on the basis of our intuition (अंतर ज्ञान) and experience and used as a mental frame of reference.
- The concept of space is associated with the notion of the position of a point P by three different lengths (in 3D) measured from a certain reference point, or origin, in three given direction.
- The time defines the ~~occurence~~ instant of occurence of event.
- Mass is a quantity of matter contained in a body.
- Force is push or pull.

The study of elementary mechanics rests on six fundamental principles based on experimental evidence.

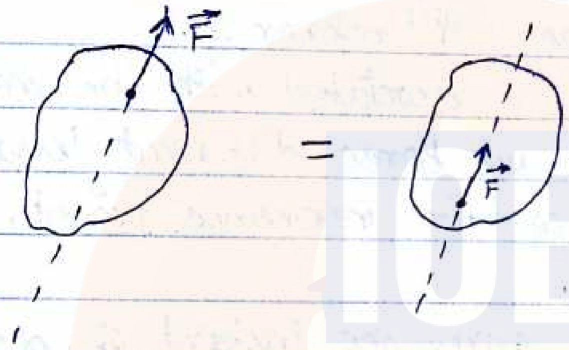
(i) Parallelogram law for Addition of forces.

- It states that two forces acting on a particle may be replaced by a single force, called their resultant, obtained by drawing the diagonal of the parallelogram which has sides equal to the given forces.



(ii) Principle of Transmissibility.

- It states that the condition of equilibrium or of motion of a rigid body will remain unchanged if a force acting at a given point of the rigid body ~~with~~ is replaced by a force of the same magnitude and same direction, but acting at a different point, provided that the two forces have the same line of action.



- This means the action of a force may be transmitted along its line of action.

Newton's Three Fundamental Laws

(iii) First Law

If the resultant force acting on a particle is zero, the particle will remain at rest (if originally at rest) or will move with constant speed in a straight line (if originally in motion).

(iv) Second Law.

If the resultant force acting on a particle is not zero, the particle will have an acceleration proportional to the magnitude of the resultant and in the direction of resultant force.

i.e. $F \propto a$.

$F = ma$. , where,

F = force acting on particle.

a = acceleration of particle.

m = mass of particle.

(v) Third Law.

The forces of action and reaction between bodies in contact have the same magnitude, same line of action and opposite sense.

(vi) Newton's Law of Gravitation.

It states that two particles of mass m_1 and m_2 are mutually attracted by equal and opposite forces F and $-F$ of magnitude F given by formula.

$$F = \frac{G.\, m_1 m_2}{d^2}$$

d = distance between two particles.

G = constant of gravitation. (universal constant).

2. BASIC CONCEPT IN STATICS AND STATIC EQUILIBRIUM

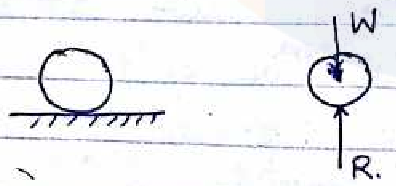
2.1 CONCEPT OF PARTICLES AND FREE BODY DIAGRAM

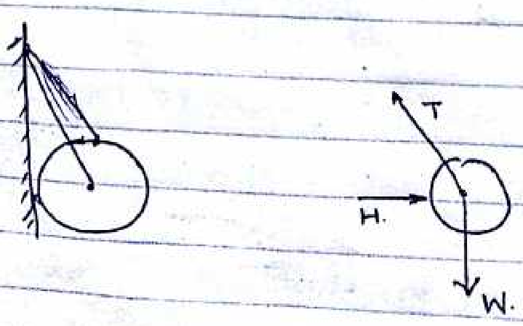
Particle.

- Particle is a material body which is so small that it can be treated as negligible in comparison to other dimension involved in the problem.
- The mass of particle is assumed to be concentrated at a point, and a particle is sometimes called a mass point.
- The use of the word particle does not imply that our study will be limited to that of small corpuscles. What it means is that the size and shape of the bodies under consideration will not affect the solution of the problems and that all the forces acting on a given body will be assumed to be applied at the same point.

Free Body Diagram. (FBD)

- FBD is a graphical illustration used to visualize the applied forces, and resulting reactions on a body in a steady state condition.



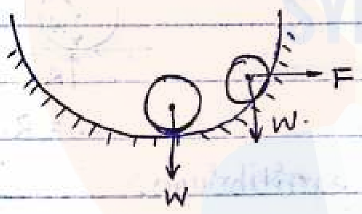


2.2 PHYSICAL MEANING OF EQUILIBRIUM AND ITS ESSENCE IN STRUCTURAL APPLICATION

When a body is acted upon by number of forces, the body is said to be in equilibrium if there is no unbalanced force acting on it i.e. the resultant of all the forces will be zero.
A structure is in equilibrium when all forces or moments acting upon it are balanced.

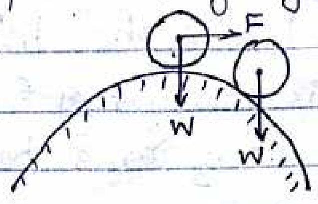
Types of Equilibrium.

a. Stable Equilibrium.
A body is said to be in stable equilibrium when it returns back to its original position if it is slightly displaced from its position of rest.



This is due to ~~the~~ the fact that additional force sets up due to displacement and brings the body back to its original position.

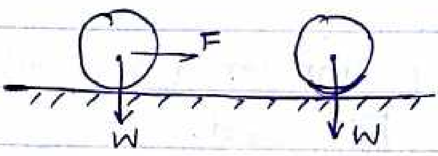
b. Unstable Equilibrium.
A body is said to be in unstable equilibrium when it does not return back to its original position and heels further if it is displaced slightly from its position of rest.



This is due to the fact that the additional force applied on it moves the body away from its position of rest.

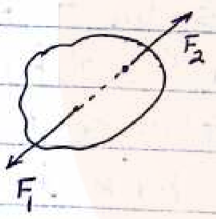
c. Neutral Equilibrium
A body is said to be in equilibrium when it occupies a new position and remains at rest in this new position if it is slightly displaced from position of rest.

This is due to the fact that no additional force sets up due to displacement

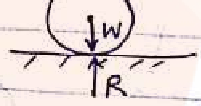


Principle of Equilibrium.

a. Equilibrium of a Two-Force Body.
It states that if a two-force body is in equilibrium, the two forces must have the same magnitude, the same line of action, and opposite sense.



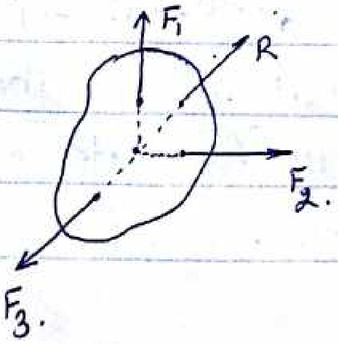
If $F_1 = F_2$, the body is in equilibrium.



R = reaction by ground.

W = R. So, ball is in equilibrium.

b. Equilibrium of a Three-Force Body.
It states that if a body acted upon by three forces is in equilibrium, the resultant of any two forces must have same magnitude, the same line of action but opposite sense (direction) with the third force.



R is resultant of F_1 and F_2.
If $R = F_3$, the body is in equilibrium

Note :- Internal forces are never shown on the free body diagram since they occur in equal but opposite collinear pairs & therefore cancel out.

2.3 EQUATION OF EQUILIBRIUM IN TWO DIMENSION

Equation of Equilibrium

A body is said to be in equilibrium, if the algebraic sum of all the external forces is zero and also algebraic sum of the moments of all the external forces about any point in their plane is zero.

i.e. $\Sigma F = 0$ & $\Sigma M = 0$.

In two dimension,

$$\Sigma F_x = 0. \quad \Sigma F_y = 0.$$
$$\Sigma M = 0.$$

These are called three equations of static equilibrium in two dimension.

PROCEDURE FOR DRAWING FBD.

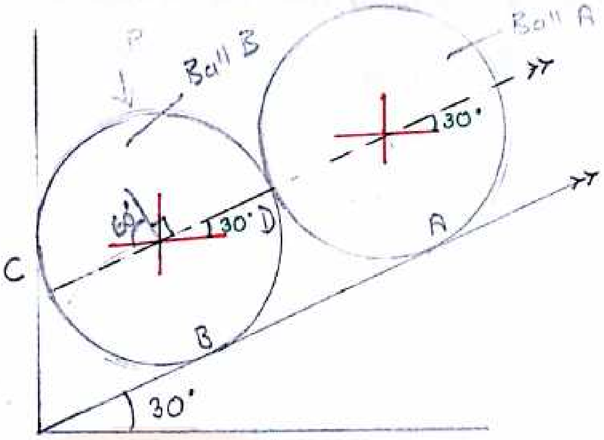
To construct FBD for rigid body / group of bodies considered as a single system, the following prodedures are followed.

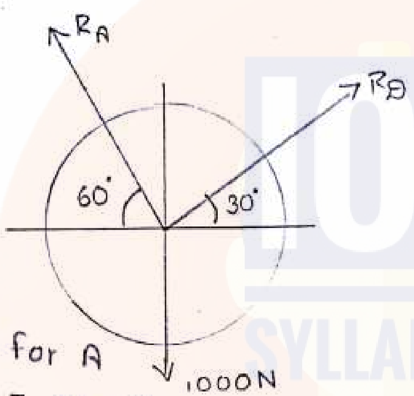
① Draw outlined shape :- Establish x, y coordinate in any suitable orientation. Imagine the body to be isolated or cut free from its constraints and connections and draw (sketch) its outlined shape.

② Show all forces & Couple Moments :- Identify all external forces / couple moments that act on the body. Those generally encountered are due to (i) applied loadings (ii) Reactions occuring at the supports or at points of contact with other bodies & (iii) the wt. of the body.

③ Identify each loading and give dimensions.
The forces & couple moments that are known should be labelled with their proper dimensions & directions.

Question No 1





FBD for A

$\xrightarrow{+}\big]$ $\Sigma F_x = 0$

$\xrightarrow{+}\big]$ $R_D \cos 30° - R_A \cos 60° = 0$

$$R_D = \frac{R_A \cos 60°}{\cos 30°} \quad \text{—} \ (1)$$

$+\uparrow\big]$ $\Sigma F_y = 0$

$+\uparrow\big]$ $R_A \sin 60° + R_D \sin 30° - 1000 = 0$

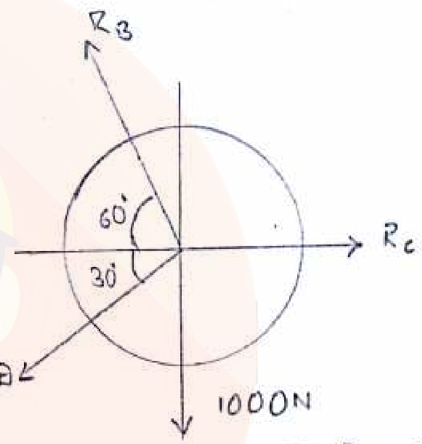
$$R_A \sin 60° + R_A \frac{\cos 60°}{\cos 30°} \times \sin 30° = 1000$$

$$R_A = 866.025 \text{ N}$$

From (1)

$$R_D = 866.025 \times \frac{\cos 60°}{\cos 30°}$$

$$= 500 \text{ N}$$



FBD for B

$\xrightarrow{+}\big]$ $\Sigma F_x = 0$

$$R_c - R_B \cos 60° - R_D \cos 30° = 0$$

$$R_c - R_B \cos 60° = \cancel{433}\ 500 \cos 30°$$

$$R_c - R_B \cos 60° = 433.0127 \quad \text{—} \ (2)$$

$+\uparrow\big]$ $\Sigma F_y = 0$

$$R_B \sin 60° - R_D \sin 30° - 1000 = 0$$

$$R_B \sin 60° - 500 \sin 30° - 1000 = 0$$

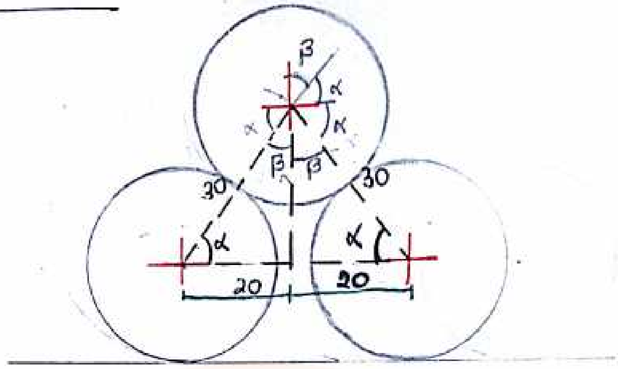
$$R_B = 1443.37 \text{ N.}$$

From (2)

$$R_c = 433.0127 + 1443.37 \cos 60°$$

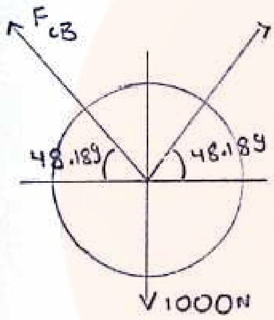
$$= 1154.7 \text{ N.}$$

Question No 2



$$\cos\alpha = \frac{b}{h} \Rightarrow \alpha = \cos^{-1}\left(\frac{20}{30}\right) = 48.189$$

$$\alpha + \beta = 90^\circ \Rightarrow \beta = 90 - \alpha = 90 - 48.189 = 41.8\ldots$$



FBD of C

$\overset{+}{\rightarrow}]\ \Sigma F_x = 0$

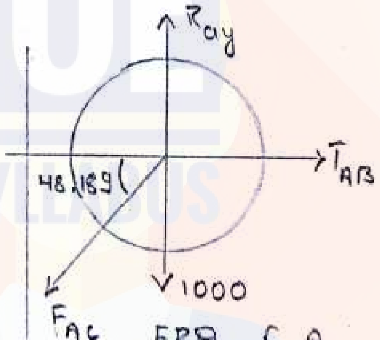
$F_{cB}\cos 48.189 - F_{cA}\cos 48.189 = 0$

$F_{cB} = F_{cA}$

$+\uparrow]\ \Sigma F_y = 0$

$F_{cB}\sin 48.189 + F_{cA}\sin 48.189 - 1000 = 0$

$F_{cB} = F_{cA} = 670.82N$



FBD of A

$F_{AC} = F_{cA} = 670.82N$

$\overset{+}{\rightarrow}]\ \Sigma F_x = 0$

$T_{AB} - F_{AC}\cos 48.189 = 0$

$T_{AB} = 670.82\cos 48.189$

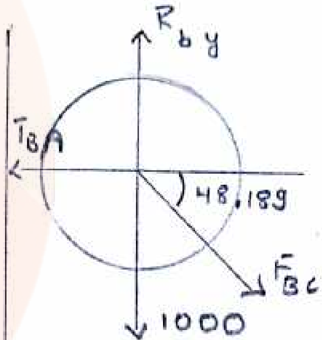
$= 447.224N.$

$+\uparrow]\ \Sigma F_y = 0$

$R_{ay} - F_{AC}\sin 48.189 - 1000 = 0$

$R_{ay} = 1499.99$

$\simeq 1500N.$



FBD of B

$F_{cB} = F_{Bc} = 670.82N$

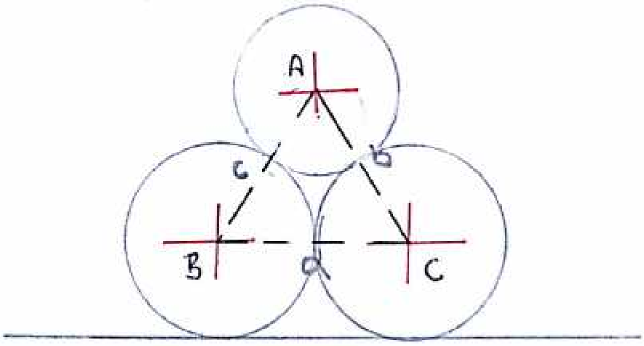
$+\uparrow]\ \Sigma F_y = 0$

$R_{by} - F_{Bc}\sin 48.189 - 1000 = 0$

$R_{by} = 1000 + 670.82\sin 48.189$

$= 1500N$

Question no 3a.



$a = 30 \qquad b = 25 \qquad c = 25$

$\cos A = \dfrac{b^2 + c^2 - a^2}{2bc}$

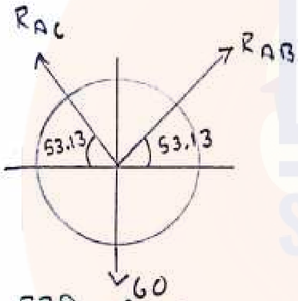
$A = 73.74$

$\cos B = \dfrac{a^2 + c^2 - b^2}{2ac}$

$B = 53.13$

$\cos C = \dfrac{a^2 + b^2 - c^2}{2ab}$

$C = 53.13$



FBD of A

$\xrightarrow{+}] \ \Sigma F_x = 0$

$R_{AB} \cos 53.13 - R_{AC} \cos 53.13 = 0$

$R_{AB} = R_{AC}$ ——— (1)

$+\uparrow] \ \Sigma F_y = 0$

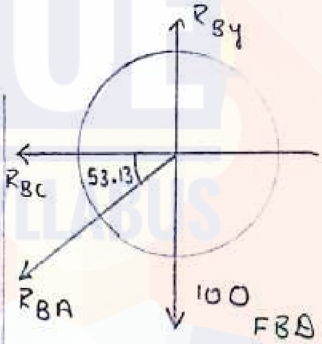
or, $R_{AC} \sin 53.13 + R_{AB} \sin 53.13 - 60 = 0$

or, $R_{AC} \sin 53.13 + R_{AC} \sin 53.13 = 60$

or, $R_{AC} = 37.5 N$

From (1)

$R_{AC} = R_{AB} = 37.5 N.$



FBD of B

$\xrightarrow{+}] \ - R_{BC} - R_{BA} \cos 53.13 = 0$

or, $R_{BC} = -37.5 \cos 53.13$

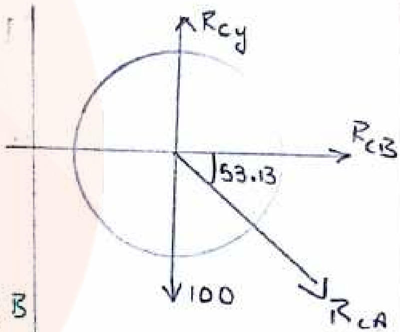
$R_{BC} = -22.5 N.$

$+\uparrow] \ R_{By} - R_{BA} \sin 53.13 - 100 = 0$

or, $R_{By} - 37.5 \sin 53.13 - 100 = 0$

or, $R_{By} = 129.99$

$\approx 130 N.$



FBD of C

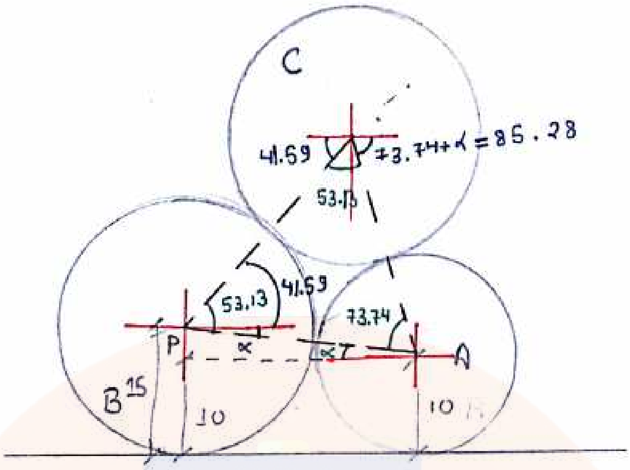
$+\uparrow] \ \Sigma F_y = 0$

$R_{cy} - R_{cA} \sin 53.13 - 100 = 0$

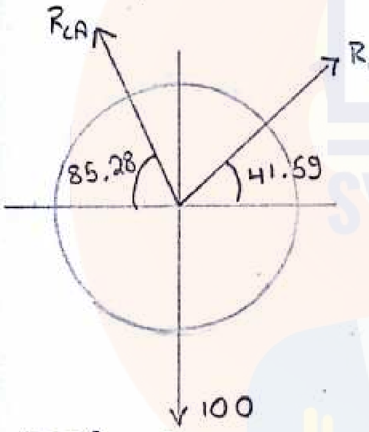
or, $R_{cy} - 37.5 \sin 53.13 - 100 = 0$

or, $R_{cy} = 130 N.$

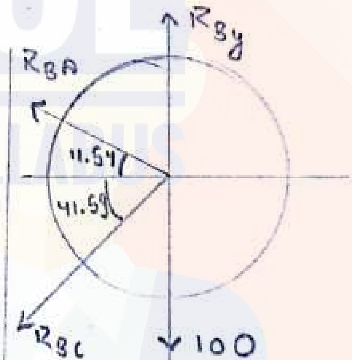
Question No 3 b



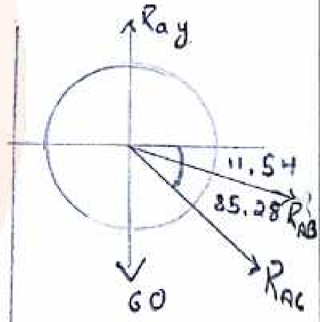
$\sin\alpha = \frac{p}{h} \Rightarrow \alpha = \sin^{-1}\left(\frac{p}{h}\right) \Rightarrow \sin^{-1}\left(\frac{15-10}{15+10}\right)$ $\therefore \alpha = 11.54$



FBD of C



FBD of B



FBD of A

FBD of C

$\xrightarrow{+}\ \Sigma F_x = 0$

or, $R_{CB}\cos 41.59 - R_{CA}\cos 85.28 = 0$

or, $R_{CB} = R_{CA}\dfrac{\cos 85.28}{\cos 41.59}$ ——— ①

$+\uparrow\ \Sigma F_y = 0$

or, $R_{CB}\sin 41.59 + R_{CA}\sin 85.28 - 100 = 0$

or, $R_{CA}\dfrac{\cos 85.28}{\cos 41.59} \times \sin 41.59 + R_{CA}\sin 85.28 = 100$

or, $R_{CA} = 93.489$ N.

From ① $R_{CB} = 10.285$ N

FBD of B

$R_{BC} = R_{CB} = 10.285$ N

$\xrightarrow{+}\ \Sigma F_x = 0$

$R_{BA}\cos 11.54 + R_{BC}\cos 41.59 = 0$

$R_{BA} = -R_{BC}\dfrac{\cos 41.59}{\cos 11.54}$

$= -7.85$ N

$+\uparrow\ \Sigma F_y = 0$

or, $R_{By} + R_{BA}\sin 11.54 - R_{BC}\sin 41.59 - 100 = 0$

or, $R_{By} - 7.85\sin 11.54 - 10.285\sin 41.59 - 100 = 0$

or, $R_{By} = 108.39$ N

FBD of A

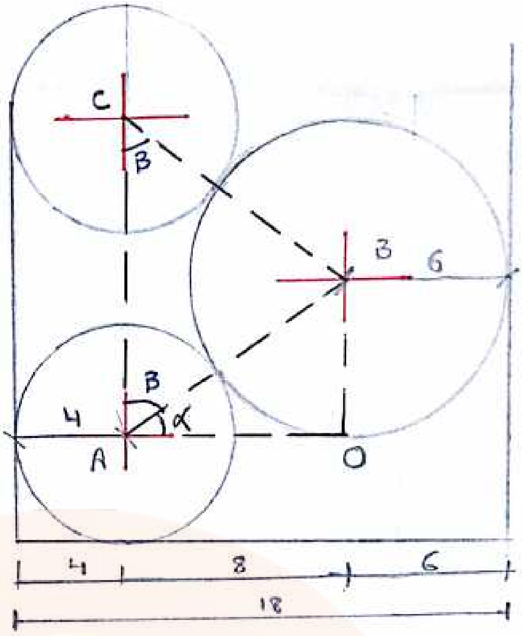
$+\uparrow\ \Sigma F_y = 0$

$R_{ay} - R_{AB}\sin 11.54 - R_{AC}\sin 85.28 - 60 = 0$

or, $R_{ay} + 7.85\sin 11.54 - 93.489\sin 85.28 - 60 = 0$

or, $R_{ay} = 154.74$ N

Question no 4



In Δ ABO

AO = 18 - radius of A - radius of B

= 18 - 4 - 6

= 8

AB = radius of A + radius of B

= 4 + 6

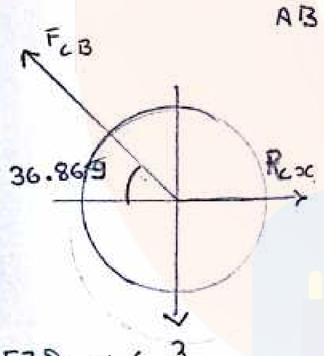
= 10

$\cos\alpha = \frac{b}{h} \Rightarrow \alpha = \cos^{-1}\left(\frac{AO}{AB}\right) = \cos^{-1}\left(\frac{8}{10}\right) \quad \therefore \alpha = 36.869°$

$\beta = 90 - \alpha = 90 - 36.869 = 53.13$

In Δ ABC

AB = CB Hence $\angle BAC = \angle BCA = \beta = 53.13$



FBD of C

$\xrightarrow{+}\sum F_x = 0$

or, $R_{cx} - F_{CB}\cos 36.869 = 0$

or, $R_{cx} = F_{CB}\cos 36.869$ — ①

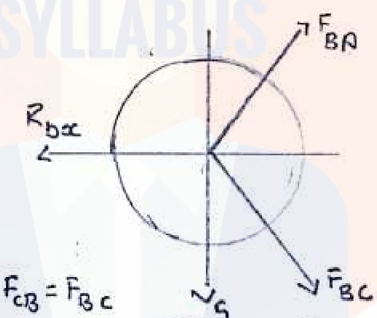
$+\uparrow \sum F_y = 0$

or, $F_{CB}\sin 36.869 - 3 = 0$

or, $F_{CB} = \frac{3}{\sin 36.869}$

or, $F_{CB} = 5\,kN$

From ① $F_{cx} = 4\,kN$



FBD of B

$+\uparrow \sum F_y = 0$

or, $F_{BA}\sin 36.869 - F_{BC}\cos 53.13 - 5 = 0$

or, $F_{BA}\sin 36.869 = 5 + 5\cos 53.13$

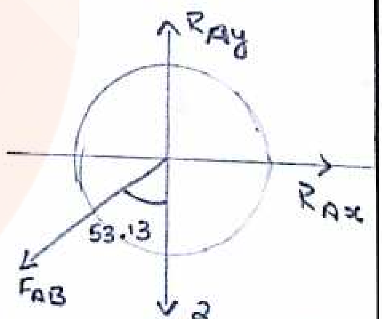
or, $F_{BA} = 13.336\,KN$

$\xrightarrow{+}\sum F_x = 0$

$F_{BA}\cos 36.869 + F_{BC}\sin 53.13 - R_{bx} = 0$

or, $R_{bx} = 13.336\cos 36.869 + 5\sin 53.13$

or, $R_{bx} = 14.667\,KN$



$F_{AB} = F_{BA}$ FBD of A

$+\uparrow \sum F_y = 0$

or, $R_{Ay} - 2 - F_{AB}\cos 53.133 = 0$

or, $R_{Ay} = 2 + 13.33\cos 53.13$

$= 10.0016\,KN$

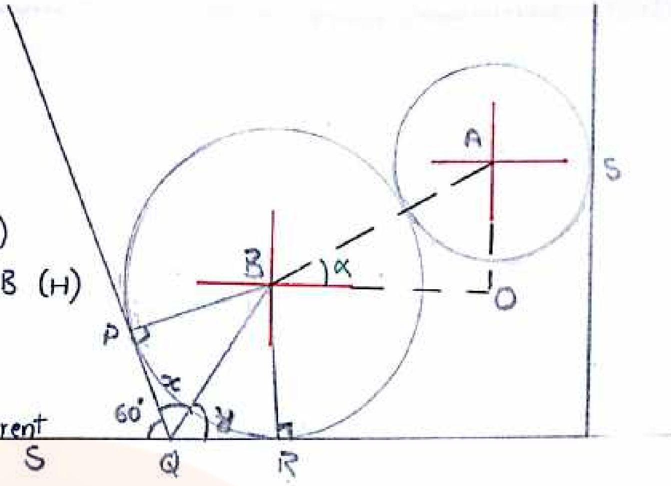
$\xrightarrow{+}\sum F_x = 0$

or, $R_{Ax} - F_{AB}\sin 53.13 = 0$

or, $R_{Ax} = 13.33\sin 53.13$

or, $R_{Ax} = 10.6687\,KN$

Question No 5



In ΔPBQ and ΔQBR

~~QB (common side)~~

$\angle BPQ = \angle BRQ = 90^\circ$ (R)

$BP = BR =$ radius of circle B (H)

$QB =$ common side (S)

By RHS theorem

ΔPBQ and ΔQBR concurent

Hence, $\angle x = \angle y$

In a straight line, $\angle PQS + \angle PQB + \angle BQR = 180^\circ$

or, $60 + x + y = 180^\circ$

or, $x + x = 180 - 60$

or, $2x = 120$

$\therefore x = 60^\circ$

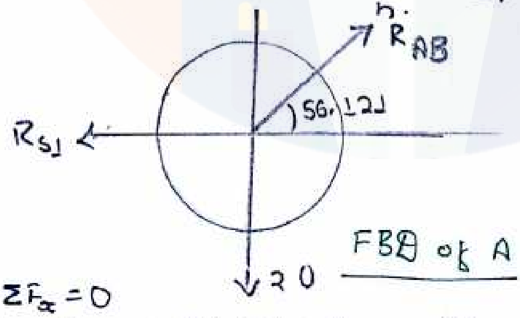
In ΔBQR $\tan 60^\circ = \dfrac{QR}{BR}$ => $QR = \tan 60 \times 9 = 5.196$

Hence $QR + BO + AS = 18$. (channel bottom)

or, $BO = 18 - 5.19 - 5 = 7.804$.

In ΔBAO

$\cos\alpha = \dfrac{b}{h}$ => $\alpha = \cos^{-1}\left(\dfrac{7.804}{14}\right) = 56.121^\circ$



FBD of A

$\xrightarrow{+}\rfloor \ \Sigma F_x = 0$

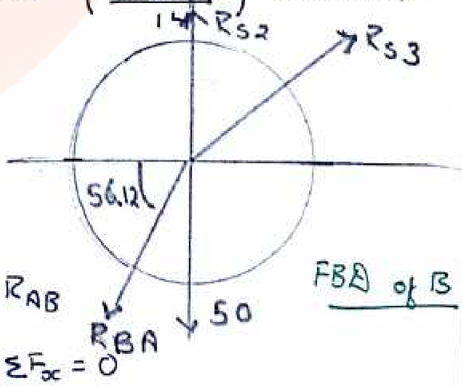
or, $R_{S1} - R_{AB}\cos 56.121 = 0$ —— ①

$+\uparrow\rfloor \ \Sigma F_y = 0$

$R_{AB}\sin 56.121 - 20 = 0$

$R_{AB} = 24.09$ kg

From eq^n ① $R_s = 13.42$ kg



FBD of B

$R_{BA} = R_{AB}$

$\xrightarrow{+}\rfloor \ \Sigma F_x = 0$

$R_{S3}\cos 30^\circ - R_{BA}\cos 56.121 = 0$

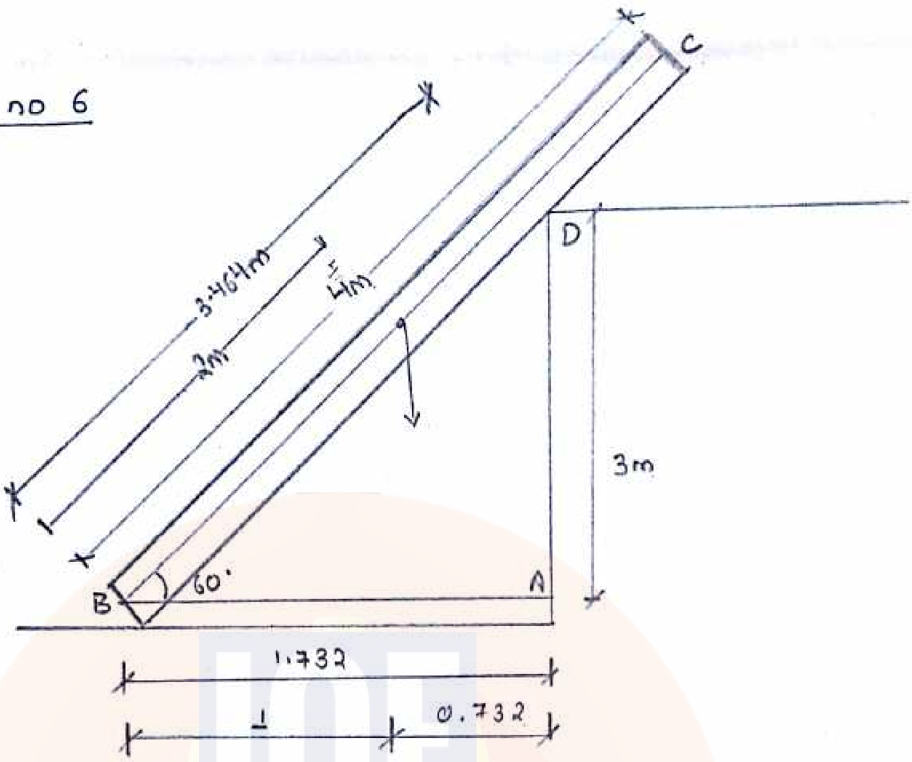
$R_{S3} = 15.506$ kg.

$+\uparrow\rfloor \ \Sigma F_y = 0$

or, $R_{S2} + R_{S3}\sin 30^\circ - R_{BA}\sin 56.121 - 50 = 0$

or, $R_{S2} = 62.2469$ kg

Question no 6



$\sin 60° = \frac{p}{h} = \frac{3}{h}$ or, $h = 3.464$ m.

$\tan 60° = \frac{p}{b} = \frac{3}{b}$ or, $b = 1.732$ m.

$\cos 60° = \frac{b}{h} = \frac{b}{2}$ or, $b = 1$ m.

weight acts from center of scale at 2m length.

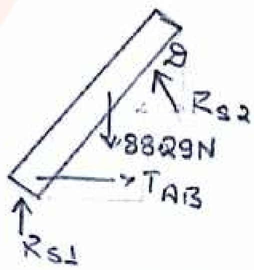
Taking moment at D

$+\circlearrowleft] \; M_D = 0$

or, $R_{S1} \times 1.732 - T_{AB} \times 3 - 88.29 \times 0.732 = 0$

or, $R_{S1} = \frac{3T_{AB} + 64.28}{1.732}$

or, $R_{S1} = 1.732T_{AB} + 37.314$



$+\uparrow] \; \Sigma F_y = 0$

or, $R_{S1} + R_{S2} \sin 30° - 88.29 = 0$

or, $1.732 T_{AB} + 37.314 + R_{S2} \sin 30° = 88.29$

or, $R_{S2} = \frac{50.976 - 1.732 T_{AB}}{\sin 30°}$

or, $R_{S2} = 101.952 - 3.464 T_{AB}$

$\xrightarrow{+}] \; \Sigma F_x = 0$

$T_{AB} - R_{S2} \cos 30° = 0$

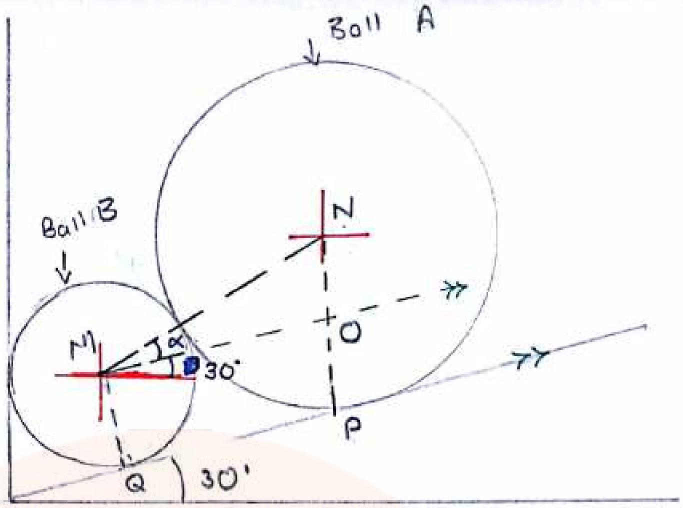
or, $T_{AB} - 101.952 \cos 30° + 3.464 T_{AB} \cos 30° = 0$

or, $T_{AB} + 2.999 T_{AB} = 88.293$

or, $3.99991 T_{AB} = 88.293$

or, $T_{AB} = 22.07$ N.

Question no' 7



In Δ MNO

$NO = NP - OP$

$= NP - MQ$

$= 150 - 100$

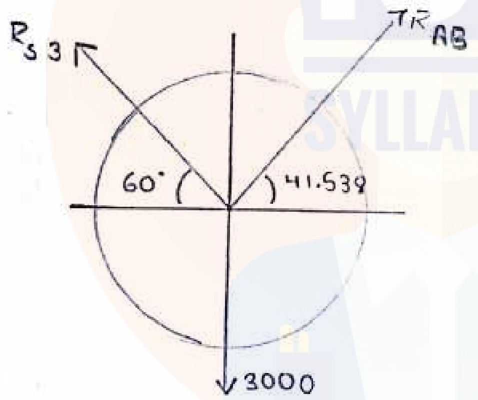
$= 50.$

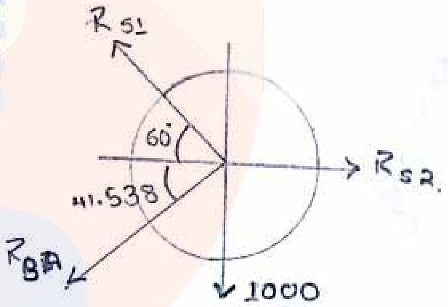
$NM = 100 + 150$

$= 250$

$\sin\alpha = \frac{NO}{MN}$. $\Rightarrow \alpha = \sin^{-1}\left(\frac{50}{250}\right)$ $\therefore \alpha = 11.53$

Hence, $\alpha + 30 = 11.53 + 30 = 41.53$.





$\pm\!\to]\ \Sigma F_x = 0$

or, $R_{AB}\cos 41.538 - R_{s3}\cos 60° = 0$

or, $R_{AB} = R_{s3}\dfrac{\cos 60°}{\cos 41.538}$ —— (1)

$+\uparrow]\ \Sigma F_y = 0$

or, $R_{AB}\sin 41.538 + R_{s3}\sin 60° - 3000 = 0$

or, $R_{s3}\dfrac{\cos 60°}{\cos 41.538} \times \sin 41.538 + R_{s3}\sin 60° - 3000 = 0$

or, $R_{s3} = 2291.861$ N.

From (1) $R_{AB} = 1530.936$ N.

$+\uparrow]\ \Sigma F_y = 0$

$R_{s1}\sin 60° - R_{BA}\sin 41.538 - 1000 = 0$

or, $R_{s1}\sin 60° = 1530.936\sin 41.538 + 1000$

or, $R_{s1} = 2326.8943$ N.

$\pm\!\to]\ \Sigma F_x = 0$

or $R_{s2} - R_{s1}\cos 60° - R_{AB}\cos 41.56 = 0$

or, $R_{s2} = 2326.8943\cos 60° + 1530.936\cos 41.56$

or, $R_{s2} = 2309.41$ N

Question no 8

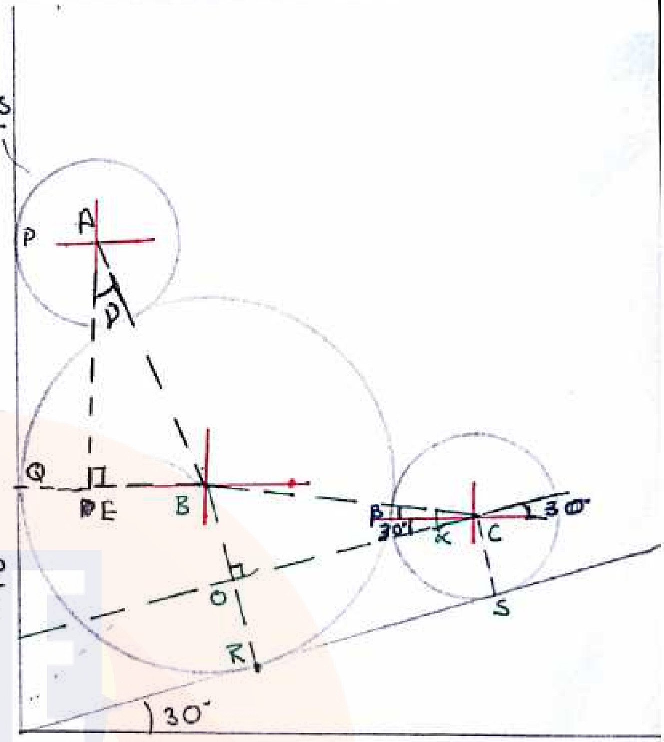
In triangle BOC

$\sin\alpha = \frac{BO}{BC} = \frac{BR-OR}{BC} = \frac{BR-CS}{BC}$

$\sin\alpha = \frac{60-40}{60+40}$

$\alpha = \sin^{-1}\left(\frac{20}{100}\right) = 11.53$

Hence $\beta = 30 - \alpha = 30 - 11.53$
$= 18.47$

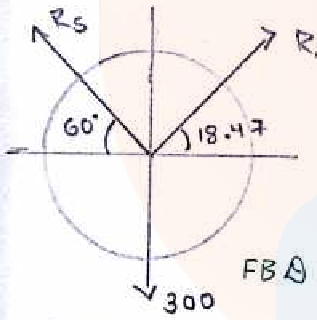


In Δ ABE

$\sin\gamma = \frac{BE}{AB} = \frac{QB-QE}{AB} = \frac{QB-AP}{AB}$

or, $\gamma = \sin^{-1}\left(\frac{60-40}{60+40}\right)$

or, $\gamma = 11.53°$



FBD of C

$\xrightarrow{+}\; \Sigma F_x = 0$

$\xrightarrow{+}\; -R_s\cos 60° + R_{CB}\cos 18.48 = 0$

$R_{CB} = R_s \frac{\cos 60°}{\cos 18.48}$ ——— (1)

$+\uparrow\; \Sigma F_y = 0$

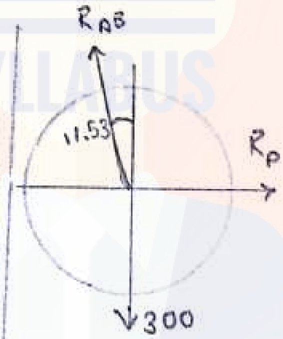
$R_s \sin 60° + R_s \frac{\cos 60°}{\cos 18.47} \times \sin 18.47$

$- 300 = 0$

$R_s = 290.4407$ N

From (1)

$R_{CB} = 153.089$ N.



