Dynamics (Solution)

(2) The motion of a particle is defined by the relation $x = t^3/3 - 3t^2 + 8t + 2$, where x is expressed in metre and t in sec. Determine (a) when the velocity is zero? (b) the position and the distance travelled when the acceleration is zero.

⇒ Sol^n

$$x = \frac{t^3}{3} - 3t^2 + 8t + 2$$

$$V = \frac{dx}{dt} = t^2 - 6t + 8 \quad \text{—— (1)}$$

(a) $t = ?$ $\quad V = 0$

or, $0 = t^2 - 6t + 8$

or, $(t-4)(t-2) = 0$

$\therefore t = 4\,\text{sec}, 2\,\text{sec}$ #

(b) Diff equation (1)

or, $a = \frac{dv}{dt} = 2t - 6$

when $a = 0$

or, $0 = 2t - 6$

$\therefore t = 3\,\text{sec.}$

$\therefore x_3 = \frac{(3)^3}{3} - 3(3)^2 + 8(3) + 2$

$= 8\,m$ #

total distance travelled

$x_3 = 8\,m$

$x_0 = 0 - 0 + 0 + 2 = 2\,m$

$x_2 = \frac{(2)^3}{3} - 3(2)^2 + 8(2) + 2 = \frac{26}{3}$

$\therefore$ total distance travelled when acceleration is zero.

$= |x_3 - x_2| + |x_2 - x_0|$

$= |8 - 8.66| + |8.66 - 2|$

$= 7.33\,m$ #

(3) The acceleration of a particle is defined by the relation $a = -2\,m/sec^2$. Initially, its velocity (v_0) = [illegible] and position (x_0) = 0 m, determine the velocity, position and the total distance travelled at the instant at 6 sec.

⇒ Sol^n

$a = -2\,m/sec^2$

or, $\frac{dv}{dt} = -2$

or, $\int dv = -2 \int dt$

or, $V = -2t + c_1$ —— (1)

when $t = 0$ (initially), $V = V_0 = 8\,m/sec$

or, $8 = -2(0) + c_1$

$\therefore c_1 = 8$ ——— (ii)

from (i) & (ii),

or, $V = -2t + 8$ ——— (iii)

or, $\frac{dx}{dt} = -2t + 8$

or, $\int dx = -2\int t\,dt + 8\int dt$

or, $x = -2\frac{t^2}{2} + 8t + c_2$

or, $x = -t^2 + 8t + c_2$ ——— (iv)

<u>when $t=0$ (initially), $x = x_0 = 0m$.</u>

or, $0 = 0 + c_2$

$\therefore c_2 = 0$ ——— (v)

from (iv) and (v),

or, $x = -t^2 + 8t$ ——— (v)(a)

or, $v = -2t + 8$ ——— (vi)

<u>when $t = 6sec$, $v = ?$</u>

$\therefore V = -2(6) + 8 = -4$ m/sec #.

<u>when $t = 6$; $x = ?$</u>

from (v)(a) equation,

$\therefore x = -(6)^2 + 8(6) = 12$ m #

<u>total distance travelled</u>

when $v = 0$, from equation (iii),

or, $0 = -2t + 8$

$\therefore t = 4$ sec.

$x_4 = -(4)^2 + 8(4) = -16 + 32 = 16m$ [eq^n (v)(a)]

$x_0 = 0$ [from eq (v)(a)].

total distance travelled is given by,

$= |x_4 - x_0| + |x_6 - x_4|$

$= |16 - 0| + |12 - 16|$

$= 16 + 4$

$= 20 m$ #.

(4) The acceleration of a particle is defined by the relation, $a = kt^2$. (a) Knowing that velocity is -32 m/sec. when time is zero second and again velocity is $+32$ m/sec when time is 4 sec, (a) determine, the value of the constant k (b) Write the equations of motion knowing also that the position of the particle is zero at the instant of 4 sec.

$\Rightarrow$ Sol^n

$a = kt^2$ ——— (i) , $V_0 = -32$ m/sec.

$V_4 = +32$ m/sec.

or, $\frac{dv}{dt} = kt^2$

or, $\int dv = k\int t^2 dt$

or, $v = k\frac{t^3}{3} + c_1$ —— (ii)

At $t=0$, $v = -32$ m/s

or, $c_1 = -32$ —— (iii)

$\therefore v = \frac{kt^3}{3} - 32$ —— (iv)

At $t = 4$ sec, $v = +32$ m/sec.

or, $32 = \frac{k(4)^3}{3} - 32$

$\therefore k = 3$ m/sec^4 #

Then eq^n (iv) becomes,

$v = 3\,t^3/3 - 32 = t^3 - 32$ —— (v)

or, $\frac{dx}{dt} = t^3 - 32$

or, $\int dx = \int t^3 dt - \int 32\, dt$

or, $x = \frac{t^4}{4} - 32t + c_2$ —— (vi)

When $t = 4$ sec, $x = 0$

or, $0 = \frac{(4)^4}{4} - 32(4) + c_2$

$\therefore c_2 = 64$

Eq (vi) becomes,

or, $x = \frac{t^4}{4} - 32t + 64$ —— (vii)

Then from equation (i), (v) and (vii), we get, eq^n of motions.

$$\left.\begin{aligned} a &= kt^2 = 3t^2 \\ v &= t^3 - 32 \\ x &= \frac{t^4}{4} - 32t + 64 \end{aligned}\right\} \#$$

5. The equation of motion of a particle is defined by the relation, $a = 0.4v$, where a is the acceleration in m/sec^2 and v is the velocity in m/sec units. Find the time for which velocity will be zero. It is known that velocity is 30 m/s at the instant of zero sec.

$\Rightarrow$ Sol^n

$a = -0.4v$

or, $\frac{dv}{dt} = -0.4v$

or, $\frac{dv}{v} = -0.4\,dt$

Integrating,

$\int \frac{dv}{v} = -0.4\int dt$

or, $\ln v = -0.4t + c$

To find c, at $t = 0$, $v = 30$ m/s.

So, $\ln 30 = C$.

$\therefore C = 3.4$

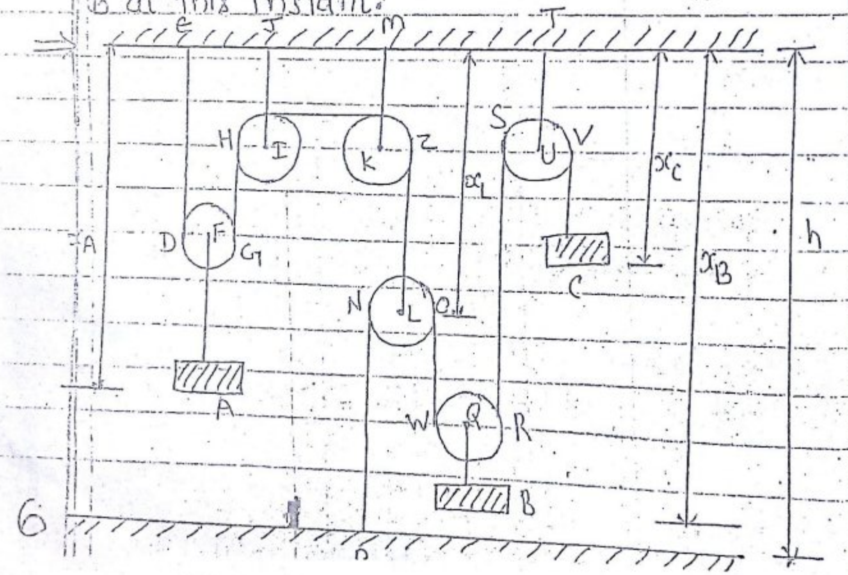
Now,

$\ln v = -0.4t + 3.4$ which is the expression for velocity.

for $v = 0$,

$\ln 0 = -0.4t + 3.4 \Rightarrow t = \infty$ #

(6) Three blocks A, B and C are connected by cords and pulleys as shown in fig 2.9. At the instant shown, the blocks A and C have $V_A = 4$ m/sec (↓), $V_C = 6$ m/sec (↑), $a_A = 3$ m/sec^2 (↑) and $a_C = 8$ m/sec^2 (↓). Determine v_B and a_B of the block B at this instant.



⇒ Sol^n

$V_A = 4$ m/sec (↓)

$V_C = 6$ m/sec (↑)

$a_A = 3$ m/sec^2 (↑)

$a_C = 8$ m/sec^2 (↓)

JI = constant, FA = constant, MK = constant

QB = constant, TU = constant, h = constant.

rope wrapped around = constant.

We have,

$ED + GH + ZL = \text{constant}$ —— (I)

$PN + OW + RS + VC = \text{constant}$ —— (II)

or, $(x_A - FA) + (x_A - JI - FA) + (x_L - MK) = \text{constant}$

or, $2x_A + x_L - \underbrace{2FA - JI - MK}_{\text{const}} = \text{const}$

$\therefore 2x_A + x_L = \text{const.}$ —— (III)

$\therefore 2V_A + V_L = 0$ —— (III)(a)

$\therefore 2a_A + a_L = 0$ —— (III)(b)

Given,

Let (↑ +ve) (↓ −ve)

$V_A = 4$ m/sec (↓) $= -4$ m/sec

$V_C = 6$ m/sec (↑) $= 6$ m/sec

$a_A = 3$ m/sec^2 (↑) $= 3$ m/sec^2

$a_C = -8$ m/sec^2 (↓) $= -8$ m/sec^2

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from equation (III)(a).

or, $2(-4)+V_L=0$

$\therefore V_L = 8\ m/sec$

from equation (III)(b).

or, $2(3)+a_L=0$

$\therefore a_L = -6\ m/sec^2$

from equation (II), we have,

$PN + OW + RS + VC = \text{constant}$.

or, $(h-x_L)+(x_B-a_B-x_L)+(x_B-a_B-TU) + (x_C-TU) = \text{constant}$.

or, $2x_B + x_C - 2x_L = \text{constant}$ —— (IV)

$\therefore 2V_B + V_C - 2V_L = 0$ —— (IV)(a)

$\therefore 2a_B + a_C - 2a_L = 0$ —— (IV)(b)

from (IV)(a), and putting value of V_L:

or, $2V_B + 6 - 2(8) = 0$

$\therefore V_B = +5\ m/sec = 5\ m/sec (\uparrow)$ #.

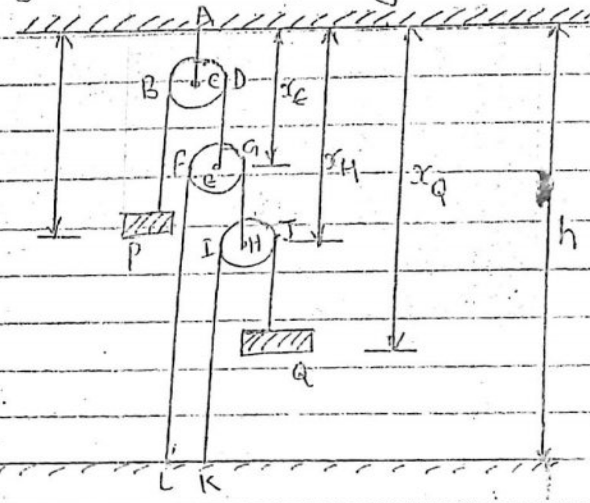
from (IV)(b), and putting value of a_L,

or, $2a_B + (-8) - 2(-6) = 0$

or, $2a_B - 8 + 12 = 0$

$\therefore a_B = -2\ m/sec^2 = 2\ m/sec^2$ #.

(7) Find the relation between the acceleration of bodies P and Q which are connected by means of in inextensible strings running on pulleys as shown in figure 2.10.



Lets consider,

$\downarrow = +ve$

$\uparrow = -ve$

From figure,

$BP + DE = \text{constant}$.

or, $x_P - \text{constant} + x_E - \text{const} = \text{const}$.

$\therefore x_P + x_E = \text{constant}$ —— (1)

Again,

$LF + GH = \text{constant}$.

or, $h - x_G + x_H - x_G = \text{constant}$

or, $x_H - 2x_G = \text{constant}$ —— (ii)

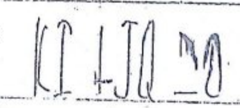
$[\because h = \text{constant}]$

$a_P + a_G = 0$ —— (iii) $\times 2 \Rightarrow$ Diff eq^n (i)

$a_H - 2a_G = 0$ —— (iv) $\Rightarrow$ Diff eq^n (ii)

$2a_P + a_H = 0$ —— (v)

Again,



or, $h - x_H + x_Q - x_H = 0$

or, $x_Q - 2x_H = 0$ —— (2)

$\therefore a_Q - 2a_H = 0$ —— (vi) [Diff eq^n (2)]

from eq^n (v) & (vi),

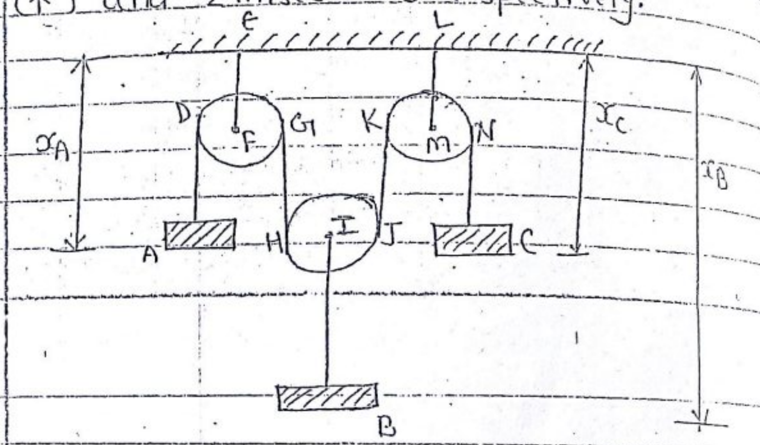
$2a_P + a_H = 0 \times 2$

$a_Q - 2a_H = 0$

$4a_P + a_Q = 0$

$\therefore \boxed{\dfrac{a_P}{a_Q} = -\dfrac{1}{4}}$ #..

(8) For the pulleys system as shown in fig 2.11, calculate the velocity and acceleration of block C. If the velocities and accelerations of blocks A and B are 2 m/sec (↓), 1 m/sec^2 (↑), 3 m/s (↑) and 2 m/sec^2 (↑) respectively.



⇒ Sol^n

$FE = \text{constant}$, $IB = \text{const.}$, $LM = \text{constant}$ and rope wrapped round the pulley is also constant. Let ↑ be +ve & ↓ be −ve.