FBD of A

$+\uparrow\; \Sigma F_y = 0$

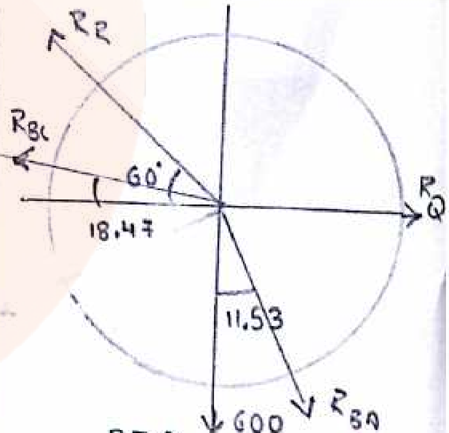
$R_{BA}\cos 11.53 - 300 = 0$

or, $R_{BA} = 306.178$ N

$\xrightarrow{+}\; \Sigma F_x = 0$

$R_{BA}\sin 11.53 - R_p = 0$

$R_p = 61.19$ N



FBD of B

$\xrightarrow{+}\; R_Q - R_{BC}\cos 18.47 + R_{BA}\sin 11.53 - R_R\cos 60° = 0$ ——— (2)

$+\uparrow\; \Sigma F_y = 0$

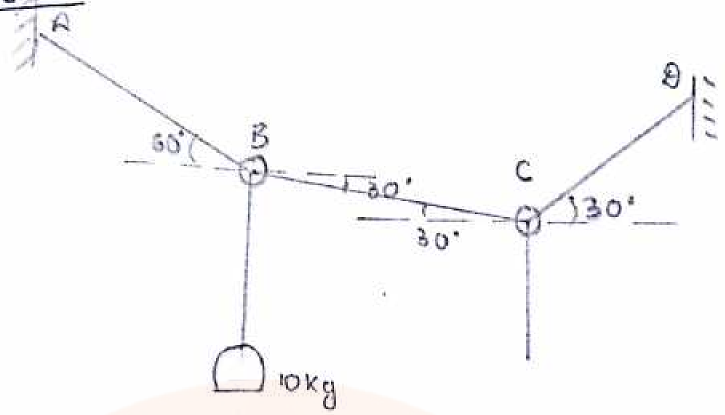
or, $R_R \sin 60° - R_{BC}\sin 18.47 - R_{BA}\cos 11.53 - 600 = 0$

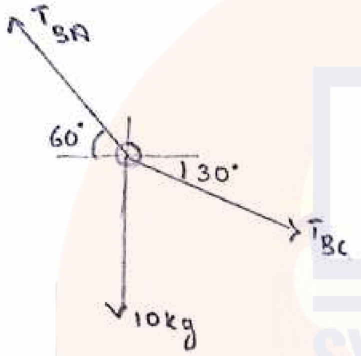
or, $R_R \sin 60° - 153.089 \sin 18.47 - 306.178\cos 11.53 - 600 = 0$

or, $R_R = 1095.23$

From (2) $R_Q = 631.62$ N

Question no. 9





$\overset{+}{\rightarrow}$ $\sum F_x = 0$

$T_{BC} \cos 30° - T_{BA} \cos 60° = 0$

$T_{BC} = T_{BA} \dfrac{\cos 60°}{\cos 30°}$ — (1)

$+\uparrow$ $\sum F_y = 0$

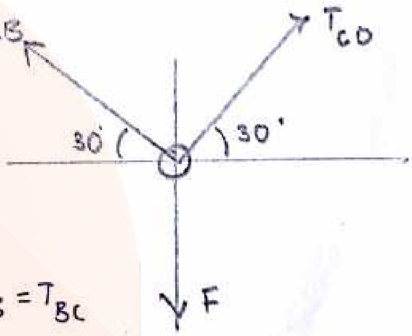
or, $T_{BA} \sin 60° - T_{BC} \sin 30° - 10 = 0$

or, $T_{BA} \sin 60° - T_{BA} \dfrac{\cos 60°}{\cos 30°} \times \sin 30° - 10 = 0$

or, $T_{BA} = 17.32$ kg

From (1)

$T_{BC} = 10$ kg.



$T_{CB} = T_{BC}$

$\overset{+}{\rightarrow}$ $\sum F_x = 0$

$T_{CD} \cos 30° - T_{CB} \cos 30° = 0$

$T_{CD} = T_{CB} = 10$ kg.

$+\uparrow$ $\sum F_y = 0$

or, $T_{CB} \sin 30° + T_{CB} \sin 30° - F = 0$

or, $T_{CB} \sin 30° + T_{CB} \sin 30° - F = 0$

or, $2 T_{CB} \sin 30° = F$

or, $F = 2 \times 10 \times \sin 30°$

or, $F = 10$ kg.

3. FORCES ACTING ON PARTICLE AND RIGID BODY

3.1 Different Types of forces (Moments).

a. External forces and Moments.

b. Reaction forces and Moments.

c. Internal forces and Moments.

a. External forces and Moments

Forces and moments that are applied on the body and are often ~~offer~~ referred to as the load on the body. They are assumed to be known in an analysis.

(i) Surface forces and Moments.

External forces (moments) that act on the surface and are transmitted ~~by contact~~ to the body by contact.

- Surface forces (moments) applied at a point are called concentrated forces (moment).
- Surface forces (moments) applied along a line or over a surface are called distributed forces (moments).

(ii) • Body forces are external forces that act at every point on the body. Body forces are not transmitted by contact. Gravitational forces & electromagnetic forces are two examples of body forces. Its unit is force per unit volume.

b. Reaction forces and Moments.

Forces and moments that are developed at supports of a body to resist movement due to external forces (moments). These reaction forces (moments) are usually not known and must be calculated.

Three principles are used to decide whether there is a reaction force (moment) at the support :

(i) If a point cannot move in a given direction, then a reaction force opposite to the direction acts at that support point.

(ii) If a line cannot rotate about an axis in a given direction, then a reaction moment opposite to the direction acts at that support.

(iii) The support in isolation but not entire body is considered in making decisions about the movement of a point / rotation of line ~~at~~ the support.

c. Internal forces (Moments)

The force that holds a body together that exists irrespective of whether or not we apply external forces is called internal forces (moment).

The material resists changes due to applied forces and moments by increasing the internal forces.

3.2 RESOLUTION AND COMPOSITION OF FORCES

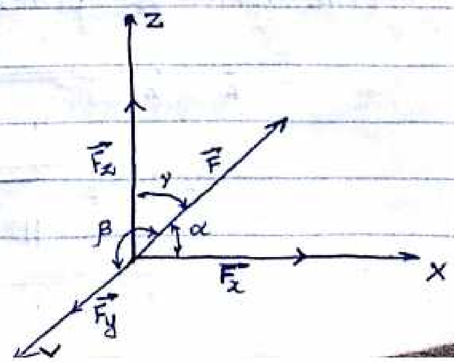
→ Resolution of a force is a process of splitting up the given force into a number of components without changing its effect on the body.

→ Generally a force is resolved into two components (in 2D), i.e. along mutually perpendicular directions.

→ The broken two (in 2D) parts of a force are known as component forces.

The force $\vec{F}$ can be resolved as,

$$\vec{F} = F_x\vec{i} + F_y\vec{j} + F_z\vec{K}$$



where F_x, F_y and F_z are the x, y and z component of F.

$$F_x = \vec{F}.\vec{i} = F\cos\alpha$$
$$F_y = \vec{F}.\vec{j} = F\cos\beta$$
$$F_z = \vec{F}.\vec{K} = F\cos\gamma$$

$$|\vec{F}| = \sqrt{F_x^2 + F_y^2 + F_z^2}$$

Here,
$\cos\alpha$, $\cos\beta$ and $\cos\gamma$ are the direction cosines of the line of action of force $\vec{F}$.

$$\cos\alpha = \frac{F_x}{|\vec{F}|} = \frac{F_x}{\sqrt{F_x^2+F_y^2+F_z^2}}$$
$$\cos\beta = \frac{F_y}{|\vec{F}|} = \frac{F_y}{\sqrt{F_x^2+F_y^2+F_z^2}}$$
$$\cos\gamma = \frac{F_z}{|\vec{F}|} = \frac{F_z}{\sqrt{F_x^2+F_y^2+F_z^2}}$$

So,
$$\vec{F} = F\cos\alpha\,\vec{i} + F\cos\beta\,\vec{j} + F\cos\gamma\,\vec{K}$$

If $\vec{F} = |\vec{F}|.\hat{u}$

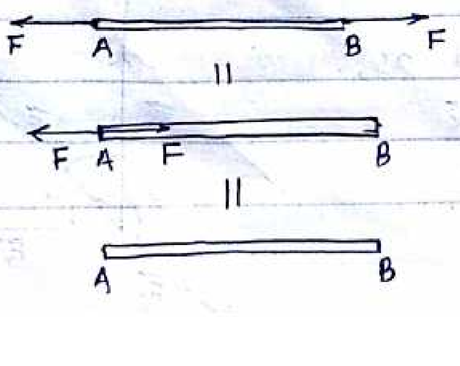
where $\hat{u} = \cos\alpha\,\hat{i} + \cos\beta\,\hat{j} + \cos\gamma\,\hat{k}$

3.3 PRINCIPLE OF TRANMISSIBILITY AND EQUIVALENT FORCES

→ Principle of Transmissibility.

- Refer Section 1.3 (ii).

Limitation of principle of Transmissibility.



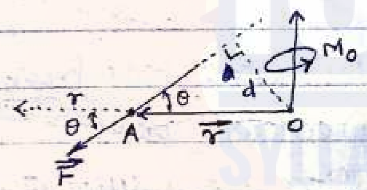
Theis principle of transmissibility is applicable only to the rigid body in which we neglect the internal effects. So we consider rigid body only in applied mechanics.

3.4 MOMENTS AND COUPLES

Moment of Force. About a Point.

- The turning effect of a force applied to a rotational system at a distance from the axis of rotation.
- The moment is equal to the magnitude of the force multiplied by the perpendicular distance between its line of action & the axis of rotation.
- The moment of a force F about point O can be expressed using vector cross product as.

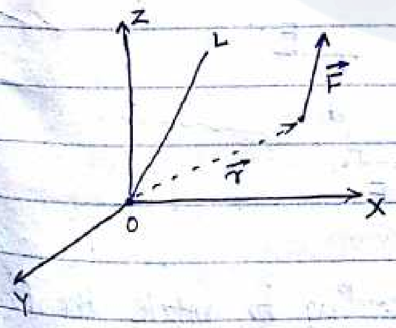
$$\vec{M}_0 = \vec{r} \times \vec{F}.$$
$$= |\vec{r}||\vec{F}| \sin\theta.$$
$$= |\vec{F}||\vec{r}| \sin\theta$$
$$= Fd. \text{ (magnitude)}.$$



Direction: The direction & sense of M_0 are determined by the right-hand rule as it applies to the vector cross product.

Moment of force About An Axis.

- Moment of force about an axis is just the projection of moment on that axis. Thus it is scalar quantity.



Let OL be any arbitrary axis through O. A force $\vec{F} = f_x\vec{i} + f_y\vec{j} + f_z\vec{K}$ is acted at a point at a position vector $\vec{r}$ as in figure.

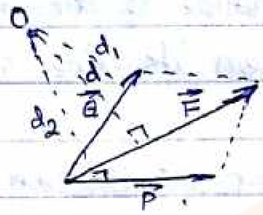
Then, moment of $\vec{F}$ about O is $\vec{M}_0 = \vec{r} \times \vec{F}$.

Moment about OL $(M_{OL}) = \vec{\lambda} \cdot \vec{M}_0$.

$\vec{\lambda} = \lambda_x\hat{i} + \lambda_y\hat{j} + \lambda_z\hat{k}$ is unit vector along OL.)

Varignon's Theorem. (Principle of Moments).

It states that the moment of a resultant of two concurrent forces about any point is equal to the algebraic sum of the moments of its components about the same point.



$\vec{F}$ is the resultant of $\vec{P}$ & $\vec{Q}$.
O is a point about which moment is being taken.

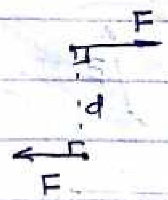
Moment of resultant force $\vec{F} = Fd$.
Moment of its components $= Pd_1 + Qd_2$.
So,

$$\boxed{Fd = Pd_1 + Pd_2}$$

Couple.

It is defined as two parallel forces that have the same magnitude, have opposite directions, and are separated by a perpendicular distance 'd'. Since the resultant force is zero, the only effect of a couple is to produce a rotation or tendency of rotation in a specified direction.

The moment produced by a couple is called couple moment.
i.e. $M = r \times F$.

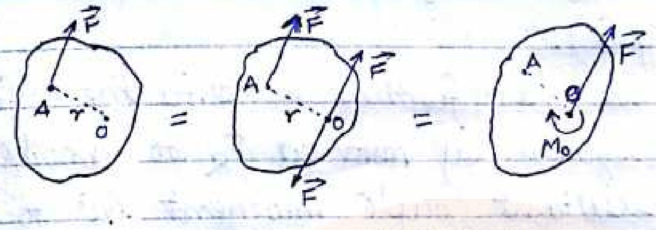


Types of Couple.

(i) Clockwise Couple :- Any couple tending to rotate the body in clockwise direction.

ii. Anticlockwise Couple :- Any couple tending to rotate the body in anticlockwise direction.

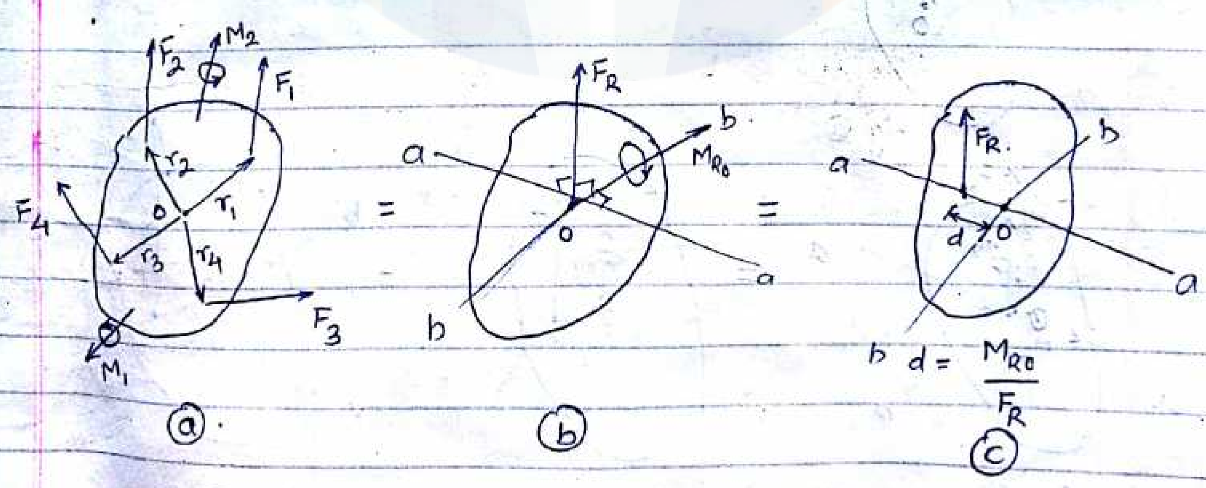
3.5 RESOLUTION OF A FORCE INTO FORCES AND A COUPLE



Any force F acting on a rigid body can be moved to an arbitrary point O provided that a couple is added whose moment is equal to the moment of F about O.

- Couple is represented by couple vector $\overrightarrow{M_0}$ perpendicular to the plane containing $\vec{r}$ and $\vec{F}$. Since couple is a free vector, it may be applied anywhere; for convenience, however the couple vector is usually attached at O, together with $\vec{F}$ and combination obtained is referred to as force couple system.
- The reverse of the theorem is also true.

3.6 RESULTANT OF FORCE AND MOMENT FOR A SYSTEM OF FORCES



Consider a special case for which the system of forces and couple moments acting on a rigid body (fig ⓐ) reduces ~~to~~ at point O to a resultant force $F_R = \Sigma F$ and resultant couple moment $M_{RO} = \Sigma M_O$, which are perpendicular to one another (fig ⓑ).

In such cases, we can further simplify the force & couple moment system by moving F_R to another point P so that no resultant couple moment has to be applied to the body (fig ⓒ).

Point P must lie on the axis which is perpendicular to both line of action of F_R and line of action of M_{RO} (i.e. b-b axis in this case).

The distance d of point P from O is :-

$$d = \frac{M_{RO}}{F_R}$$

REDUCTION TO A WRENCH

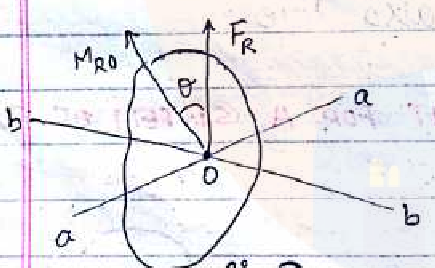
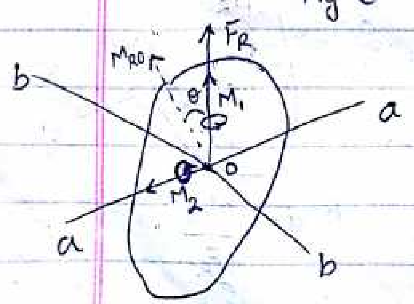


Fig ⓐ



Fib ⓑ

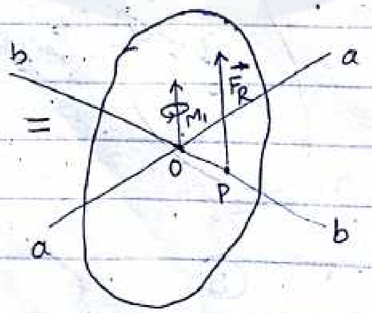


Fig ⓒ

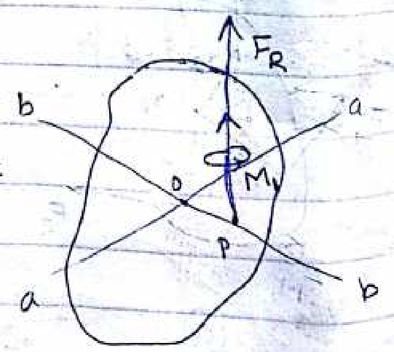
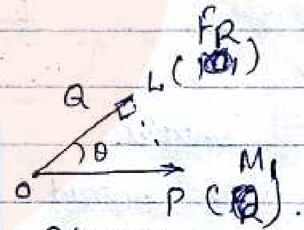


Fig ⓓ.

The resultant force F_R and couple moment M_{RO} at O are not perpendicular in general cases. They act at an angle θ. (fig ⓐ).
M_{RO} is resolved into two components : one perpendicular (M_2) and the other parallel (M_1) to line of action of F_R. (fig ⓑ).
As we know, the perpendicular moment (M_2) may be eliminated by moving F_R to point P (fig ©).
The point P lies at distance $d = \frac{M_2}{F_R}$ and perpendicular to both M_1 and F_R (i.e. b-b axis).
Since M_1 is a free vector, it may be moved to P so that it is collinear with F_R

- Thus, combination of collinear force and couple moment is called wrench or screw.
- The axis of the wrench has the same line of action as the force. Hence wrench tends to cause both a translational along and a rotation about this axis.
- The ratio $p = \frac{M_1}{F_R}$ is called pitch of wrench.

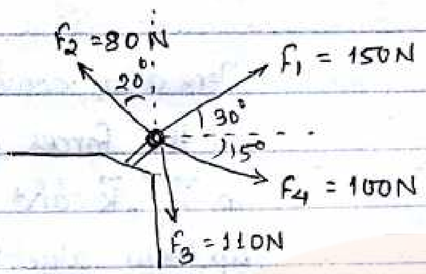
$M_1 = \frac{F_R \cdot M_O^R}{F_R}$. i.e. $p = \frac{F_R \cdot M_O^R}{F_R^2}$.



$P \cdot Q = PQ\cos\theta$.
$= QP\cos\theta$

$\frac{P \cdot Q}{Q} = P_{OL}$.

1. Four forces act on bolt as shown. Determine the resultant of the forces on the bolt.



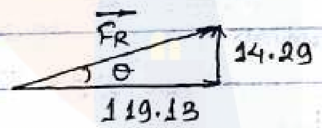
Note :- To determine resultant of forces or moments, write them in terms of $\vec{i}, \vec{j}, \vec{k}$ and then add them algebraically

Solution :-

Resultant force $(\vec{F_R}) = (150\cos 30° - 80\sin 20° + 100\cos 15°)\vec{i} + (150\sin 30° + 80\cos 20° - 110 - 100\sin 15°)\vec{j}$

$$\boxed{\vec{F_R} = 199.13\vec{i} + 14.29\vec{j}}$$ Answer.

Further if we are interested in magnitude and direction of the $\vec{F_R}$, visualize this way :-



from pythagoras rule,

$$\vec{F_R} = \sqrt{119.13^2 + 14.29^2}$$

$$= 119.984 \text{ N (magnitude)}.$$

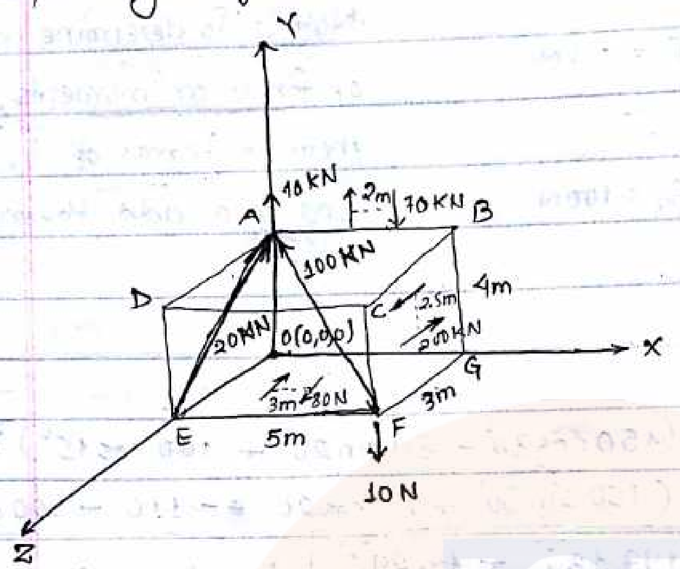
$$\theta = \tan^{-1}\left(\frac{14.29}{119.13}\right) \quad \left(\text{since } \tan\theta = \frac{p}{b}\right).$$

$\theta = 4.1°$ with horizontal direction (direction).

But,

$\vec{F_R} = 199.13\vec{i} + 14.29\vec{j}$ is sufficient for solution.

2. Determine the resultant force and moment of the following system about point 'O'.



Here also, convert or write all the forces and moments in $\vec{i}, \vec{j}, \vec{k}$ and then sum up them algebraically.

Here,

$\vec{F_A} = 10\vec{j}$
$\vec{F_F} = -10\vec{j}$ [Since they are parallel to Y-axis.]

$$\vec{F_{EA}} = 20 \times \underbrace{\frac{\vec{EA}}{|\vec{EA}|}}_{\text{magnitude} \times \text{unit vector along } \vec{EA}}$$

$$= 20 \times \frac{\vec{OA} - \vec{OE}}{|\vec{EA}|}$$

coordinate of A, coordinate of E.

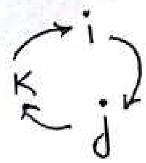
$$= \frac{20 \times [(4\vec{j}) - 3\vec{K}]}{\sqrt{4^2 + 3^2}}$$

$$\boxed{\vec{F_{EA}} = 16\vec{j} - 12\vec{K}}$$

$$\vec{F_{AFA}} = 100 \times \frac{\vec{AFA}}{|\vec{AFA}|}$$

$$= 100 \times \frac{\vec{OA} - \vec{OF}}{|\vec{FA}|}$$

$$= 100 \times \frac{[4\vec{j} - (5\vec{i} + 3\vec{K})]}{\sqrt{(-1)^2 + 3^2}} = -31.62\vec{i} + 94.87\vec{K}$$



$$\boxed{\vec{F}_{FA} = -31.62\vec{i} + 94.87\vec{k}}$$

Now, to write couples in terms of $\vec{i}, \vec{j}, \vec{k}$, we have to use 'Right Hand Thumb Rule'.

'Right Hand Thumb Rule'

- Place four curling fingers of right hand in the direction where body is rotating or tending to rotate. Then the direction of thumb gives direction of moment.

E.g.

Couple moment $= 70 \times 2 = 140$ Nm is in XY plane and rotating in clockwise direction. So place curling fingers of right hand in XY plane. While doing this, thumb of right hand points towards $-$ve Z-axis. So, the couple moment $(\vec{C_1}) = -140\vec{k}$

Similarly, using right hand thumb rule

$\vec{C_2} = 500\vec{i}$

$\vec{C_3} = -240\vec{j}$.

So,

Resultant force $(\vec{F_R}) = 10\vec{j} - 10\vec{j} + 16\vec{j} - 12\vec{k} - 31.62\vec{i} + 94.87\vec{k}$

$= (-31.62\vec{i} + 16\vec{j} + 82.87\vec{k})$ KN <u>Ans</u>.

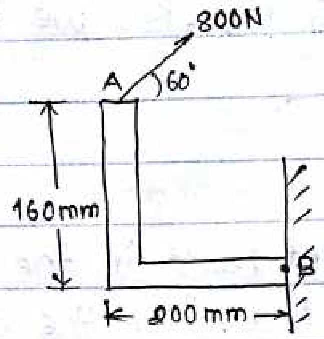
Resultant moment at O $(\vec{M_O^R}) = -140\vec{k} + 500\vec{i} - 240\vec{j} + (\vec{r_{OE}} \times \vec{F_{EA}}) + (\vec{r_{OF}} \times \vec{F_{FA}}) + \vec{r_{OF}} \times \vec{F_C}$

or, $\vec{M_O^R} = -140\vec{k} + 500\vec{i} - 240\vec{j} + 3\vec{k} \times (16\vec{j} - 12\vec{k}) + (5\vec{i} + 3\vec{k}) \times (-31.62\vec{i} + 94.87\vec{k}) + (5\vec{i} + 3\vec{k}) \times (-10\vec{j})$

or, $\vec{M_O^R} = -140\vec{k} + 500\vec{i} - 240\vec{j} - 48\vec{i} - 474.35\vec{j} - 158.1\vec{j} - 50\vec{k} + 30\vec{i}$

$\vec{M_O^R} = (482\vec{i} - 872.45\vec{j} - 190\vec{k})$ KNm. <u>Ans</u>

3. A force of 800N acts on the bracket as shown. Determine the moment of force about B.



200mm = 0.2m.

Coordinate of A is $-0.2\vec{i} + 0.16\vec{j}$

So, $\vec{r}_{A/B} = -0.2\vec{i} + 0.16\vec{j}$.

Now,

$$\vec{M}_B = \vec{r}_{A/B} \times \vec{F}$$

$$= (-0.2\vec{i} + 0.16\vec{j}) \times (800\cos60^\circ\vec{i} + 800\sin60^\circ\vec{j})$$

$$= -138.564\vec{K} - 64\vec{K}$$

$$\boxed{\vec{M}_B = -202.564\vec{K}.}$$ Ans

Understanding!

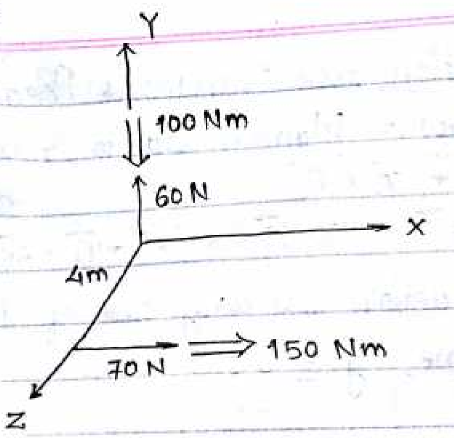
Moment is also a vector. So, it has both magnitude and direction. It's magnitude is 202.564 Nm and direction is $-\vec{K}$ i.e. perpendicular to plane of paper inwards.

Using Right Hand Thumb Rule, pointing thumb of right hand towards $-\vec{K}$ direction, the four curling finger is in ↻ direction. That means the moment $\vec{M}_B$ rotates or tends to rotate the bracket in clockwise (↻) direction.

Conclusion :- All these informations are hidden inside $\boxed{M_B = -202.564\vec{K}}$.

To determine line of action of resultant, or position of resultant, always use Varignon's Theorem.

4. V. Imp



Two wrenches are shown in figure, determine the equivalent wrench and indicate it's line of action.

Solution :-

Step 1 :-

$\vec{M} = \vec{r} \times \vec{F}$

Resultant of forces $(\vec{F_R}) = 70\vec{i} + 60\vec{j}$

Resultant of moments about O $(\vec{M_0^R}) = -100\vec{j} + 150\vec{i} + (4\vec{k} \times 70\vec{i})$

$= 180\vec{j} + 150\vec{i}$

Step 2 :-

$$\text{pitch } (p) = \frac{\vec{F_R} \cdot \vec{M_0^R}}{|\vec{F_R}|^2}$$

Resultant को magnitude को वर्ग square

$$= \frac{(70\vec{i} + 60\vec{j}) \cdot (180\vec{j} + 150\vec{i})}{\left(\sqrt{70^2 + 60^2}\right)^2}$$

$$= \frac{10500 + 10800}{8500}$$

$$= 2.51$$

Since pitch is a distance, if -ve value is encountered, proceed with its ~~va~~ +ve value.

Step 3 :-

Couple moment $(\vec{M_1}) = p . \vec{F_R}$

$= 2.51 . (70\vec{i} + 60\vec{j})$

$= 175.7\vec{i} + 150.6\vec{j}$.

So, resultant wrench is $\boxed{\begin{aligned} \vec{F_R} &= 70\vec{i} + 60\vec{j} \\ \& \ \vec{M_1} &= 175.7\vec{i} + 150.6\vec{j} \end{aligned}}$ Ans

Step 4:

To determine line of action, use Varignon's Theorem,

Σ(Moment due to component) = Moment due to resultant.

$$\vec{M_0^R} = \vec{M_1} + \vec{r} \times \vec{F_R}$$

$$150\vec{i} + 180\vec{j} = (175.7\vec{i} + 150.6\vec{j}) + (x\vec{i} + y\vec{j} + z\vec{k}) \times (70\vec{i} + 60\vec{j})$$

Let us determine line of action on any one of the plane. say XZ plane. for XZ plane, $y = 0$.

So,

$$(x\vec{i} + z\vec{k}) \times (70\vec{i} + 60\vec{j}) = -25.7\vec{i} + 29.4\vec{j}$$

or, $$60x\vec{k} + 70z\vec{j} - 60z\vec{i} = -25.7\vec{i} + 29.4\vec{j}.$$

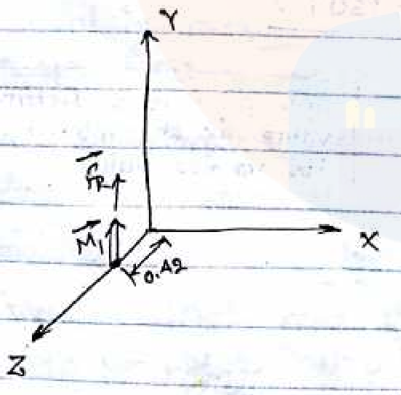
For these vectors to be equal, the corresponding coefficients should be equal.

i.e.

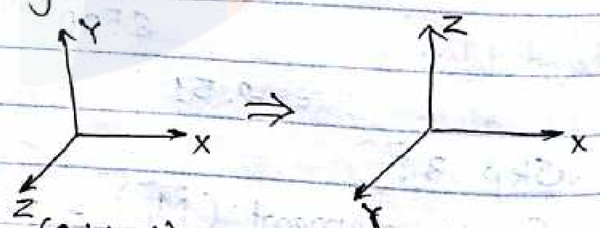
$60x = 0$	$70z = 29.4$	$-60z = -25.7$
$\boxed{x = 0}$	$\boxed{z = 0.42}$	$\boxed{z = 0.42}$

These two values must be equal for accuracy.

So, resultant wrench passes through $0.42\vec{k}$.



Note: If orientation of X, Y and Z axis changes,

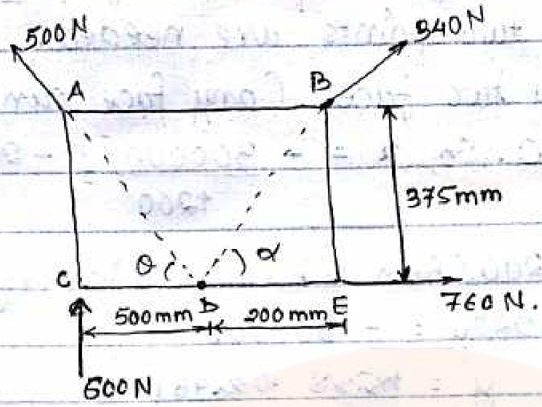


(Original) $\Longrightarrow$ Changed.

then,

This circular rule is no longer valid.

5. Four forces acting on a plate are shown in figure. Determine the resultant force and indicate the line of action of resultant.



Solution :-

Let two angles be θ and α.

$$\theta = \tan^{-1}\left(\frac{375}{500}\right) = 36.87^\circ$$

$$\alpha = \tan^{-1}\left(\frac{375}{200}\right) = 61.93^\circ$$

$$\text{Resultant force } (\vec{F_R}) = (-500\cos 36.87^\circ + 340\cos 61.93^\circ + 760)\vec{i} + (500\sin 36.87^\circ + 340\sin 61.93^\circ + 600)\vec{j}$$

$$= (520\vec{i} + 1200\vec{j})\ N \ \underline{\text{Ans}}.$$

Let $\vec{F_R}$ acts at any point $(x\vec{i} + y\vec{j})$ from D. (We considered D as origin here because the moment of force 500 & 340 about D becomes zero. So calculation will be easy. But you can assume any point as origin).

Using Varignon's Theorem

Moment due to resultant about D = Σ(moment due to components about D)

or, $(x\vec{i} + y\vec{j}) \times (520\vec{i} + 1200\vec{j}) = (-500\vec{i} \times 600\vec{j}) + 0$

or, $1200x\vec{k} - 520y\vec{k} = -300000\vec{k}$

Equating like vectors.

$$\boxed{1200x - 520y = -300000}$$

This is the equation of line of action of resultant force.

To draw a line, two points are needed.

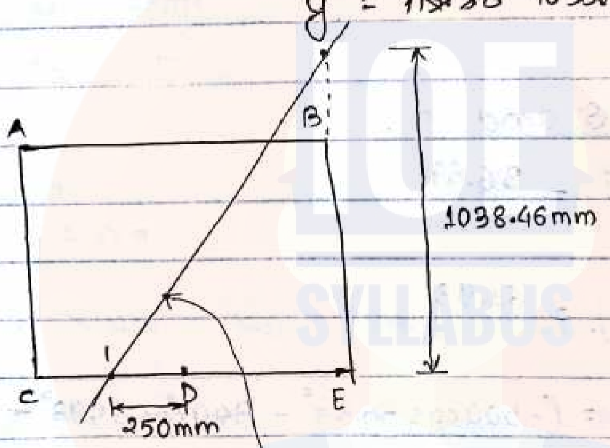
So, lets take any two faces (any face can be taken).

At face CE, $y = 0$. So, $x = -\frac{300000}{1200} = -250$ mm

At face BE, $x = 200$. (from D because D is origin).

So, $1200 \times 200 - 520y = -300000$

$y = \cancel{115.38}$ 1038.46 mm



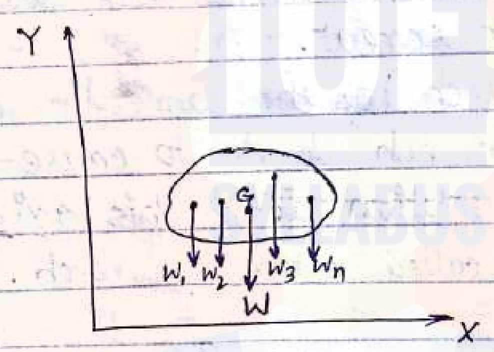
line of action of resultant force.

4. CENTER OF GRAVITY, CENTROID AND MOMENT OF INERTIA

4.1 CONCEPTS AND CALCULATION OF CENTRE OF GRAVITY AND CENTROID

Centre of Gravity. (C.G.)

- The centre of gravity is the point through which the resultant of the system of forces formed by the weight of all particles forming the body passes.
- In other words, it is a unique point through which the whole weight of the body acts, irrespective to the position of body.
- Every body has one & only C.G. within or outside the body.

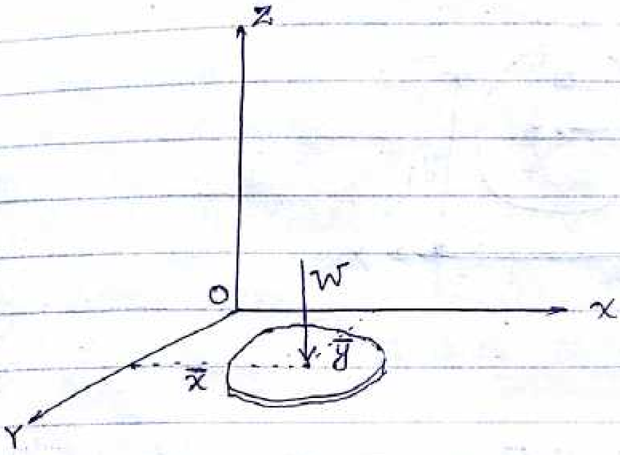


Centroid.

The centroid C ~~of~~ is a point which defines the geometric center of an object. It is the point at the centre of any shape, sometimes called centre of area or centre of volume.

- For a body of uniform density (i.e. homogeneous), the centroid coincides with C.G. in a uniform gravitational field.

DERIVATION OF CG



Consider a flat horizontal plate of thickness t as shown in figure. Let the plate be divided into n small elements. The co-ordinates of each element are denoted by x_i and y_i and force exerted by earth on the i^{th} element of plate will be denoted by ΔW_i. The total weight of plate $W = \Sigma \Delta W$.

The resultant magnitude W of these forces can be expressed as,

$$\Sigma F_z = W = \Delta W_1 + \Delta W_2 + \Delta W_3 + \dots + \Delta W_n.$$

To obtain the co-ordinates $\bar{x}$ and $\bar{y}$ of point G (point of application of W), we use Varignon's Theorem,

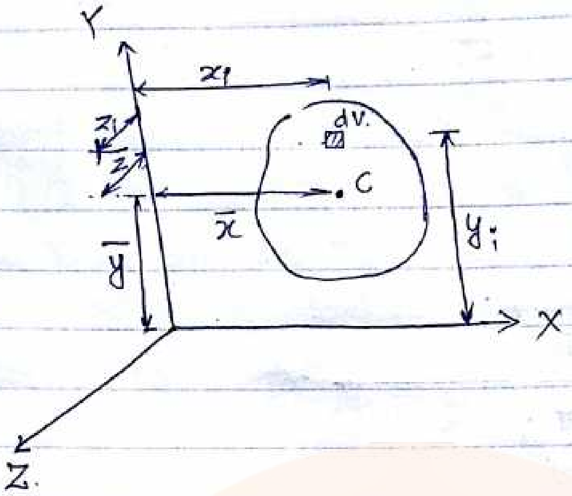
$$\Sigma M_y ; \bar{x} W = x_1 \Delta W_1 + x_2 \Delta W_2 + x_3 \Delta W_3 + \dots + x_n \Delta W_n$$

$$\Sigma M_x ; \bar{y} W = y_1 \Delta W_1 + y_2 \Delta W_2 + y_3 \Delta W_3 + \dots + y_n \Delta W_n$$

If we increase the number of elements into which the plate is divided and simultaneously decrease the size of each element, we obtain in the limit of following expressions,

$$W = \int dW.$$

So, $\bar{x} W = \int \bar{x}\, dW$ & $\bar{y} W = \int \bar{y}\, dW$



If an object is subdivided into volume elements dV, the location of the centroid $C(\bar{x}, \bar{y}, \bar{z})$ for the volume of object can be determined by computing the 'moments' of the elements about each of the coordinate axes as,

$$\bar{x} = \frac{\int_V x_i \, dV}{\int_V dV} ; \quad \bar{y} = \frac{\int_V y_i \, dV}{\int_V dV} ; \quad \bar{z} = \frac{\int_V z_i \, dV}{\int_V dV}$$

Similarly, the centroid of the surface area of an object, (such as plate or shell) can be found by subdividing the area into differential elements dA & computing moments of these area elements about each of the coordinate axes as,

$$\bar{x} = \frac{\int_A x_i \, dA}{\int_A dA} ; \quad \bar{y} = \frac{\int_A y_i \, dA}{\int_A dA} ; \quad \bar{z} = \frac{\int_A z_i \, dA}{\int_A dA}.$$

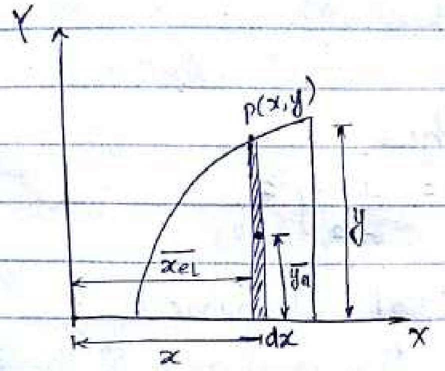
Similarly, for line,

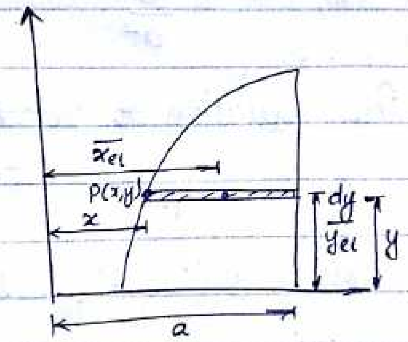
$$\bar{x} = \frac{\int_L x_i \, dL}{\int_L dL} ; \quad \bar{y} = \frac{\int_L y_i \, dL}{\int_L dL} ; \quad \bar{z} = \frac{\int_L z_i \, dL}{\int_L dL}.$$

DIFFERENCE BETWEEN CG AND CENTROID

- C.G. applies to bodies with mass and weight whereas centroid is purely a geometrical thing. C.G. and centroid is same for objects with uniform density.
- C.G. of a body is a point through which the resultant gravitational force (weight) acts for any orientation of the body whereas centroid is a point in a plane area such that the moment of area about any axis through that point is zero.
- Imagine a 2D shape made from cardboard. The point in the middle where you can balance it on the tip of a pin is its C.G. + centroid. But if you then replace part of the shape with steel, keeping the same shape, the centroid is the same but C.G. has shifted i.e. it won't balance at the same point anymore.

DETERMINATION OF CENTROIDS BY INTEGRATION





Here,

$\bar{x}_{el} = x$.

$\bar{y}_{el} = y/2$

$dA = y\,dx$

Here, $\bar{x}_{el} = \frac{a+x}{2}$

$\bar{y}_{el} = y$.