We have,

$AD + GH + JK + NC = \text{const.}$

or, $(x_A - EF) + (x_B - IB - EF) + (x_B - IB - LM) + (x_C - LM) = \text{const.}$

or, $x_A + 2x_B + x_C = \text{const.}$

$\therefore V_A + 2V_B + V_C = 0$ —— (a)

$\therefore a_A + 2a_B + a_C = 0$ —— (b)

from equation (a),

$V_A + 2V_B + V_C = 0$

or, $-2 + 2.3 + V_C = 0.$

$\therefore v_C = -4\ m/sec = 4\ m/sec\ (\downarrow)$ #

from eq^n (B),

$a_A + 2a_B + a_C = 0$

or: $1 + 2\cdot 2 + a_C = 0$

$\therefore a_C = -5\ m/sec^2 = 5\ m/sec^2 (\downarrow)$ #

(9) A ball is thrown vertically upward from the 12 m level in an elevator shaft, with an initial velocity of 18 m/sec. At the same instant an open-platform elevator passes the 5 m level, moving upward with a constant velocity of 2 m/sec. Determine (a) when and where the ball will hit the elevator, (b) the relative velocity of the ball with respect to the elevator when the ball hits the elevator.

⇒ Sol^n

Let 't' be the time after which the ball will strike the elevator.

Distance travelled by ball in t sec,

$s_b = u_b t - \frac{1}{2} g t^2$

$\therefore s_b = 18t - 4.9t^2$ —— (i)

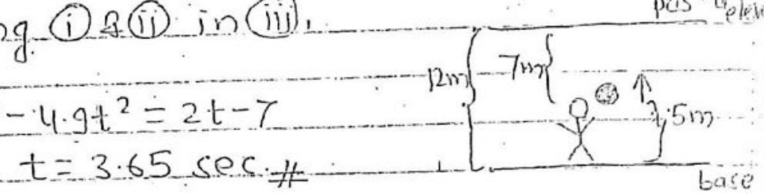
Distance travelled by elevator in t sec,

$S_e = u_e t = 2t$ —— (ii)

From question,

$S_b = S_e - 7$ $(\because 12 - 5 = 7)$ —— (iii)

Putting (i) & (ii) in (iii),



$18t - 4.9t^2 = 2t - 7$

$\therefore t = 3.65$ sec. #

Distance travelled by elevator.

$= 2t = 7.3 m$.

From base it is: $7.3 + 5 = 12.3$ m.

At the instant of striking.

velocity of ball $= u - gt$

$= 18 - 9.8 \times 3.652$

$\uparrow\uparrow$ $V_R = V_A - V_B$ $= -17.789$ m/s.

$\therefore$ Relative velocity of ball with respect to elevator when the ball hits the elevator is,

$-17.789 - (0.2) = -19.789$ m/sec.

(10) The acceleration of a particle is given by a relation, $a = v^3$. It is known that at time $t = 0$, position is -2m and velocity is 1 m/sec. Find the displacement position, velocity and acceleration at instant of 1/4 sec.

⇒ Sol^n

$a = v^3$

or, $v\dfrac{dv}{dx} = v^3$

or, $\int \dfrac{dv}{v^2} = \int dx$

or, $\dfrac{v^{-2+1}}{-2+1} = x + c_1$

or, $-\frac{1}{v} = x + c_1$

To find c_1, at $t=0$, $x=-2m$ & $v = 1 m/sec$

$-\frac{1}{1} = -2 + c_1 \Rightarrow c_1 = 1$

$\therefore -\frac{1}{v} = x + 1$ —— (x)

or, $-\dfrac{1}{dx/dt} = x+1$

or, $-\dfrac{dt}{dx} = x+1$

or, $\int -dt = \int x\,dx + \int dx$

or, $-t = \frac{x^2}{2} + x + c_2$

To find c_2, at $t=0$; $x=-2m$.

$\therefore c_2 = 0$

So,

$-t = \dfrac{x^2}{2} + x$ —— (y)

from (y), at $t = \frac{1}{4}$ sec.

$-0.25 = \dfrac{x^2}{2} + x$

or, $x^2 + 2x + 0.5 = 0$

$\therefore x = -1.707$

from (x), at $t = \cancel{0.0}\ 0.25$ sec.

$-\frac{1}{v} = -1.707 + 1$

$\Rightarrow v = 1.414$ m/sec.

Again,

$a = v^3 = (1.414)^3 = 2.82$ m/sec^2 #.

displacement $= (-1.707) - (-2)$

$= 0.293$ m #.

(11) The acceleration of a particle is defined by the relation $a = 21 - 12x^2$, where a is expressed in m/sec^2 and x in meters. The particle starts with no initial velocity at the position $x = 0$. Determine (a) the velocity when x = ... (b) the position where the velocity is again zero. (c) the position where the velocity is ...

maximum.

⇒ Sol^n

$a = 21 - 12x^2$

or, $v\frac{dv}{dx} = 21 - 12x^2$

or, $\int v\,dv = \int 21\,dx - 12\int x^2\,dx$

or, $\frac{v^2}{2} = 21x - 12\,x^3/3 + c_1$

To find c_1, at $x = 0 \Rightarrow v = 0$

$\therefore 0 = 0 + 0 + c_1 \Rightarrow c_1 = 0$

or, $\frac{v^2}{2} = 21x - 4x^3$

$\therefore v^2 = 42x - 8x^3$

(a) for $x = 1.5$ m.

$v^2 = 42(1.5) - 8(1.5)^3$

or, $v = \pm 6$ m/sec ##

(b) for $v = 0$

$0 = 42x - 8x^3$

$x = 0$, $x = \pm 2.29$ m

Thus, velocity is again zero at $x = \pm 2.29$ m

CO for v = max.

We have,

$a = 21 - 12x^2$

$\frac{dv}{dt} = 21 - 12x^2$

For maxima,

$\frac{dv}{dt} = 0$

$\therefore 0 = 21 - 12x^2$

or, $x = \pm 1.323$ m $= 1.323$ m (taking +ve)

At $x = 1.323$ m, $\frac{dv}{dt}$ is −ve.

hence maximum at this point.

(12) A motorist is travelling at 45 mile/hr when he observes that a traffic light 800 ft. ahead of him turns red. The traffic light is timed to stay red for 15 sec. If the motorist wishes to pass the light without stopping just as it turns green again, determine (a) the required uniform retardation of the car, (b) the speed of the car as it passes the light.

⇒ Sol^n

$u = 45$ miles/hr

$= \frac{45 \times 5280}{60 \times 60} = 66$ ft/sec

[$\because$ 1 mile = 1.6 km, 1 km = 1000 m, 1 m = 3.3 ft]

$S = 800$ ft.

$t = 15$ sec.

We have,

$S = ut + \frac{1}{2}at^2$

or, $800 = 66 \times 15 + \frac{1}{2} \times a \times (15)^2$

$\Rightarrow a = -1.689$ ft/sec^2 #

Now,

$v = u + at$

$= 66 + (-1.689) \times 15$

$= 40.7$ ft/sec #

(13) Two automobiles A and B are travelling in the same direction in adjacent highway lanes. Automobile B is stopped when it is passed by A, which travels at a constant speed of 36 km/hr. Two seconds later automobile B starts and accelerates at a constant rate of 1.5 m/sec^2. Determine (a) when and where B will overtake A, (b) the speed of B at that time.

$\Rightarrow$ Sol^n

For automobile A

12

$u = 36$ km/hr $= 10$ m/sec

$\therefore S = ut = 10t$ —— (I)

For automobile B

$S = ut + \frac{1}{2}at^2$

$= 0 + \frac{1}{2} \times 1.5\ (t-2)^2$ —— (II)

It is noted that if automobile A travels for t seconds, then B travels $(t-2)$ seconds.

Comparing equation (I), (II)

$10t = 0.75\ (t-2)^2$

(a) $\therefore t = 17.01$ sec (neglecting -ve sign)

$S = ut = 10(17.01) = 170.1$ m. #

(b) $v_B = u_B + at_b$

$= 0 + 1.5\ (17.01 - 2)$

$= 22.60$ m/sec

$= 81.5$ km/hr #

* ~ *

CHAPTER 3: Curvilinear Motion of Particles.

① The motion of a particle is defined by the equations. $x = t^3/2 - 2t^2$ and $y = t^2/2 - 2t$, where x and y are expressed in metre and t in sec. Determine the velocity and acceleration when $t = 1$ sec.

⇒ Solⁿ

$x = t^3/2 - 2t^2 \Rightarrow V_x = \frac{3}{2}t^2 - 4t$ — (i)

$y = t^2/2 - 2t \Rightarrow V_y = t - 2$ — (ii)

At $t = 1$ sec,

$V_x = 3/2 - 4 = -5/2$ m/sec.

$V_y = 1 - 2 = -1$ m/sec.

$\therefore V = \sqrt{V_x^2 + V_y^2} = \sqrt{(-5/2)^2 + (-1)^2}$

$= 2.7$ m/sec #.

$\theta = \tan^{-1}(V_y/V_x) = \tan^{-1}(-1/-5/2)$

$= 180° + 21.8°$

$= 201.8°$ ($\because$ 3rd quadrant)

Differentiating equation (i) & (ii), we have.

$a_x = 3t - 4$, $a_y = 1$.

At $t = 1$ sec,

$a_x = -1$, $a_y = +1$

$\therefore a = \sqrt{a_x^2 + a_y^2} = \sqrt{(-1)^2 + (1)^2}$

$= 1.41$ m/sec #.

$\alpha = \tan^{-1}(a_y/a_x) = \tan^{-1}\left(\frac{1}{-1}\right)$

$= 180° - 45°$ ($\because$ 2nd quadrant)

$= 135°$ #.

② The motion of a particle is defined by the equations $x = (t+1)^2$ & $y = (t+1)^{-2}$, where x & y are expressed in feet and t in seconds. Show that the path of the particle is a rectangular hyperbola & determine the velocity and acceleration when $t = 0$ sec.

⇒ Solⁿ

$x = (t+1)^2$

$y = (t+1)^{-2}$

$\therefore xy = (t+1)^2 (t+1)^{-2}$

$= (t+1)^{2-2}$

$= 1.$

$\therefore \boxed{xy = 1}$ which suggests that the path of the particle is a rectangular hyperbola.

Now,

$V_x = \frac{dx}{dt} = \frac{d}{dt}(t+1)^2 = \frac{d(t+1)^2}{d(t+1)} \frac{d(t+1)}{dt}$

$= 2(t+1)$ — (i)

Similary, $V_y = -2(t+1)^{-3}$ — (ii)

At $t = 0$ sec,

$V_x = 2,\ V_y = -2$

$\therefore V = \sqrt{V_x^2 + V_y^2} = \sqrt{4+4} = 2.83$ ft/sec.

$\theta = \tan^{-1}\dfrac{V_y}{V_x} = \tan^{-1}\dfrac{-2 \rightarrow y}{2 \rightarrow x}$ (4th quadrant)

$= \tan^{-1}(-1)$

$= 360° - 45°$

$= 315°$

Differentiating equation (i), (ii).

$a_x = \dfrac{dV_x}{dt} = 2$

$a_y = \dfrac{dV_y}{dt} = \dfrac{d\,{-2}(t+1)^{-3}}{dt}$

$a_y = -2\dfrac{d(t+1)^{-3}}{d(t+1)} \times \dfrac{d(t+1)}{dt}$

$= -2 \times -3\,(t+1)^{-3-1}$

$= 6\,(t+1)^{-4}$.

At $t = 0$ sec,

$a_x = 2,\ a_y = 6$.

$a = \sqrt{a_x^2 + a_y^2} = \sqrt{4+36}$

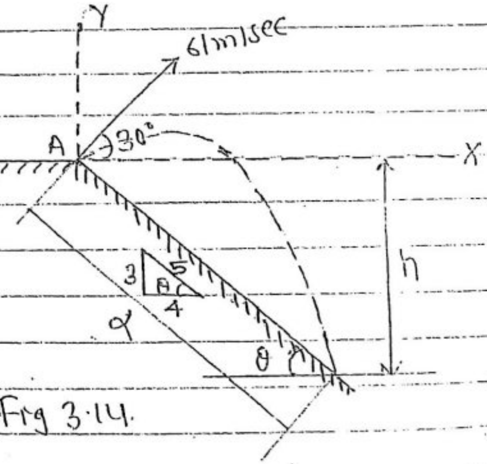
$= 6.32$ ft/sec^2 #

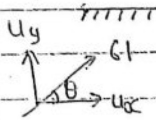
$\alpha = \tan^{-1}\dfrac{a_y}{a_x}$

$= \tan^{-1}(6/2)$

$= \tan^{-1} 3 = 71.57°$ #

(3) A particle is projected at an angle of 30° with an initial velocity of 61 m/sec as shown in fig 3.14. Find the slopping distance covered by the projectile.





$u_x = 61\cos 30°$

$u_y = 61\sin 30°$

Fig 3.14.

$\Rightarrow$ Sol^n

We have,

$-h = (u_0)_y t - \frac{1}{2} g t^2$

or, $-h = (61\sin 30°)\,t - \frac{1}{2} \times 9.8\,t^2$

or, $h = 4.9t^2 - 30.5t$ —(1)

Again,

$R = (u_0)_x t = (61\cos 30°)t$

Also, we have,

$\tan\theta = h/R$

or, $3/4 = \dfrac{4.9t^2 - 30.5t}{61\cdot\cos 30° t}$

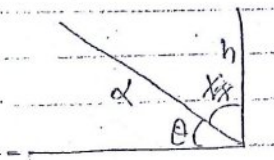
or, $39.62t = 4.9t^2 - 30.5t$

$\therefore t = 0$ or 14.29 sec.

$\therefore h = 4.9(t)^2 - 30.5t$

$= 4.9(14.29)^2 - 30.5 \times 14.29 = 566.39$ m #

Again,

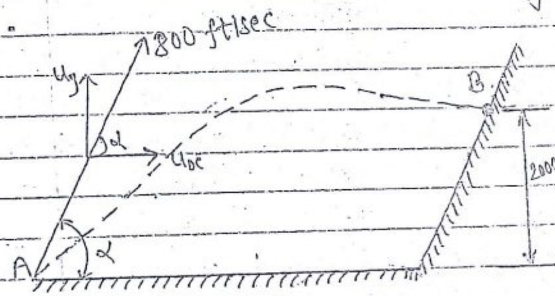


$\sin\theta = h/\alpha$

or, $\frac{3}{5} = h/\alpha \Rightarrow \frac{3}{5} = \frac{566.39}{\alpha}$

$\therefore \alpha = 944$ m #

(4) A projectile is fired with an initial velocity of 800 ft/sec at a target B located 2000 ft above the gun A and at a horizontal distance of 12,000 ft. Neglecting air resistance, determine the value of firing angle α.