$dA = (a-x)\,dy$.

Then,

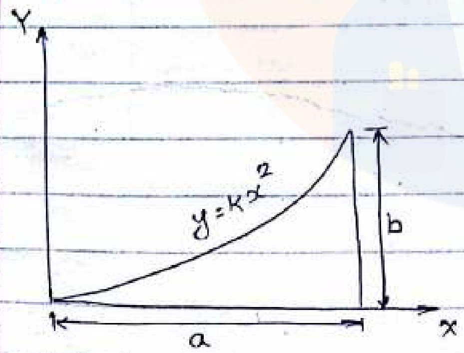
apply.

$Q_y = \bar{x}A = \int \bar{x}_{el}\,dA$.

$Q_x = \bar{y}A = \int \bar{y}_{el}\,dA$.

For explanation, refer Page 232 (Bear & Johnston's Book).

Determine by direct integration the location of the centroid of a parabolic sprandel.



Solution:-

Determination of constant k: The value of constant k is determined by substituting $x = a$ and $y = b$ into the given equation.

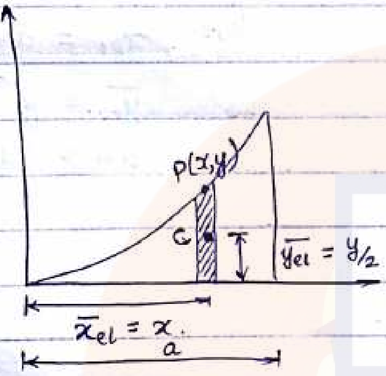
i.e. $b = ka^2$.

or, $K = \frac{b}{a^2}$.

The equation of curve is thus,

$$y = \frac{b}{a^2}x^2 \quad \text{or,} \quad x = \frac{a}{b^{1/2}}y^{1/2}$$

Method 1 (Vertical Differential Element)



The total area of the spandrel is :-

$$A = \int dA = \int y\,dx$$
$$= \int_0^a \frac{b}{a^2}x^2 dx$$
$$= \frac{b}{a^2}\left[\frac{x^3}{3}\right]_0^a$$
$$= \frac{ab}{3}.$$

The first moment of the differential element w.r.t. Y axis is $\int \overline{x_{el}}\,dA$. So, the first moment of the entire area w.r.t. Y axis is :-

$$Q_y = \int \overline{x_{el}}\,dA$$
$$= \int xy\,dx$$
$$= \int_0^a x\left(\frac{b}{a^2}x^2\right)dx$$
$$= \frac{b}{a^2}\cdot\left[\frac{x^4}{4}\right]_0^a$$
$$= \frac{a^2b}{4}$$

Since $Q_y = \bar{x}A$, we have

$$\bar{x}A = \int \overline{x_{el}}\,dA \text{ i.e. } \bar{x}\frac{ab}{3} = \frac{a^2b}{4} \quad \therefore \boxed{\bar{x} = \frac{3}{4}a}$$

Similarly, the first moment of the differential element w.r.t. x-axis is $\bar{y}_{el} \cdot dA$. So, the first moment of the entire area w.r.t X-axis is

$$Q_x = \int \bar{y}_{el} \, dA = \int \frac{y}{2} \cdot y \, dx = \int_0^a \frac{1}{2}\left(\frac{b}{a^2}x^2\right)^2 dx$$

$$= \frac{b^2}{2a^4}\left[\frac{x^5}{5}\right]_0^a = \frac{ab^2}{10}.$$

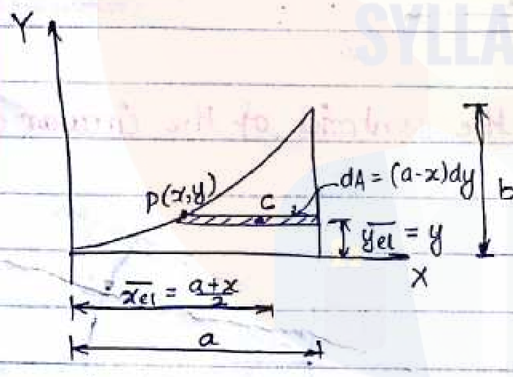
Since $Q_x = \bar{y}A$, we have,

$$\bar{y}A = \int \bar{y}_{el} \, dA$$

i.e. $\bar{y}\frac{ab}{3} = \frac{ab^2}{10}$

$$\therefore \boxed{\bar{y} = \frac{3}{10}b}$$

Method 2 (Horizontal Differential Element).



The first moment of the entire area w.r.t. Y-axis is :-

$$Q_y = \int \bar{x}_{el} \, dA$$

$$= \int_0^b \frac{a+x}{2} \cdot (a-x) \, dy$$

$$= \int_0^b \frac{a^2 - x^2}{2} dy$$

$$= \frac{1}{2}\int_0^b \left(a^2 - \frac{a^2}{b}y\right) dy$$

$$= \frac{a^2 b}{4}.$$

$$\therefore \boxed{\bar{x} = \frac{3}{4}a}$$

The first moment of the entire area w.r.t. X-axis is :-

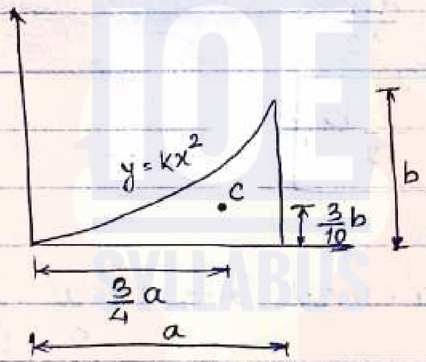
$$Q_x = \int \bar{y}_{el}\, dA = \int y.(a-x)\, dy = \int_0^b y.\left(a - \frac{a}{b^{1/2}} y^{1/2}\right) dy$$

$$= \int_0^b \left(ay - \frac{a}{b^{1/2}} y^{3/2}\right) dy$$

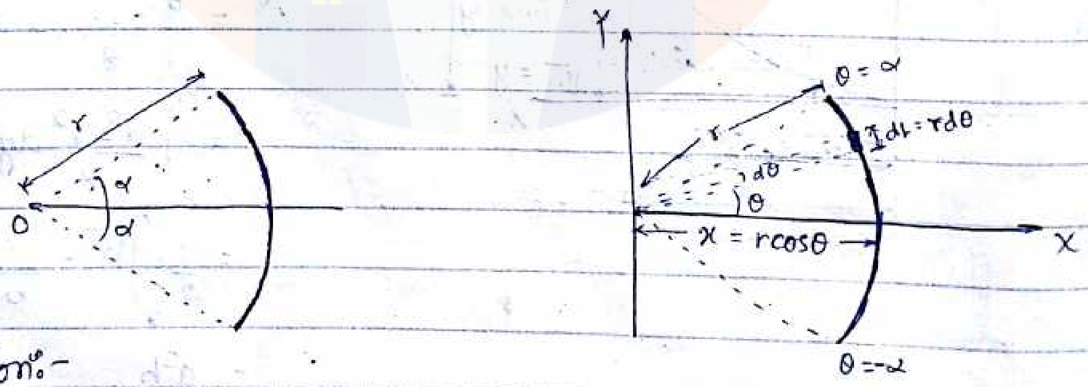
$$= \frac{ab^2}{10}.$$

$$\therefore \boxed{\bar{y} = \frac{3b}{10}}$$

Hence,



Determine the location of the centroid of the circular arc shown.



Solution:-

Since the arc is symmetric w.r.t. X-axis, $\bar{y} = 0$.

A differential element is chosen as shown and the length of the arc is determined by integration as,

$$L = \int dL = \int_{-\alpha}^{\alpha} r\, d\theta = r\int_{-\alpha}^{\alpha} d\theta = 2r\alpha.$$

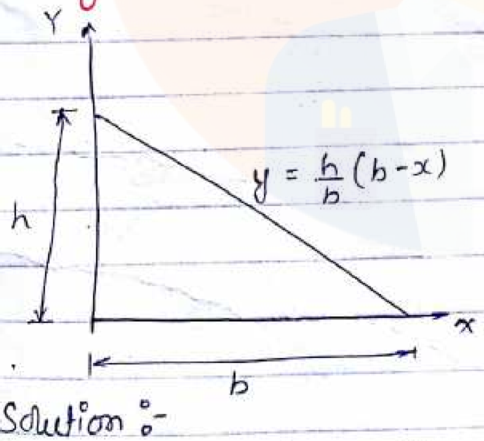
The first moment of the arc w.r.t. Y-axis is

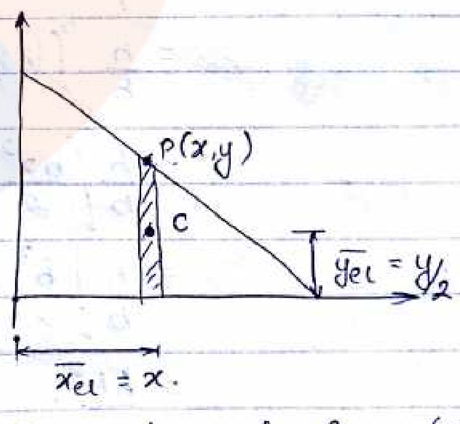
$$Q_y = \int x\,dL$$
$$= \int_{-\alpha}^{\alpha} (r\cos\theta).(r\,d\theta)$$
$$= \int_{-\alpha}^{\alpha} r^2 \cos\theta\, d\theta$$
$$= r^2 [\sin\theta]_{-\alpha}^{\alpha}$$
$$= 2r^2\sin\alpha$$

Since $Q_y = \bar{x}L$, we have,

$$\bar{x}L = 2r^2\sin\alpha$$
$$\bar{x} = \frac{2r^2\sin\alpha}{2r\alpha}$$
$$\therefore \boxed{\bar{x} = \frac{r\sin\alpha}{\alpha}}$$

Determine by direct integration the centroid of the area of triangle as shown.





Solution :-
Consider a vertical differential element as shown in figure (ii).
(Note :- In most cases, it is easy to solve the problem by considering vertical differential element).

The area of the triangle is :-

$$\text{Area } (A) = \int_0^b y\,dx$$

$$= \int_0^b \frac{h}{b}(b-x)\,dx$$

$$= \frac{h}{b}\int_0^b (b-x)\,dx$$

$$= \frac{h}{b}\left[bx - \frac{x^2}{2}\right]_0^b$$

$$= \frac{1}{2}bh.$$

The first moment of the differential element w.r.t. Y axis is $\bar{x}_{el}dA$. So, the first moment of the entire area w.r.t. Y axis is :- $Q_y = \int \bar{x}_{el}dA$.

i.e. $Q_y = \int_0^b x.y\,dx$

$$= \int_0^b x.\frac{h}{b}(b-x)\,dx$$

$$= \frac{h}{b}\int_0^b (bx - x^2)\,dx$$

$$= \frac{h}{b}\left[\frac{bx^2}{2} - \frac{x^3}{3}\right]_0^b$$

$$= \frac{h}{b}\left[\frac{b^3}{2} - \frac{b^3}{3}\right]$$

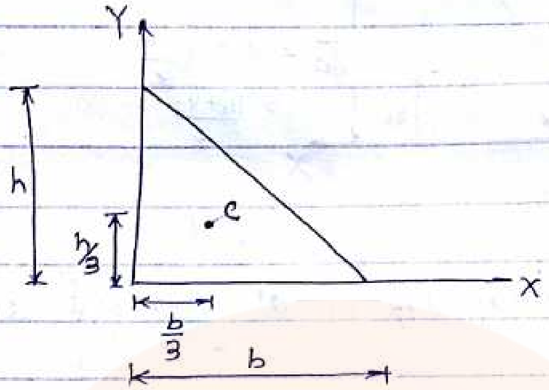
$$= \frac{hb^2}{6}.$$

Since $Q_y = \bar{x}A$, we have

$\bar{x}A = \frac{hb^2}{6}$. So, $\boxed{\bar{x} = \frac{1}{3}b}$

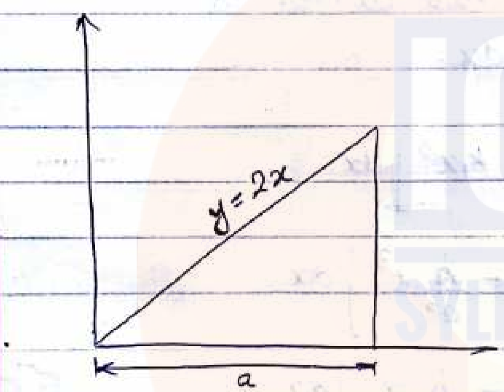
Similarly,

$$\boxed{\bar{y} = \frac{1}{3}h}$$



C ⇒ centroid of triangular shape.

\#



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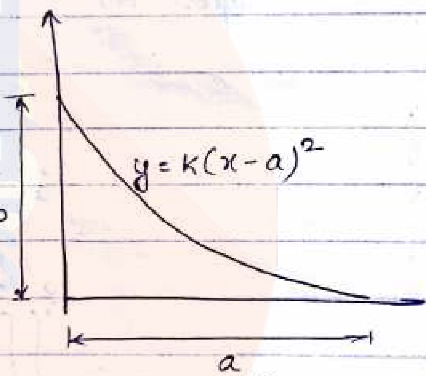
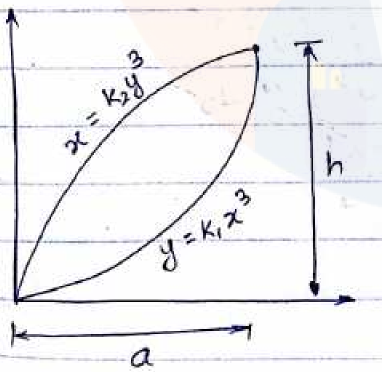


(Ans: $\frac{a}{4}, \frac{3b}{10}$).

\#

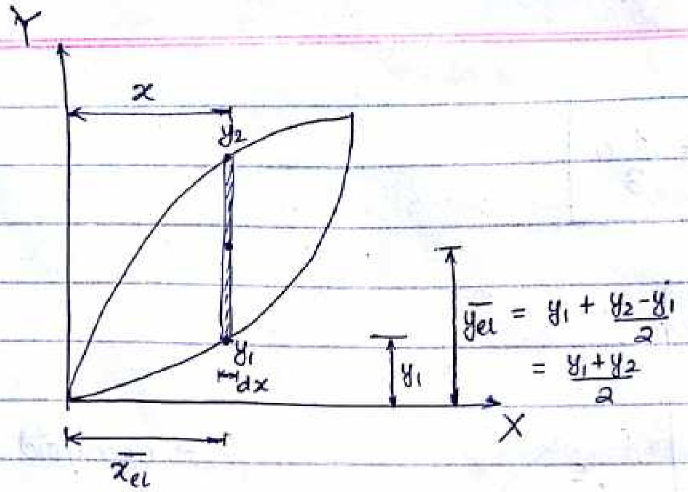


Solution :-

Determination of value of k :-

For curve $x = k_2 y^3$; $a = k_2 h^3$ i.e. $k_2 = a/h^3$.

For curve, $y = k_1 x^3$; $h = k_1 a^3$ i.e. $k_1 = h/a^3$.



Consider a vertical differential element as shown in figure.

The area formed by two curves is :-

$$\text{Area (A)} = \int_0^a (y_2 - y_1)\, dx.$$

$$= \int_0^a \left[\left(\frac{x}{k_2}\right)^{1/3} - k_1 x^3\right] dx$$

$$= \int_0^a \left[\left(\frac{xh^3}{a}\right)^{1/3} - \frac{h}{a^3} x^3\right] dx$$

$$= \int_0^a \left[x^{1/3} \cdot \frac{h}{a^{1/3}} - \frac{h}{a^3} \cdot x^3 \right] dx$$

$$= \frac{h}{a^{1/3}} \left[\frac{x^{4/3} \cdot 3}{4}\right]_0^a - \frac{h}{a^3} \cdot \left[\frac{x^4}{4}\right]_0^a$$

$$= \frac{3}{4}\frac{h}{a^{1/3}} \cdot a^{4/3} - \frac{h}{a^3} \cdot \frac{a^4}{4}$$

$$= \frac{3}{4} ha - \frac{ha}{4}$$

$$\therefore \text{Area (A)} = \frac{ha}{2}.$$

The first moment of differential element w.r.t. Y-axis is $\overline{x_{el}} \cdot dA$. So, the first moment of the whole area is :-

$$Q_y = \int \overline{x_{el}} \cdot dA$$
$$= \int_0^a x \cdot (y_2 - y_1) \cdot dx \quad \left(\text{Since } \overline{x_{el}} = x \text{ from figure}\right).$$
$$= \int_0^a x \cdot \left(\frac{h}{a^{1/3}} \cdot x^{1/3} - \frac{h}{a^3} \cdot x^3 \right) dx$$
$$= \int_0^a \left(\frac{h}{a^{1/3}} \cdot x^{4/3} - \frac{h}{a^3} \cdot x^4 \right) dx$$
$$= \frac{h}{a^{1/3}} \cdot \left[\frac{x^{7/3} \cdot 3}{7} \right]_0^a - \frac{h}{a^3} \left[\frac{x^5}{5} \right]_0^a$$
$$= \frac{3}{7} \cdot \frac{h}{a^{1/3}} \cdot a^{7/3} - \frac{h}{5a^3} a^5$$
$$= \frac{3}{7} ha^2 - \frac{ha^2}{5}$$
$$= \frac{8}{35} ha^2.$$

Since $Q_y = \bar{x}A$, we have,

$$\bar{x}A = \int \overline{x_{el}}\, dA$$

i.e. $\bar{x} A = \frac{8}{35} ha^2$

$$\therefore \boxed{\bar{x} = \frac{16\,a}{35}}$$

The first moment of differential element w.r.t. X-axis is $\overline{y_{el}}\, dA$. So, the first moment of the whole area w.r.t. X-axis is,

$$Q_x = \int \overline{y_{el}}\, dA.$$
$$= \int_0^a \left(\frac{y_1 + y_2}{2} \right) \cdot (y_2 - y_1) \cdot dx \quad \left(\text{since } \overline{y_{el}} = \frac{y_1 + y_2}{2} \right).$$

or, $Q_x = \frac{1}{2}\int_0^a (y_2^2 - y_1^2)\,dx$

$= \frac{1}{2}\int_0^a \left(\frac{xh^3}{a}\right)^{2/3} - \left(\frac{h}{a^3}x^3\right)^2 dx$

$= \frac{1}{2}\int_0^a \left(\frac{h^2}{a^{2/3}}\cdot x^{2/3} - \frac{h^2}{a^6}\cdot x^6\right)dx$

$= \frac{1}{2}\cdot\left[\frac{h^2}{a^{2/3}}\cdot\frac{3}{5}\left[x^{5/3}\right]_0^a - \frac{h^2}{a^6}\cdot\left[\frac{x^7}{7}\right]_0^a\right.$

$= \frac{1}{2}\left[\frac{h^2}{a^{2/3}}\cdot\frac{3}{5}\cdot a^{5/3} - \frac{h^2}{a^6}\cdot\frac{a^7}{7}\right]$

$= \frac{3h^2a}{10} - \frac{h^2a}{14}$

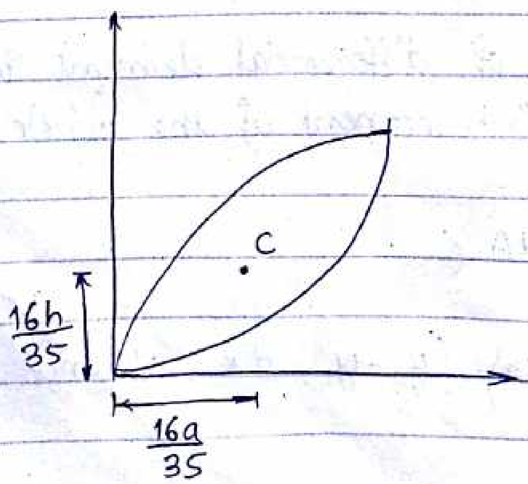
$\therefore Q_x = \frac{8}{35}h^2a.$

Since $Q_x = \bar{y}A$, we have,

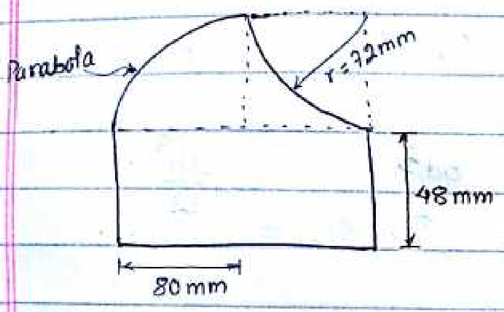
$\bar{y}A = \int \bar{y}_{el}\,dA$

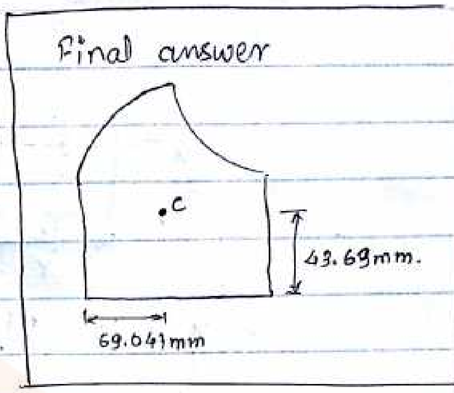
$\bar{y}\cdot\frac{ha}{2} = \frac{8}{35}h^2a$

$\therefore \boxed{\bar{y} = \frac{16h}{35}}$

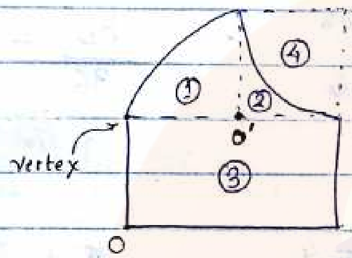


Q) Locate the centroid of the composite figure as shown in figure.





Solution :-



O' → origin of parabola.

Vertex of semi parabola should be defined in question else we should suppose it. Then it is $\frac{3b}{5}$ towards origin of parabola & it is $\frac{3a}{8}$ towards other direction.

The total area is obtained by summing up the areas of parabola ①, rectangle ③, rectangle ② and subtracting the quarter circular part ④.

S.N.	Components	Area (A_i) mm^2	$\bar{x}_i$ (mm)	$\bar{y}_i$ (mm)	$\bar{x}_i A_i$ (mm^3)	$\bar{y}_i A_i$ (mm^3)
1.	Parabola	$\frac{2}{3}\times80\times72$ $= 3840$	$\frac{3\times80}{5}$ $= 48$	$\frac{3\times72}{8}+48$ $= 27+48 = 75$	184320	~~103680~~ 288000
2.	Rectangle (Square)	72×72 $= 5184$	$80+\frac{72}{2}$ $= 116$	$48+\frac{72}{2}$ $= 84$	601344	435456
3.	Rectangle	152×48 $= 7296$	$\frac{152}{2}$ $= 76$	24	554496	175104
4.	Quarter Circle (-ve)	$-\frac{\pi\times72^2}{4}$ $= -4071.5$	$152-\frac{4\times72}{3\pi}$ $= 121.44$	$120-\frac{4\times72}{3\pi}$ 89.44	-494442.96	-364154.96
		$\Sigma A_i = 12248.5$			$\Sigma A_i\bar{x}_i = 845717.04$	$\Sigma A_i\bar{y}_i =$ ~~350085.04~~ 534405.04

Hence, $\bar{x} = \frac{\Sigma A_i\bar{x}_i}{\Sigma A_i} = \frac{845717.04}{12248.5} = 69.047$ mm.

$\bar{y} = \frac{\Sigma A_i \bar{y}_i}{\Sigma A_i} = \frac{\cancel{350085.04}\ 534405}{12248.5} =$ ~~28.582 mm~~ 43.63 mm.

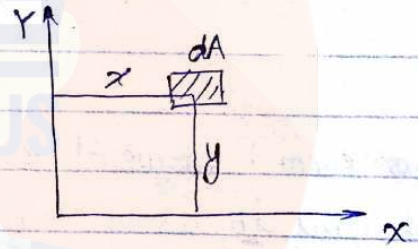
4.2 CALCULATION OF SECOND MOMENT OF AREA / MOMENT OF INERTIA AND RADIUS OF GYRATION

First moment of area

- The integral $\int x\,dA$ is called first moment of area A w.r.t. Y axis and is denoted by Q_y. Similarly, integral $\int y\,dA$ defines the first moment of area A w.r.t. X axis & denoted by Q_x.

i.e. $Q_x = \int y\,dA$

$Q_y = \int x\,dA$.



So,

$$\bar{x} = \frac{Q_y}{A} \text{ and } \bar{y} = \frac{Q_x}{A}.$$

The first moment of area are useful to determine the centroid of plane figures.

SECOND MOMENT OF AREA

The product of area and its perpendicular distance from an axis is known as first moment of area. If this quantity is again multiplied by the same distance, the quantity obtained is called the second moment of ~~inertia~~. area/ moment of inertia.

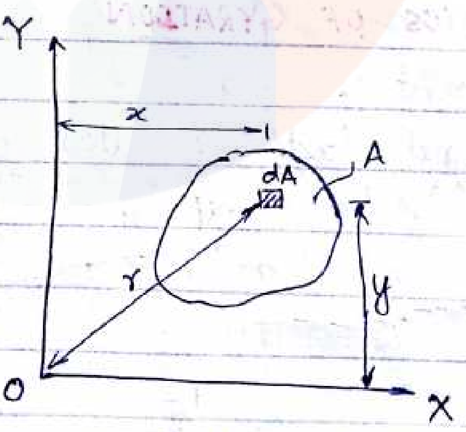
We have,

$$Q_y = A\bar{x} = \int x\,dA \;\&\; Q_x = A\bar{y} = \int y\,dA$$

Thus second moment of area are $\int x^2 dA$ & $\int y^2 dA$.
They are also called axial moment of inertia.
Consider the area A lies in X-Y plane.
By definition, the moment of inertia (M.O.I.) for the differential area dA about X-axis is :- $dI_x = y^2 dA$ and
about Y-axis is :- $dI_y = x^2 dA$
For the entire area the MOI are determined by integration

$$I_x = \int_A y^2 dA$$

$$I_y = \int_A x^2 dA$$

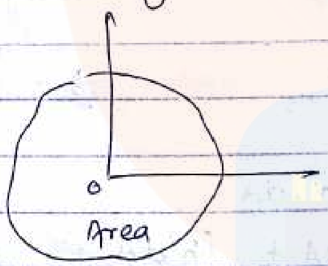


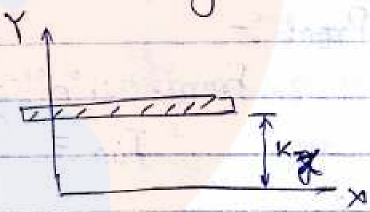
The second moment of dA about the pole (O) /or z axis referred to as polar moment of inertia.
It is defined as, $dJ_0 = r^2 dA$, where r is perpendicular distance from the pole (z-axis) to the element dA.
For entire area, the polar MOI is,

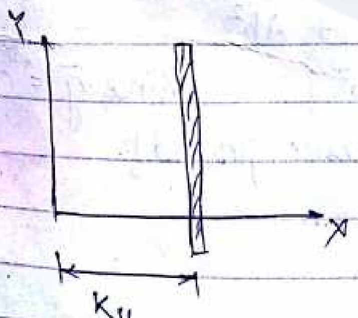
$$J_0 = \int_A r^2 dA = I_x + I_y. \quad (\text{Since } r^2 = x^2 + y^2)$$

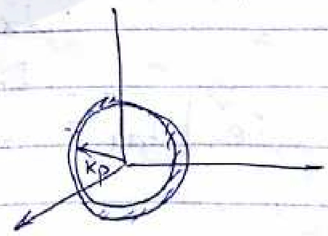
RADIUS OF GYRATION

- The radius of gyration of an area with respect to a particular axis is the square root of the quotient of the moment of inertia divided by the area.
- It is the distance at which the entire area must be assumed to be concentrated in order that the product of the area and the square of this distance will equal the moment of inertia of the actual area about the given axis.
- The radius of gyration describes the way in which the total cross-sectional area is distributed around the centroidal axis.
- If more area is distributed further from the axis, it will have greater resistance to buckling.









$$I_{xx} = k_x^2 A \Rightarrow k_x = \sqrt{\frac{I_{xx}}{A}}$$

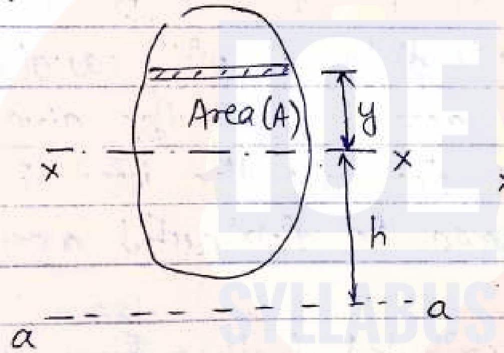
$$I_{yy} = k_y^2 A \Rightarrow k_y = \sqrt{\frac{I_{yy}}{A}}.$$

$$I_p = k_p^2 A$$

$$k_p = \sqrt{\frac{I_p}{A}}$$

Parallel Axis Theorem

- The moment of inertia of an area with respect to any given axis is equal to the moment of inertia with respect to the centroidal axis plus the ~~centroid~~ product of the area and the square of the distance between the two axes.
- The parallel axis theorem is used to determine the moment of inertia of composite sections.



xx is centroidal axis.

$$I_{aa} = I_{xx} + Ah^2$$

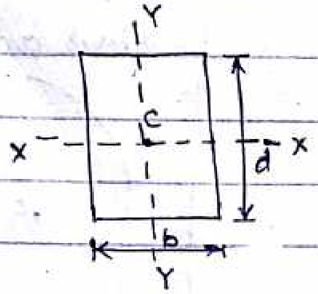
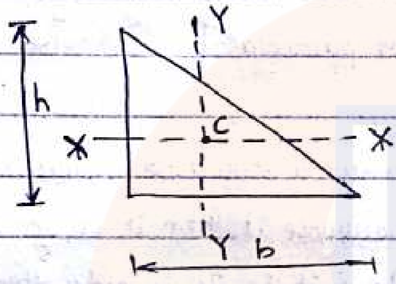
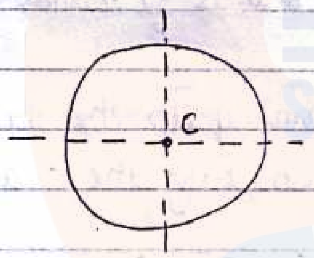
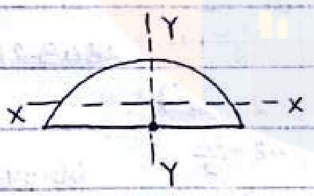
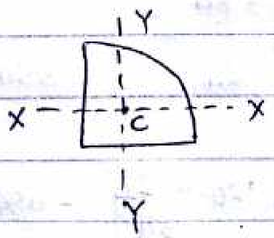
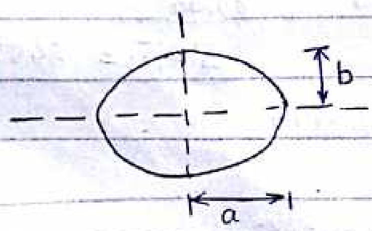
Proof :-

From definition of MOI,

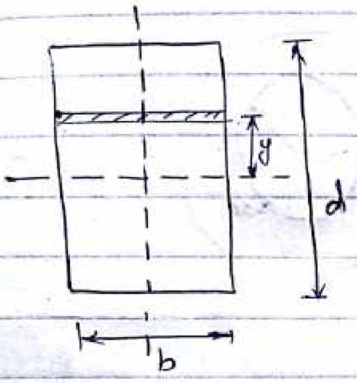
$$\begin{aligned} I_{aa} &= \int (h+y)^2 dA \\ &= \int (y^2 + 2hy + h^2) dA \\ &= \int y^2 dA + \int 2hy\, dA + \int h^2 dA \\ &= I_{xx} + 2h\bar{y}.A + Ah^2 \\ &= I_{xx} + 0 + Ah^2 \qquad (\text{Since } \bar{y} = 0) \end{aligned}$$

i.e. $\boxed{I_{aa} = I_{xx} + Ah^2}$ Hence, proved!

Moment of Inertia of different Sections

S.N.	Shape	I_{xx}	I_{yy}
1.		$\frac{bd^3}{12}$	$\frac{db^3}{12}$
2.		$\frac{bh^3}{36}$	$\frac{hb^3}{36}$
3.		$\frac{\pi r^4}{4}$ $\left(\frac{\pi d^4}{64}\right)$	$\frac{\pi r^4}{4}$ $\left(\frac{\pi d^4}{64}\right)$
4.		$0.11r^4$	$\frac{\pi r^4}{8}$
5.		$0.055r^4$	$0.055r^4$
6.		$\frac{\pi ab^3}{4}$	$\frac{\pi a^3 b}{4}$

- Determine MOI of rectangular section about centroidal axis.



Solution:-

Area of strip(dA) = $b\,dy$

MOI of strip(dI_x) = $y^2 dA$

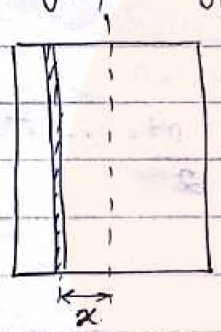
$$= y^2 . b\,dy.$$

$$I_{xx} = \int_{-d/2}^{d/2} y^2 b\,dy$$

$$= b.\left[\frac{y^3}{3}\right]_{-d/2}^{d/2}$$

$$= \frac{bd^3}{12}.$$

Similarly, for I_{yy}.



$$dA = d.dx.$$

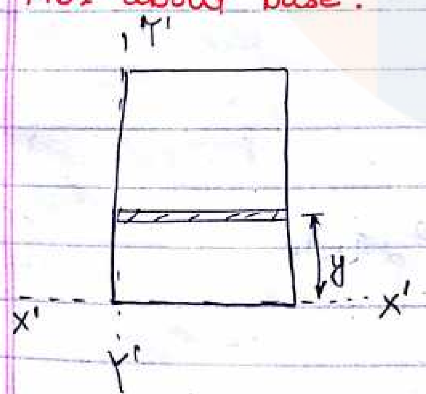
$$dI_y = x^2 d.dx.$$

$$I_y = \int dI_y$$

$$= \int_{-b/2}^{b/2} x^2 d.dx$$

$$= \frac{db^3}{12}.$$

MOI about base.



$$dA = b\,dx.$$

$$dI_{x'x'} = y^2 . b\,dx$$

$$I_{x'x'} = \int dI_{x'x'}$$

$$= \int_0^d y^2 b\,dx$$

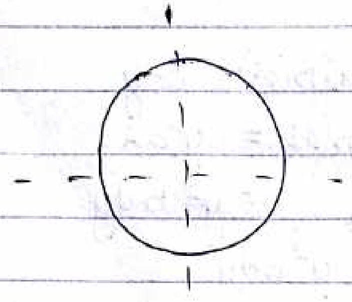
$$= b\left[\frac{y^3}{3}\right]_0^d$$

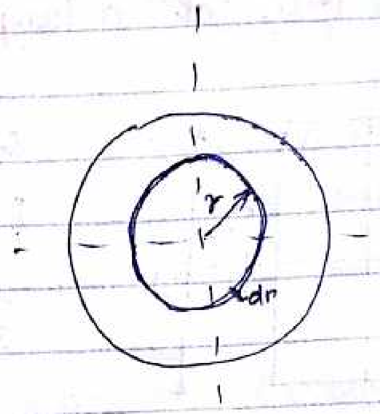
$$I_{x'x'} = \frac{bd^3}{3}.$$

Similarly

$$I_{y'y'} = \frac{db^3}{3}$$

MOI of circular area.





Area of strip $(dA) = 2\pi r\,dr$

MOI of strip $(dI) = r^2 2\pi r\,dr$

MOI of whole area $(I)_{zz} = \int_0^R 2\pi r^3 dr$

$= 2\pi \left[\frac{r^4}{4}\right]_0^R$

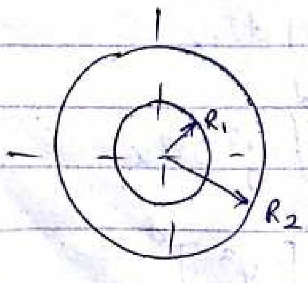
$= 2\pi \frac{R^4}{4} = \frac{\pi R^4}{2}$

Since $I_{zz} = I_{xx} + I_{yy}$

As $I_{xx} = I_{yy}$.

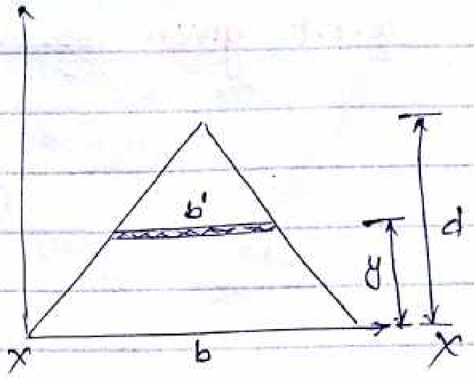
$I_{xx} = I_{yy} = \frac{I_{zz}}{2} = \frac{\pi R^4}{4}$.

For hollow circular area,



$I_{zz} = \int_{R_1}^{R_2} r^2 2\pi r\,dr$

MOI of triangular area



Since $\frac{b'}{d-y} = \frac{b}{d}$

$$b' = \frac{b}{d}(d-y)$$

Area of strip (dA) $= b'dy$

$$= \frac{b}{d}(d-y)\,dy$$

MOI of strip $(dI_x) = y^2 dA$

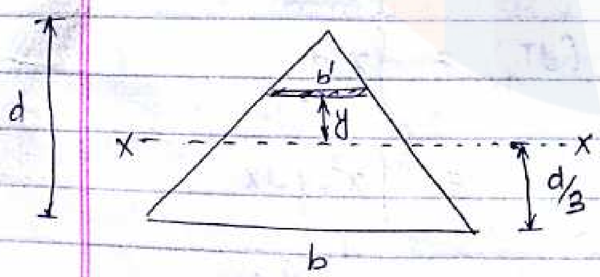
$$= y^2 \frac{b}{d}(d-y)\,dy.$$

MOI of whole area $(I_{xx}) = \int y^2 dA$
about X-X axis

$$= \int_0^d y^2 \frac{b}{d}(d-y)\,dy$$

$$= \frac{bd^3}{12} \text{ (about base)}$$

MOI of triangular area about centroidal axis.



Since,

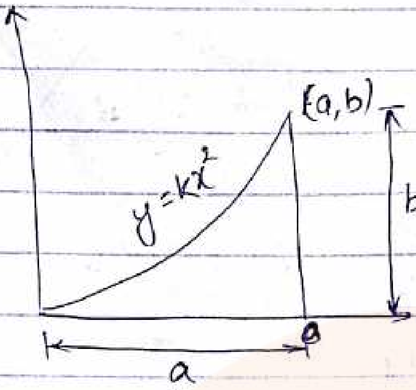
$$\frac{b'}{\left(\frac{2}{3}d - y\right)} = \frac{b}{d}.$$

$$b' = \frac{b}{d}\left(\frac{2}{3}d - y\right)$$

$$I_{xx} = \int y^2 dA$$

$$= \int y^2 b' dy = \frac{b}{d}\int_{-d/3}^{2d/3} \left(\frac{2}{3}d - y\right) y^2 dy = \frac{bd^3}{36}.$$

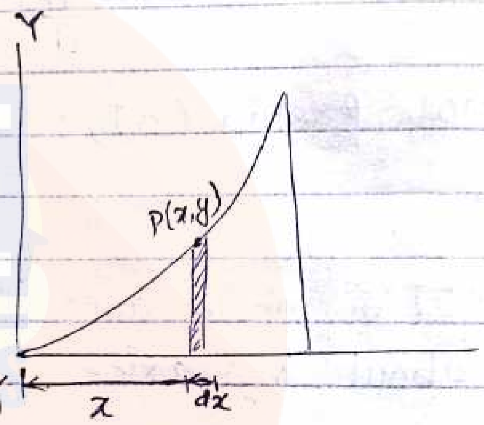
Determine the moment of inertia of the area shown w.r.t given axes & find radius of gyration.



Here, $y = kx^2$

$b = ka^2$

$$\boxed{k = \frac{b}{a^2}}$$



Area of differential element (dA) = $y\,dx$.

Total area of the curve (A) = $\int_0^a y\,dx$

$$= \int_0^a \frac{b}{a^2} x^2\,dx = \frac{ab}{3}$$

MOI of differential element (dI_y) about Y-Y axis = $\int_0^a x^2\,dx$

$$= \int_0^a x^2 \cdot y\,dx$$

$$= \frac{b}{a^2}\int_0^a x^4\,dx$$

$$= \frac{b}{a^2}\cdot\frac{a^5}{5} = \frac{a^3 b}{5}$$

Again, taking horizontal strip:
MOI of differential strip (dI_{xx}) $= \oint y^2 dA$
about X-axis
So, MOI of the whole area (I_{xx}) $= \int_0^b y^2(a-x)dy$

$$= \int_0^b y^2\left(a - \left(\frac{y a^2}{b}\right)^{0.5}\right) dy.$$

$$= \int_0^b \left(ay^2 - \frac{a}{b^{1/2}} \cdot y^{5/2}\right) dy$$

$$= \left[a\frac{y^3}{3} - \frac{a}{b^{1/2}} \cdot y^{7/2} \cdot \frac{2}{7}\right]_0^b$$

$$= \frac{ab^3}{3} - \frac{2ab^{7/2}}{7b^{1/2}}$$

$$= \frac{ab^3}{3} - \frac{2ab^3}{7}$$

$$\boxed{I_{xx} = \frac{ab^3}{21}}$$

or, the same vertical strip can also be used for I_{xx}.
Since all the portions of this element are not at the same distance from X-axis, we must treat the element as a thin rectangle, then,

$$dI_{xx} = \frac{1}{3}y^3 \cdot dx \qquad ; \quad \frac{y^3dx}{12} + A\left(\frac{y}{2}\right)^2$$

$$I_{xx} = \int_0^a \frac{y^3}{3} dx \qquad = \frac{y^3dx}{12} + \frac{y^3dx}{4}$$

$$= \int_0^a \frac{b^3}{3a^6} x^6 \, dx \qquad = \frac{y^3dx}{3}.$$

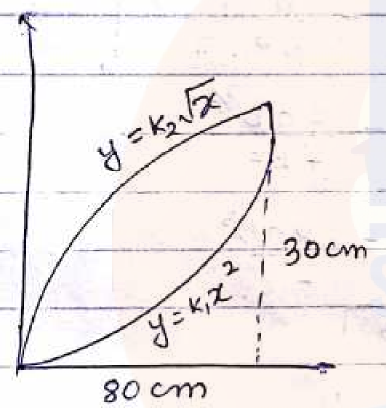
$$= \frac{ab^3}{21}.$$

Now, radius of gyration k_x and k_y,

$$k_x = \sqrt{\frac{I_{xx}}{A}} = \sqrt{\frac{ab^3/21}{ab/3}} = \frac{b}{\sqrt{7}} \text{ units.}$$

$$\& \quad k_y = \sqrt{\frac{I_{yy}}{A}} = \sqrt{\frac{a^3b}{5 \cdot ab/3}} = \sqrt{\frac{3}{5}}\, a \text{ units.}$$

Find I_{xx}, I_{yy} and k_x, k_y of the following figure :-

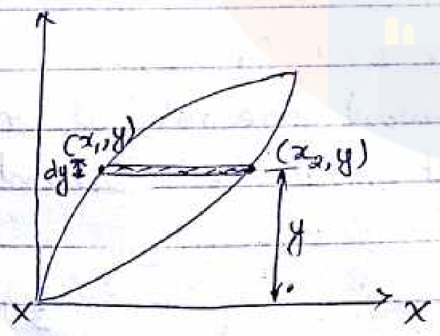


Solution :-

Determination of value of k.

For curve, $y = k_2\sqrt{x}$, $30 = k_2\sqrt{80} \Rightarrow k_2 = 3.354$

For curve, $y = k_1x^2$, $30 = k_1 80^2 \Rightarrow k_1 = 0.0047$



Considering horizontal differential element.

Area of diff element $(dA) = (x_2 - x_1)\, dy$.

MOI of diff element about X-X axis $(dI_{xx}) = y^2 dA$

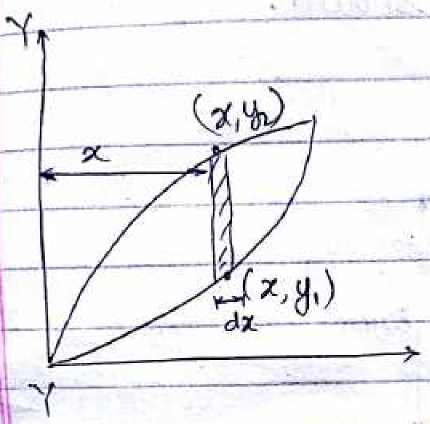
So, MOI of whole area about X-X axis

$$(I_{xx}) = \int dI_{xx}$$

$$= \int_0^{30} y^2 (x_2 - x_1)\, dy.$$

$$= \int_0^{30} y^2 \left\{ \left(\frac{y}{0.0047}\right)^{1/2} - \left(\frac{y}{3.354}\right)^{2} \right\} dy.$$

or, $I_{xx} = 184295.375 \text{ cm}^4$ (from calculator).



Considering vertical differential element.

Area of differential element (dA)

$= (y_2 - y_1).dx$.

MOI of differential element about Y-Y axis $(dI_{yy}) = x^2 dA$

So, MOI of whole area about Y-Y axis

$$(I_{YY}) = \int dI_{yy}$$

$$= \int x^2 dA$$

$$= \int_0^{80} x^2 (y_2 - y_1) dx$$

$$= \int_0^{80} x^2 (3.354\sqrt{x} - 0.0047x^2) dx.$$

$$= 1308246.012 \text{ cm}^4.$$

Now,

$$k_x = \sqrt{\frac{I_{xx}}{A}} = \sqrt{\frac{184295.375}{797.818}} = 15.2 \text{ cm}.$$

$$k_y = \sqrt{\frac{I_{yy}}{A}} = \sqrt{\frac{1308246.012}{797.818}} = 40.494 \text{ cm}.$$

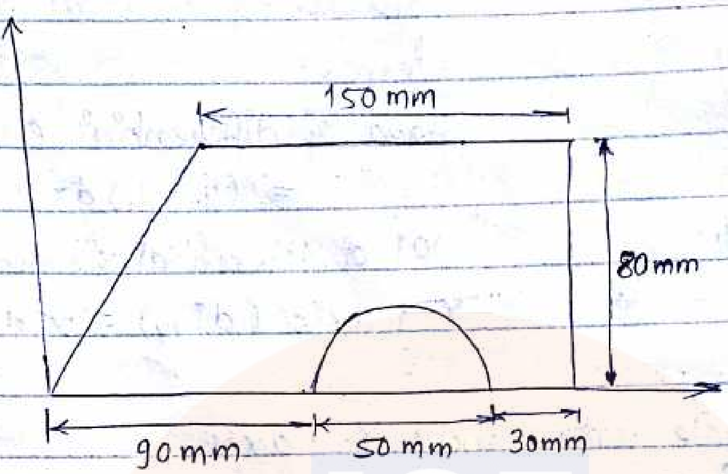
Here,

$$\text{Area}(A) = \int dA = \int (y_2 - y_1) dx$$

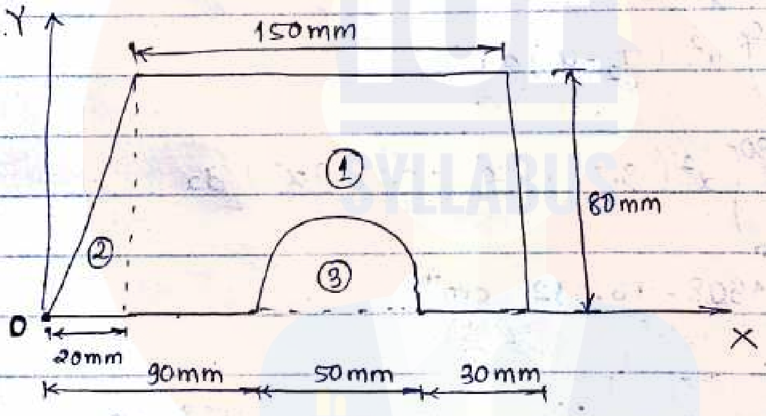
$$= \int_0^{80} (3.354\sqrt{x} - 0.0047x^2) dx$$

$$= 797.818 \text{ cm}^2.$$

Find the moment of inertia and radius of gyration about the centroidal axes for the section shown.



Solution :-



Considering O as origin, the values are tabulated as :-

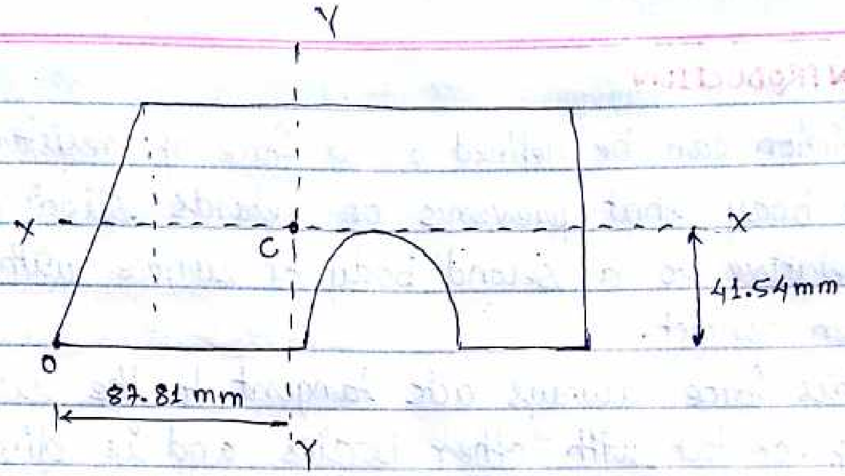
S.N.	Components	Area (A_i) mm^2	$\bar{x}_i$ (mm)	$\bar{y}_i$ (mm)	$A_i\bar{x}_i$ (mm^3)	$A_i\bar{y}_i$ (mm^3)
1.	Rectangle	12000	95	40	1140000	480000
2.	Triangle	800	13.33	26.67	10664	21336
3.	Semicircle (-ve)	- 981.75	115	10.61	- 112901.25	-10416.37
		ΣA_i = 11818.25			$\Sigma A_i\bar{x}_i$ = 1037762.75	$\Sigma A_i\bar{y}_i$ = 490919.6

So,

$$\bar{x} = \frac{\Sigma A_i \bar{x}_i}{\Sigma A_i} = \frac{1037762.75}{11818.25} = 87.81 \text{ mm}$$

$$\bar{y} = \frac{\Sigma A_i \bar{y}_i}{\Sigma A_i} = \frac{490919.63}{11818.25} = 41.54 \text{ mm}.$$

i.e.



C is the centroid of the section shown in figure.

Moment of inertia about X-X axis is :-

$$I_{xx} = (I_{xx})_{rectangle} + (I_{xx})_{triangle} - (I_{xx})_{semi\text{-}circle}$$

$$= \left[\frac{150 \times 80^3}{12} + (150 \times 80) \times (41.54 - 40)^2\right] +$$

$$\left[\frac{20 \times 80^3}{36} + \left(\frac{1}{2} \times 20 \times 80\right) \times \left(41.54 - \frac{1}{3} \times 80\right)^2\right] -$$

$$\left[0.11 \times 25^4 + \left(\frac{\pi \times 25^2}{2}\right) \times \left(41.54 - \frac{4 \times 25}{3\pi}\right)^2\right]$$

$$= 5.908 \times 10^6 \ mm^4$$

Moment of inertia about Y-Y axis is :-

$$I_{YY} = (I_{YY})_{rectangle} + (I_{YY})_{triangle} - (I_{YY})_{semi\text{-}circle}$$

$$= \left[\frac{80 \times 150^3}{12} + 12000 \times (95 - 87.81)^2\right] + \left[\frac{80 \times 20^3}{36} + 800 \times (87.81 - 13.33)^2\right] -$$

$$\left[\cancel{0.11 \times} \ \frac{\pi \times 25^4}{8} + 981.75 \times (115 - 87.81)^2\right]$$

$$= 2.67 \times 10^7 \ mm^4.$$

Radius of gyration $(k_x) = \sqrt{\frac{I_{xx}}{A}} = \sqrt{\frac{5.908 \times 10^6}{11818.25}} = 22.36 \ mm.$

Similarly, $k_y = 47.53 \ mm.$

5. Friction

INTRODUCTION

- Friction can be defined as a force of resistance acting on a body that prevents or retards slipping of the body relative to a second body or surface with which it is in contact.
- This force always acts tangent to the surface at points of contact with other bodies and is directed so as to oppose the possible or existing motion of the body i.e. friction is a self adjusting force.
- There are two types of friction :-
  (i) Fluid friction
  (ii) Dry friction (Coulomb friction).

Characteristics of force of friction

(i) It always acts in a direction opposite to the direction of the motion.

(ii) It always acts tangent to the surface of contact.

(iii) It acts only when a body tries to move relative to another body in contact.

(iv) It depends upon the nature of surfaces in contact but not upon the areas of surfaces in contact.

5.1 LAWS OF FRICTION, STATIC & DYNAMIC COEFFICIENT OF FRICTION, ANGLE OF FRICTION

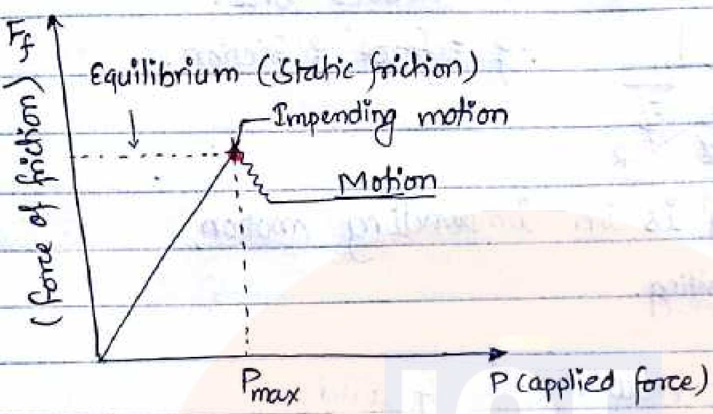
Laws of Friction

ⓐ The force of friction is directly proportional to the applied load.

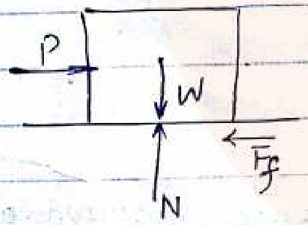
ⓑ The force of friction is independent of the apparent area of contact.

Ⓒ Kinetic friction is independent of the velocity.

Static and Dynamic ~~force~~ Coefficient of friction.



Coefficient of friction is defined as the ratio of limiting force of friction (F) to the normal reaction (N) between two bodies.



W = wt. of block.

P = applied force

F_f = force of friction.

Experimentally (or from figure),

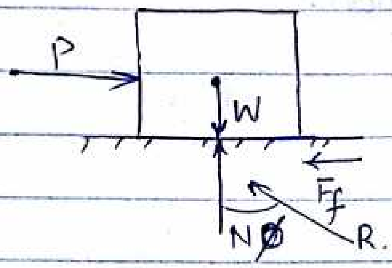
$F_f \propto N$.

$F_f = \mu N$. $\quad \mu \rightarrow$ coefficient of friction.

There are two values of μ. When there is limiting equilibrium, it is called coefficient of static friction (μ_s) and when the bodies are in relative motion, it is called kinetic (dynamic) coefficient of friction (μ_K).

$$\boxed{\mu_s > \mu_K}$$

Angle of friction.



Here,

W = wt. of body

P = applied force.

F_f = force of friction.

When the body is in impending motion,

$P = F_f = F_{limiting}$.

Let R be the resultant of F_f and N.

The angle (ϕ) made by resultant (R) with normal reaction (N) is called angle of friction.

$$\tan\phi = \frac{F_{limiting}}{R} = \mu_s.$$

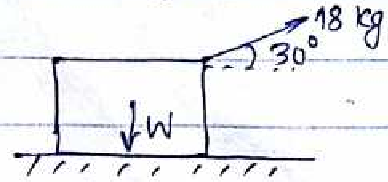
$$\therefore \phi = \tan^{-1}\left(\frac{F_{limiting}}{R}\right) = \tan^{-1}(\mu_s).$$

NUMERICALS :

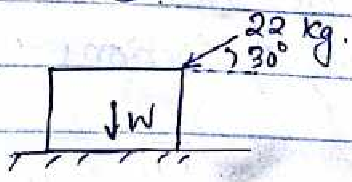
1. A body resting on a rough horizontal plane required a pull of 18 kg inclined at 30° to the plane just to move it. It was found that a push of 22 kg inclined at 30° to the plane just moved the body. Determine the weight of the body and the coefficient of friction.

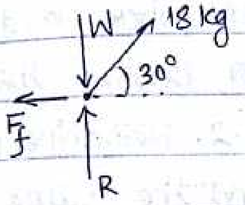
Solution :-

Case ⓐ.

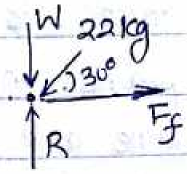


Case ⓑ





FBD of case ⓐ.



F_f - force of friction

FBD of case ⓑ.

Case ⓐ.

$\Sigma F_x = 0$

$18\cos 30° = F_f \Rightarrow F_f = 15.588$ kg.

$\Sigma F_y = 0$

$18\sin 30° + R = W$

or, $R = W - 9$

But, $F_f = \mu R$

$15.588 = \mu (W - 9)$ ----- (i)

Case ⓑ.

$\Sigma F_x = 0$

$F_f = 22\cos 30° = 19.05$ kg.

$\Sigma F_y = 0$

$22\sin 30° + W = R$

But,

$F_f = \mu R$

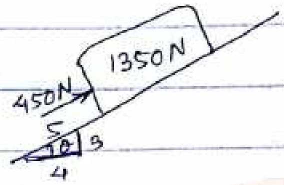
$19.05 = \mu (11 + W)$. --- (ii)

Solving (i) and (ii).

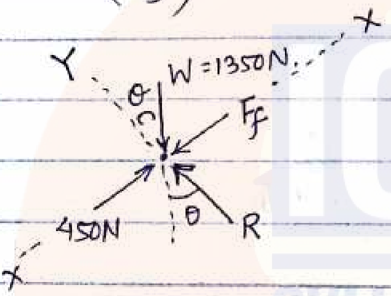
$W = 99.05$ kg.

$\mu = 0.173$

2. A 450 N force acts as shown on a 1350 N block placed on an inclined plane. The coefficients of friction between the block and the plane are $\mu_s = 0.25$ and $\mu_k = 0.2$. Determine whether the block is in equilibrium, and find the value of the friction force.



$$\theta = \sin^{-1}\left(\frac{3}{5}\right) = 36.87^\circ$$



F_f = force of friction
R = normal reaction.
W = wt. of block.

Force required for equilibrium :-
The value of force of friction (F_f) is determined that maintains equilibrium. Assuming the direction of F_f as shown.

$+\nearrow \Sigma F_x = 0.$

$450 - F_f - 1350\,\overset{\sin}{\cancel{\cos}}\,36.87^\circ = 0.$

$F_f = -360N.$ (-ve sign indicates the direction of F_f is opposite to our assumed direction).

So, $F_f = 360N \nearrow$

$+\nwarrow \Sigma F_y = 0$

$1350 \cos 36.87^\circ = R$

$R = 1080N \nwarrow$

The force F_f required to maintain equilibrium is an 360N force directed up & to right ($\nearrow$) i.e. the tendency of block is to move down the plane.

Maximum friction Force :

The magnitude of maximum friction force which can be developed is :-

$F_{limiting} = \mu_s R$

$= 0.25 \times 1080$

$= 270$ N

Since the value of force required to maintain equilibrium (360N) is larger than the maximum value which can be developed (270N), equilibrium will not be maintained & the block will slide down the plane.

Actual Value of friction force (F_{actual}) $= \mu_K R$

$= 0.2 \times 1080$

$= 216$ N

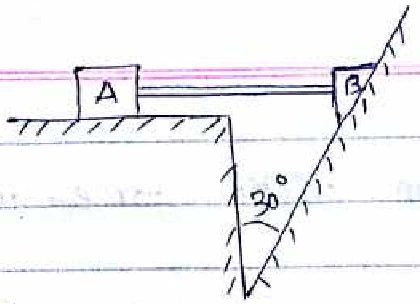
The direction of this force is opposite to the direction of motion i.e. $F_{actual} = 216 N \nearrow$

It should be noted that the forces acting on the block are not balanced. The resultant is

$R = 1350 \cos 36.87° - 216 - 450$

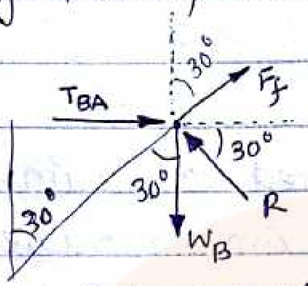
$R = 144 N \swarrow$

3. Two blocks connected by a horizontal link AB are supported on two rough planes as shown in figure. The coefficient of friction for the block on the horizontal plane is 0.4. The limiting angle of friction for block B on the inclined plane is 20°. What is the smallest weight W of the block A for which equilibrium of the system can exist if wt. of block B is 5 KN?



Solution :-

Drawing FBD of both block A and B.



FBD of B block.

W_B = wt. of block B

R = normal reaction.

T_{BA} = tension on B due to A.

F_f = force of friction.

$\cancel{\Sigma F_x = 0}$

$\cancel{T_{BA} + F_f \sin 30^\circ}$

We know,

$\tan 20^\circ = \frac{F_f}{R} \Rightarrow F_f = R \tan 20^\circ$

$+\uparrow \Sigma F_y = 0$

$R \sin 30^\circ + F_f \cos 30^\circ = W_B.$

$R \sin 30^\circ + R \tan 20^\circ \cos 30^\circ = 5$

$R = 6.133\ KN$

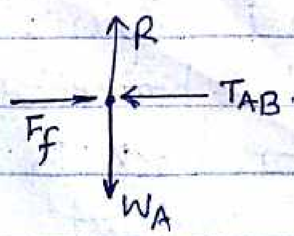
So, $F_f = 6.133 \tan 20^\circ = 2.232 KN$

$\xrightarrow{+} \Sigma F_x = 0$

$T_{BA} + F_f \sin 30^\circ = R \cos 30^\circ$

$T_{BA} = 6.133 \cos 30^\circ - 2.232 \sin 30^\circ$

$T_{BA} = 4.196\ KN.$



$\overset{+}{\rightarrow} \Sigma f_x = 0$

$f_f = T_{AB} = 4.196 \text{ kN } (\leftrightarrow).$

We know,

$f_f = \mu R$

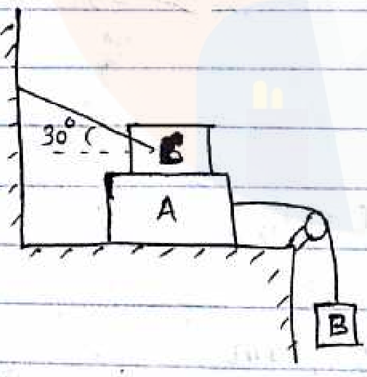
$R = \frac{f_f}{\mu} = \frac{4.196}{0.4} = 10.49 \text{ kN}.$

$\Sigma f_y = 0$

$W_A = R = 10.49 \text{ kN}.$

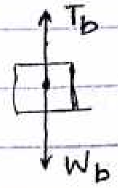
$\therefore \boxed{W_A = 10.49 \text{ kN}}$

A block A of 15 kg mass is connected to another block B of mass 10 kg by a string passing over a frictionless pulley as shown in figure. Determine the minimum mass of the block C (connected to the wall by a string CD) which must be placed over the block A to keep it from sliding. Take coefficient of friction between all contact surfaces to be 0.25. Also determine the tension in the string CD.



Solution:-

As block A moves towards the right, the frictional force act on it towards left on both of its faces. Hence the friction force acting on block C must be opposite direction i.e. towards right.



FBD of B.

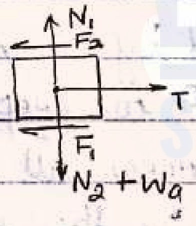
For block B,

$\Sigma F_y = 0$

$T_b = W_b = 10 \times 10$

$T_b = 100\ N$

Since the pulley is frictionless, tension in left and right of the rope is same. (i.e. $T = T_b$).



FBD of A.

For block A,

$\Sigma F_x = 0.$

$T - F_1 - F_2 = 0$

$F_1 + F_2 = 100 \quad (\because T = T_b)$

$\mu_1 N_1 + \mu_2 N_2 = 100$

$N_1 + N_2 = \dfrac{100}{\mu} = \dfrac{100}{0.25} = 400$

$N_1 + N_2 = 400 \quad \text{.....(i)}$

$\Sigma F_y = 0$

$N_1 - N_2 - W_a = 0$

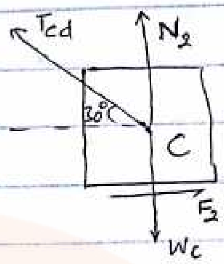
$N_1 - N_2 = 150 \quad \text{....(ii)}$

Solving (i) and (ii).

$N_1 = 275\ N$

$N_2 = 125\ N$

For block C.



FBD of C

$\Sigma f_x = 0$

$T_{cd} \cos 30^\circ = f_2$

$\frac{f_2}{\cos 30^\circ} = T_{cd}$

$$T_{cd} = \frac{\mu N_2}{\cos 30^\circ} = \frac{0.25 \times 125}{\cos 30^\circ} = 36.084\ N$$

$\Sigma f_y = 0$

$N_2 + T_{cd} \sin 30^\circ - W_c = 0$

$W_c = N_2 + T_{cd} \sin 30^\circ$

$= 125 + 36.084 \sin 30^\circ$

$= 143.042\ N$

Hence, mass of block C $(m_c) = \frac{W_c}{g} = \frac{143.042}{10} = 14.3\,kg$.

6. ANALYSIS OF BEAMS & FRAMES

6.1 Introduction to Structures : Discrete and Continuum

i. Continuum

In this idealization the elements are assumed to be continuous over an edge or surface. Solutions in continuum structure are represented by partial differential equations. It is realistic idealization and more complex which require in depth knowledge.

ii. Discrete (Skeletal)

Discrete idealization assumes the structural elements are composed of finite number of small elements. According to the resources and time availibility and accuracy intended, we divide the structure into number of small components like line element (1-D element), triangle, rectangle element (2-D element), cube or parallelopiped (3-D solid element).
Discrete idealization is less time consuming and makes our analysis simpler.

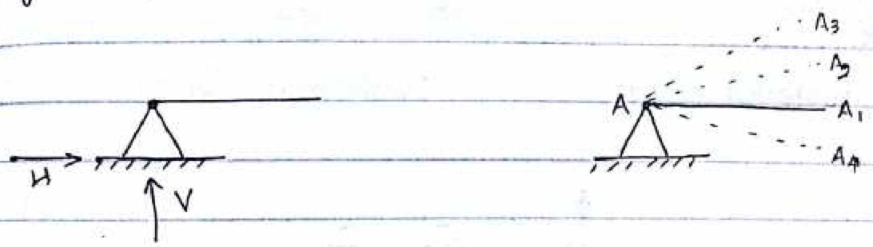
6.2 CONCEPT OF LOAD ESTIMATING & SUPPORT IDEALIZATIONS

TYPES OF SUPPORT

- All the structural members are landed on certain supports (bases). The whole external loads, self weight are transferred through these support to the ground.
- Reaction forces are developed in these supports. Reaction forces are developed in the direction where the motion is restricted or restrained.
- The main types of support are :-

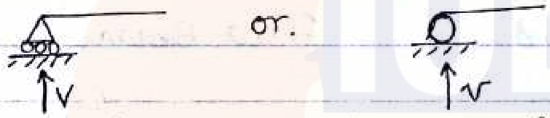
a. Hinge Support
b. Roller Support
c. Fixed Support.

a. Hinge Support.



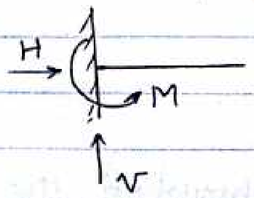
- Both horizontal and vertical reactions are developed.
- As it is free to rotate about pin-joint, no moment is developed.

b. Roller Support.



- As the member is free to more in horizontal direction, no horizontal reaction is developed.
- Only vertical reaction is developed.

c. Fixed Support.

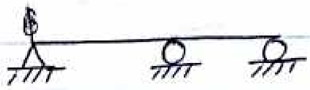


- Vertical as well as horizontal reactions are developed.
- Moment is developed since it isn't free to rotate.

· Beams are classified according to the way in which they are supported. Several types of beam are :-

Simply Supported Beam

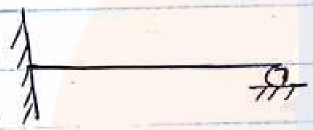
Overhanging Beam



Cantilever Beam

Continuous Beam

~~Fixed~~



Beam fixed at one end and simply supported at other end.

Fixed Beam.

Types of load.

Loads are generally the external forces that are applied to the structures. Structures are designed to resist these external loads.

Based on Design Purpose

a. Dead Load

These load do not change their position throughout the life of the structure. E.g. self weight of the structure, floor finishing, etc.

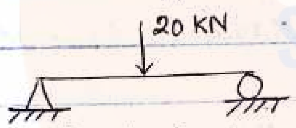
b. Live Load

These load change their position and magnitude. E.g. load of furnitures, load due to crowd, etc.

c. Dynamic load
Load due to ground motion, vibration, E.g. Earthquake load, due to moving load in bridges.

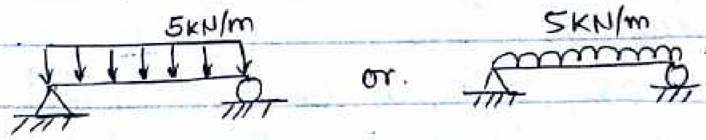
d. Wind load
e. Temperature load.
load due to variation of temperature.
f. Seismic load:- Earthquake load.

Based on Loading Pattern
a. Point load / Concentrated load
Load that is applied in negligible area compared to the contact plane area of member.
or.
Load that is applied or concentrated at a point of contact.



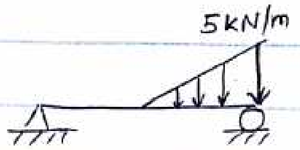
Point Load

b. Uniformly Distributed load (UDL)
These loads are distributed uniformly over the length of loaded member. E.g. load of wall on beam.
Its unit is KN/m.



Uniformly Distributed Load.

c. Uniformly Varying Load (UVL)
Loads that vary in linear, parabolic or cubic manner.
E.g. load due to waterpressure on tank surface.



Uniformly Varying Load.

6.3 USE OF BEAMS / FRAMES IN ENGINEERING : CONCEPT OF RIGID JOINTS

Beam

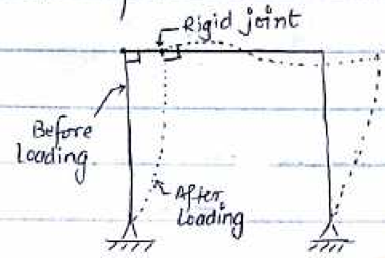
- Beam is a structural element that is capable of withstanding load primarily by resisting bending.
- Beam generally carries vertical gravitational forces but can also be used to carry horizontal loads like loads due to earthquake, wind.
- The horizontal members in a building are beams. The loads carried by a beam are transferred to columns, walls, girders which finally gets transferred to ground.

Frame

- Frame is a network of straight members like beams and columns joined together to carry the loads and transfer it to the supports.
- If the joints are rigid, it is rigid jointed frame and if the joints are hinge connections, it is pin jointed frame (truss).

Rigid Joints

- A joint is said to be rigid if it maintains same angle between the connecting members when it is acted by the external forces.



6.4 CONCEPT OF STATICALLY / KINEMATICALLY DETERMINATE AND INDETERMINATE BEAMS AND FRAMES

- Statically Determinate Structures
  - Those structures, whose reaction components and internal stresses can ~~not~~ be fully analyzed by using the equations of static equilibrium, are called statically determinate structures.
  - Statically indeterminate conditions arise when more supports than needed are used to support a structure.

Total static indeterminacy (I_s) = external static indeterminacy (I_{se}) + internal static indeterminacy (I_{si})

- For beam,

  External static indeterminacy (I_{se}) = $r - (3 + c)$

  $r \rightarrow$ number of reactions

  $c \rightarrow$ Special conditions (no.)

  3 → number of static equilibrium equation.

- For frame,

  External static indeterminacy (I_{se}) = $r - (3 + c)$

  Internal static indeterminacy (I_{si}) = 3 × total number of closed loops.

Determine static indeterminacy for following beams & frames

a.

Here,

$r = 4$

$c = 0$

So, $I_s = I_{se} + I_{si}$

$= r - (3+c) + 0$

$= 4 - (3+0) + 0$

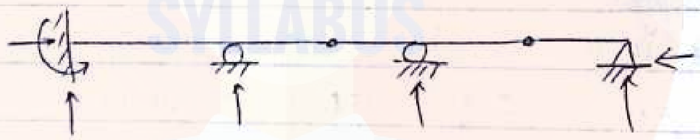
$\therefore \boxed{I_s = 1.}$ i.e. it cannot be solved by equation of static equilibrium.

b.

Here,

$r = 7$

$c = 2$



So,

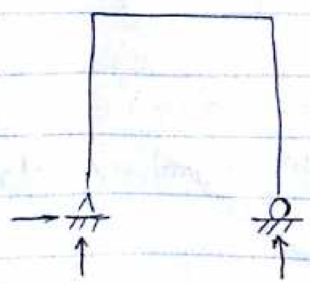
$I_s = I_{se} + I_{si}$

$= r - (3+c) + 0$

$= 7 - (3+2) + 0$

$\therefore \boxed{I_s = 2}$

c.



Here, $r = 3$, $c = 0$.

Number of closed loops = 0.

So,

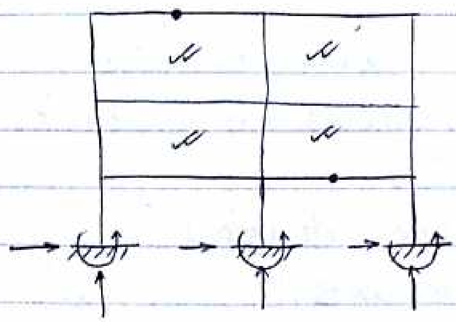
$I_s = I_{se} + I_{si}$

$= r - (3+c) + 0$

$= 3 - (3+0) + 0$

$\therefore \boxed{I_s = 0}$

d.



Here,

$r = 9.$

$c = 2.$

No. of closed loops = 4.

So,

$I_s = I_{se} + I_{si}$

$= r - (3+c) + 3 \times$ no. of closed loops.

$= 9 - (3+2) + 3 \times 4$

$= 4 + 12$

$\therefore \boxed{I_s = 16}$

- **Kinematic Indeterminacy**

Kinematic Indeterminacy (I_K) is the total number of unknown joint displacement i.e. total degree of freedom in the structure.

E.g.

For roller, it is free to rotate about Z-axis and also free to translate ~~about~~ in X axis. i.e. DOF = 2.

For hinge, it is free to rotate about Z-axis i.e. DOF = 1.

For fixed support, DOF = 0.

- For beam

- Count the number of freedoms at the joints.

- for frame,

$$I_K = 3j - r \quad \text{if members are extensible}$$
$$= 3j - (r+m) \quad \text{if members are inextensible}$$

$j \rightarrow$ total number of joints
$r \rightarrow$ total number of reactions developed
$m \rightarrow$ total number of members.

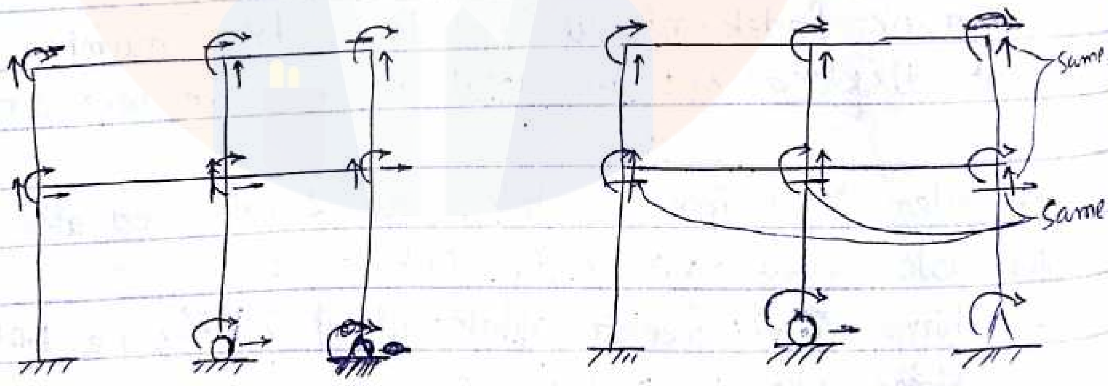
Determine degree of kinematic indeterminacy for following beams and frames.

a.

$$DKI = (DOF)_{hinge} + (DOF)_{roller} + (DOF)_{roller}$$
$$= 1 + 2 + 2$$
$$= 5$$

If we consider the beam as axially rigid enough, DKI reduces to 3 (i.e. translation motion of both rollers in neglected).

b.



$I_K = 3 \times 9 - 6 = 21$.
$(3j - r)$

Members are extensible.

$I_K = 3j - (r+m)$
$= 3 \times 9 - (6+10)$
$= 11$.

Members are inextensible.

6.5 & 6.6 CALCULATION OF AXIAL FORCE, SHEAR FORCE & BENDING MOMENT FOR DETERMINATE BEAMS & FRAMES & DRAWING THEIR DIAGRAMS

Axial force (AF)

- It is the algebraic sum of all forces acting parallel to the longitudinal axis on either side of the beam.
- A tensile axial force is positive while compressive axial force is negative.

Tensile (+ve) Compressive (-ve).

Shear force (SF)

- It is the algebraic sum of all forces acting transverse to the beam on either side of the section of beam.

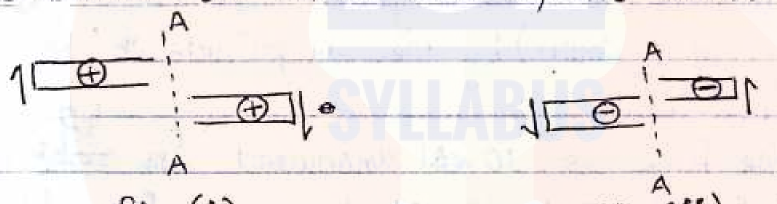


Fig (i) Fig (ii)

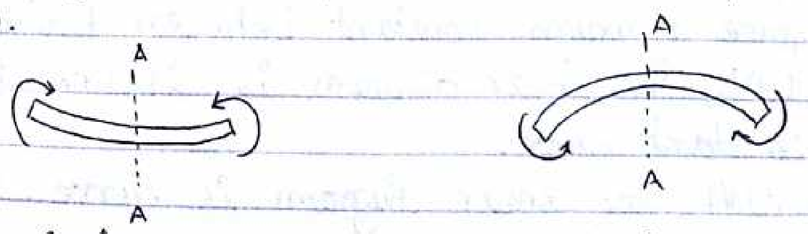
~~Bending Moment~~

Suppose we are doing calculation at section A - A,

- If upward shear is encountered at left side, it is +ve.
- If downward shear is encountered at right side, it is +ve.

Bending Moment (BM).

It is the algebraic sum of the moments of all forces at a section of beam.



Sagging Hogging.

- Suppose we are doing calculation at section A-A,
- If clockwise moment at left side is encountered, it is +ve.
- If anticlockwise moment at right side is encountered, it is +ve.

OR

If sagging moments are encountered, it is +ve.

Shear Force Diagram and Tips to draw SFD

- Shear force Diagram (SFD) is a graphical representation of the ~~shear~~ variation of shear force at every point along the length of beam or a member.
- According to me, SFD is a graph obtained by pushing the point (up or down) by the magnitude of force conside proper direction.

E.g. if the force is 10 KN downward in ~~right~~ left side, the point is pushed 10 KN downward from its origin position.

Tips to draw SFD

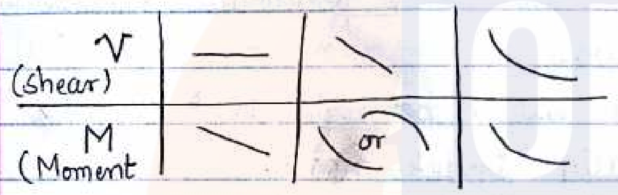
- Proper sign convention should be followed.
- Shear at support = reaction at support.
- Since the entire beam is in equilibrium, the algebr sum of beam must be zero.
- Shear force remains constant between two point load
- For a UDL, the shear diagram is inclined line with constant slope.
- For a UVL, the shear diagram is curve.

Bending Moment Diagram & Tips To Draw BMD

- BMD is a graphical representation of the moment at every point along the length of the member or beam.

Tips To Draw BMD.

- Moment at hinge = 0 (zero).
- Since the beam is in equilibrium, $\Sigma M = 0$
- Horizontal line in SFD changes to inclined line in ~~SFD~~ BMD.
- Inclined line in SFD changes to curve in BMD.
- Curve in SFD remains curve in BMD.

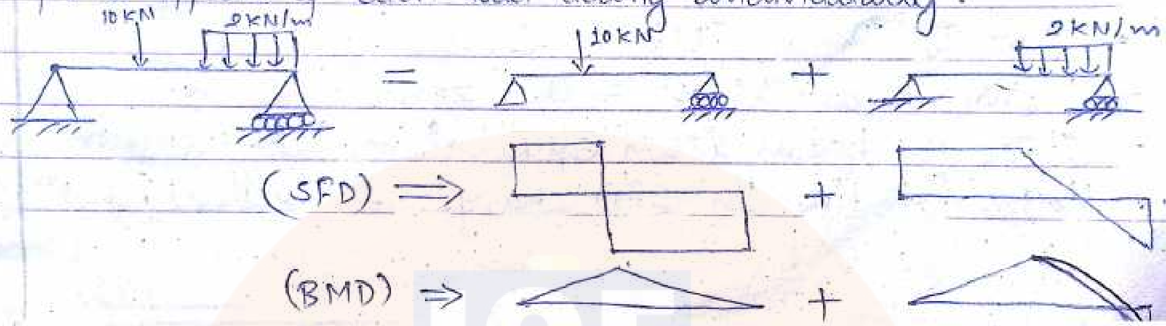


Point of Contraflexure (Salient feature).

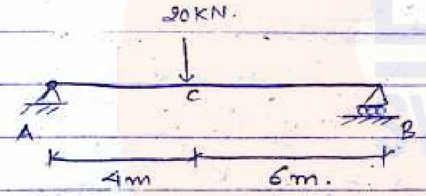
- It is the point where bending moment is zero & is a salient feature in beam and frame.

Principle Of Superposition.

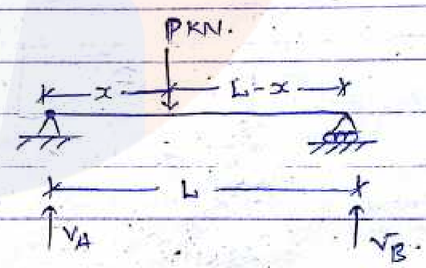
- The combined effect of several loads acting simultaneously on a linear elastic structure is equal to the "algebraic" sum of the effects of each load acting individually.



Calculate reactions and draw SFD and BMD.



Solution.



$\Sigma M_A = 0.$

$V_B \times 10 = 20 \times 4.$

$V_B = \dfrac{20 \times 4}{10}$

$V_B = 8$ KN. (↑)

$\Sigma M_A = 0.$

$V_B \times L = P \times x$

$$\boxed{V_B = \dfrac{P \times x}{L}}$$

Similarly,

$$V_A = 20 \times \frac{(10-4)}{10}$$

$$\boxed{V_A = \frac{Px\,(L-x)}{L}}$$

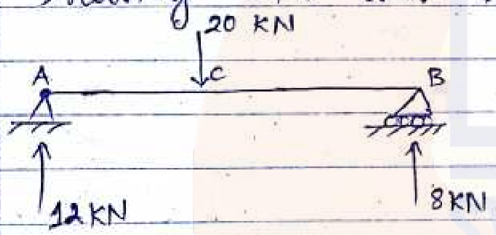
Important Expression for BCE

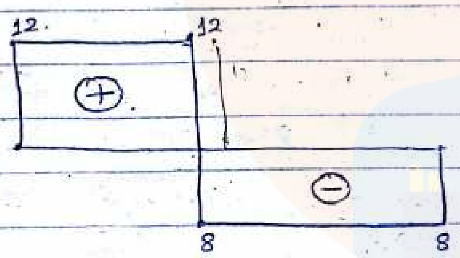
$$V_A = 12\ KN\ (\uparrow)$$

or,

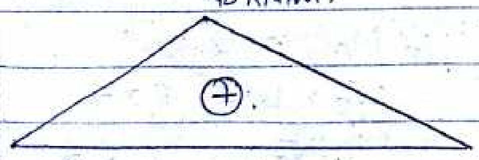
$$\boxed{\begin{aligned} \Sigma V &= 0. \\ V_A + V_B &= 20. \\ V_A &= 20 - V_B \\ &= 20 - 8 \\ &= 12\ KN\ (\uparrow) \end{aligned}}$$

- Drawing SFD and BMD.





SFD.



BMD.

Explanation.

Original Position = 0.

V_A = 12KN (↑) pushes the point 12KN in upward direction. So, SF at A = 12 KN.