⇒ Sol^n

$R = u_x t$ —(1)

$h = u_y t - \frac{1}{2}gt^2$ —(11)

here,

$u = 800$ ft/sec $= 240$ m/sec.

$R = 12000$ ft $= 3600$ m

$h = 2000$ ft $= 600$ m

$u_x = 240\cos\alpha$

$u_y = 240\sin\alpha$

From equation (1),

$3600 = (240\cos\alpha)t$

$\therefore t = \frac{15}{\cos\alpha}$

From eq^n (ii),

$600 = (240 \sin\alpha)\, t - 4.9t^2$

or, $600 = (240 \sin\alpha) \frac{15}{\cos\alpha} - 4.9\left(\frac{15}{\cos\alpha}\right)^2$

or, $600 = 240 \times 15 \tan\alpha - 4.9 \times (15)^2 \sec^2\alpha$.

or, $600 = 240 \times 15 \tan\alpha - 4.9 \times (15)^2 (1 + \tan^2\alpha)$

Solving for $\tan\alpha$, we have,

$\tan\alpha = 0.5657$ & 2.74747

$\alpha = 29.5°$ & $70°$ #

(5) The relation for r and θ for the motion of a particle is given by $r = \theta^2$ and $\theta = t^2$ where r is in metres, θ is in radians & t is in sec. Find the velocity and acceleration when $\theta = 0.2$ radian.

Sol^n

$r = \theta^2$ & $\theta = t^2$

$r = (t^2)^2 = t^4$ $\qquad \therefore \dot{\theta} = 2t$

$\therefore \dot{r} = 4t^3$ $\qquad \therefore \ddot{\theta} = 2$

$\therefore \ddot{r} = 12t^2$.

For $\theta = 0.2$ radian,

velocity and acceleration is calculated as,

$\vec{v} = \dot{r}\,\hat{e}_r + r\dot{\theta}\,\hat{e}_\theta$

$= 4t^3\,\hat{e}_r + t^4 (2t)\,\hat{e}_\theta$ — (x)

At $\theta = 0.2^c$,

$\theta = t^2 \Rightarrow t = \sqrt{\theta} = \sqrt{0.2} = 0.447$ sec.

$\therefore$ eq^n (x) is,

$\vec{v} = 4(0.447)^3\,\hat{e}_r + 2(0.447)^5\,\hat{e}_\theta$

$= 0.3572\,\hat{e}_r + 0.0357\,\hat{e}_\theta$

$\therefore v = |\vec{v}| = \sqrt{(0.3572)^2 + (0.0357)^2}$

$= 0.36$ m/sec. #

Also,

$\vec{a} = (\ddot{r} - r\dot{\theta}^2)\,\hat{e}_r + (r\ddot{\theta} + 2\dot{r}\dot{\theta})\,\hat{e}_\theta$

$= [12t^2 - t^4 (2t)^2]\,\hat{e}_r + [t^4(2) + 2(4t^3)(2t)]\,\hat{e}_\theta$

$= [12t^2 - 4t^6]\,\hat{e}_r + [2t^4 + 16t^4]\,\hat{e}_\theta$

At $\theta = 0.2^c$ i.e. $t = 0.447$ sec.

$\vec{a} = 2.3681\,\hat{e}_r + 0.71862\,\hat{e}_\theta$

$\therefore a = |\vec{a}| = \sqrt{(2.3681)^2 + (0.71862)^2}$

$= 2.51$ m/sec^2 #

⑥ The rectangular components of acceleration for a particle are $a_x = 3t$ and $a_y = (30 - 10t)$. where a is in m/sec². If the particle starts from rest at the origin, find the radius of curvature of the path at the instant of 2 sec.

$\Rightarrow$ Sol^n

$a_x = 3t$

$a_y = (30 - 10t)$

Now,

$a_x = 3t$

or, $\frac{dV_x}{dt} = 3t$

or, $\int dV_x = \int 3t\,dt$

or, $V_x = \frac{3t^2}{2} + C_1$

At $t = 0$, $V_x = 0$

$\therefore C_1 = 0$

$\therefore V_x = \frac{3t^2}{2}$ —— (i)

$a_y = (30 - 10t)$

or, $\frac{dV_y}{dt} = (30 - 10t)$

or, $\int dV_y = \int (30 - 10t)\,dt$

or, $V_y = 30t - \frac{10t^2}{2} + C_2$

At $t = 0$, $V_y = 0$

$\Rightarrow C_2 = 0$

$\therefore V_y = 30t - 5t^2$ —— (ii)

Radius of curvature of the path is,

$$\rho = \frac{\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{3/2}}{\frac{d^2y}{dx^2}} \quad \text{(x)}$$

Here,

$\frac{dy}{dx} = \frac{dy}{dt} \times \frac{dt}{dx}$

$\therefore$ Taking equation (i).

$V_x = \frac{dx}{dt} = \frac{3t^2}{2}$

Taking equation (ii).

$V_y = \frac{dy}{dt} = 30t - 5t^2$

$\therefore \frac{dy}{dx} = (30t - 5t^2) \times \frac{2}{3t^2}$

$= \frac{20}{t} - \frac{10}{3}$

$$\frac{d^2y}{dx^2} = \frac{d}{dx}\left(\frac{dy}{dx}\right)$$

$$= \frac{d}{dx}\left(\frac{20}{t} - 10/3\right)$$

$$= \frac{d}{dx}(20/t) - \cancel{\frac{d}{dx}(10/3)}^{\,0}$$

$$= \frac{d(20/t)}{dt} \times \frac{dt}{dx}$$

$$= 20 \cdot -t^{-2} \cdot \frac{2}{3t^2}$$

$$= -\frac{40}{3t^4}$$

At $t = 2$ sec,

$$\frac{dy}{dx} = \frac{20}{2} - 10/3 = 20/3$$

$$\frac{d^2y}{dx^2} = \frac{-40}{3t^4} = \frac{-40}{3(2)^4} = 0.8333$$

from equation (x),

$$\rho = \frac{\left[1 + (dy/dx)^2\right]^{3/2}}{d^2y/dx^2}$$

$$= \frac{\left[1 + (20/3)^2\right]^{3/2}}{0.8333}$$

$\therefore \rho = 36.8$ m #

7) A particle moves along a path $y = (x^2 - 4x + 100)$ with initial velocity, $\vec{V_0} = (4\hat{i} - 16\hat{j})$ m/s. If x-component of velocity is constant determine y-component of velocity and acceleration at $x = 10$ m.

Soln

$$\vec{V_0} = (4\vec{i} - 16\vec{j})$$

which is in the form of

$$\vec{V_0} = (V_x)_0 \hat{i} + (V_y)_0 \hat{j}$$

$\therefore (V_x)_0 = 4$ m/s, $(V_y)_0 = -16$ m/sec.

Also,

$$y = x^2 - 4x + 100.$$

So,

$$\frac{dy}{dt} = 2x\frac{dx}{dt} - 4\frac{dx}{dt}$$

or, $\frac{dy}{dt} = 2xV_x - 4V_x \Rightarrow V_y = 2xV_x - 4V_x$ — (i)

At $x = 10$ m,

$$\therefore V_y = 2 \times 10 \times 4 - 4 \times 4$$

$$= 64 \text{ m/sec} \#$$

To find acceleration a_y,
Differentiating equation (i),

$$\frac{dV_y}{dt} = 2\frac{d}{dt}(x V_x) - 4\frac{dV_x}{dt}$$

or, $a_y = 2\left[V_x \cdot \frac{dx}{dt} + x \cdot \frac{dV_x}{dt}\right] - 4a_x$

or, $a_y = 2\left[V_x \cdot V_x + x a_x\right] - 4a_x$

$\therefore a_y = 2V_x^2 + 2x a_x - 4a_x$

Here,

$V_x = V_{x_0} = 4$ m/s (Given, x component of velocity is const.)