Between A and C, there is no forces to push neither (↑) nor (↓). So, SF between A and C is constant.

At C, 20KN (↓) pushes 20KN in downward direction. So, new position = 12 − 20 = − 8 KN.

Similarly, between C and B, no forces. So, constant SF. At B, 8 KN (↑) pushes 8KN in upward direction resulting zero SF at B.

Note:- Point load भएको ठाउँमा SF left and right Calculate गर्नु पर्छ

र

Point moment भएको ठाउँमा BM left and right calculate गर्नुपर्छ।

Explanation for BMD. किन यसो गर्नु पर्छ?

Since,

$$M = \int V\,dx.$$

Moment = Area of shear diagram.

Moment at C = Area of shear between A and C
$= 12 \times 4$ (length × breadth).
$= 48$ KNm

Moment at B = Area of shear between A and B
= Area of shear between A and C + Area of shear between C and B
$= (12 \times 4) + (-8 \times 6)$
$= 48 - 48$
$= 0$ KNm.

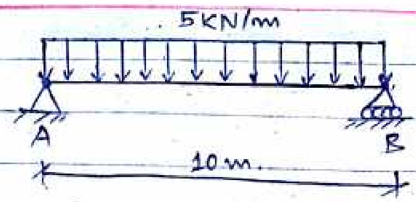
Or,

Moment at C = Force × distance
$= V_A \times x$
$= 12 \times 4$
$= 48$ KNm.

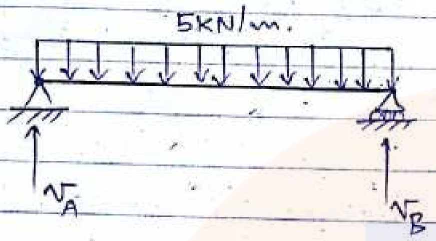
Moment at B $= 12 \times 10 - 20 \times 6$
$= 0$ KNm.

किनभने. e.g. माथिको numerical मा A मा SF = 0 छ initially. जब 12KN बराबरको V_A आउँछ तब SF@A 12 KN हुन्छ। त्यसो भए SF@A=? भनेर कसैले सोध्यो भने यसो भन्नु :- $(SF_A)_L = 0$ & $(SF_A)_R = 12$ KN. त्यसैले एउटै ठाउँमा दुईवटा value हुने भएकोले गर्दा left र right calculate गर्नु परेको हो। अब $(BM)_L$ & $(BM)_R$ को आफै हेर्नु।

◆)



Solution :-



$\Sigma M_A = 0.$

$V_B \times 10 = (5\times10)\times 5$

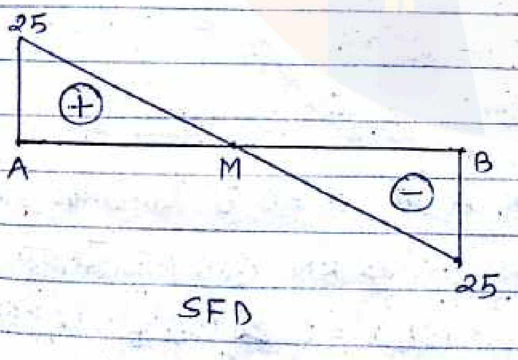
$V_B = 25\ KN\ (\uparrow)$

Similarly,

$\Sigma V = 0$

$V_A + V_B = (5\times10)$

$V_A = 25\,KN(\uparrow).$



SFD

Explanation.

What's 5KN/m?

Intensity of load for 1m = 5KN.

Intensity of load for 10m = 5×10

= 50 KN.

Since it is uniformly distributed, the reaction is also equal on both supports i.e. $\frac{50}{2} = 25$ KN.

$\boxed{V_A = \frac{Wl}{2}}$ $\boxed{V_B = \frac{Wl}{2}}$.

SFD

$V_A = 25KN(\uparrow)$. So, SF at A = 25.

Now, UDL = 5KN/m ($\downarrow$) means as we moves 1m from left to right UDL pushes 5KN downwards. So, as we move 10m, UDL pushes 5×10 = 50 KN downwards.

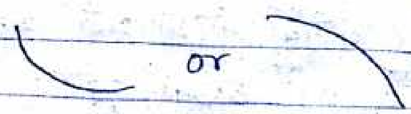
So,

SF at B = 25 − 50 = −25 KN.

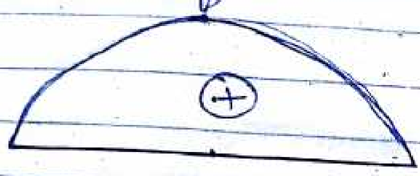
Finally V_B pushes 25KN upward, So, SF at B becomes zero.

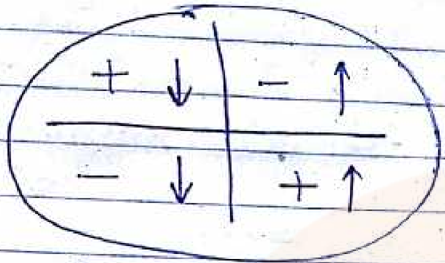
BMD.

Since, SFD is an inclined line, BMD is a curve. But is it



$\frac{Wl^2}{8} = \frac{5 \times 10^2}{8} = 62.5$ KNm → Imp. Expression.





"Ellipse Rule"

I found a useful sign convention for this. The nature of curve is discussed below. Now, for value, calculate area of SFD.

BM at M = Area of SFD betwⁿ A & M.

$= \frac{1}{2} \times 25 \times 5 = 62.5$ KNm.

BM at B = Area of SFD betwⁿ A & M + Area of SFD betwⁿ M and B.

$= \frac{1}{2} \times 25 \times 5 + \left(-\frac{1}{2} \times 25 \times 5\right)$

$= 0$ KNm.

How to use this sign convention?

- First of all, you must be sure that the diagram is a curve.
- Then, for SFD, look at load diagram.
- for BMD, look at SFD.
- Load pointing upward is +ve.
- Load pointing downward is −ve.
- Positive SF is +ve.
- Negative SF is −ve.

E.g.

For BMD, look at SFD. From A to M, SFD is positive and decreasing. i.e. + ↓. So, curve is type.

Again,

From M to B, SFD is ~~posi~~ negative and increasing i.e. − ↑. So, curve is .

load distance of CG.

or

$$BM \text{ at } M = V_A \times 5 - \underbrace{(5\times5)}_{\text{load}} \times \underbrace{\left(\frac{5}{2}\right)}_{\text{distance of CG}}$$

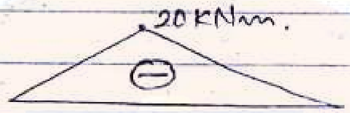
$$= 25\times5 - 5\times5\times2.5$$

$$= 62.5 \text{ KNm}.$$

Note :-

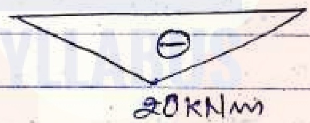
Some have habit of drawing -ve value moment above the axis of beam.

E.g.



Stop this habit, if you want to use "Ellipse Rule."

Draw like this :-



If -ve value is drawn above the axis of beam then, a mirror image of the curve obtained by Ellipse rule about the longitudinal axis of beam is obtained.
E.g. from Ellipse rule, if ⌒ is obtained, drawing -ve above axis of beam means ⌣ curve is the answer.

Conclusion.

Draw +ve moment above axis of beam.

Draw -ve moment below axis of beam.

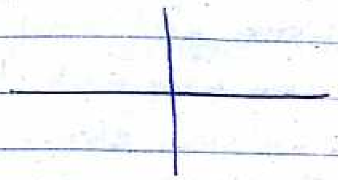
Assignment.

Refer. Books and tally whether "Ellipse Rule" is correct or not. Remember sign convention and everything that I mentioned above.

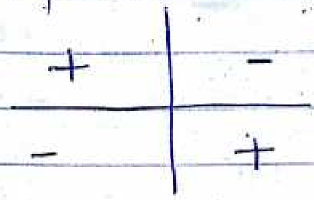
If -ve moment is drawn above axis of beam, as done is Dr. Rajan Suwal's Book, "Ellipse Rule can't be applied.

How to Draw Ellipse Rule Diagram.

Step 1:



Step 2:

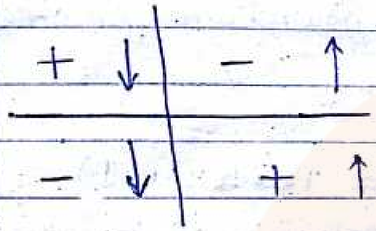


As In matrix.

$$\begin{bmatrix}1 & 5\\ 2 & 3\end{bmatrix} = 1\times3 - 5\times2$$

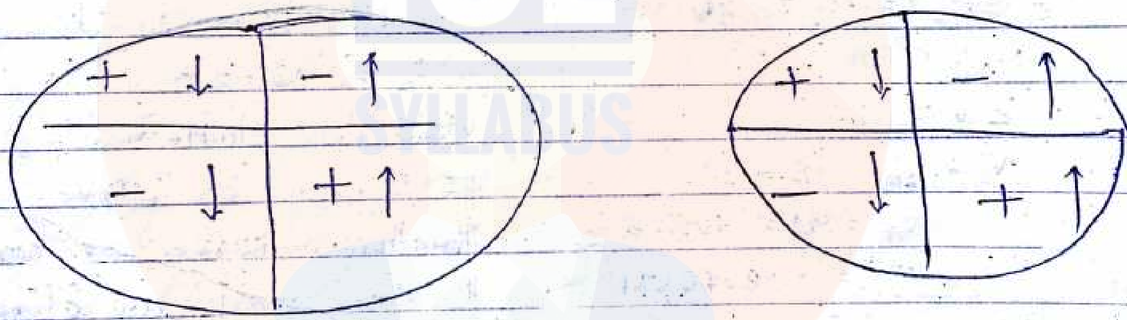
Determinant

Step 3:



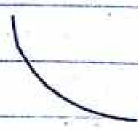
⇐ start from here to give arrows.

Step 4: Finally, draw an ellipse.



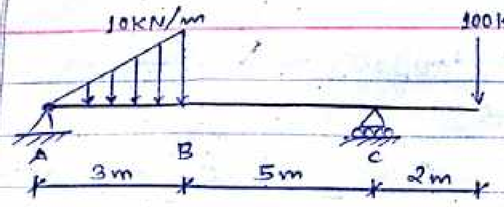
E.g.

If. SF is −ve and decreasing, BM becomes.

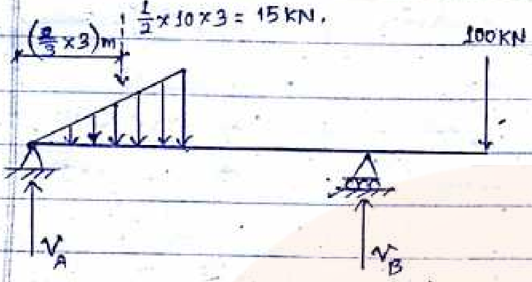


If SF is + and increasing, BM becomes

◈



Solution :-



$\Sigma M_A = 0.$

$V_B \times 8 = 15\times 2 + 100\times 10$

$V_B = 128.75\ KN\ (\uparrow)$

Now,

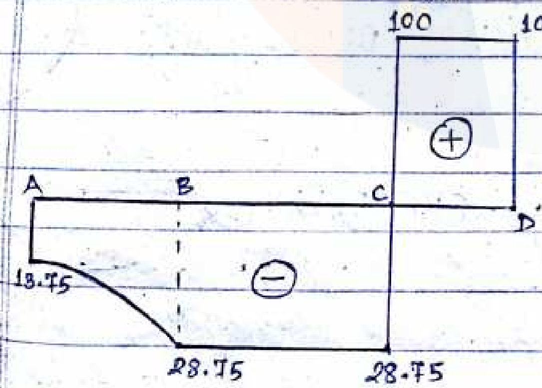
$\Sigma V = 0$

$V_A + V_B = 15 + 100$

$V_A = 115 - 128.75$

$V_A = -13.75 KN$

i.e. $V_A = 13.75\ KN\ (\downarrow)$.

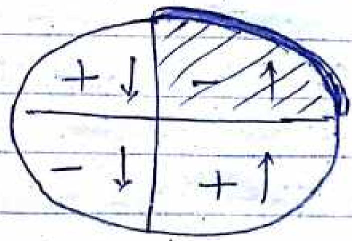


Explanation :-

While doing $\Sigma M = 0$,
Always write clockwise moments in one side and anti-clockwise moments in other sides.
Similarly, while doing $\Sigma V = 0$,
Write upwards forces in one side and downward in other side.

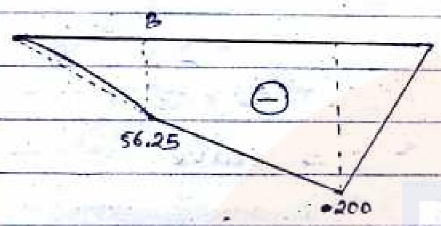
⇒ SFD.

Since $V_A = 13.75\ KN\ (\downarrow)$, go down 13.75 KN. i.e. $SF_A = -13.75\ KN$.
Now, UVL pushes downward by the amount equal to its area. So,
$SF_B = -13.75 - 15 = -28.75\ KN$.
But, load diagram is inclined. So, SFD must be curve. To find the nature of curve (i.e. concave upward or downward), use 'Ellipse Rule'.
- The load in downward dir^n is −ve.
- As we move from left to right, it's increasi
So, −↑ in 'Ellipse Rule' is :



So, curve becomes  nature.

No forces between B and C. So, straight line between B and C. Now, at C, $V_B = 128.75$ KN (↑) pushes upwards by 128.75 unit. So, $SF_c = -28.75 + 128.75 = +100$ KN. ~~Now,~~ Again, no forces between C and D. So, straight line. Finally go down 100 KN resulting $SF_D = 0$.



⇒ BMD.

Here, calculation of area of curve is tedious. So, calculate by usual method.

$$BM_B = -13.75 \times 3 - 15 \times \frac{1}{3} \times 3 = -56.25$$

i.e. $BM_B = 56.25$ KNm (↻).

Since, in SFD between A to B there is no st. line. So, BMD is curve between A and B. Again use "Ellipse Rule" to determine nature of curve. Between A and B, SF is -ve and it's increasing i.e. $-\uparrow$. So, from 'Ellipse Rule', the nature is

Now, $BM_C = -56.25 - \underbrace{28.75 \times 5}_{\text{Area betw}^n \text{ B and C.}}$

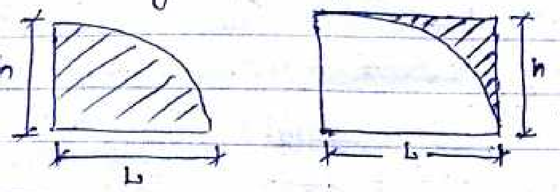
$= -200$ KNm.

$BM_D = -200 + \underbrace{(100 \times 2)}_{\text{Area of SFD between C and D.}}$

$= 0$ KNm.

Since, in SFD, B and C is connected by a horizontal st. line, BMD is a inclined line. Same for between C and D.

Calculating Area of Curve.



Area $= \frac{2}{3} \times h \times L$. Area $= \frac{1}{3} \times h \times L$.

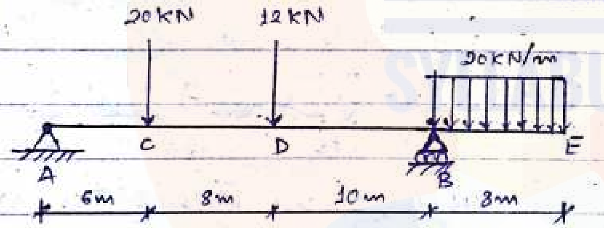
At the previous question,

BM_B = Area ~~between~~ of SFD between A and B

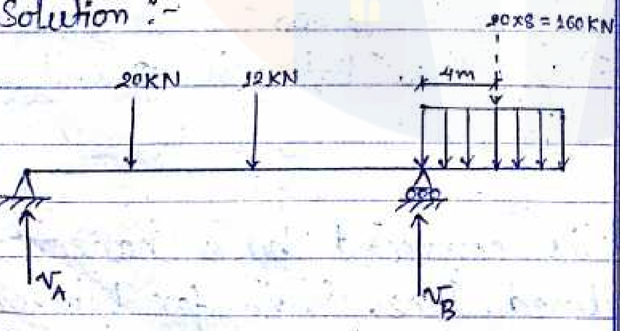
= Area of rectangle + Area of curve.

$= 13.75 \times 3 + \frac{1}{3} \times 15 \times 3$

$BM_B = -56.25$ KNm.



Solution :-



$\Sigma M_A = 0$.

$V_B \times 24 = 20 \times 6 + 12 \times 14 + 160 \times 28$

$V_B = 198.67$ KN (↑).

for $\Sigma M_A = 0$, write clockwise moment on one side and anticlockwise on other side. Also for $\Sigma V = 0$, write upward forces on one side and downward forces on another side.

⇒ SFD

$- V_A = 6.67$ KN (↓). So, $SF_A = -6.67$ KN.

No forces betw^n A and C. So, horizontal st. line i.e. constant SF. 20 KN force pushes 20 KN downwards.

So, $SF_C = -6.67 - 20 = -26.67$ KN.

Again 12 KN downwards i.e.

$SF_D = -26.67 - 12 = -38.67$ KN.

No forces between D and B.

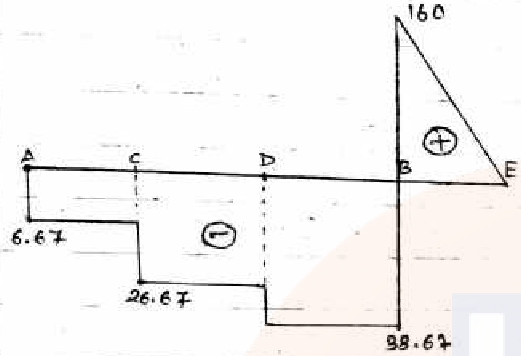
$\Sigma V = 0$

So,

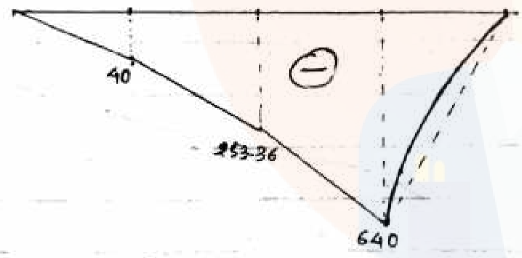
$V_A + V_B = 20 + 12 + 160$

$V_A = -6.67$ kN

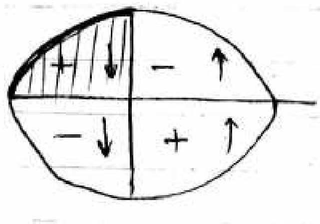
i.e. $V_A = 6.67$ kN $(\downarrow)$



SFD



BMD.



So, constant SF. $V_B = 198.67 (\uparrow)$ pushes upwards by 198.67 kN. So, $SF_B = -38.67 + 198.67 = 160$ kN.

Now, UDL of 20 kN/m $(\downarrow)$.

As we move 1 m from left to right, it pushes 20 kN downwards.

∴ As we move 8m from left to right, it pushes $20 \times 8 = 160$ kN downwards.

So, $SF_E = 160 - 160 = 0$.

⇒ BMD.

First calculate ~~and ma~~ the values and mark the points.

BM_C = Area of SFD between A and C

$= 6.67 \times 6 = -40$ kNm.

$BM_D = -40 - (26.67 \times 8) = -253.36$ kNm.

$BM_B = -253.36 - (38.67 \times 10)$

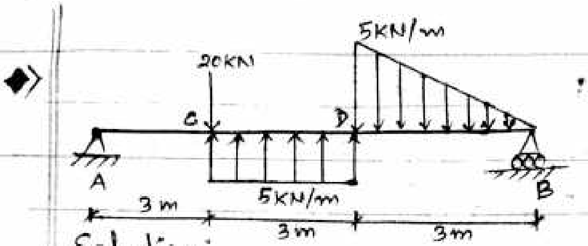
$= -640$ kNm.

$BM_E = -640 + \underbrace{\frac{1}{2} \times 160 \times 8}_{\text{Area of SFD betw}^n \text{ B and E}}$

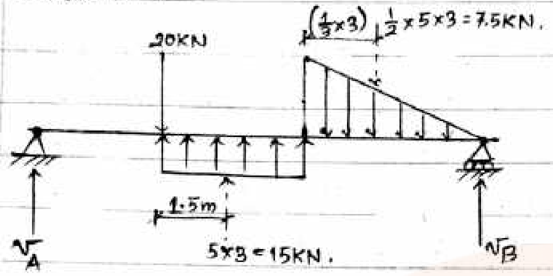
$= 0$ kNm.

Since in SFD, A and C, C and D, D and B are connected by horizontal st. line, BMD becomes inclined line. But B and E are connected by inclined line in SFD. So, it will be a curve in BMD and use "Ellipse Rule" to find nature of curve.

+ and ↓. So, ⌒ nature.



Solution:



$\Sigma M_A = 0.$

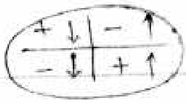
$V_B \times 9 + 15 \times 4.5 = 20 \times 3 + 7.5 \times 7$

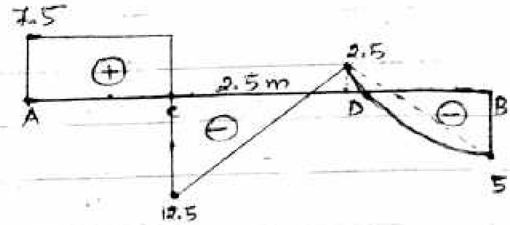
$V_B = 5\ KN\ (\uparrow)$

$\Sigma V = 0.$

$V_A + V_B + 15 = 20 + 7.5$

$V_A = 7.5\ KN\ (\uparrow)$





$SF_A = V_A = 7.5\ KN.$

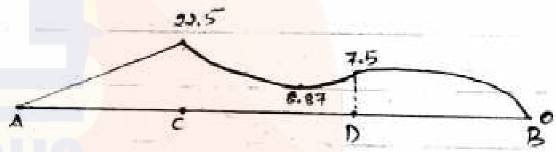
$SF_C = 7.5 - 20 = -12.5\ KN.$

$SF_D = -12.5 + 15 = 2.5\ KN.$

$(SF_B)_L = 2.5 - 7.5 = -5\ KN.$

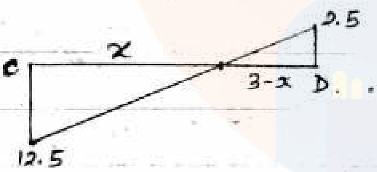
$(SF_B)_R = -5 + V_B = -5 + 5 = 0.$

$\Rightarrow$ BMD.



BMD.

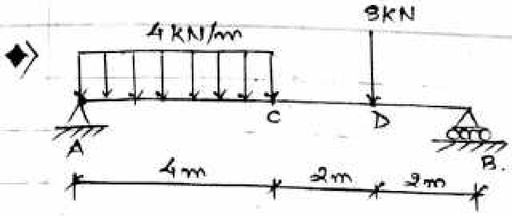
Calculating position of salient feature.



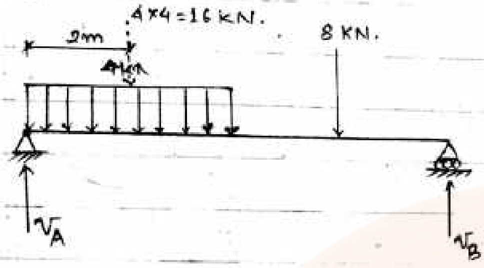
Using property of similar triangle,

$$\frac{x}{12.5} = \frac{3-x}{2.5}$$

$$x = 2.5m.$$



Solution :-



$\Sigma M_A = 0.$

$V_B \times 8 = 8 \times 6 + 16 \times 2$

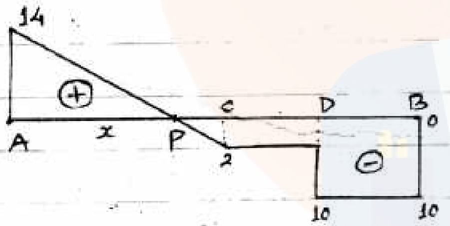
$V_B = 10 \text{ KN } (\uparrow).$

Similarly,

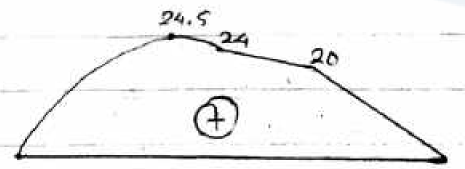
$\Sigma V = 0.$

$V_A + V_B = 16 + 8$

$V_A = 14 \text{ KN } (\uparrow).$



SFD



Explanation,

Since, $V_A = 14\text{KN} (\uparrow)$, $SF_A = 14 \text{ KN}$.

UDL pushes 4KN downwards per meter.

∴ In 4m, UDL pushes $4 \times 4 = 16$ KN downwards.

∴ $SF_C = 14 - 16 = -2 \text{ KN}$.

No forces between C & D so, horizontal st. line joins C & D. 8 KN (↓) pushes 8 KN downwards.

∴ $SF_D = -2 - 8 = -10$.

Again, join D and B by horizontal st. line. Finally, $V_B = 10 \text{ KN } (\uparrow)$ pushes 10 KN upwards.

∴ $SF_B = -10 + 10 = 0$.

⇒ BMD ..

Using property of similar triangles,

$$\frac{x}{14} = \frac{4-x}{2}$$

$$x = 3.5\text{m}.$$

∴ Area upto P $= \frac{1}{2} \times 14 \times 3.5$

$= 24.5$ kNm.

∴ $BM_P = 24.5$ KNm.

$BM_C = 24.5 - \frac{1}{2} \times 2 \times 0.5 = 24 \text{ KNm}.$

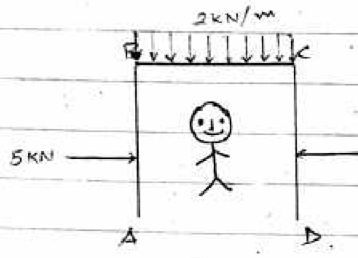
$BM_D = 24 - 2 \times 2 = 20 \text{ KNm}.$

$BM_B = 20 - 10 \times 2 = 0 \text{ KNm}.$

From A to C, SFD has inclined line. So, BMD from A to C has curve.

-- Frame --

- Frame is a combined beam.
- Each members of frame is solved separately as beam by transferring reactions, moments at the joints.



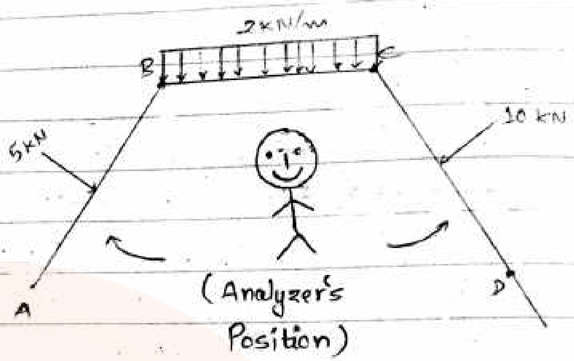


Fig (i)

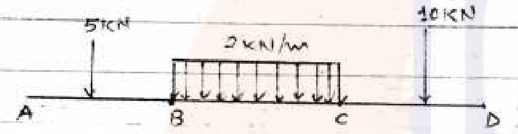


Fig (iii)

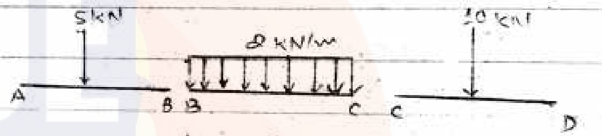
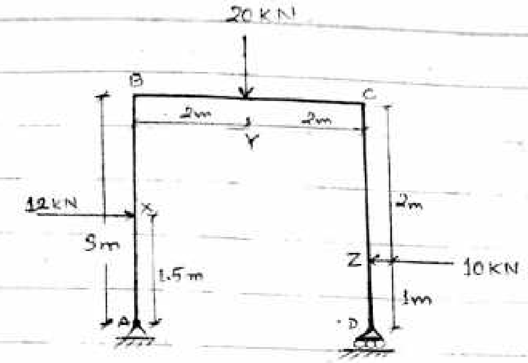


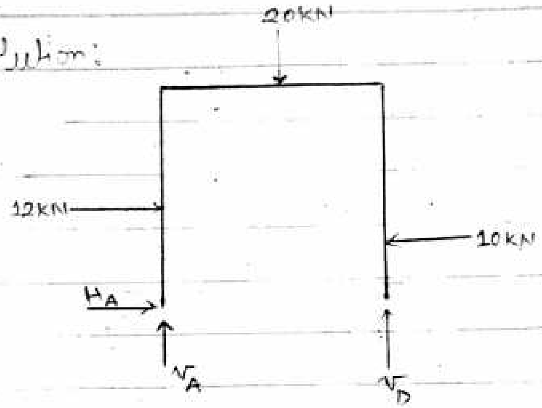
Fig (iv)

→ Holding member AB with left hand and member CD with right hand, try to stretch the frame apart as in figure (ii).

→ As in fig (iv), the frame is solved as three different beams AB, BC, CD provided that reactions and moments at joints must be transferred.



Solution:



→ At point A, there is hinged support. So, both horizontal and vertical reactions are developed. So, V_A and H_A are developed.

→ At point D, there is roller support. So, only vertical force is developed i.e. V_D

→ Now, calculation of reactions,

$\Sigma M_A = 0$

$12 \times 1.5 + 20 \times 2 = 10 \times 1 + V_D \times 4$

$\therefore \boxed{V_D = 12 \text{ KN } (\uparrow)}$

$\Sigma F_y = 0$

$V_A + V_D = 20$

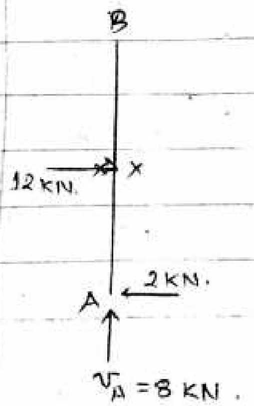
$\therefore \boxed{V_A = 8 \text{ KN } (\uparrow)}$

$\xrightarrow{+} \Sigma H = 0$

$H_A + 12 = 10$

$\therefore \boxed{H_A = 2 \text{ KN } (\leftarrow)}$

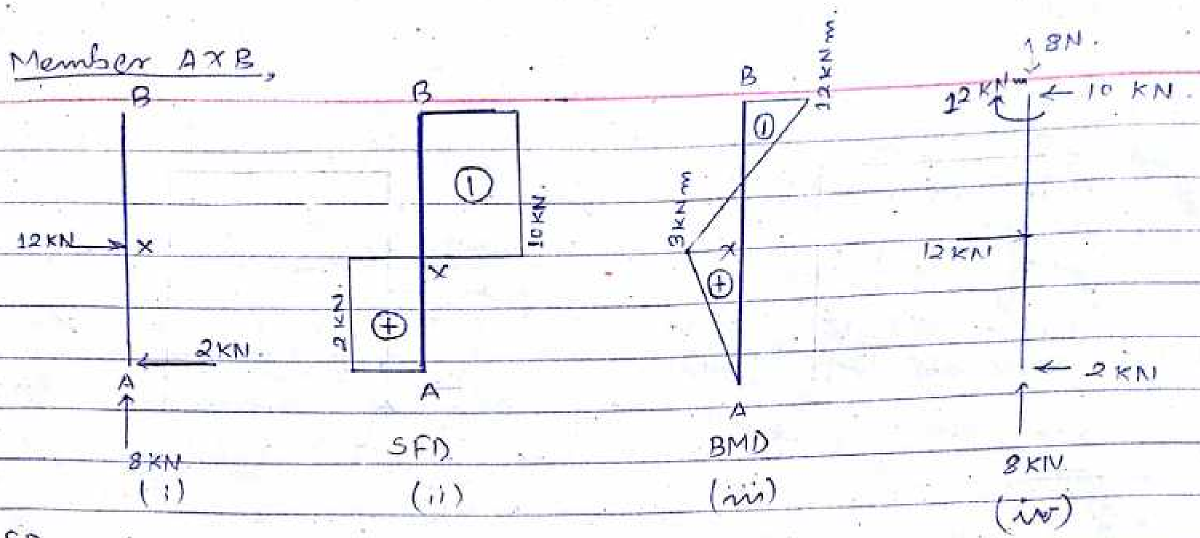
Now, starting from member AXB.



In member AXB, calculated reactions H_A and V_A are applied in ~~right~~ correct direction.

Now, our aim is to first, draw SFD and BMD.

Member AXB,



◆ SFD

At A, go 2 KN upwards. Between A and x, there are no forces. So, go constant i.e. a line straight to the member AXB. At x, 12 KN pushes 12 KN downwards resulting 2 - 12 = -10 KN. So, SF at x = -10 KN. No forces between x and B. So, constant SF. Now, our main objective is to balance the member. To balance the member or to make SF zero, we need 10 KN force is left direction. So, add 10 KN force in left direction as in figure (iv). For balancing the member, we can add forces by ~~simply~~ simple calculation. E.g. in this case, 10 + 2 from right side balances 12 KN from left side. So, 10 KN should be added. 8 KN should be added at B in downward direction to balance.

◆ BMD.

Calculate area of SFD

Moment at x = Area of SFD upto x.

= 2 × 1.5

= 3 KNm

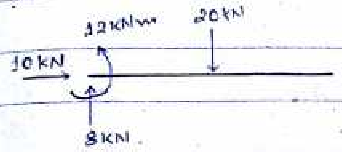
Moment at B = Area of SFD between A and x + Area of SFD between x and B

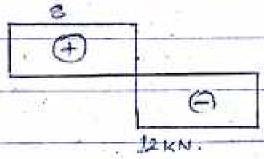
= 2 × 1.5 - 10 × 1.5

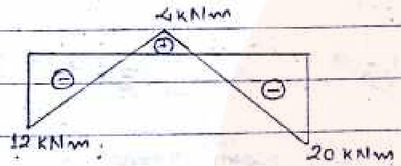
= -12 KNm.

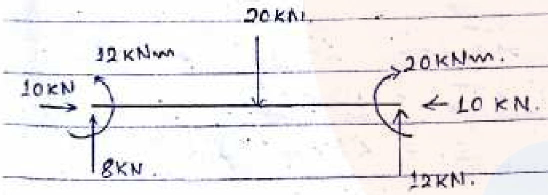
To balance the member add 12 KNm positive (i.e. clockwise) moment at B as in figure (iv).

Member BYC.









At B, transfer reactions and moments from point B of previous member AXB. But the direction becomes opposite. Now, draw SFD and BMD and balance the member BYC.

- SFD

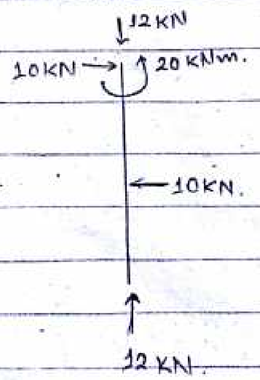
  After drawing SFD, we conclude that 12 KN force in upward direction should be added at point C. [See fig (iv).]

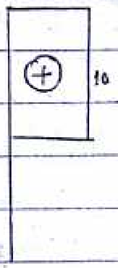
- BMD

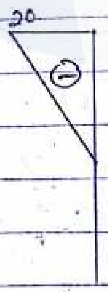
  After drawing BMD, we conclude that +ve 20 KNm moment i.e. clockwise should be added at point C of member BYC

  Re-check if the beam is in equilibrium or not from every angle. (Horizontal, vertical as well as moments)

Member CZD.







Note:

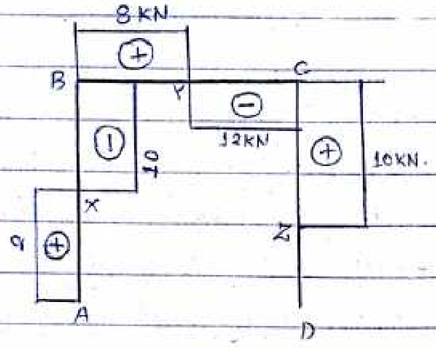
If our calculations are correct, last member (CZD in this case) should balanced without adding any reaction and moments.

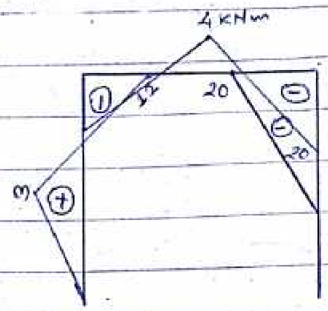
In horizontal direction $10 = 10$ (Balanced)

In vertical direction $12(\downarrow) = 12(\uparrow)$ (Balanced)

$M_Z = -20 + 10\times 2 = 0$ (Balanced).

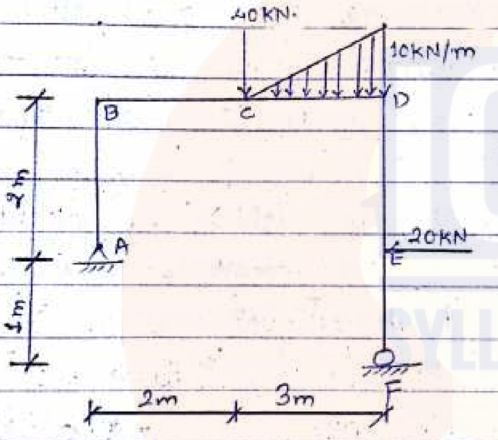
Final answer :-





SFD

◆>



$\Sigma M_A = 0$

$40\times2 + \left(\frac{1}{2}\times10\times3\right)\times\left(2+\frac{2}{3}\times3\right) = 5\times V_F$

$\boxed{V_{F_y} = 28\,KN\ (\uparrow)}$

$\Sigma V = 0$

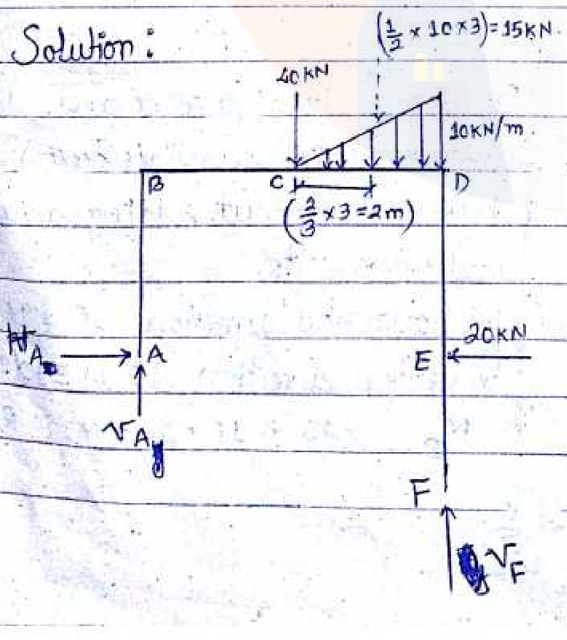
$V_A + V_F = 40 + 15$

$\boxed{V_A = 27\,KN\ (\uparrow)}$

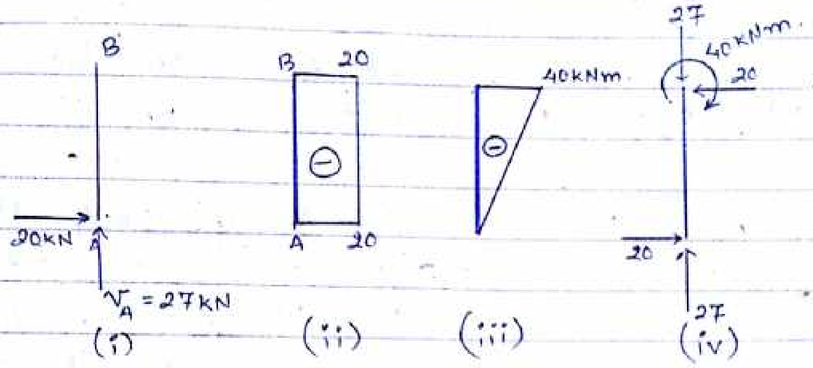
$\Sigma H = 0.$

$\boxed{H_A = 20\,KN\ (\rightarrow)}$

Solution :



Member AB,



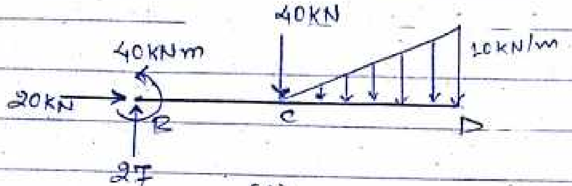
Our main objectives are to draw SFD, BMD and also to balance each individual members.

If it is easy to balance a member by simply looking at it, then we can do it. E.g. in above case if we add 27 kN(↓) at B, the member balances in vertical direction. Also, adding 20 kN(←) at B balances member AB.

But if, it becomes difficult to balance by eye estimation, first draw SFD and BMD. Then the member balances itself. E.g. in above case, first draw SFD. After drawing SFD, it evaluated that $SF_B = -20$ kN which means we have to add 20 kN (←) in order to balance the member. Similarly, BM at B = −40 kNm which means 40 kNm (↺) moment should be added at B to balance the member.

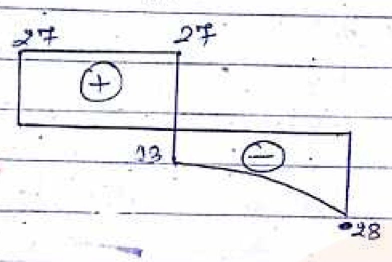
Hence, it's our choice whether to first draw BM, SF and balance the member later on or first balance the member and draw SF, BM. But, I prefer ~~first~~ drawing BM, SF and balancing the member later on as in figure above.

Member BCD:



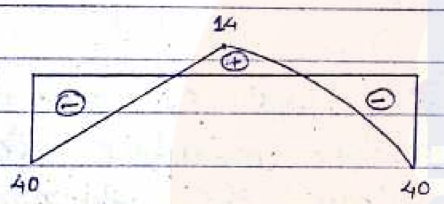
(i)

Point B is the common point for member AB and BCD. So, forces and moments should be transferred at point B as in fig (i).



(ii) SFD

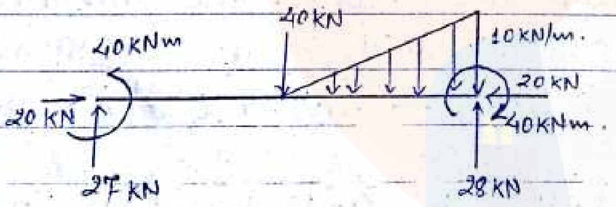
Draw SFD. For curve, follow 'Ellipse Rule'. $SF_D = -28$ KN. So, 28 KN (↑) should be applied to balance member BCD vertically.



(iii) BMD

Draw BMD. −40 KNm is initial moment at B. Then BM at D = −40 KNm. So, 40 KNm (↺) should be added to balance member BCD.