$a_x = \frac{dV_x}{dt} = 0$

$\therefore a_y = 2 \times (4)^2 + 0 = 32$ m/sec^2 #

(8) Automobile A is travelling east at the constant speed of 25 km/hr. As automobile A crosses the intersection shown, automobile B starts from rest 30m north of the intersection and moves south with a constant acceleration of 1.2 m/sec^2. Determine the position, velocity and acceleration of B relative to A, five seconds later (after) A crosses the intersection. (see fig 3.16).

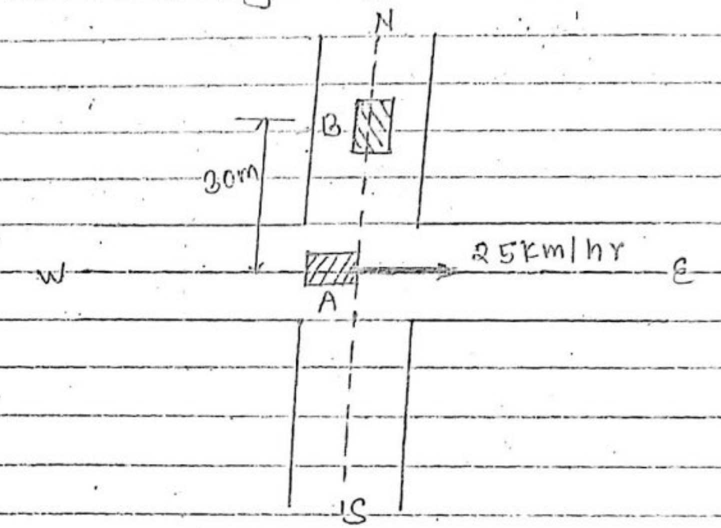


Fig. 3.16

For automobile A,

$V_A = 25$ km/hr $= 6.94$ m/sec.

$\vec{V_A} = 6.94$ m/sec $(\longrightarrow)$ rightward.

$\vec{a_A} = 0$.

$x_A = (x_A)_0 + V_A t = 0 + 6.94 \times 5 = 34.72$m

$\therefore \vec{x_A} = 34.72$ m $(\rightarrow) \Rightarrow \vec{r_A} = 34.72$ m $(\rightarrow)$

For automobile B,

$a_B = -1.2\ m/sec^2 = 1.2\ m/sec^2\ (\downarrow)$

$V_B = (V_B)_0 + at = 0 - 1.2 \times 5 = -6\ m/sec$
$= 6\ m/sec\ (\downarrow)$

$y_B = (y_B)_0 + (V_B)_0 t + \frac{1}{2} a_B t^2$

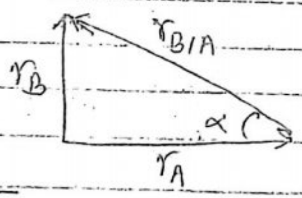
$= 30 + 0 \times t + \frac{1}{2}(-1.2) \times (5)^2$

$= 15\ m$

$\therefore \vec{y_B} = 15\ m \uparrow$.

$\Rightarrow \vec{r_B} = 15\ m\ (\uparrow)$

Motion of B relative to A.

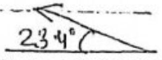


$\therefore \vec{r_{B/A}} = \vec{r_B} - \vec{r_A}$
$= 15\hat{j} - 34.72\hat{i}$

So,

$r_{B/A} = \sqrt{(15)^2 + (-34.72)^2}$

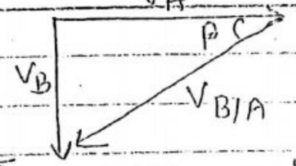
$= 37.82\ m$ #

$\alpha = \tan^{-1}\left(\frac{15}{34.72}\right) = 23.4^\circ$ ()

Again,

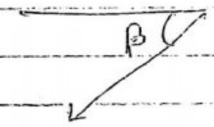
$\vec{V_{B/A}} = \vec{V_B} - \vec{V_A}$

$= -6\hat{j} - 6.94\hat{i}$



$V_{B/A} = \sqrt{(-6)^2 + (-6.94)^2} = 9.17\ m/sec^2$ #

$\beta = \tan^{-1}\left(\frac{6}{6.94}\right) = 40.84^\circ$ #



Also,

$\vec{a_{B/A}} = \vec{a_B} - \vec{a_A}$
$= -1.2\hat{j} - 0$
$= -1.2\hat{j}$

$\therefore a_{B/A} = \sqrt{(-1.2)^2} = 1.2\ m/sec^2$

$\gamma = \tan^{-1}\left(\frac{1.2}{0}\right) = 90^\circ$ i.e. $\downarrow$ #

(9) A car is travelling on a curved section of the road of radius 915 m at the speed of 50 km/hr. The brakes are suddenly applied causing the car to slow down to the speed of 32 km/hr after 6 sec. Calculate the acceleration of the car immediately after the brakes have been applied.

⇒ Sol^n

$\vec{a} = a_n \hat{e}_n + a_t \hat{e}_t$

Given,

$v_0 = 50 km/hr$

$= 13.88 m/sec.$

$v_1 = 32 km/hr$

$= 8.88 m/sec.$

Here,

$a_n = \frac{v^2}{\rho} = \frac{13.88^2}{915} = 0.210 \, m/sec^2$

$a_t = \frac{v_1 - v_0}{\Delta t} = \frac{8.88 - 13.88}{6} = -0.833 \, m/sec^2$

$\therefore \vec{a} = a_n \hat{e}_n + a_t \hat{e}_t$

$= 0.210 \hat{e}_n - 0.833 \hat{e}_t$

$\therefore a = |\vec{a}| = \sqrt{(0.210)^2 + (-0.833)^2}$

$= 0.856 \, m/sec^2$ #..

$\theta = \tan^{-1}\left(\frac{0.210}{-0.833}\right)$;

$= 14.15^\circ$

#..

(10) The rotation of the 0.9m arm OA about O is defined by the relation $\theta = 0.15t^2$, where θ is expressed in radians and t in seconds. Collar B slides along the arm in such a way that its distance from O is $r = 0.9 - 0.12t^2$, where r is expressed in metres and t in seconds. After the arm OA has rotated through 30°, determine (a) the total velocity of the collar (b) the total acceleration of the collar, (c) the relative acceleration of the colar with respect to the arm.