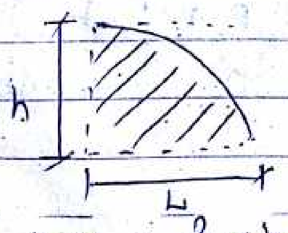
Finally 20 KN (←) at D should be applied to balance member BCD horizontally.



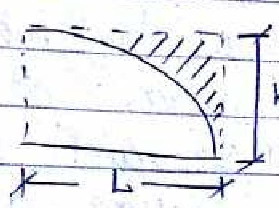
BM_D = area of SFD upto D.

$= -40 + 27\times 2 - 13\times 3 - \frac{1}{3}\times 3\times 1$

$= -40$ KNm.

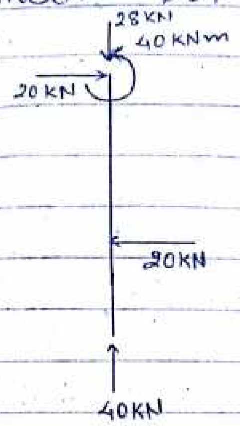


Area $= \frac{2}{3}\times h\times L$.

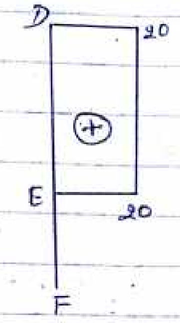


Area $= \frac{1}{3}\times h\times L$.

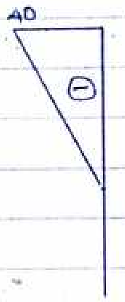
Member DEF.



(i)



(ii)
SFD



(iii)
BMD

Any comments and suggestions are welcomed.
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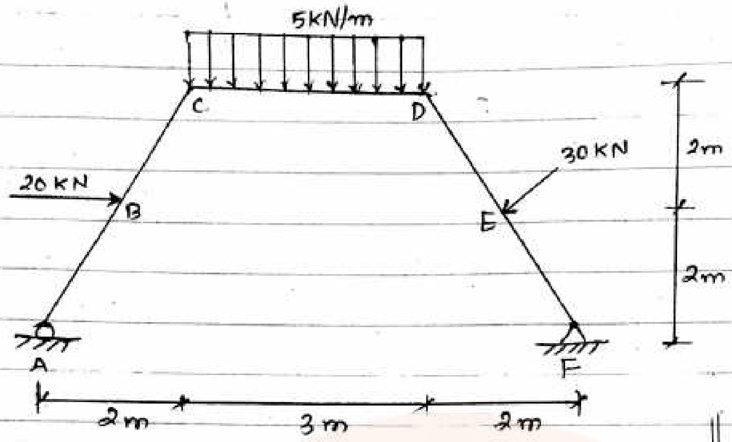
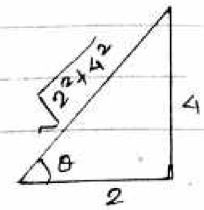
- Bibek Tamang
- 068' Batch
- Kantipur Eng. College
- Dhapakhel, Lalitpur.

Member DEF is the last member of the given frame. So, at last member, no forces, no moments are needed to balance it. It should balance ~~automatically~~ itself as in above case. If it happens, the frame analysis is correct else, check for mistakes.

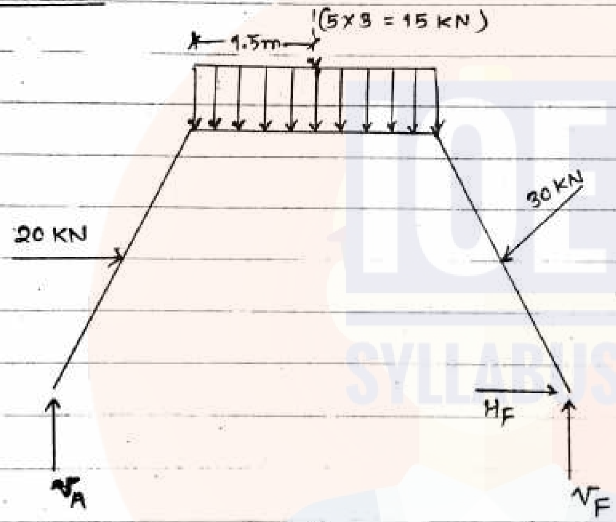
Combine all the members and draw SFD and BMD for final answer.

Note :- (for BCE II/I)
The most tough question that can be asked is 'frame with UVL and having an inclined member'. It is most probable question.
So, practise 'Frame with UVL and Inclined Member'.





Solution:



$p = 4\,m.$

$b = 2\,m$

$h = \sqrt{2^2+4^2}$

So,

$\sin\theta = \dfrac{p}{h} = 0.894$

$\cos\theta = \dfrac{b}{h} = 0.447$

$\boxed{\sqrt{1^2+2^2} = 2.236\,m.}$

$\Sigma M_F = 0.$

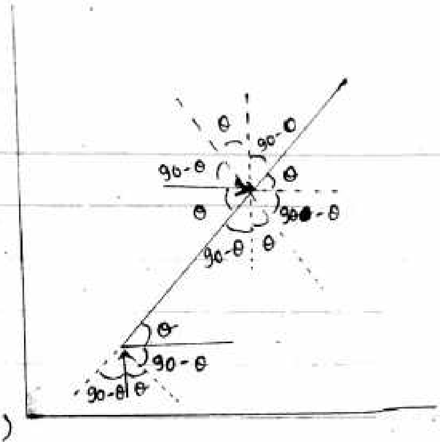
$V_A \times 7 + 20 \times 2 = 15 \times 3.5 + 30 \times 2.236$

$V_A = 11.369\ KN\ (\uparrow).$

$\Sigma V = 0.$

$11.369 + V_F = 15 + 30 \times 0.447$

$V_F = 17.041\ KN\ (\uparrow).$

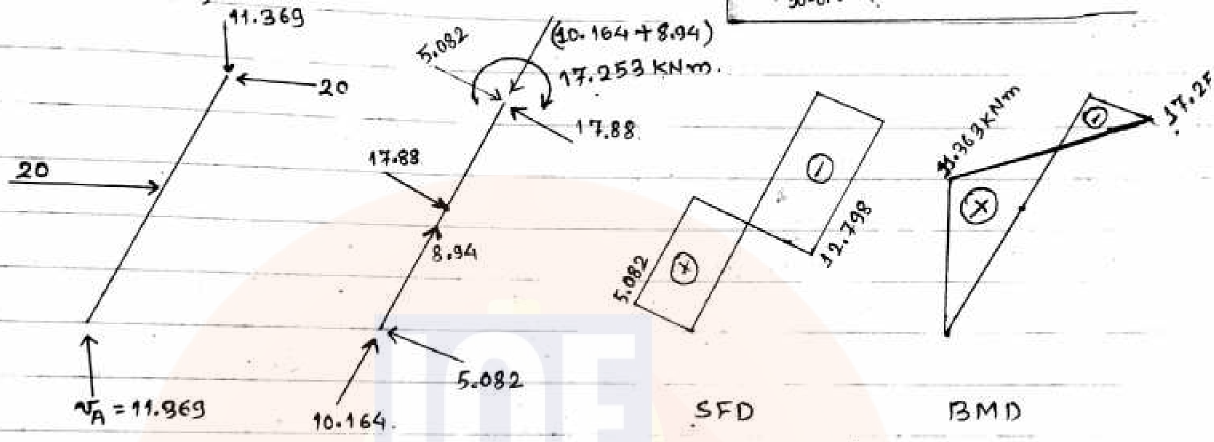


$$\Sigma H = 0$$

$$H_F + 20 = 30 \times 0.894$$

$$H_F = 6.82 \text{ KN } (\rightarrow)$$

Member ABC,



$$BM_B = \text{Area of SFD between A and B}$$

$$= 5.082 \times 2.236$$

$$= 11.363 \text{ KNm}.$$

$$BM_C = 11.363 - 12.798 \times 2.236$$

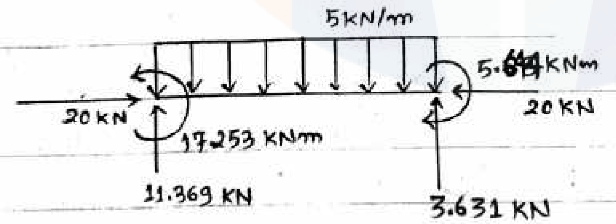
$$= -17.253 \text{ KNm}.$$

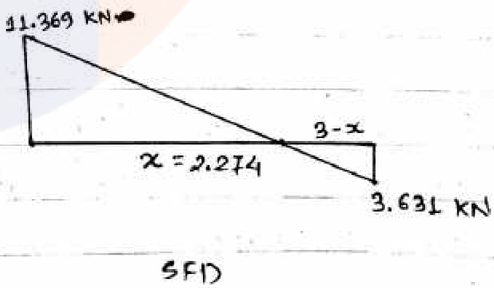
Solve,

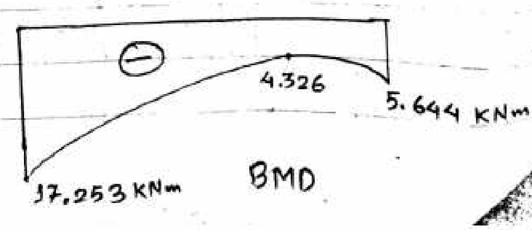
$$\frac{x}{3-x} = \frac{11.369}{3.631}$$

$$x = 2.274 \text{ m}$$

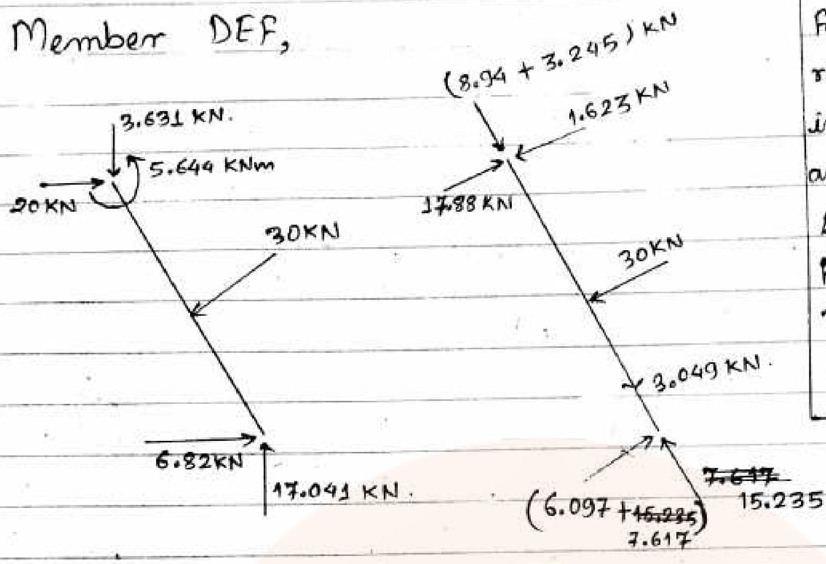
Member CD,



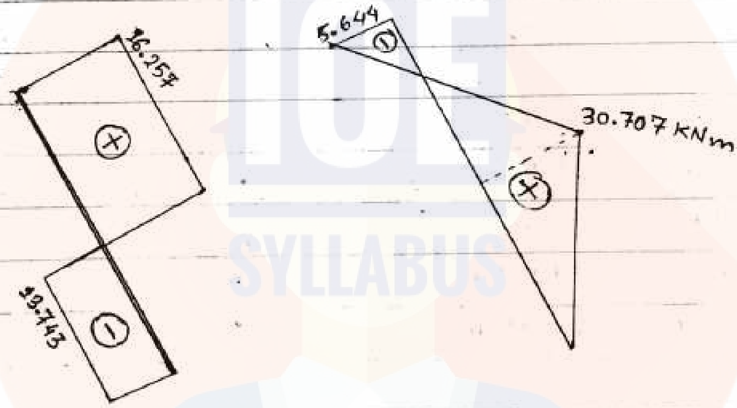




Member DEF,



Note:-
for inclined members, resolve all the forces, reaction in the direction perpendicular and parallel to the member. Axial Force Diagram (AFD) has not been drawn. Draw it yourself.

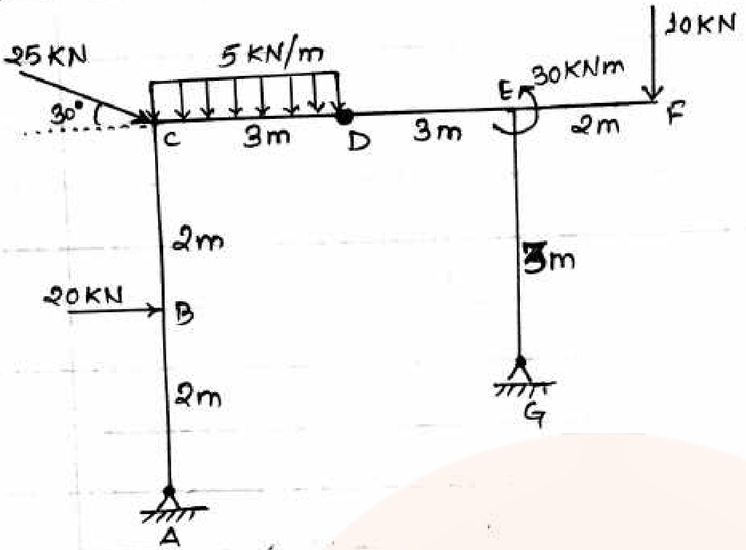


SFD

for last member DEF of given frame, no forces or moments are added i.e the last member balances itself. That means our calculation is correct.

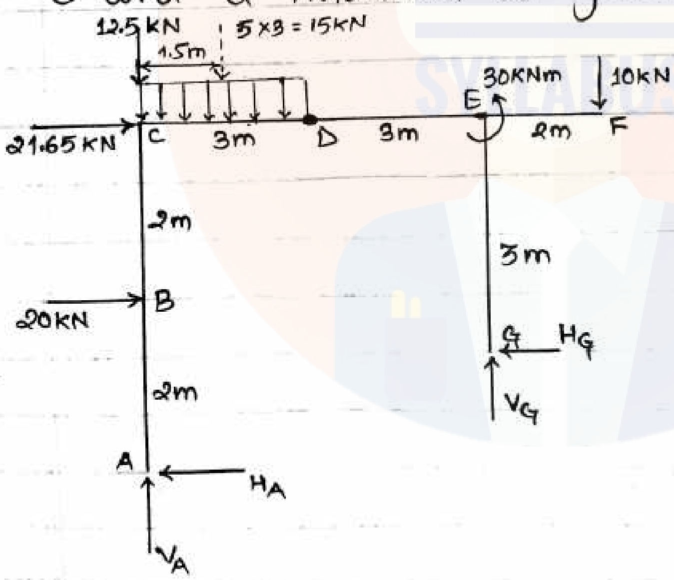
-- The End--

V. Imp



Note: This is not a solution of the problem but explanation.

The main objective of this numerical is to teach students to tackle problems that has forces and moments at joints as there is a force at joint C and a moment at joint E.



$25 \sin 30^\circ = 12.5\text{KN}$

$25 \cos 30^\circ = 21.65\text{ KN}$

$$\Sigma M_A = 0$$

$$V_G \times 6 + H_G \times 1 + 30 = 10 \times 8 + 15 \times 1.5 + 21.65 \times 4 + 20 \times 2$$

$$\therefore\ 6V_G + H_G = 199.1 \quad — \quad (1)$$

$(\Sigma M_D)_{right} = 0$ [Since D is a internal hinge].

$3H_G + 10\times 5 = 3V_G + 30$

$\therefore 3H_G - 3V_G = -20$ — (ii)

Solving (i) and (ii),

$V_G = 29.4 \text{ KN } (\uparrow)$

& $H_G = 22.73 \text{ KN } (\leftarrow)$.

Now,

$\Sigma V = 0$

$V_A + V_G = 10 + 15 + 12.5$

$\therefore \boxed{V_A = 8.1 \text{ KN } (\uparrow)}$

$\Sigma H = 0$

$H_A + H_G = 20 + 21.65$

$\therefore \boxed{H_A = 18.92 \text{ KN } (\leftarrow)}$

Proceed with any one of the methods that you like. Another method can be used to cross check your accuracy.

OR

$\Sigma M_G = 0$

$H_A \times 1 + V_A \times 6 + 20\times 1 + 21.65\times 3 + 10\times 2 = 30 + 15\times 4.5 + 12.5\times 6$

$\therefore H_A + 6V_A = 67.55$ — (iii)

$(\Sigma M_D)_{left} = 0$

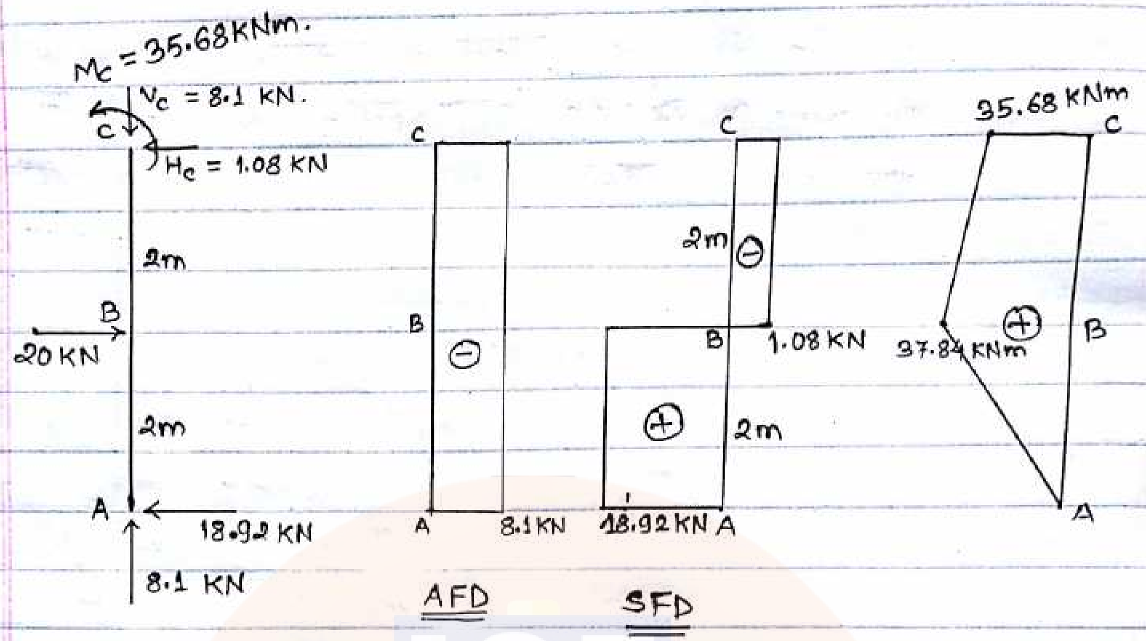
$H_A \times 4 + V_A \times 3 = 20\times 2 + 12.5\times 3 + 15\times 1.5$

$4H_A + 3V_A = 100$ — (iv)

Solving (iii) and (iv)

$$\boxed{\begin{aligned} H_A &= 18.92 \text{ KN } (\leftarrow) \\ V_A &= 8.1 \text{ KN } (\uparrow) \end{aligned}}$$

Member ABC,



Member ABC is cut at joint C. So member ABC is now unbalanced. Our aim is to balance the member or to determine the forces at C (V_c, H_c and M_c) such that the member ABC is balanced.

- In vertical direction, adding a force 8.1 KN (↓) balances the member ABC. (or you can use $\Sigma V = 0$).
- Applying 1.08 KN (←) balances the member ABC in horizontal direction (or you can use $\Sigma H = 0$).
- For moment (M_c), it becomes difficult or confusing to directly say the value of M_c. So, lets use $\Sigma M_c = 0$.

$$\Sigma M_c = 0$$
$$M_c + 20 \times 2 = 18.92 \times 4$$
$$M_c = 35.68 \text{ KNm } (\circlearrowleft).$$

This means applying 35.68 KNm (↺) moment at joint C balances member ABC against rotation (as same as H_c and V_c).

Then draw AFD, SFD and BMD.

For AFD, the member ABC is under compression from A to C by a force equal to 8.1 KN and compression is negative. It is represented graphically as shown in figure above.

SFD.

- At A, initially SF = 0 i.e. $(SF_A)_L = 0$
- Then, 18.92 KN (←) pushes the point in left direction (or +ve direction) by 18.92 KN. i.e. $(SF_A)_R = +18.92$ KN.
- From A to B, there are no forces in ← or → direction. So SF is constant from A to B. i.e. $(SF_B)_L = +18.92$ KN.
- As we reach point B, a force 20 KN (→) pushes the graph by 20KN in → direction and the graph reaches to value equal to $(18.92 - 20 = -1.08)$. i.e. $(SF_B)_R = -1.08$ KN.
- Similarly, there are no forces between B and C in ← or → direction. So, the graph is constant upto C i.e. $(SF_C)_L = -1.08$ KN
- As we reach point C, a force 1.08 KN (←) [or H_C] pushes the graph by 1.08 KN in ← direction and the graph reaches to $(-1.08 + 1.08 = 0)$ i.e. $(SF_C)_R = 0$
- All these activities can be represented graphically as shown in figure above.

BMD.

- At A, there is hinge support i.e. $(BM)_A = 0$
- At B, $(BM)_B$ = area of SFD between A and B.

$= 18.92 \times 2 = +37.84$ KNm.

i.e. $(BM)_B = +37.84$ KNm.

- At C, $(BM_C) = (BM)_B$ + area of SFD between B and C

-ve sign comes bcoz SFD between B&C is -ve

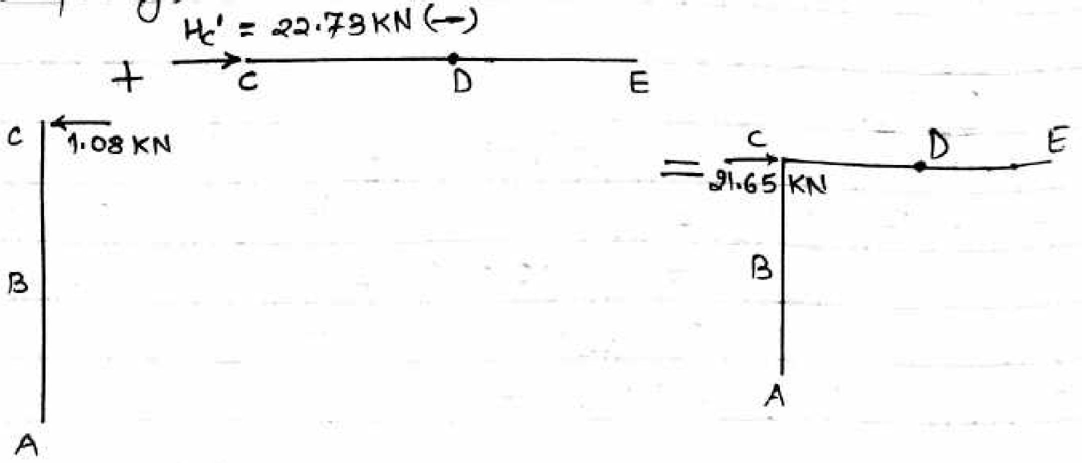
$$(BM)_C = +37.84 - 1.08 \times 2$$
$$= 35.68 \text{ KNm}.$$

There is point moment (M_C) at point C. So, there will be two values of BM at C i.e. left & right.
i.e. $(BM_C)_{left} = +35.68$ KNm
Then, at point C there is another moment $M_C = 35.68$ KNm (↺). When working from left to right, ↺ direction is negative. So, $(M_C)_R = +35.68 - 35.68 = 0$.
All these activities are shown in diagram above.

Joint C is a bit different in this numerical because a force 25KN (30°) is acting at joint C or a force equal to 12.5KN (↓) and 21.65KN (→) is acting at joint C.

In other numericals, we transferred forces and moment equal in magnitude but opposite in direction because they would cancel each other and become zero. But we can't do that here.

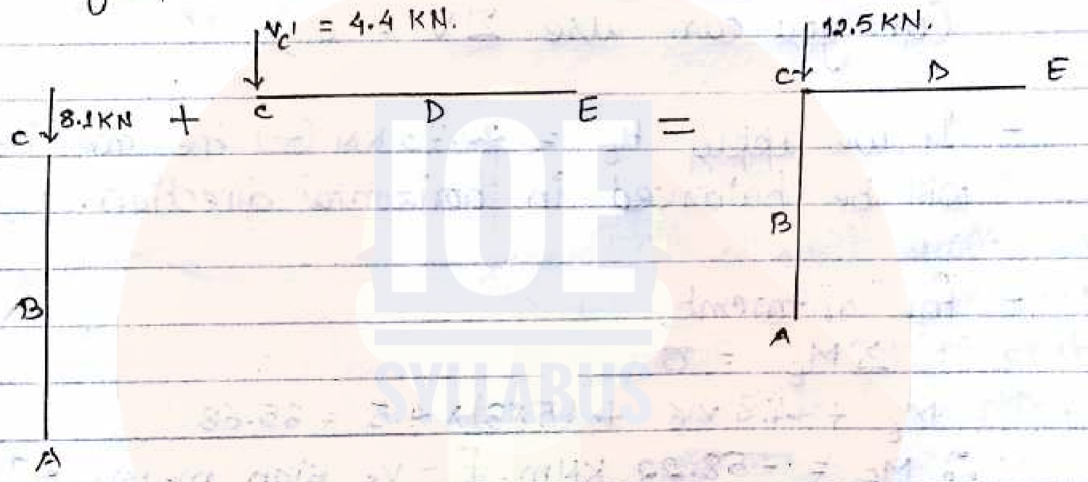
Lets see joint C by zooming in (on horizontal forces for simplicity).



- Our main aim is to obtain 21.65 KN ($\rightarrow$) by adding (or subtracting) H_c (1.08 KN ($\leftarrow$)) and H_c'.
- If we apply $H_c' = 21.65 + 1.08 = 22.73$ KN ($\rightarrow$), then adding H_c and H_c' will equal 21.65 KN ($\rightarrow$).

  $\therefore H_c' = 22.73$ KN ($\rightarrow$).

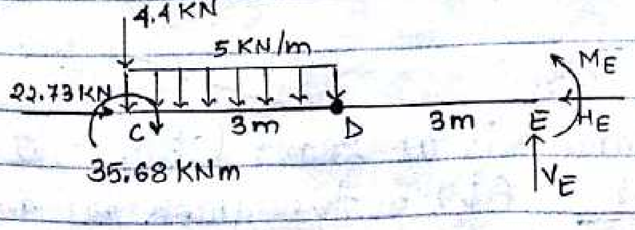
Similarly for vertical direction,



Applying $V_c' = 4.4$ KN ($\downarrow$) with $V_c = 8.1$ KN ($\downarrow$) equals 12.5 KN ($\downarrow$). So, $V_c' = 4.4$ KN ($\downarrow$).

But there is no moment at joint C. So, we can transfer equal and opposite moment at joint C such that they cancel each other.

So, finally, <u>Member CDE</u>,



- Member CDE is cut at E and it becomes unbalanced. Our aim is to balance the member or to find the forces (M_E, H_E, V_E) in order that member CDE is balanced.
- In vertical direction,
  If we apply $V_E = 4.4 + (5\times3) = 19.4$ KN (↑) the member will balance in vertical direction.
  (or you can use $\Sigma V = 0$).

- If we apply $H_E = 22.73$ KN (←) the member CDE will be balanced in horizontal direction.

- for moment,

$$\Sigma M_E = 0$$

$$M_E + 4.4\times6 + (5\times3)\times4.5 = 35.68$$

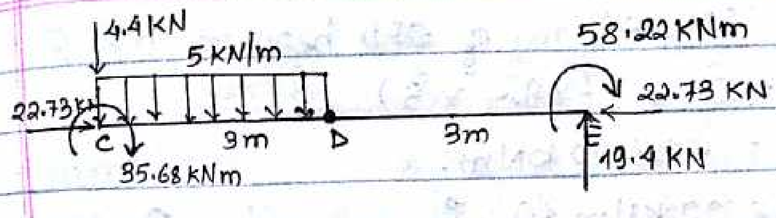
$\therefore M_E = -58.22$ KNm. [−ve sign means :].
I considered M_E is in ↺ direction. But I was wrong.
So, $M_E = 58.22$ KNm (↻).
i.e. Applying a moment 58.22 KNm (↻) will balance member CDE against rotation.

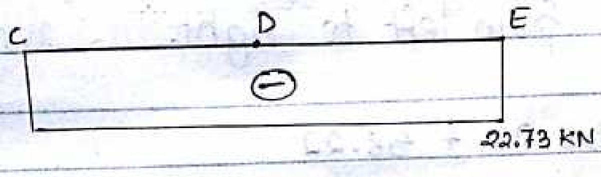
AFD

Member is under compression by force equal to 22.73 KN i.e. graph is -ve since compression is -ve.

SFD.

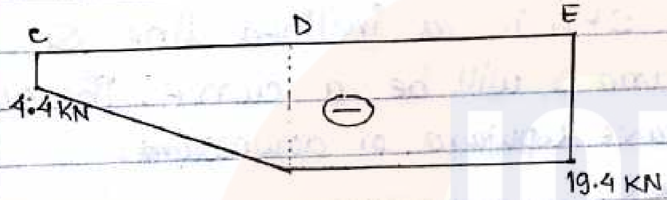
- Initially, graph is at zero. $(SF_C)_L = 0$.
- Then a 4.4 KN (↓) pushes graph by 4.4 KN in ↓ direction i.e. $(SF_C)_R = 0 - 4.4 = -4.4$ KN.

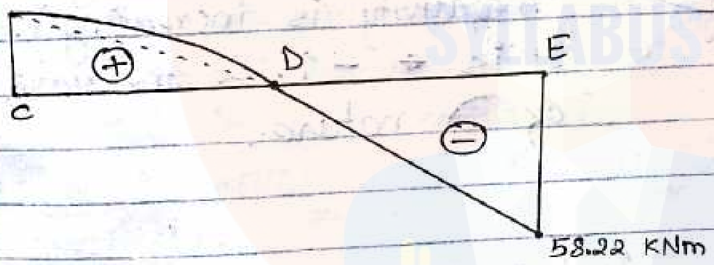




AFD

- from C to D, a UDL 5KN/m (5 KN per m) pushes graph by 5KN per meter in ↓ direction So, graph reaches $-4 - 5\times3 = -19.4$ KN at D. So, $SF_D = -19.4$ KN.
- No forces between between D and E. So, $(SF_E)_L = -19.4$ KN.
- At E, 19.4 KN (↑) pushes graph in ↑ direction resulting $(SF_E)_R = 0$.





BMD

Initially, $(BM_C)_L = 0$ KNm. $M_E' = 35.68$ KNm makes $(BM_C)_R = +35.68$ KNm

Then, $BM_D = BM_C +$ Area of SFD between C & D

$= +35.68 - \left(\underbrace{\frac{1}{2}\times 15\times 3}_{\text{(Area of }\Delta)} + \underbrace{4.4\times 3}_{\text{Area of }\square}\right)$

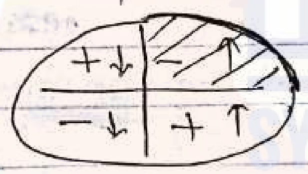
$= -0.02 \approx 0$ (This error occurs due to round off errors).

- BM_D must be zero. Because it is an internal hinge. We proceeded by considering $(\Sigma M_D)_{left} = 0$ and $(\Sigma M_D)_{right} = 0$ and it must be zero from calculation also.

Now, $((BM)_E)_{left}$ = BM_D + area of SFD between D & E.
$= 0 + (-19.4 \times 3)$
$= -58.22$ KNm.

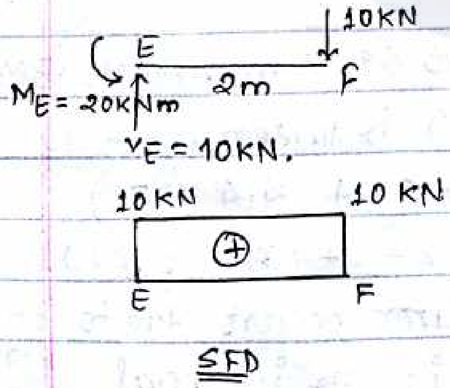
At E, $M_E = 58.22$ KNm (↻) is present. ↻ means +ve while working from left to right.
So,
$(BM_E)_{right} = -58.22 + 58.22$
$= 0$.

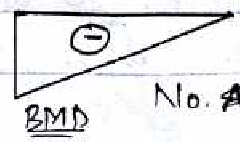
Between C and D, SFD is a inclined line. So, BMD between C and D will be a curve. The curve will either be concave upward or downward.
Using Ellipse Rule,



Between C & D, SFD is −ve and value is increasing. (or its negativity is increasing).
It is −↑. So the curve is of ⌒ nature.

Member EF,

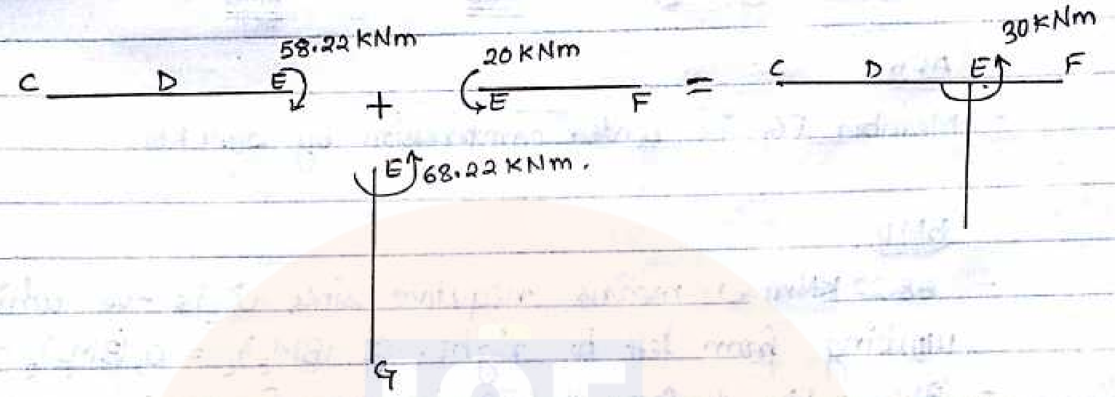




To balance the member apply 10KN (↑) at E.
The 10KN (↓) at F applies 20KNm moment in (↻) direction at point E. So applying 20KNm in ↺ direction will balance member against rotation. (or use $\Sigma M_E = 0$).
For SFD and BMD, work from left to right always. Do it yourself!!

No. AFD since no forces in axial direction of the member.

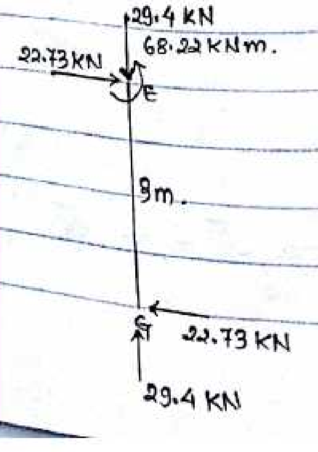
Again, in joint E, 30 KNm (↺) moment is present in question. As joint E is a junction of three members viz. CDE, EF and EG. So, we have to obtain 30 KNm (↺) by adding (or subtracting) moments of these three members.



i.e. Applying 68.22 KNm (↺) in joint E of member EG will result in 30 KNm (↺) ~~moment~~ resultant moment at joint E. Similarly for horizontal and vertical forces, applying 29.4 KN (↓) will result in zero vertical force at E since 19.4 KN (↑) and 10 KN (↑) forces are acting at joint E of CDE & EF member respectively.

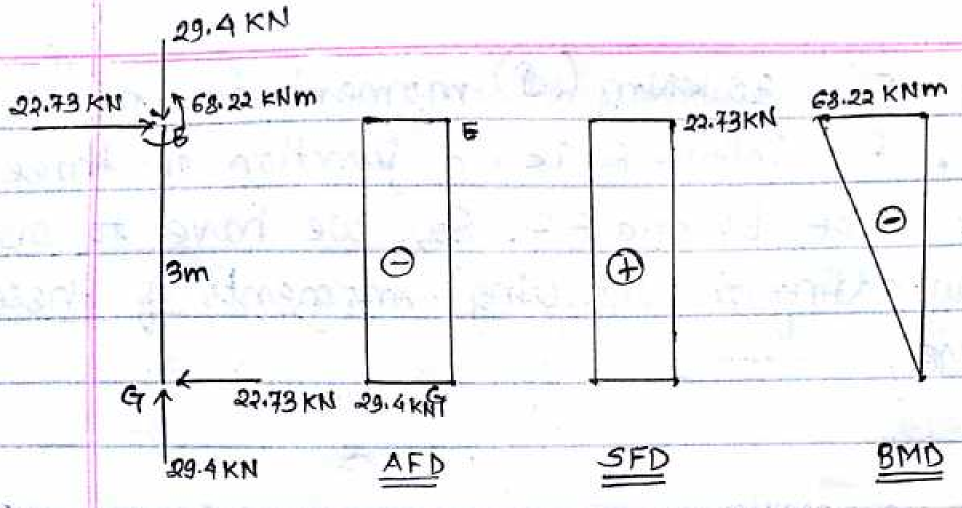
Similarly, applying 22.73 KN (→) at joint E of EG member result in zero resultant force in horizontal direction.

i.e.



OR.

- First draw EG member independently. As H_G = 22.73 KN (←) and V_G = 29.4 KN (↑). Now, to balance these H_G and V_G, we should apply 22.73 KN (→) and 29.4 KN (↓) at joint E and moment 22.73 × 3 = 68.2 ≈ 68.22 KNm (↺) at joint E to balance EG against rotation.



AFD

- Member EG is under compression by 29.4 KN.

BMD.

- 68.22 KNm (↺) means negative since ↺ is -ve while working from left to right. So, $(BM_E)_L = 0, (BM_E)_R = -68.22$ KNm
- $BM_G = BM_E$ + Area of SFD between E and G

$= -68.22 + 22.73 \times 3$

$= -0.03 \approx 0$ (This small error is due to round off error).

7. ANALYSIS OF TRUSS

7.1 USE OF TRUSSES IN ENGINEERING

Truss

- a structure made up of one or more triangular units made from slender members that are pin jointed at ends.
- Truss are mainly used in bridges, long roof spans, buildings etc. Truss members are connected at their extremities only. All loads are applied to the joints.

Degree of Static Indeterminacy.

- If $m = 2j - r$, truss is said to be perfect or rigid & it is statically determinate.
- If $m > 2j - r$, truss is said to be redundant or over rigid & it is statically indeterminate.
- If $m < 2j - r$, truss is said to be deficient or imperfect truss & it is unstable.

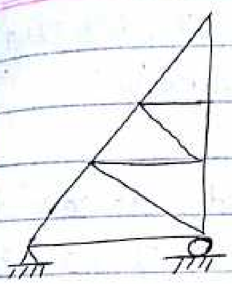
- Total static indeterminacy (TSI) = External static indeterminacy + Internal static indeterminacy

$$= I_{se} + I_{si}$$
$$= (r - 3) + [m - (2j - 3)]$$
$$= m + r - 2j.$$

$m \rightarrow$ no. of members

$r \rightarrow$ no. of unknown reaction.

$j \rightarrow$ no. of joints.



$j = 7$
$m = 11$
$r = 3.$

So,

$I_{se} = r - 3 = 3 - 3 = 0$

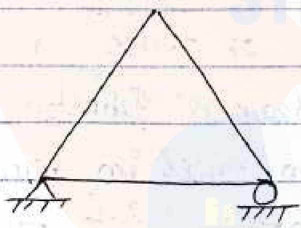
$I_{si} = m - (2j - 3) = 11 - (2 \times 7 - 3)$
$= 0.$

So, $TSI = I_{se} + I_{si}$
$= 0.$

Degree of Kinematic Indeterminacy (DKI).

- It is the total number of freedom that are allowed for t joints in a structure.

$DKI = 2j - r$
$= 2 \times$ no. of joints $-$ no. of support reactions.



$j = 3.$
$r = 3.$

$DKI = 2j - r$
$= 2 \times 3 - 3$
$= 3.$

7.2 CALCULATION OF MEMBER FORCES OF TRUSS BY METHOD OF JOINTS (Refer Page 292 Bear & Johnston)

Procedure :-

(i) At first, unknown reactions are determined by consider the whole truss is in equilibrium. ($\Sigma M = 0, \Sigma V = 0$ & ΣH=

(ii) At each joints, the forces in members and applied load are concurrent & two independent equation of equilib can be formed at each joints. i.e. $\Sigma H = 0$, $\Sigma V = 0$.
So, each joint is selected that has maximum number of unknowns equal to two.

(iii) Similarly, other joints are solved one by one by using equation of equilibrium.

- Method of joints is used when forces on each (all) members of truss is required.

7.3 CALCULATION OF MEMBER FORCES OF TRUSS BY METHOD OF SECTION (Refer Page 304 Bear & Johnston).

- Method of section is used when forces in few members are required and when method of joints fail to ~~solv~~ give the solution.

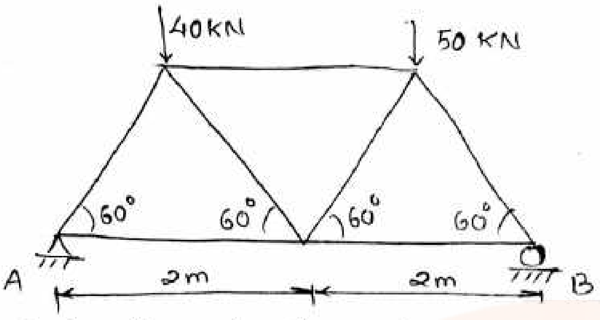
Procedure :-

(i) At first, all unknown reactions are determined by considering the whole truss is in equilibrium ($\Sigma M = 0$, $\Sigma V = 0$ & $\Sigma H = 0$).

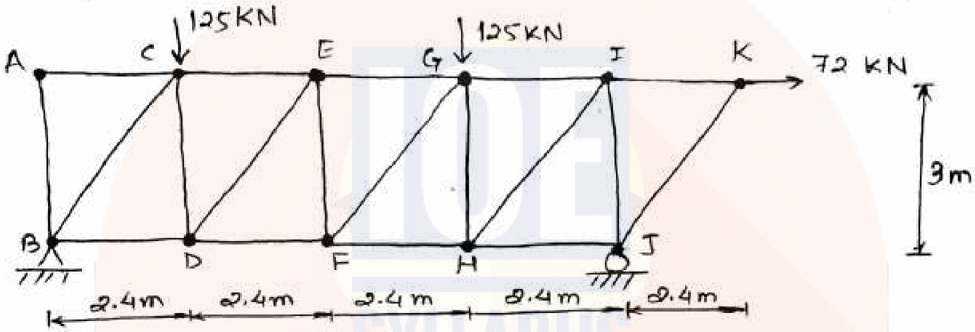
(ii) Truss is now cut into section by a line that passes through maximum three members of the truss. Then use three equations of equilibrium to find three unknown forces.
such that truss is divided into two completely separate parts.
Now, both parts obtained can be used as free body.