⇒ Sol^n

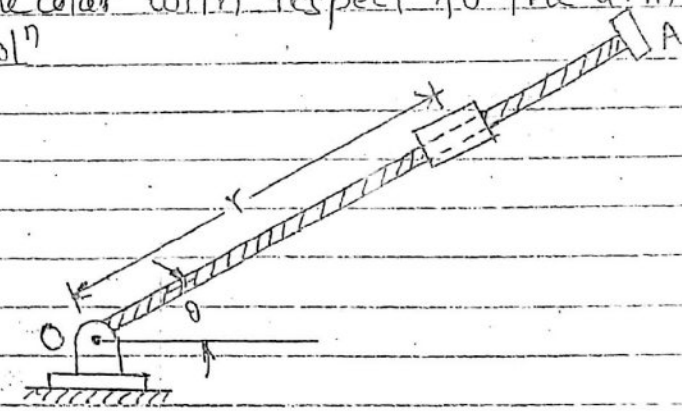


fig: 3.17

$\theta = 0.15t^2$ $\qquad r = 0.9 - 0.12t^2$

$\therefore \dot{\theta} = 0.3t$ $\qquad \dot{r} = -0.24t$

$\ddot{\theta} = 0.3$ $\qquad \ddot{r} = -0.24$

Given,

$\theta = 30^\circ = \pi/6 = \frac{3.141516}{6} = 0.5235$

$\theta = 0.15 t^2$

or, $0.5235 = 0.15 t^2$

$\therefore t = 1.868$ sec.

At $t = 1.868$ sec

$\dot{\theta} = 0.3(1.86) = 0.558 = 0.56$

$\ddot{\theta} = 0.3$

$r = 0.9 - 0.12(1.86)^2 = 0.484$

$\dot{r} = -0.24 \times 1.86 = -0.446$

$\ddot{r} = -0.24$

So,

$v_r = \dot{r} = -0.446$

$v_\theta = r\dot{\theta} = 0.481 \times 0.56 = 0.269$.

Now,

$\vec{v} = v_r \hat{e}_r + v_\theta \hat{e}_\theta$

$= -0.446\, \hat{e}_r + 0.269\, \hat{e}_\theta$

$\therefore v = |\vec{v}| = \sqrt{(-0.446)^2 + (0.269)^2}$

$= 0.522$ m/sec.

$\alpha = \tan^{-1}\left(\frac{v_\theta}{v_r}\right) = \tan^{-1}\left(\frac{0.269}{0.496}\right)$

$= 31.095^\circ$

Again,

$a_r = \ddot{r} - r\dot{\theta}^2 = (-0.24) - (0.481)(0.56)^2$

$= -0.39$ m/sec^2.

$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta}$

$= 0.481 \times 0.3 + 2 \times 0.446 \times 0.56$

$= -0.357$ m/sec^2.

$\therefore \vec{a} = a_r \hat{e}_r + a_\theta \hat{e}_\theta = -0.39\, \hat{e}_r + (-0.357)\, \hat{e}_r$

$a = |\vec{a}| = \sqrt{(-0.39)^2 + (-0.357)^2}$

$= 0.531$ m/sec^2

$\beta = \tan^{-1}\left(\frac{a_\theta}{a_r}\right)$

$= \tan^{-1}\left(\frac{0.357}{0.39}\right) = 42.4^\circ$

Acceleration of B with respect to arm OA.

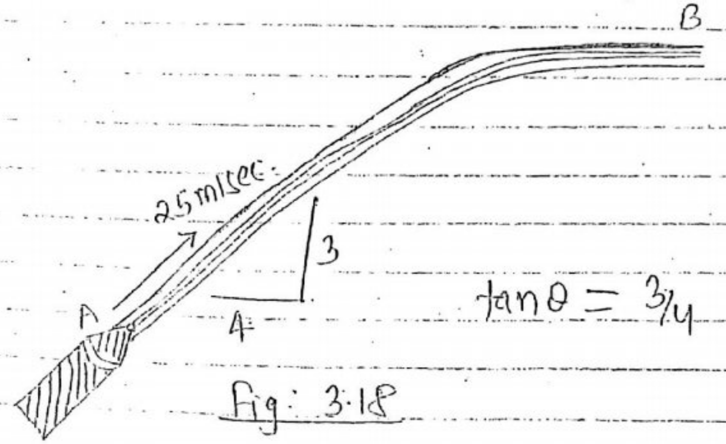
The motion of the collar w.r.t. arm is rectilinear and is defined by co-ordinate r.

$\therefore a_{B/A} = \ddot{r} = -0.24 \text{ m/sec}^2$

$= 0.24 \text{ m/sec}^2$ towards O.

#.

(ii) A nozzle discharges a stream of Walter in the direction shown in fig 3.18 with an initial velocity of 25 m/sec. Determine the radius of curvature of the stream (a) as it levels the nozzle, (b) at the maximum height of the stream.



$\tan\theta = 3/4$

Fig: 3.18

⇒ Sol^n

We have,

$$\rho = \frac{\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{3/2}}{d^2y/dx^2}$$

Here,

$$\frac{dy}{dx} = \frac{dy/dt}{dx/dt}$$

$V_x = V \cdot \cos\theta = 25 \times 4/5 = 20$ m/sec.

$V_y = V \sin\theta = 25 \times 3/5 = 15$ m/sec.

Considering y-direction.

$y = (v\sin\theta)t - \frac{1}{2}gt^2$

$\frac{dy}{dt} = v\sin\theta - \frac{1}{2} \times 2gt$

or, $\frac{dy}{dt} = v\sin\theta - gt$ ——— (i)

Again,

$V_x = U_x = \frac{dx}{dt} = 20$ m/sec ——— (ii)

eq^n (i) & (ii) suggests vertical velocity has influence of gravity while horizontal doesn't have.

Combining eqn (i) & (ii),

$$\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{V\sin\theta - gt}{20}$$

$$= \frac{15 - gt}{20}$$

$$\therefore \frac{d^2y}{dx^2} = \frac{d(dy/dx)}{dx} = \frac{d}{dx}\left(\frac{15-gt}{20}\right)$$

$$= \frac{1}{20}\frac{d}{dx}(15-gt)$$

$$= \frac{1}{20}\left(\cancel{\frac{d}{dx}(15)}^{0} - \frac{d}{dx}gt\right)$$

$$= \frac{1}{20}\left[0 - g\frac{dt}{dx}\right]$$

$$= \frac{1}{20}\left[-9.8 \times \frac{1}{20}\right]$$

$$= 0.0245.$$

(i) At t=0 sec i.e. as it leaves the nozzle.

$$\frac{dy}{dx} = \frac{15 - g\times 0}{20} = 3/4$$

24

$$\frac{d^2y}{dx^2} = -0.0245$$

$$\therefore \rho = \frac{\left[1+\left(\frac{dy}{dx}\right)^2\right]^{3/2}}{d^2y/dx^2} = \frac{\left[1+(3/4)^2\right]^{3/2}}{-0.0245}$$

$$\therefore \rho = -79.71\text{ m.}$$

$$= 79.71\text{ m}$$ #

(ii) At the maximum height of the stream,

$$\frac{dy}{dx} = 0$$

$$\therefore \rho = \frac{[1+0^2]^{3/2}}{-0.0245}$$

$$= 40.8\text{ m}$$ #.

* ~ *

① A 900 N block rests on a horizontal plane as shown in fig 4.26. Find the magnitude of the force P required to give the block an acceleration of 3 m/sec² to the right. The coefficient of friction between the block and the plane is 0.25.