Determine all member forces of following truss.

a.



Determine the force in member EF & GJ of following truss.



$\Sigma M_B = 0$

$V_J \times 9.6 = 125 \times 2.4 + 125 \times 7.2 + 72 \times 3$

$\therefore \boxed{V_J = 147.5 \text{ KN } (\uparrow)}$

$\Sigma H = 0$

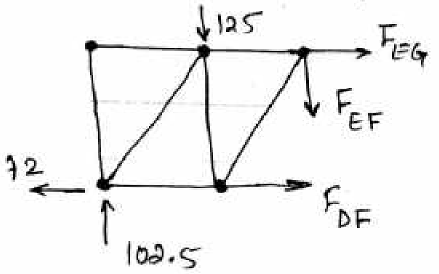
$\therefore \boxed{H_B = 72 \text{ KN } (\leftarrow)}$

$\Sigma V = 0$

$V_B + V_J = 125 + 125$

$\therefore \boxed{V_B = 102.5 \text{ KN } (\uparrow)}$

force in member EF :



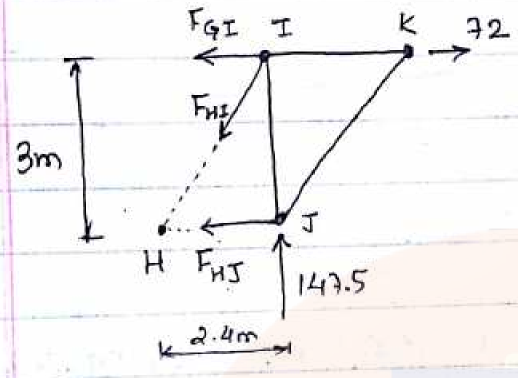
$\Sigma V = 0$

$F_{EF} + 125 = 102.5$

$F_{EF} = -22.5 \text{ KN}$

i.e. $\boxed{F_{EF} = 22.5 \text{ KN } (\uparrow)}$

Force in member GI



$\Sigma M_H = 0$

$F_{GI} \times 3 + 147.5 \times 2.4 = 72 \times 3$

$F_{GI} = -46$ KN

i.e. $\boxed{F_{GI} = 46 \text{ KN } (\rightarrow)}$

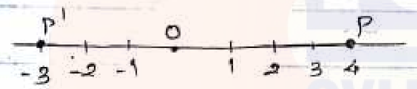
8. Kinematics of Particles & Rigid Body.

8.1. RECTILINEAR KINEMATICS : CONTINUOUS MOTION

- A particle is said to be in rectilinear motion if it moves along a straight line. So the study of dynamics by discussing the kinematics of a particle that moves along a straight line is called rectilinear kinematics.
- Rectilinear kinematics of a particle is characterized by specifying the particle's position, velocity and acceleration at any instant.

Position

The particle will occupy a certain position at any instant of time t. To define the position (P) of the particle, we choose a fixed origin O on the straight line and a direction.



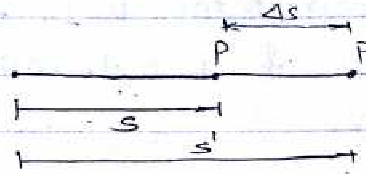
$OP = 4m$

$OP' = -3m.$

Here, P and P' are called position coordinates and $\vec{OP}$ and $\vec{OP'}$ are called position vector.

Displacement

The displacement of the particle is defined as the change in its position.



If particle moves from P to P', the displacement is $\Delta s = s' - s$

Velocity

If the particle moves through a displacement Δs from P to P' during the time interval Δt, the average velocity of the particle during this time interval is.

$$v_{avg} = \frac{\Delta s}{\Delta t}.$$

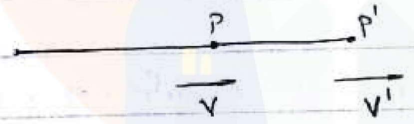
By chosing shorter & shorter time intervals Δt, instantaneous velocity (v) of the particle is obtained.

$$\text{Instantaneous velocity } (v) = \lim_{\Delta t \to 0} \frac{\Delta s}{\Delta t}.$$

$$\therefore v = \frac{ds}{dt}. \quad — (a).$$

Acceleration :-

The rate of change of velocity is acceleration.



$$a_{avg} = \frac{\Delta v}{\Delta t}.$$

where, $\Delta v \rightarrow$ difference in velocity during time interval Δt.

i.e. $\Delta v = v' - v$.

The instaneous acceleration at time t is found by taking smaller & smaller values of Δt and corresponding smaller & smaller values of Δv.

So,

$$\text{Instantaneous acceleration } (a) = \lim_{\Delta t \to 0} \frac{\Delta v}{\Delta t}$$

Substituting value of v from equation ⓐ.

$$v = \frac{d^2s}{dt^2}.$$

Since,

$$v = \frac{dx}{dt}, \quad dt = \frac{dx}{v}.$$

$$\text{So,} \quad a = \frac{dv}{dt} = \frac{dv}{\frac{dx}{v}} = \frac{v\,dv}{dx} \quad \text{i.e.} \quad \boxed{a = \frac{v\,dv}{dx}}$$

8.3 DETERMINATION OF MOTION OF PARTICLE & RIGID BODY

The condition of motion will be specified by the type of acceleration that the particle possess. The acceleration of the particle may be expressed as the function of one or more of the variables x, v and t.

ⓐ When acceleration is a function of time [i.e. $a = f(t)$. e.g. $a = 2t^2 - 5$]

We know, $a = \frac{dv}{dt} \Rightarrow dv = a\,dt \Rightarrow dv = f(t)\,dt$

Integrating,

$$\int dv = \int f(t)\,dt$$

When $t = 0$, $v = v_0$ (initial velocity)

When $t = t$, $v = v$ (final velocity)

So,

$$\int_{v_0}^{v} dv = \int_0^t f(t)\,dt$$

$$v - v_0 = \int_0^t f(t)\,dt.$$

$$\therefore \boxed{v = v_0 + \int_0^t f(t)\,dt} \quad \text{— ①.}$$

Again, $v = \frac{dx}{dt}$

$dx = v dt$

Integrating,

$\int dx = \int v dt$

At $t = 0$, $x = x_0$.

At $t = t$, $x = x$.

So,

$$\int_{x_0}^{x} dx = \int_{0}^{t} v\,dt$$

Substituting value of v from equation ①.

$$\text{or, } x - x_0 = \int_0^t \left[v_0 + \int_0^t f(t)\,dt \right] dt$$

$$\therefore \boxed{x = x_0 + \int_0^t \left[v_0 + \int_0^t f(t)\,dt \right] dt}$$

i.e. In order to determine x in terms of t, it is necessary to perform two successive integration.

② When acceleration is a function of position [i.e. $a = f(x)$. eg. $a = x^2$

We know, $a = \frac{dv}{dt}$

Also, $a = \frac{v\,dv}{dx}$

or, $v\,dv = a\,dx$

or, $v\,dv = f(x)\,dx$

Integrating,

At $t = 0$, $x = x_0$, $v = v_0$. (Initial velocity)

At $t = t$, $x = x$, $v = v$ (Final velocity).

So,

$$\int_{v_0}^{v} v\,dv = \int_{x_0}^{x} f(x)\,dx$$

or, $\left[\frac{v^2}{2}\right]_{v_0}^{v} = \int_{x_0}^{x} f(x)dx$.

$$\frac{v^2}{2} - \frac{v_0^2}{2} = \int_{x_0}^{x} f(x)dx$$

$$\boxed{v^2 = v_0^2 + 2\int_{x_0}^{x} f(x)dx}$$

or, $v = \left[v_0^2 + 2\int_{x_0}^{x} f(x)dx\right]^{1/2} \rightarrow (1)$.

Again,

$$v = \frac{dx}{dt}$$

$$dx = vdt.$$

Integrating,

$$\int dx = \int vdt$$

$$\int_{x_0}^{x} dx = \int_0^t vdt.$$

Substituting value of v from eqn (1).

$$x - x_0 = \int_0^t vdt.$$

$$\therefore \boxed{x = x_0 + \int_0^t \left[v_0^2 + 2\int_{x_0}^{x} f(x)dx\right]^{1/2} dt.}$$

(c) When acceleration is a function of velocity [i.e. $a = f(v)$, e.g. $a = 8v - 4$]

We know, $a = \frac{dv}{dt}$.

Also, $a = v\frac{dv}{dx}$.

or, $dx = \frac{v\,dv}{f(v)}$

At $t = 0$, $x = x_0$, $v = v_0$

At $t = t$, $x = x$, $v = V$

Integrating,

$$\int_{x_0}^{x} dx = \int_{v_0}^{v} \frac{v\,dv}{f(v)}$$

$$x - x_0 = \int_{v_0}^{v} \frac{v\,dv}{f(v)}$$

$$\therefore \boxed{x = x_0 + \int_{v_0}^{v} \frac{v\,dv}{f(v)}}$$

(d) Uniformly rectilinear motion.

→ particle is moving is constant velocity in a straight line.

i.e.

$$V = \frac{dx}{dt} = \text{constant} = u \text{ (say)}.$$

$$\frac{dx}{dt} = u$$

$$dx = u\,dt.$$

$$\int dx = \int u\,dt$$

$$x = ut + c_1$$

At $t = 0$, $x = x_0$.

$$c_1 = x_0.$$

$$\therefore \boxed{x = x_0 + ut}$$

© Uniformly accelerated rectilinear motion.

→ Particle is moving with constant acceleration in a straight line.

i.e. $\frac{dv}{dt}$ = Constant = a.

$dv = adt$.

$\int dv = \int adt$.

$v = at + c_1$

At $t = 0$, $v = v_0$ (Initial velocity).

$\boxed{v = v_0 + at}$ — ①. ($v = u + at$)

similar.

Also,

$v = \frac{dx}{dt}$.

$\frac{dx}{dt} = v_0 + at$.

$dx = (v_0 + at)dt$.

$\int dx = \int (v_0 + at)dt$.

$x = v_0 t + \frac{at^2}{2} + c$

At $t = 0$, $x = x_0$

$c = x_0$.

Similar.

So, $\boxed{x = x_0 + v_0 t + \frac{1}{2}at^2}$ ($s = ut + \frac{1}{2}at^2$)

Also, $a = v\frac{dv}{dx}$.

$vdv = adx$.

$\int vdv = \int adx$.

$\frac{v^2}{2} = ax + c$.

At $t = 0$, $v = v_0$, $x = x_0$.

$$\frac{v_0^2}{2} = a x_0 + c_1$$

So, $$c_1 = \frac{v_0^2}{2} - a x_0.$$

$$\frac{v^2}{2} = a x + \left(\frac{v_0^2}{2} - a x_0\right) \quad \text{(Substituting value of c).}$$

$$\frac{v^2}{2} - \frac{v_0^2}{2} = a(x - x_0).$$

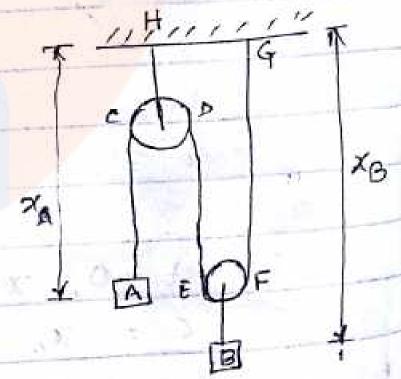
$$\therefore \boxed{v^2 = v_0^2 + 2a(x - x_0)} \qquad \boxed{v^2 - u^2 = 2as}$$

Similar

Dependent Motion

When position of a particle depends upon the position of another particle(s), such motion is called dependent motion.

Consider a pulley system as shown in figure. Here, the position of block B depends upon the position of block A.



In this pulley system, the length of rope ACDEFG is constant. It just changes its orientation.

from figure,

HC = Constant & EB = Constant.

length of rope ACDEFG = Constant.

i.e. AC + CD + DE + EF + FG = Constant.

Since, CD and EF are alway constant,

AC + DE + FG = Constant. — (1)

Here, x_A and x_B represents position of block A and B from the support.

So,

$$x_A = AC + HC = AC + \text{Constant.} \quad -(II)$$

$$x_B = BF + FG = FG + \text{Constant.} \quad -(III)$$

Multiplying equation (III) by 2.

$$2x_B = 2FG + \text{Constant.} \quad -(IV)$$

Now, adding (II) and (IV).

$$\begin{aligned} x_A + 2x_B &= AC + 2FG + \text{Constant.} \\ &= AC + FG + FG + \text{Constant.} \\ &= AC + FG + (HD + DE) + \text{Constant.} \\ &= AC + FG + DE + \text{Constant.} \\ &= \text{Constant.} \quad (\text{from } (I)). \end{aligned}$$

i.e. $x_A + 2x_B = \text{Constant.}$

Differentiating w.r.t. 't',

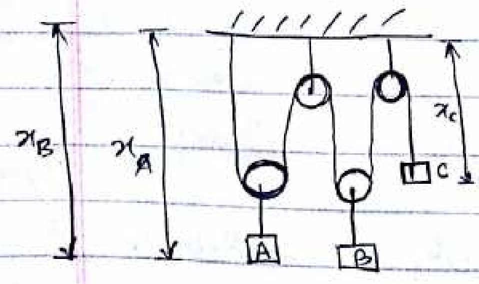
$$v_A + 2v_B = 0.$$

$$\boxed{v_A = -2v_B} \text{ and } \boxed{a_A = -2a_B}.$$

Conclusion :-

The velocity & acceleration of block A is twice that of block B & in opposite direction.

Multiple Degree of freedom Dependent Motion.



The position of a block depends on position of two blocks at any instance. So it has got DOF = 2.

from figure, we can derive,

$$x_A + 2x_B + x_c = \text{Constant.}$$

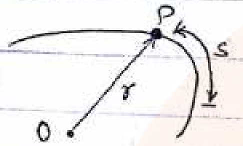
i.e. $v_A + 2v_B + v_c = 0.$

i.e. $a_A + 2a_B + a_c = 0.$

8.6 CURVILINEAR MOTION

- Curvilinear motion occurs when a particle moves along a curved path.
- Since this path is described in three dimensions, vector analysis is used to formulate the particle's position, velocity and acceleration.

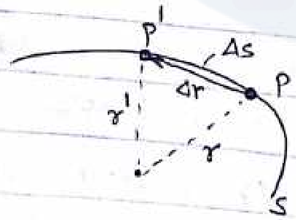
• Position.



The position of the particle is measured from a fixed point O and is designated by the position vector $r = r(t)$. This vector is a function of time since, in gener its magnitude and direction change with time.

• Displacement

Consider that during a small time interval Δt the particle moves a distance Δs along the curve to a new position P defined by $r' = r + \Delta r$, as shown in figure. The displacement Δr represents the change in the particle's position & is determine by vector subtraction; i.e. $\Delta r = r' - r$.



~~The instantaneous~~ v

• Velocity.

During time Δt, the average velocity of particle is defined as $V_{avg} = \frac{\Delta r}{\Delta t}$.

The instantaneous velocity is determined from this equation by letting $\Delta t \to 0$, and consequently the direction of Δr approaches the tangent to the curve at point P.

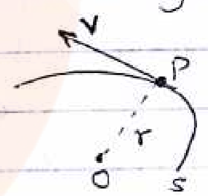
Hence,

$$v = \lim_{\Delta t \to 0} \frac{\Delta r}{\Delta t}$$

$$\text{or, } v = \frac{dr}{dt}. \longrightarrow (a)$$

Since dr will be tangent to the curve at P, the direction of v is also tangent to the curve, as in figure. The magnitude of v, which is called the speed, is obtained by noting that the magnitude of the displacement Δr is the length of the straight line segment from P to P'.

Realizing that this length, Δr, approaches the arc length Δs as $\Delta t \to 0$, we have



$$v = \lim_{\Delta t \to 0} \frac{\Delta r}{\Delta t} = \lim_{\Delta t \to 0} \frac{\Delta s}{\Delta t}$$

$$\text{or, } \boxed{v = \frac{ds}{dt}}$$

Thus, the speed can be obtained by differentiating the path function s with respect to time.

- Acceleration

If the particle has a velocity v at time t and a velocity $v' = v + \Delta v$ at $t = t + \Delta t$, then average acceleration of the particle during the time interval Δt is

$$a_{avg} = \frac{\Delta v}{\Delta t}. \quad \text{where } \Delta v = v' - v.$$

As $\Delta t \to 0$, instantaneous acceleration,

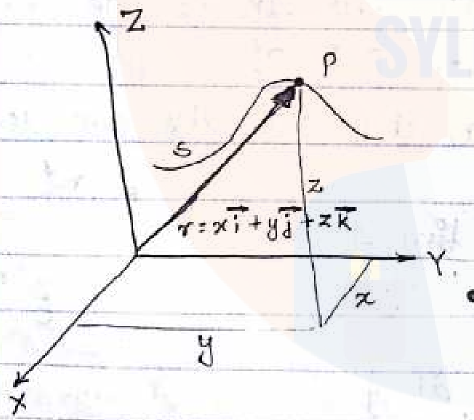
$$a = \lim_{\Delta t \to 0} \frac{\Delta v}{\Delta t}$$

or, $$\boxed{a = \frac{dv}{dt}}.$$

Substituting eqn @ into this result,

$$\therefore \boxed{a = \frac{d^2 r}{dt^2}}$$

◆ Rectangular Components



Here,

$$\vec{r} = x\vec{i} + y\vec{j} + z\vec{k} \longrightarrow (*)$$

Magnitude of $\vec{r}$ is

$$r = \sqrt{x^2 + y^2 + z^2}$$

- Velocity :-

$$v = \frac{d\vec{r}}{dt} = \frac{d}{dt}(x\vec{i}) + \frac{d}{dt}(y\vec{j}) + \frac{d}{dt}(z\vec{k})$$

So, $$\boxed{\vec{v} = v_x\vec{i} + v_y\vec{j} + v_z\vec{k}} — @$$

where,

$$v_x = \dot{x}, \quad v_y = \dot{y}, \quad v_z = \dot{z}$$

The "dot" notation $\dot{x}, \dot{y}, \dot{z}$ represents the first time ~~dire~~ derivatives of the parametric equations $x = x(t)$, $y = y(t)$, $z = z(t)$, respectively.

The magnitude of velocity.

$$v = \sqrt{v_x^2 + v_y^2 + v_z^2}$$

and direction is given by components of unit vector $u_v = \frac{\vec{v}}{v}$

This direction is always tangent to the path.

- Acceleration

The acceleration of the particle is obtained by taking the first time derivative of equation ⓐ (or second time derivative of ⊛)

$$a = \frac{dv}{dt} = a_x\vec{i} + a_y\vec{j} + a_z\vec{k}$$

where,

$$a_x = \dot{v}_x = \ddot{x}$$
$$a_y = \dot{v}_y = \ddot{y}$$
$$a_z = \dot{v}_z = \ddot{z}$$

The acceleration has a magnitude defined by the positive value of

$$a = \sqrt{a_x^2 + a_y^2 + a_z^2}$$

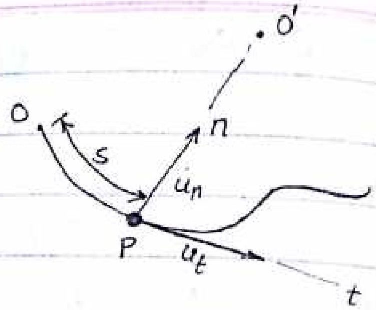
and direction is given by components of unit vector

$$u_a = \frac{\vec{a}}{a}$$

◆ Normal and Tangential Components.

When the path along which a particle is moving is known, then it is often convenient to describe the motion using n and t coordinates which acts normal & tangent to the path, respectively, and at the instant considered have their origin located at the particle.

- The t axis is tangent to the curve (path) at P & is positive in the direction of increasing s. We will designate this positive direction with unit vector u_t.



- The normal axis (n) is perpendicular to the t axis & is directed from P toward the centre of curvature. This positive direction, which is always on the concave side of the curve, will be designated by unit vector

• Velocity.
The particle's velocity $\vec{v}$ has a direction that is always tangent to the path and magnitude is determined by time time derivative of path function s.

i.e. $v = \frac{ds}{dt}$.

Hence,

$$\vec{V} = v u_t \longrightarrow ①$$

where,

$$\boxed{v = \dot{s}} \longrightarrow ②$$

• Acceleration
The acceleration of the particle is the time rate of change of the velocity. Thus,

$$\overline{a} = \dot{\vec{v}} = \dot{v} u_t + v \dot{u}_t$$

In order to determine $\dot{u}_t$, ~~consider~~ note that as the particle moves along the arc ds in time dt, u_t preserves its magnitude of unity ; however, its direction changes and becomes u_t', as in figure (ii).

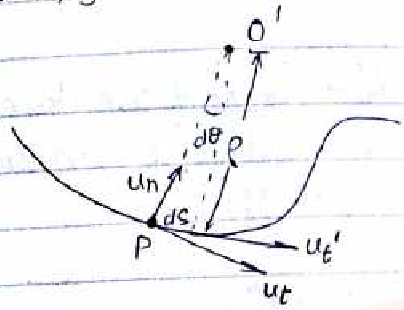


Figure (ii)

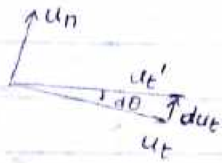


Figure (iii)

from figure (iii), $u_t' = u_t + du_t$

Here, du_t stretches between the arrowheads of u_t and u_t', which lie on an infinitesimal arc of radius $u_t = 1$.

Hence, the magnitude of $du_t = (1)d\theta$

and its direction is defined by u_n.

So,

$$du_t = d\theta u_n$$

Hence, $\dot{u}_t = \dot{\theta} u_n$ — ③

Since, $ds = \rho d\theta$ (from figure (ii)).

$$\dot{\theta} = \frac{\dot{s}}{\rho} \quad — ④$$

$\therefore \dot{u}_t = \frac{\dot{s}}{\rho} u_n = \frac{v}{\rho} u_n$ (from ③ & ④).

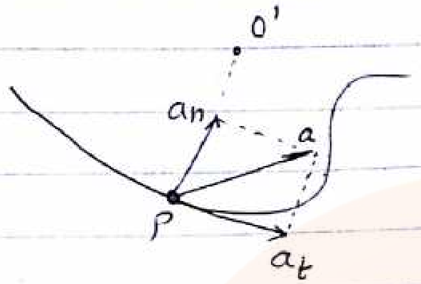
Acceleration can be written as the sum of its two component as,

$$a = a_t u_t + a_n u_n$$

where,

$$\boxed{a_t = \dot{v}} \quad — ⓐ$$

and $\boxed{a_n = \frac{v^2}{\rho}}$ — (b).



O' → centre of curvature

ρ → radius of curvature.

$$\rho = \frac{\left[1+\left(\frac{dy}{dx}\right)^2\right]^{3/2}}{\frac{d^2y}{dx^2}}$$

These two mutually perpendicular components (a_t & a_n) are shown in figure above & the magnitude of acceleration is the positive value of

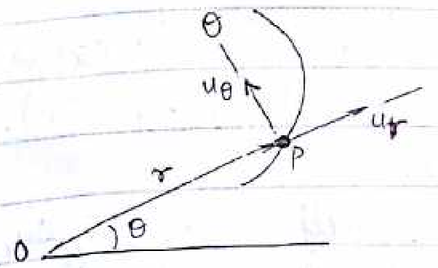
$$\boxed{a = \sqrt{a_t^2 + a_n^2}}$$

Summary :-

- If the particle moves along a straight line, $\rho \to \infty$ & from equation (b), $a_n = 0$. Thus $\vec{a} = a_t = \dot{v}$ & we conclude that the tangential component of acceleration represents the time rate of change in the magnitude of velocity.

- If the particle moves along a curve with a constant speed, $a_t = \dot{v} = 0$ & $\vec{a} = a_n = \frac{v^2}{\rho}$. Therefore, the normal component of acceleration represents the time rate of change in the direction of the velocity. Since a_n always acts towards the centre of curvature, this component is also referred to as centripetal acceleration.

Radial and Transverse Components (Cylindrical Components)



- We can specify the location of particle P using radial component r (which extends outward from the fixed origin and transverse coordinate θ (which is counterclockwise an between a fixed reference line & r axis).
- The positive direction of the r & θ coordinates are defined by the unit vectors u_r and u_θ, respectively.
Here, u_r (or the radial direction) +r extends from P alon increasing r, when θ is held fixed & u_θ or +θ exte from P in a direction that occurs when r is held fixed & θ is increased.
Note that these directions are perpendicular to each ot

- Position
At any instant the position of the particle P is defined by the position vector.
$$\vec{r} = r u_r.$$

- Velocity.
The instantaneous velocity $\vec{v}$ is obtained by taking time derivative of $\vec{r}$.
$$\vec{v} = \dot{\vec{r}} = \dot{r} u_r + r \dot{u}_r$$

To evaluate $\dot{u}_r$, notice that u_r changes only its direction maintaining its magnitude of unity.
During the time Δt, a change $\Delta\theta$ will cause u_r to become u_r', where $u_r' = u_r + \Delta u_r$ (fig (ii)). Thus, the time change in u_r is Δu_r.

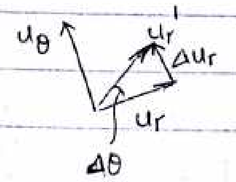


fig (ii)

For small angles $\Delta\theta$, $\Delta u_r \approx 1(\Delta\theta)$ & it acts in the direction of u_θ.
Therefore,

$$\overrightarrow{\Delta u_r} = \Delta\theta . u_\theta .$$

So,

$$\dot{u}_r = \lim_{\Delta t \to 0} \frac{\overrightarrow{\Delta u_r}}{\Delta t} = \left(\lim_{\Delta t \to 0} \frac{\Delta\theta}{\Delta t} \right) u_\theta .$$

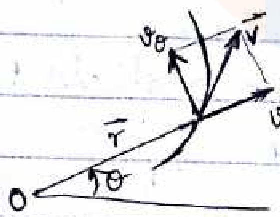
$$\dot{u}_r = \dot{\theta} u_\theta .$$

Substituting into the above equation for $\vec{v}$,

$$\vec{v} = v_r u_r + v_\theta u_\theta \rightarrow ⓐ$$

where,

$$\boxed{\begin{aligned} v_r &= \dot{r} \\ v_\theta &= r\dot{\theta} \end{aligned}}$$



The radial component v_r is a measure of the rate of increase or decrease in the length of radial coordinate, whereas the transverse component v_θ can be interpreted as the rate of motion along the circumference of a circle having a radius r.
The term $\dot{\theta} = \frac{d\theta}{dt}$ is angular velocity.

Since v_r & v_θ are mutually perpendicular, magnitude of velocity $v = \sqrt{v_r^2 + v_\theta^2}$

or, $\boxed{v = \sqrt{(\dot{r})^2 + (r\dot{\theta})^2}}$

and the direction of v is, obviously, tangent to the path at point P.

- Acceleration.

Taking time derivative of equation ⓐ, we obtain particle's instantaneous acceleration.

$$a = \dot{v} = (\ddot{r}u_r + \dot{r}\dot{u}_r) + (\dot{r}\dot{\theta}u_\theta + r\ddot{\theta}u_\theta + r\dot{\theta}\dot{u}_\theta)$$

To evaluate the term involving $\dot{u}_\theta$, u_θ changes its directio only maintaining magnitude of one.
During time Δt, a change $\Delta\theta$ will cause u_θ to become u where $u'_\theta = u_\theta + \Delta u_\theta$ (fig (iii)).

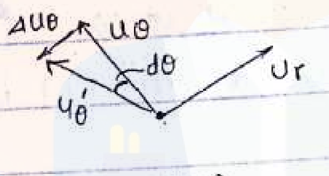


fig (iii).

The time change in u_θ is thus Δu_θ.
For small angles this vector (Δu_θ) has a magnitud $\Delta u_\theta \approx 1(\Delta\theta)$ and acts in the $-u_r$ direction i.e.

$$\Delta u_\theta = -\Delta\theta u_r.$$

Thus,

$$\dot{u}_\theta = \lim_{\Delta t \to 0} \frac{\Delta u_\theta}{\Delta t} = -\left(\lim_{\Delta t \to 0} \frac{\Delta\theta}{\Delta t}\right) u_r$$

$$\dot{u}_\theta = -\dot{\theta} u_r.$$

Substituting this value in above equation for a, acceleration in component form as,

$$a = a_r u_r + a_\theta u_\theta .$$

where,

~~$a_r \rightarrow a_r u_r +$~~

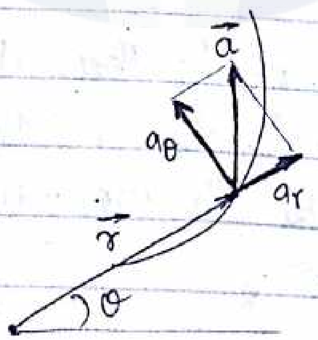
$$\boxed{\begin{aligned} a_r &= \ddot{r} - r\dot{\theta}^2 \\ a_\theta &= r\ddot{\theta} + 2\dot{r}\dot{\theta} \end{aligned}}$$

The term $\ddot{\theta} = \dfrac{d^2\theta}{dt^2}$ is called angular acceleration since it measures the change made in the angular velocity during an instant of time. Unit is rad/s^2.

Since a_r & a_θ are always perpendicular, the magnitude of acceleration,

$$a = \sqrt{(\ddot{r} - r\dot{\theta}^2)^2 + (r\ddot{\theta} + 2\dot{r}\dot{\theta})^2}$$

The direction is determined from vector addition of its two components. In general, a will not be tangent to the path (fig (iv)).



Numericals

1. The motion of particle is defined by the relation $x = t^2 - 10t + 20$, where x is expressed in meter and t in second. Determine

   a) when the velocity is zero?

   b) the position and the total distance travel when t = 8 seconds.

Solution:

$$x = t^2 - 10t + 20$$

$$v = \frac{dx}{dt} = 2t - 10$$

If v = 0;

$$2t - 10 = 0$$

$$\therefore \quad t = 5 \text{ sec}$$

At t = 0; $\quad x_0 = 20$ m

At t = 5; $\quad x_5 = -5$ m

At t = 8; $\quad x_8 = 4$ m

x_8 is the position at t = 8 sec.

$\therefore \; x_8 = 4$ m

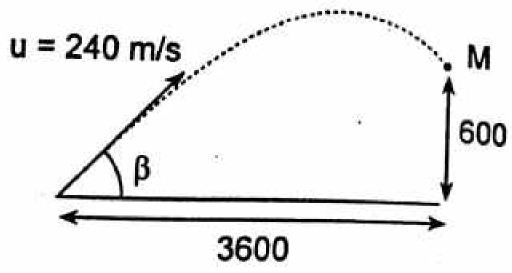
Total distance travelled when t = 8 sec in s_8

$$s_8 = |x_5 - x_0| + |x_8 - x_5|$$

$$= |-5 - 20| + |4 - (-5)|$$

$$= 25 + 9 = 34 \text{ m}$$

2. A projectile is fired with an initial velocity of 240 m/s at a target M located 600 m above a gun G and at a horizontal distance of 3600 m. Neglecting air resistance, determine the value of firing angle β.



Solution:

For projectile motion

$$y = x \tan\beta - \frac{1}{2}\, g \frac{x^2}{u^2}(1 + \tan^2\beta)$$

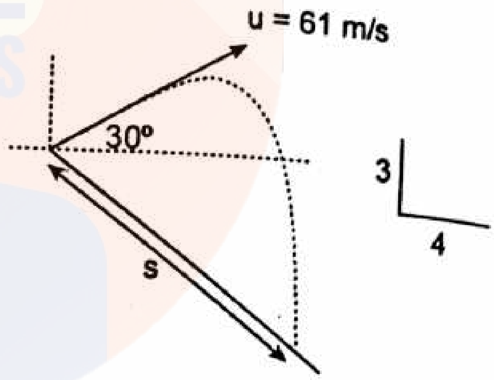
or, $$600 = 3600 \tan\beta - 4.905 \frac{(3600)^2}{(240)^2}(1 + \tan^2\beta)$$

or, $$600 = 3600 \tan\beta - 1103.6 - 1103.6 \tan^2\beta$$

or, $$1103.6 \tan^2\beta - 3600 \tan\beta + 1703.6 = 0$$

$\therefore$ $$\beta = 69.58 \text{ and } 29.85$$

3. A particle is projected at an angle of 30° with an initial velocity of 61 m/s as shown in figure. Find the slopping distance covered by the projectile.



Solution:

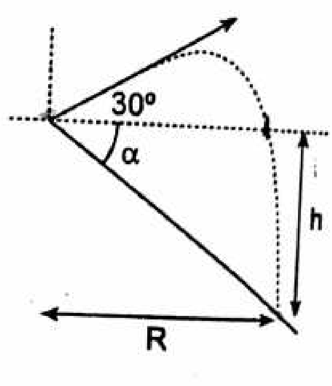
Initial velocity u = 61 m/s

Projection angle $\theta = 30°$

Let α be the angle of inclination. Then,

$$\tan\alpha = \frac{3}{4}$$

$$\alpha = 36.87°$$



Then,

$$\tan\alpha = \frac{h}{R}$$

$$\frac{3}{4} = \frac{h}{R}$$

$$R = \frac{4}{3}h \qquad \text{and} \qquad h = \frac{3}{4}R$$

Consider a vertical motion of projectile;

$$s_y = v_y.t + \frac{1}{2}a_y t^2$$

$$-h = u \sin\theta \;.\; t - \frac{1}{2}gt^2$$

$$-h = 61 \sin 30\, t - \frac{1}{2} \times 9.81 \times t^2$$

$$-h = 30.5\, t - 4.905\, t^2 \text{......(i)}$$

Again,

$$R = u_x \times t$$

$$\frac{4h}{3} = u \cos\theta \times t$$

$$\frac{4h}{3} = 61 \cos 30 \times t$$

$$h = 39.62t \qquad \text{......(ii)}$$

From eq. (i) and (ii)

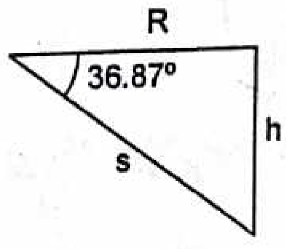
$$-39.62t = 30.5t - 4.905t^2$$

$$4.905t^2 = 70.12t$$

$$\therefore \quad t = 14.31 \text{ sec}$$

$$\therefore \quad h = 39.62 \times 14.31 = 566.97 \text{ m}$$

To find the slopping distance s



$$\sin 36.87 = \frac{h}{s}$$

$$\therefore s = \frac{h}{\sin 36.87}$$

$$= \frac{566.97}{0.6} = 944.94 \text{ m}$$

4. The rectangular component of acceleration for a particle are $a_x = 3t$, $a_y = 30 - 10t$ where a is in m/s^2 and t in seconds.

If the particle starts from rest at the origin, find the radius of curvature of the path at instant of 2 seconds.

Solution:

$$a_x = 3t$$

$$v_x = \frac{3t^2}{2} + c_1$$

At $t = 0$, $v_x = 0$, $v_y = 0$

$\therefore \quad c_1 = 0$

$\therefore \quad v_x = \frac{3t^2}{2}$

Also, $a_y = 30 - 10t$

$$v_y = 30t - 5t^2 + c_2$$

At $t = 0$, $v_x = 0$, $v_y = 0$

$\therefore \quad c_2 = 0$

$\therefore \quad v_y = 30t - 5t^2$

Again,

$$\frac{dy}{dx} = \frac{dy}{dt} \cdot \frac{dt}{dx}$$

$$= \frac{dy}{dt} \times \frac{1}{\frac{dx}{dt}}$$

$$\frac{dy}{dx} = \frac{v_y}{v_x}$$

$$\frac{dy}{dx} = \frac{30t - 5t^2}{\frac{3t^2}{2}}$$

$$\frac{dy}{dx} = \frac{60t - 10t^2}{3t^2}$$

$$\frac{dy}{dx} = \frac{20}{t} - \frac{10}{3}$$

$$\frac{d^2y}{dx^2} = \frac{d}{dx}\left[\frac{20}{t} - \frac{10}{3}\right]$$

$$= \frac{d}{dt}\left(\frac{20}{t} - \frac{10}{3}\right) \cdot \frac{dt}{dx}$$

$$= \frac{-20}{t^2} \cdot \frac{1}{v_x}$$

$$= \frac{-20}{t^2} \cdot \frac{2}{3t^2}$$

$$\therefore \frac{d^2y}{dx^2} = -\frac{40}{3t^4}$$

At t = 2 sec

$$\frac{dy}{dx} = \frac{20}{t} - \frac{10}{3}$$

$$= 10 - \frac{10}{3} = \frac{20}{3}$$

$$\frac{d^2y}{dx^2} = -\frac{40}{3t^4}$$

$$= -\frac{40}{3.2^4} = -\frac{40}{48}$$

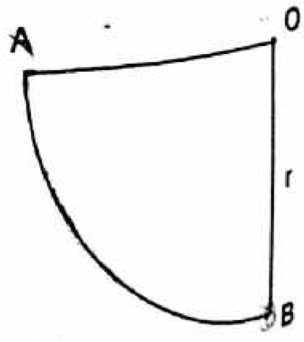
Radius of curvature

$$\delta = \frac{\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{3/2}}{\frac{d^2y}{dx^2}}$$

$$= \frac{\left[1 + \left(\frac{20}{3}\right)^2\right]^{3/2}}{\frac{-40}{48}} = -367.62$$

$\therefore$ radius of curvature = 368 m.

5. An automobile enters a curved road at 30 km/hr and then leaves at 48 km/hr. The curbed road is in the form of quarter of circle and has a length of 400 m. If the car travels at constant acceleration along the curve, calculate resultant acceleration at both ends of curve.



Solution:

$$P = \frac{2\pi r}{4}$$

$$400 = \frac{2\pi \times r}{4}$$

$\therefore\ r = 254.6$ m

Initially, $v_A = 30$ km/hr $= 8.33$ m/s

Finally, $v_B = 48$ km/hr $= 13.33$ m/s

Again, $v_B{}^2 = v_A{}^2 + 2.a_t.s$

$$(13.33)^2 = (8.33)^2 + 2 \times a_t \times 400$$

$\therefore\ a_t = 0.1353$ m/s^2

At point A:

$$a_n = \frac{v_A{}^2}{\delta} = \frac{(8.33)^2}{254.6} = 0.6979 \text{ m/s}^2$$

At point B:

$$a_n = \frac{v_B{}^2}{\delta} = \frac{(13.33)^2}{254.6} = 0.6979 \text{ m/s}^2$$

Resultant acceleration at

a) Point A

$$(a_R)_A = (a_n)_A + a_t$$
$$= \sqrt{(a_n)_A{}^2 + at^2}$$
$$= \sqrt{(0.2725)^2 + (0.1353)^2}$$
$$= 0.304 \text{ m/s}$$

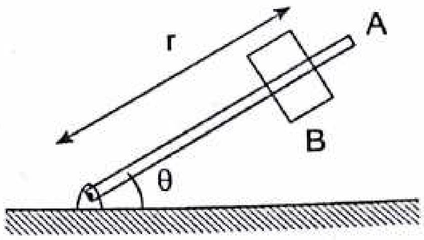
$$\tan\theta = \frac{at}{a_n}$$

$\therefore\ \theta = 26.40°$

b) Point B:

$$(a_R)_B = (a_n)_B + a_t$$
$$= \sqrt{(a_n)_B{}^2 + at^2}$$
$$= \sqrt{(0.6979)^2 + (0.1353)^2}$$
$$= 0.7108 \text{ m/s}^2$$

6. The rotation of the 0.9 m arm OA about O is defined by the relation $\theta = 0.15t^2$ where θ is expressed in radian and t in second. Collar B slides along the arm in such a way that its distance from O is $r = 0.9 - 0.12t^2$, r in meters and t in secs. After the arm OA has rotated through 30°, determine



a) total velocity of the collar

b) total acceleration of collar.

Solution:

Given, $\theta = 30°$

$$\theta = 30 \times \frac{\pi}{180} = 0.524^c$$

Also, $\theta = 0.15t^2$ (by question)

$$0.524 = 0.15\, t^2$$

or, $t^2 = 3.4906$

$\therefore$ $t = 1.868$ sec.

Now, $r = 0.9 - 0.12t^2$; $\theta = 0.15 \times (1.868)^2$

At t = 1.868 sec

$r = 0.9 - 0.12 \times (1.868)^2$; $\theta = 0.15 \times (1.868)^2$

$r = 0.481$ m ; $\theta = 0.524^c$

$\dot{r} = -2 \times 0.12\, t$; $\dot{\theta} = 0.3t$

$= -0.24 \times 1.868$; $= 0.3 \times 1.868$

$= -0.448$ m/s ; $= 0.56$ rad/sec

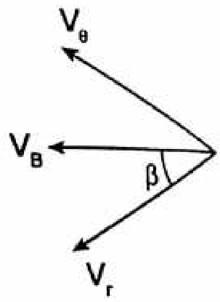
$\ddot{r} = -0.24 \text{ m/s}^2$; $\ddot{\theta} = 0.3 \text{ rad/sec}^2$

$v_r = \dot{r} = -0.448$ m/s

$v_\theta = r\dot{\theta} = 0.481 \times 0.56 = 0.270$ m/s

$v_B = \sqrt{v_r^2 + v_\theta^2}$

$= \sqrt{(0.448)^2 + (0.270)^2} = 0.523$ m/s



$\tan\beta = \dfrac{v_\theta}{v_r}$

$= \dfrac{0.270}{0.448}$

$\therefore \beta = 31.076°$

$a_r = \ddot{r} - r\dot{\theta}^2$

$= -0.24 - 0.481 \times (0.561)^2$

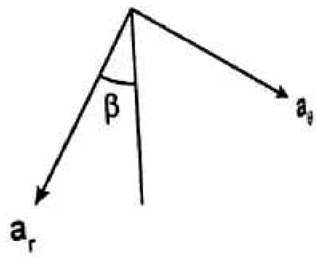
$= -0.391 \text{ m/s}^2$

$a_\theta = r\ddot{\theta} + 2\dot{r}\,\dot{\theta}$

$= 0.481 \times 0.3 + 2 \times (-0.449) \times 0.561$

$= -0.359 \text{ m/s}^2$

$\vec{a}_B = \vec{a}_r + \vec{a}_\theta$



$\therefore a_B = \sqrt{a_r^2 + a_\theta^2}$

$= \sqrt{(0.391)^2 + (0.359)^2} = 0.531 \text{ m/s}^2$

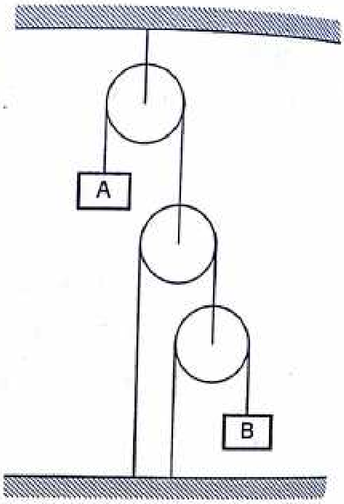
$\tan\beta = \dfrac{a_\theta}{a_r}$

$= \dfrac{0.359}{0.391}$

$\therefore \beta = 42.55°$

Acceleration of B with respect to OA is $a_{B/OA} = \ddot{r} = -0.240 \text{ m/s}^2$

7. Find the acceleration of body B if the acceleration of A is 4 m/s² (↓) for the following connection.



Solution:

$ab + bc + cd = k$

$x_A + k + x_2 = k$

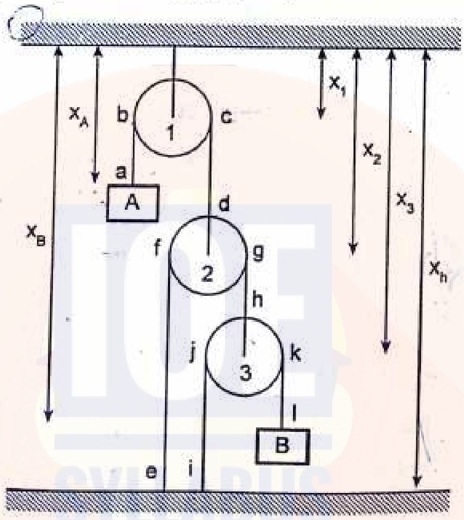
$x_A + x_2 = k$ ……(i)

$ef + fg + gh = k$

or, $x_h - x_2 + k + x_3 - x_2 = k$

or, $x_h - 2x_2 + x_3 = k$

or, $x_3 - 2x_2 = k$ ……(ii) $[x_n = k]$



$ij + jk + kl = \text{constant} = k$

or, $x_h - x_3 + k + x_B - x_3 = k$

or, $x_h - 2x_3 + x_B = k$

or, $x_B - 2x_3 = k$ ……(iii)

Multiply eq. (i) by 4, eq. (ii) by 2

$4x_A + 4x_2 = k$ ……(iv)

$2x_3 - 4x_2 = k$ ……(v)

$x_B - 2x_3 = k$ ……(vi) from eq. (iii)

Adding all (iv), (v) and (vi)

$4x_A + x_B = k$

or, $4v_A + v_B = 0$

or, $4a_A + a_B = 0$

$\therefore\ a_B = -4.a_A$

$= -4 \times 4 = -16 \text{ m/s}^2 (\downarrow) \Rightarrow 16 \text{ m/s}^2 (\uparrow)$

9. Kinetics of Particles & Rigid Body.

Kinetics is the study of relation existing between the forces acting on a body, the mass of body and the motion of body. It is used to predict the motion caused by given forces or to determine the forces required to produce a given motion.

9.1 NEWTON'S SECOND LAW AND MOMENTUM

- Newton's Second Law

It states that, "If the resultant force acting on a particle is not zero, the particle will have an acceleration proportional to the magnitude of the resultant & in the direction of this resultant force."

i.e.

$$F \propto a.$$

or,
$$\frac{F_1}{a_1} = \frac{F_2}{a_2} = \text{Constant}$$

Therefore when particle is subjected to several forces,

$$\Sigma F = ma.$$

or,
$$\Sigma F_x = ma_x$$

or,
$$\Sigma F_y = ma_y.$$

- Linear Momentum.

From Newton's Second law,

$$\Sigma \vec{F} = m\vec{a}$$

$$\Sigma \vec{F} = m\frac{d\vec{v}}{dt}.$$

$$\Sigma \vec{F} = \frac{d}{dt}(m\vec{v}).$$

The term $m\vec{v}$ is linear momentum. It has the same direction as the velocity and magnitude equal to product of mass & velocity. The linear momentum (L) is

$$\vec{L} = m\vec{v}$$

So,

$$\Sigma \vec{F} = \frac{d}{dt}(\vec{L})$$

$$\boxed{\Sigma \vec{F} = \dot{\vec{L}}}$$

i.e. The resultant of the forces acting on a particle is equal to the rate of change of linear momentum of the particle.

9.2 EQUATION OF MOTION & DYNAMIC EQUILIBRIUM

Equation of motion:

(a) Rectangular Component.

$$\Sigma f_x = ma_x \; ; \; \Sigma f_y = ma_y \; ; \; \Sigma f_z = ma_z.$$

(b) Tangential & Normal Component.

$$\Sigma f_t = ma_t \; ; \; \Sigma f_n = ma_n.$$

$$\Sigma f_t = m.\frac{dv}{dt} \; ; \; \Sigma f_n = m.\frac{v^2}{g}.$$

(c) Radial & ~~Normal~~ Tranverse Component.

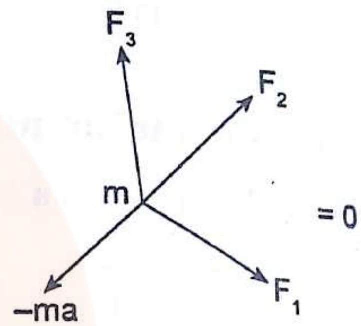
$$\Sigma f_r = ma_r \; ; \; \Sigma f_\theta = ma_\theta.$$

9.2.2 Dynamic equilibrium

From Newton's second law

$$\Sigma \vec{F} = m\,\vec{a}$$

or, $$\Sigma \vec{F} - m\,\vec{a} = 0$$



The vector $-m\vec{a}$, of magnitude ma and direction opposite to that of acceleration is called inertia vector.

So, when the particle is considered to be in equilibrium under the given forces and inertia vector $-m\vec{a}$; it is said to be in dynamic equilibrium. The inertia vector measures the resistance that particle offer when we try to set them in motion or when we try to change the condition of their motion.

$$\Sigma F_x - ma_x = 0 \qquad \Sigma F_t - ma_t = 0$$

$$\Sigma F_y - ma_y = 0 \qquad \Sigma F_n - ma_n = 0$$

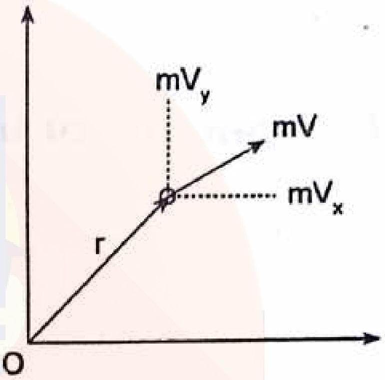
$$\Sigma F_z - mz_z = 0 \qquad \Sigma F_r - ar_r = 0$$

$$\Sigma F_\theta - a_\theta = 0$$

9.3 Angular momentum and rate of change

9.3.1 Angular momentum

The moment of the vector $m\vec{v}$ about a reference point say (O) is called angular momentum of the particle.



$$H_O = \vec{r} \times m\vec{v}$$

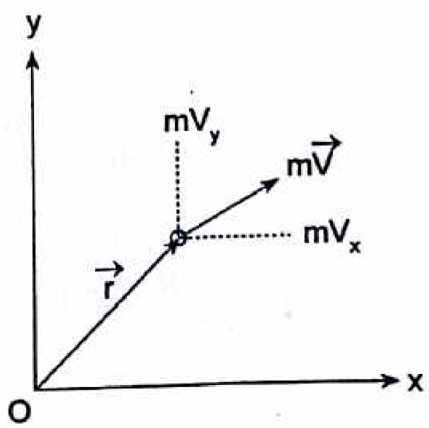
$$\vec{r} = x\vec{i} + y\vec{j} + z\vec{k}$$

$$\vec{v} = v_x\vec{i} + v_y\vec{j} + v_z\vec{k}$$

$$H_O = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ x & y & z \\ mv_x & mv_y & mz_z \end{vmatrix}$$

9.3.2 Rate of change of angular momentum

Consider a particle of mass m moving in the xy plane as shown in figure.



$$H_O = \vec{r} \times m\vec{v}$$

$$= m\,v_y.x - mv_x.y$$

$$\frac{d}{dt}(H_O) = \frac{d}{dt}(mv_y.x - mv_x.y)$$

$$\dot{H}_O = m[\dot{x}\,v_y + x\dot{v}_y - \dot{v}_x y - \dot{y}\,v_x]$$

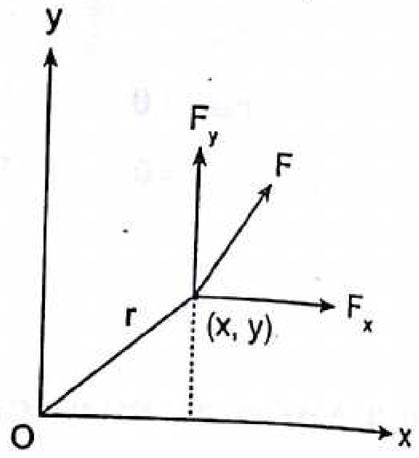
$$\dot{H}_O = [v_x.v_y + xa_y - ya_x - v_yv_x]m$$

$$= m(xa_y - ya_x)$$

$$= xma_y - yma_x$$

$$= xF_y - yF_x$$

$$= \text{moment of force about O}$$



So, rate of change of angular momentum of a particle about any point at any instant is equal to the moment of force ($\vec{F}$) about that point.

9.4 Principle of impulse and momentum

Consider a particle of mass m acted upon by a force F, then from Newton's second law

$$\vec{F} = \frac{d}{dt} m\vec{v}$$

where $m\vec{v}$ is the linear momentum.

$$\vec{F}\,dt = d(m\vec{v})$$

Integrating both sides;

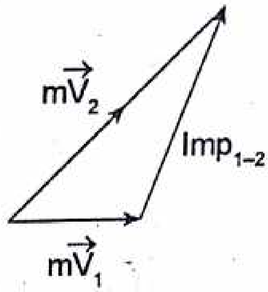
$$\int_{t_1}^{t_2} \vec{F}\,dt = \int_{v_1}^{v_2} d(m\vec{v})$$

$$\int_{t_1}^{t_2} \vec{F}\,dt = m\vec{v_2} - m\vec{v_1}$$

$$\int_{t_1}^{t_2} \vec{F}\,dt = m\vec{v_2} - m\vec{v_1}$$

$$\therefore \int_{t_1}^{t_2} \vec{F}\,dt + m\vec{v_1} = m\vec{v_2}$$

Here the $\int_{t_1}^{t_2} \vec{F}\, dt$ is called linear impulse or simply impulse.



$$\therefore\ Imp_{1-2} = \int_{t_1}^{t_2} \vec{F}\, dt$$

$$= \vec{i} \int_{t_1}^{t_2} F_x dt + \vec{j} \int_{t_1}^{t_2} F_y dt + \vec{k} \int_{t_1}^{t_2} F_z dt$$

$$\therefore\ m\vec{v_1} + Imp_{1-2} = m\vec{v_2}$$

So, when a particle is acted upon by a force F during a given time interval, the final momentum $m\vec{v_2}$ of the particle can be obtained by adding vectorically its initial momentum and the impulse of the force $\vec{F}$ during the time interval considered.

When several forces acts on particle

$$m\vec{v_1} + \Sigma Imp_{1-2} = m\vec{v_2}$$

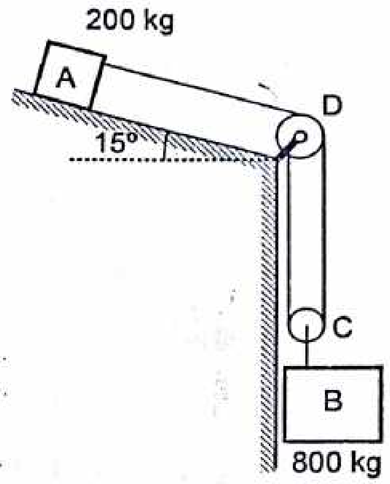
If problem involves two or more particles

$$\Sigma m\vec{v_1} + \Sigma Imp_{1-2} = \Sigma m\vec{v_2}$$

If no external forces act

$$\Sigma m\vec{v_1} = \Sigma m\vec{v_2}$$

1. Two blocks start from rest. The pulley are frictionless and having no mass. If u_k between block A and inclined plane is 0.4. Determine the acceleration of each block and tension in each cord.



Solution:

$x_A \rightarrow$ position of block A

$x_B \rightarrow$ position of block B

$x_C \rightarrow$ position of block C

$x_C + k = x_B$

$\therefore\ x_B = x_C$

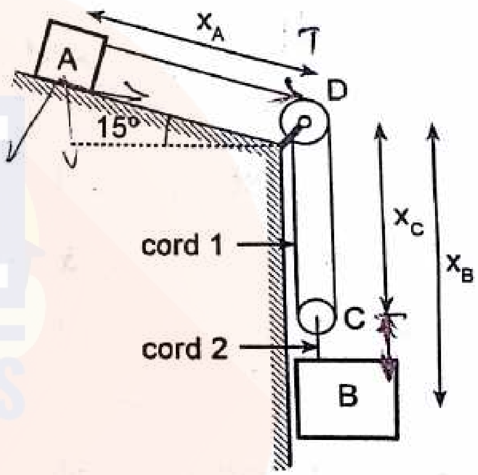
ADCO is a continuous cord.

$AD + DC + CO = k$

$x_A + 2x_C = k$

$x_A + 2x_B = k$

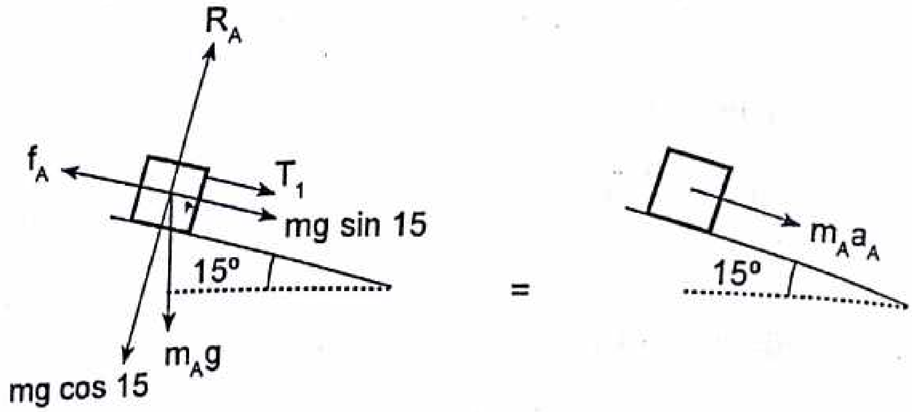
$v_A = -2v_B$ and $a_A = -2a_B$



Block A:

$\Sigma F_x = ma$

$T_1 + m_A.g \sin 15 - f_A = m_A a_A$



$$T_1 + 507.8 - \mu_k R_A = 200a_A$$

$$T_1 + 507.8 - 0.4\ m_A\ g\cos 15 = 200a_A$$

$$T_1 + 507.8 - 758.05 = 200a_A$$

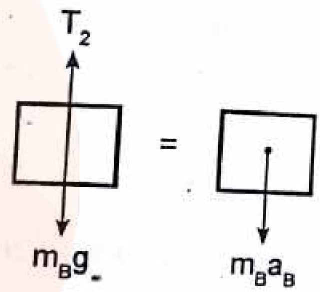
$$T_1 - 200a_A - 250.25 = 0\ \ldots\ldots\text{(i)}$$

Block B:

$\Sigma F_y = ma$

or, $\quad m_B.g - T_2 = m_B.a_B$

or, $\quad 7848 - T_2 = 800a_B\ \ldots\ldots\text{(ii)}$



Pulley C

$T_2 = 2T_1\ \ldots\ldots\text{(iii)}$

Use $a_A = 2a_B$ and $T_2 = 2T_1$ in eq. (i)

$$\frac{T_2}{2} - 200(2a_B) - 250.25 = 0$$

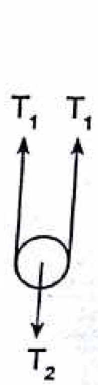
$$T_2 - 800a_B - 500.5 = 0\ \ldots\ldots\text{(iv)}$$

From eq. (ii) and eq. (iv)

$$T_2 - 800a_B - 500.5 = 0$$

$$\underline{-T_2 - 800a_B + 7848 = 0}$$

$$-1600a_B = -7347.5$$



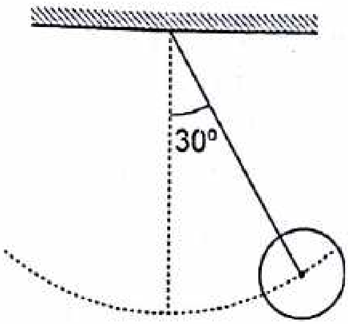
$\therefore\ a_B = 4.592\ m/s^2\ (\downarrow)$

$a_A = 9.184\ m/s^2$

$\therefore\ T_1 = 2087.125\ N$

$\therefore\ T_2 = 4174.25\ N$

2. The bob of 6 m pendulum describe an arc of a circle in a vertical plane. If the tension in the cord is 2 times the weight of bob for the position when the bob is displaced through an angle 30° from its mean position. Find the velocity and acceleration of bob for this situation.



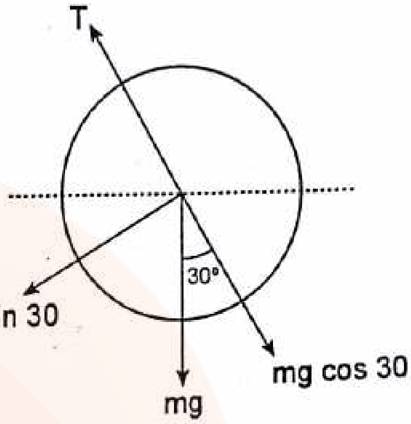
Solution:

Consider a tangential direction,

$$mg \sin 30 = ma_t$$

$$g \sin 30 = a_t$$

$$\therefore \ a_t = 4.905 \text{ m/s}^2$$



Normal component is directed towards centre

$$T - mg \cos 30 = ma_n$$

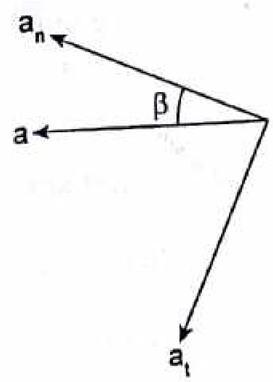
$$2mg - mg \cos 30 = ma_n$$

$$2g - g \cos 30 = a_n$$

$$\therefore \ a_n = 11.12 \text{ m/s}^2$$

$$\therefore \ \vec{a} = \vec{a_t} + \vec{a_n}$$

$$a = \sqrt{a_t^2 + a_n^2} = \sqrt{(4.905)^2 + (11.12)^2} = 12.15 \text{ m/s}^2$$



$$\tan \beta = \frac{a_t}{a_n} = \frac{4.905}{11.12}$$

$$\therefore \ \beta = 23.78°$$

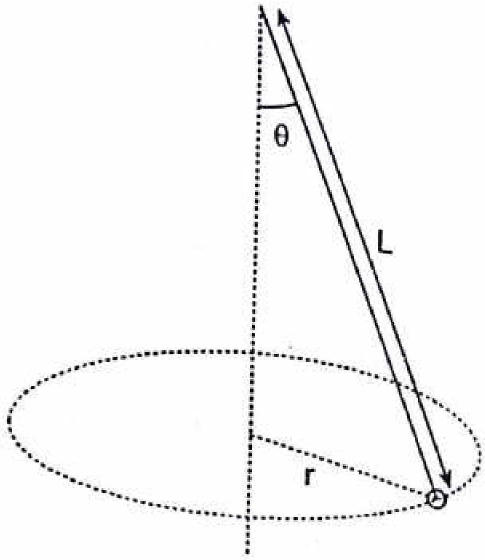
$$\text{Again, } a_n = \frac{v^2}{\delta}$$

$$11.12 = \frac{v^2}{6}$$

$$v^2 = 66.745$$

$$\therefore \ v = 8.16 \text{ (along } a_t\text{)}$$

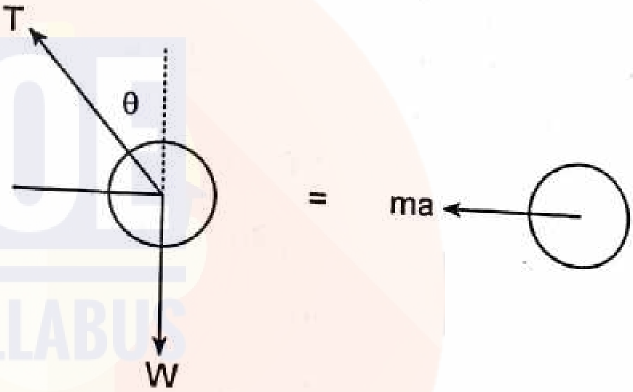
3. A 2 kg ball revolves in a horizontal circle as shown at a constant speed of 1.5 m/s knowing that L = 600 mm. Determine a) θ, b) tension.



Solution:

$$\text{Radius } r = L\cos(90-\theta)$$
$$= L\sin\theta$$
$$= 600\sin\theta$$

$$\Sigma F_y = ma_y$$
$$T\cos\theta - W = 0$$
$$T\cos\theta = 2g \text{ ......(i)}$$
$$\Sigma F_x = ma_x$$
$$T\sin\theta = ma$$



$$\frac{2g}{\cos\theta}\sin\theta = 2a$$
$$g\tan\theta = a \text{ ......(ii)}$$
$$\text{Also } a_n = \frac{v^2}{\delta}$$

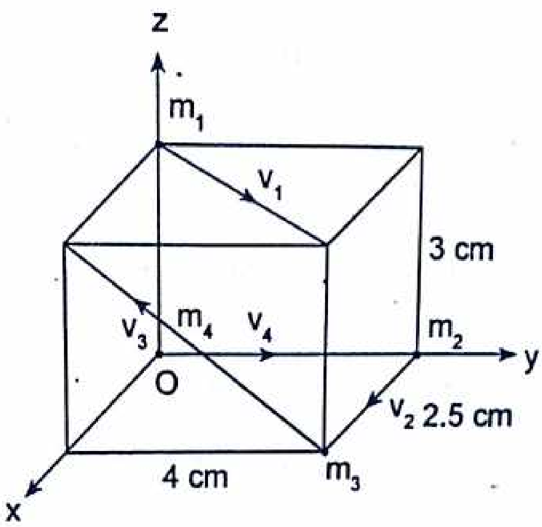
For uniform circular motion

$$a_n = a$$
$$\therefore\ g\tan\theta = \frac{v^2}{\left(\frac{600}{1000}\right)\sin\theta}$$
$$\tan\theta\sin\theta = \frac{(1.5)^2}{9.81\times 0.6}$$
$$\tan\theta\sin\theta = 0.38226$$

solving, $\theta = 34.21°$

$$\text{Tension } T = \frac{2g}{\cos\theta} = \frac{2\times 9.81}{\cos 34.21} = 23.72\text{ N}$$

4. The system of particle at a time t is shown in the figure below



$v_1 = 7$ m/s	$m_1 = 0.5$ kg
$v_2 = 6$ m/s	$m_2 = 1.5$ kg
$v_3 = 5$ m/s	$m_3 = 1$ kg
$v_4 = 1.5$ m/s	$m_4 = 0.5$ kg

Determine:

a) total linear momentum of the system.

b) angular momentum of system about O.

c) angular momentum of system about O.

Solution:

Write the coordinate of all point and find the direction of v_1, v_2, v_3 and v_4.

$m_1(0, 0, 3)$ $m_2(0, 4, 0)$ $m_3(2.5, 4, 0)$ $m_4(0, 0, 0)$

$O(0, 0, 0)$ $a(2.5, 4, 3)$ $b(2.5, 0, 3)$

$$\text{Unit vector along } v_1 = \hat{n}_1 = \frac{\overrightarrow{m_1 a}}{|\overrightarrow{m_1 a}|} = \frac{\vec{a} - \vec{m_1}}{|\overrightarrow{m_1 a}|}$$

$$= \frac{(2.5, 4, 3) - (0, 0, 3)}{|\overrightarrow{m_1 a}|}$$

$$= \frac{2.5\vec{i} + 4\vec{j}}{\sqrt{(2.5)^2 + 4^2}}$$

$$= \frac{2.5\vec{i} + 4\vec{j}}{4.71}$$

$$\hat{n}_1 = 0.529\vec{i} + 0.847\vec{j}$$

$$\therefore \vec{v_1} = \hat{n}_1 . v_1$$

$= 7(0.529\,\vec{i} + 0.847\,\vec{j})$

$= 3.703\,\vec{i} + 5.929\,\vec{j}$

Unit vector along $v_2 = \vec{i}$

$\vec{v}_2 = v_2.\,\vec{i} = 6\,\vec{i}$

Unit vector along $v_3 = \hat{n}_3 = \dfrac{\vec{m_3b}}{|\vec{m_3b}|} = \dfrac{\vec{b} - \vec{m}_3}{|\vec{m_3b}|}$

$= \dfrac{(2.5, 0, 3) - (2.5, 4, 0)}{|\vec{m_3b}|}$

$= \dfrac{-4\,\vec{j} + 3\,\vec{k}}{\sqrt{4^2 + 3^2}}$

$\hat{n}_3 = -0.8\,\vec{j} + 0.6\,\vec{k}$

$\therefore\ \vec{v}_3 = v_3.\hat{n}_3$

$= 5(-0.8\,\vec{j} + 0.6\,\vec{k})$

$= -4\,\vec{j} + 3\,\vec{k}$

Unit vector along $v_4 = \vec{j}$

$\vec{v}_4 = v_4.\,\vec{j} = 1.5\,\vec{j}$

Total linear momentum

$\vec{L} = \Sigma m\,\vec{v}$

$= m_1\vec{v}_1 + m_2\vec{v}_2 + m_3\vec{v}_3 + m_4\vec{v}_4$

$= 0.5(3.703\,\vec{i} + 5.929\,\vec{j}) + (1.5)\,6\,\vec{i} + 1.(-4\,\vec{j} + 3\,\vec{k}) + (0.5).1.5\,\vec{j}$

$$= 10.85\ \vec{i} - 0.29\ \vec{j} + 3\ \vec{k}\ \text{kg m/s}$$

$$|\vec{L}| = \sqrt{(10.85)^2 + (0.29)^2 + 3^2} = 11.2608\ \text{kg m/s}$$

Angular momentum about O

$$\vec{H}_O = \Sigma(\vec{r} \times m\vec{v})$$

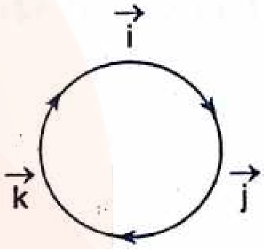
$$r_1 = r_{m1/O} = m_1 - O = (0, 0, 3) - (0, 0, 0) = 3\ \vec{k}$$

$$r_2 = r_{m2/O} = m_2 - O = (0, 4, 0) - (0, 0, 0) = 4\ \vec{j}$$

$$r_3 = r_{m3/O} = m_3 - O = (2.5, 4, 0) - (0, 0, 0) = 2.5\ \vec{i} + 4\ \vec{j}$$

$$r_4 = r_{m4/O} = m_4 - O = (0, 0, 0) - (0, 0, 0) = 0$$

$$\vec{H}_O = \Sigma\ \vec{r} \times m\vec{v}$$



$$= 3\ \vec{k} \times 0.5(3.703\ \vec{i} + 5.929\ \vec{j})$$

$$+ 4\ \vec{j} \times (1.5)\ 6\ \vec{i} + (2.5\ \vec{i} + 4\ \vec{j})$$

$$\times 1.(-4\ \vec{j} + 3\ \vec{k}) + 0$$

$$= 5.55\ \vec{j} - 8.89\ \vec{i} - 36\ \vec{k} - 10\ \vec{k} - 7.5\ \vec{j} + 12\ \vec{i}$$

$$= 3.106\ \vec{i} - 1.95\ \vec{j} - 46\ \vec{k}$$

$$|\vec{H}_O| = \sqrt{(3.106)^2 + (1.95)^2 + (46)^2} = 46.146\ \text{kg m}^2/\text{s}^2$$

Angular momentum about a

$$\vec{H}_a = \Sigma\ \vec{r} \times m\vec{v}$$

$$r_1 = r_{m1/a} = m_1 - a = (0, 0, 3) - (2.5, 4, 3) = -2.5\ \vec{i} - 4\ \vec{j}$$

$$r_2 = r_{m2/a} = m_2 - a = (0, 4, 0) - (2.5, 4, 3) = -2.5\ \vec{i} - 3\ \vec{k}$$

$$r_3 = r_{m3/a} = m_3 - a = (2.5, 4, 0) - (2.5, 4, 3) = -3\ \vec{k}$$

$$r_4 = r_{m4/a} = m_4 - a = (0, 0, 0) - (2.5, 4, 3) = -2.5\ \vec{i} - 4\ \vec{j} - 3\ \vec{k}$$

$$H_a = \Sigma\ \vec{r} \times m\,\vec{v}$$

$$= (-2.5\ \vec{i} - 4\ \vec{j}\ ) \times (0.5)\,(3.703\ \vec{i} + 5.929\ \vec{j}\ )$$

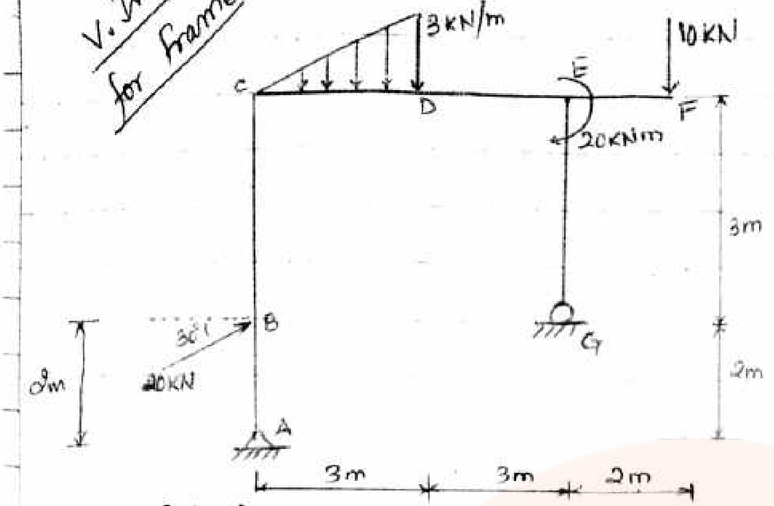
$$+ (-2.5\ \vec{j} - 3\ \vec{k}\ ) \times (1.5) .\ \vec{i} + -3\ \vec{k} \times 1.(-4\ \vec{j} + 3\ \vec{k}\ )$$

$$+ (-2.5\ \vec{i} - 4\ \vec{j} - 3\ \vec{k}\ ) \times (0.5)\,(1.5\ \vec{j}\ )$$

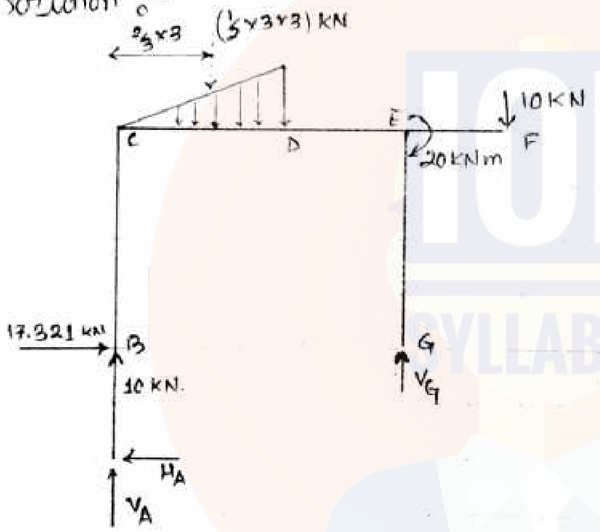
$$\vec{H}_a = -9.75\ \vec{i} - 27\ \vec{j} - 1.875\ \vec{k}$$

$$|\vec{H}_a| = \sqrt{(9.75)^2 + 27^2 + (1.875)^2} = 28.767 \text{ kg m}^2/\text{s}^2$$

V. Imp Question for Frame



Solution :-



$$\Sigma M_A = 0$$

$$V_G \times 6 = 20 + 10\times 8 + \left(\tfrac{1}{2}\times 3\times 3\right)\times\left(\tfrac{2}{3}\times 3\right) + 17.321\times 2$$

$$\boxed{V_G = 23.94 \text{ KN } (\uparrow)}$$

$$\Sigma V = 0$$

$$V_A + V_G + 10 = \tfrac{1}{2}\times 3\times 3 + 10$$

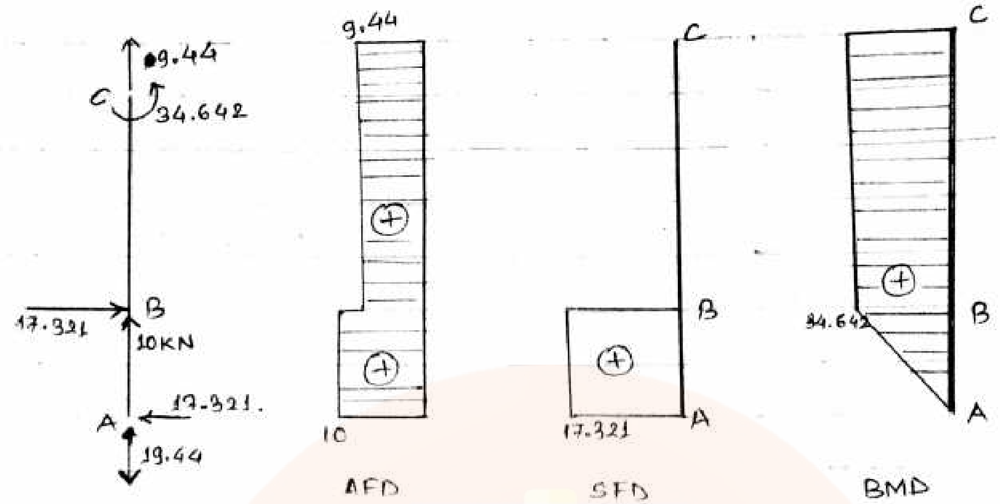
$$V_A = -19.44$$

i.e. $\boxed{V_A = 19.44 \text{ KN } (\downarrow)}$

$$\Sigma H = 0$$

$$\boxed{H_A = 17.321 \text{ KN } (\leftarrow)}$$

Member ABC,



SFD

$(SF_A)_L = 0$ $\quad$ $(SF_A)_R = 10$ KN

$(SF_B)_L = 10$ KN $\quad$ $(SF_B)_R = 0.$

BMD

$M_A = 0$

$M_B = 34.642$ KNm

$(M_C)_L = 34.642$ KNm

$(M_C)_R = 0.$

for member EDEF,

SFD.

$(SF_C)_L = 0$ $\quad$ $(SF_C)_R = -9.44$ KN

$(SF_D) = -13.94$ KN

$(SF_E)_L = -13.94$ KN $\quad$ $(SF_E)_R = +10$ KN

$(SF_F)_L = +10$ KN $\quad$ $(SF_F)_R = 0.$

BMD.

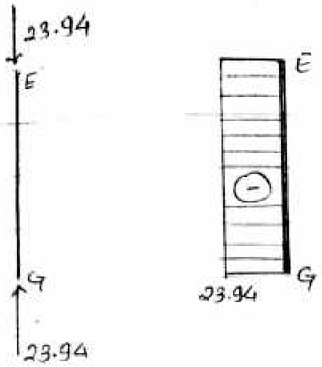
$(BM_C)_L = 0$ $\quad$ $(BM_C)_R = +34.642$ KNm

$BM_D = +1.822$ KNm

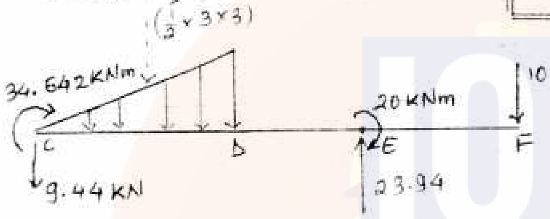
$(BM_E)_L = -40$ KNm $\quad$ $(BM_E)_R = -20$ KNm

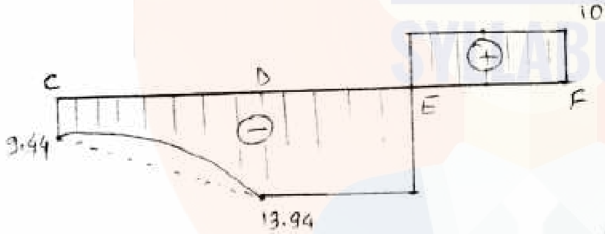
$BM_F = 0.$

Member EG.

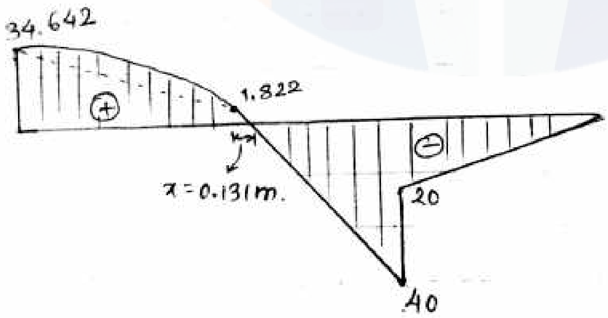


Member CDEF

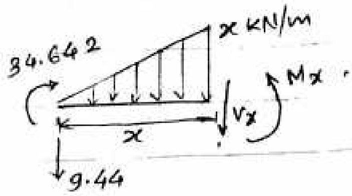




SFD.



for $(0 \le x \le 3)$m,



$$\Sigma V = 0$$

$$V_x + \frac{1}{2} \times x \times x + 9.44 = 0$$

i.e. $V_x = -9.44 - 0.5x^2$.

i.e. $V_x = -9.44$ @ $x = 0$m.

i.e. $V_x = -13.94$ @ $x = 3$m

$$\Sigma M_x = 0$$

$$M_x + \left(\frac{1}{2} \times x \times x\right) \times \frac{x}{3} + 9.44x = 34.642$$

$$M_x = 34.642 - 9.44x - \frac{x^3}{6}.$$

$\therefore M_x = 34.642$ KNm @ $x = 0$ m.

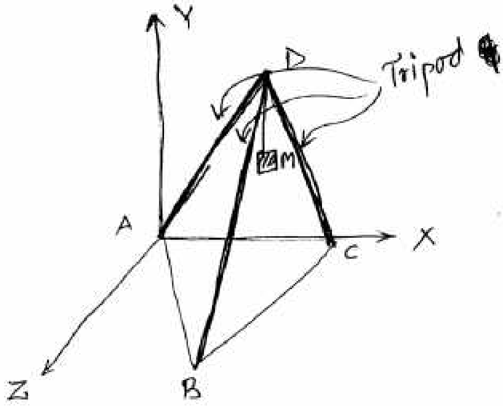
$M_x = 1.822$ KNm @ $x = 3$ m.

BM = 0 भएको ठाउँलाई point of contraflexure or point of inflection भनिन्छ।

i.e. $\frac{x}{1.822} = \frac{3-x}{40}$

$x = 0.131$m.

⑨ Solution :-



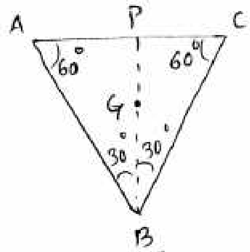


Fig : Top view.

Let G is the centroid of ΔABC. Let x be the length of sides of ΔABC.

from geometry,

$$\boxed{AP = \frac{1}{2}x.}$$

$$BP = AB \cos 30^\circ$$

or, $BP = x \cos 30^\circ$

$$\therefore \boxed{BP = 0.866x}$$

$PG = \frac{1}{3} \times BP$ (Centroid सम्मको distance).

$$= \frac{1}{3} \times 0.866x$$

$$\therefore \boxed{PG = 0.2887x}$$

Now,

$AG^2 = AP^2 + PG^2$ (Pythagoras Theorem).

$$= (0.5x)^2 + (0.2887x)^2$$

$$\therefore \boxed{AG = 0.577x}$$

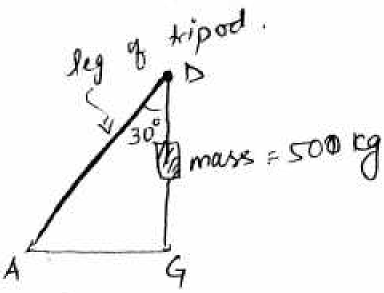


Fig: front view

In Δ AGD,

$$\tan 30^\circ = \frac{AG}{DG}.$$

$$\therefore DG = \frac{AG}{\tan 30^\circ} = \frac{0.577x}{\tan 30^\circ} = x.$$

अहिले सम्म जे-जे गरियौं, त्यो सबै A, B, C र D को coordinate निकाल्नलाई गरियौं। अब सबै points (A, B, C र D) लाई $\vec{i}$, $\vec{j}$ र $\vec{k}$ को term मा लेखौं न त !

A (0, 0, 0) C ($x\vec{i}$)

B ($0.5x\vec{i} + 0.866x\vec{k}$) D ($0.5x\vec{i} + x\vec{j} + 0.2887x\vec{k}$)

Since triangle is equilateral and makes an angle 30° with the mass, the load sharing by each of them is equal. Let the magnitude of the load distributed on each tripod leg be F. Then,

Unit vector along AD is,

$$\hat{n}_A = \frac{0.5x\vec{i} + x\vec{j} + 0.2887x\vec{k}}{\sqrt{(0.5x)^2 + x^2 + (0.2887x)^2}}$$

$$= 0.433\vec{i} + 0.866\vec{j} + 0.25\vec{k}$$

Similarly,

Unit vector along BD is,

$$\hat{n}_B = \frac{0\vec{i} + x\vec{j} + (-0.577x\vec{k}}{\sqrt{x^2 + (0.577x)^2}}$$

$$= 0.866\vec{j} - 0.5\vec{k}$$

Unit vector along CD is,

$$\hat{n}_c = -0.433\vec{i} + 0.866\vec{j} + 0.25\vec{k}$$ [Calculate it yourself].

For equilibrium,

$$\Sigma\vec{F} = 0$$

i.e. $\vec{F}_{AD} + \vec{F}_{BD} + \vec{F}_{CD} + \vec{W} = \vec{0}$.

or, $|\vec{F}|(\hat{n}_A + \hat{n}_B + \hat{n}_c) + \vec{W} = \vec{0}$

or, $|\vec{F}|[(0.433 - 0.433)\vec{i} + (0.866 + 0.866 + 0.866)\vec{j} + (0.25 - 0.5 + 0.25)\vec{k}] = -\vec{W}$

$$|\vec{F}|(2.598) = -500 \times 9.81 \quad (W = mg)$$

$$|\vec{F}| = \frac{-4905}{2.598}$$

$$= -1887.99$$

$$F \approx -1888\ N.$$

-ve sign represents the direction of force is opposite to our assumed direction or opposite to the direction of weight of the body hung.

Best Of Luck ☺