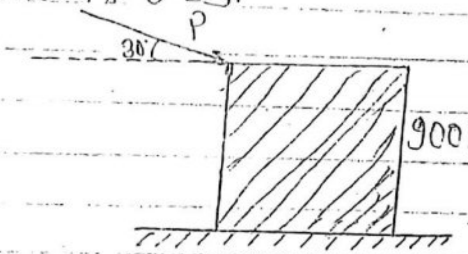
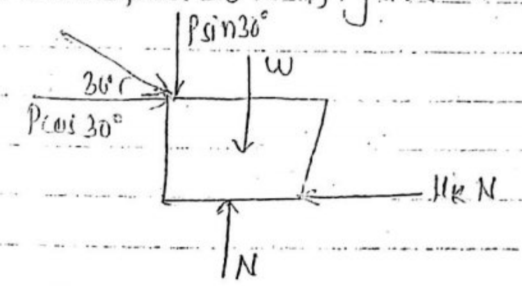


Fig: 4.26

⇒ Solⁿ

FBD of above figure is,



$m = \frac{W}{g} = \frac{900}{9.81} = 91.74$ kg.

$(\overset{+}{\rightarrow}) \sum F_x = ma_x$

or, $P\cos 30° - \mu_k N = ma_x$

or, $P\cos 30° - 0.25N = 91.74 \times 3$

or, $0.1542P - 0.25N = 275.22$ — ①

$(\uparrow +) \sum F_y = 0$

or, $N - W - P\sin 30° = 0$

or, $N - 900 - 0.5P = 0$ - - - ⓘⓘ

Solving equation ① & ⓘⓘ,

$P = 675.07$ N #
unit

② A pendulum composed of a thin wire and a bob of mass 5kg swings in a vertical plane. If the bob has a speed of 6 m/sec in the position shown in fig. 4.27 determine the tension in the wire.

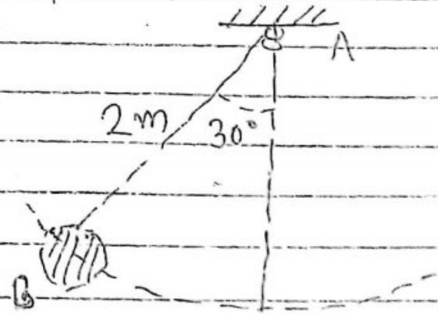
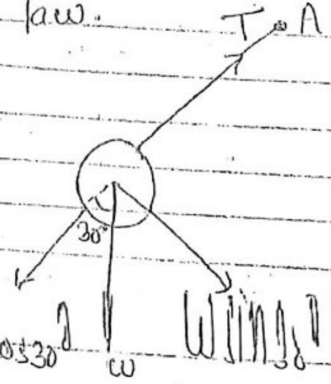


Fig: 4.27

⇒ Sol^n A

Applying Newton's 2nd law.

$(\nearrow +) \sum F_n = m a_n$



or, $T - w\cos 30° = 5 a_n$ — (i)

But, $a_n = \frac{v^2}{\rho} = \frac{36}{2}$

$= 18 \ m/sec^2$

from equation (i),

$T - 5 \times 9.8 \times \cos 30° = 5 \times 18$

$\therefore T = 132.8 N$ #

(3) A small bob of mass 5 kg is attached to a string of length 2m as shown in fig. 4.28. It is allowed to revolve in a horizontal circle at a constant speed of v_0. Determine (a) the tension in the string (b) speed v_0 of the bob.

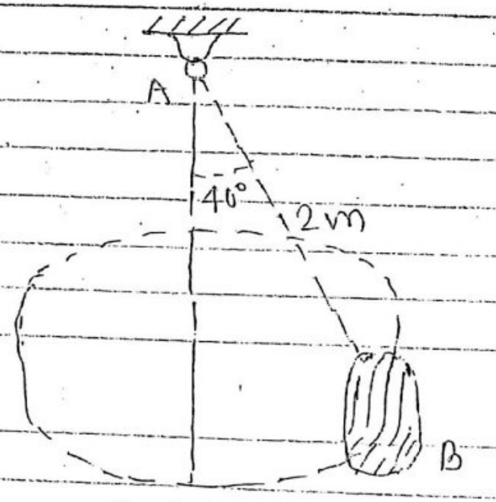
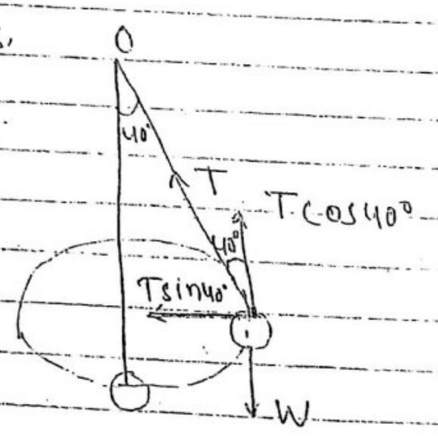


Fig: 4.28

⇒ Sol^n

FBD is,



Using Newton's 2nd law,

$(\uparrow +) \sum F_y = m a_y$

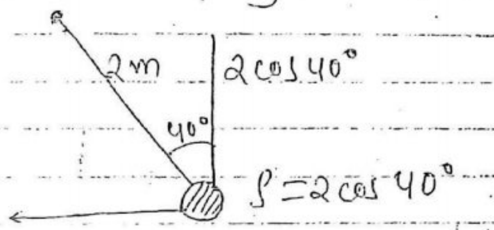
or, $T\cos 40^\circ - W = m \times 0$

or, $T = \frac{W}{\cos 40^\circ} = \frac{5 \times 9.81}{\cos 40^\circ} = 64\,N$

$(\overset{+}{\leftarrow}) \sum F_n = m a_n$

or, $T \sin 40^\circ = 5 a_n$

$\therefore a_n = \frac{64 \sin 40^\circ}{5} = 8.23\ m/sec^2$



But,

$a_n = \frac{v^2}{\rho}$

$\therefore v = \sqrt{a_n \cdot \rho}$

$= \sqrt{8.23 \times 2\cos 40^\circ}$

$= 3.5\ m/sec$ #

④ Two blocks shown in fig. 4.29 start from rest. The pulleys are frictionless and having no mass. The kinetic coefficient of friction between the block A and the inclined plane is 0.4. Determine the acceleration of each block and tension in each cord.

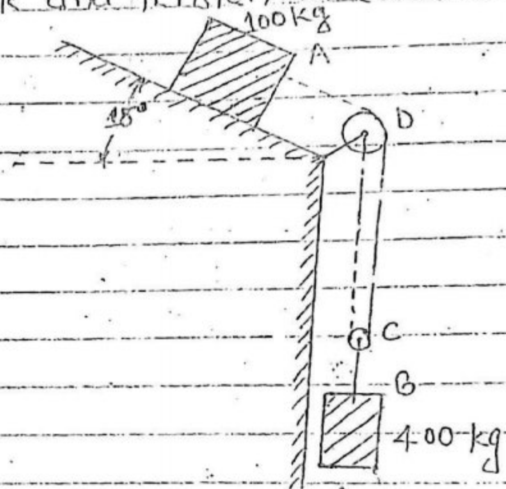
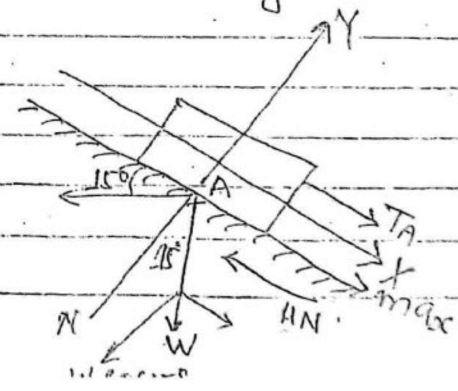


fig: 4.29

⇒ Sol^n

FBD of above fig. is,



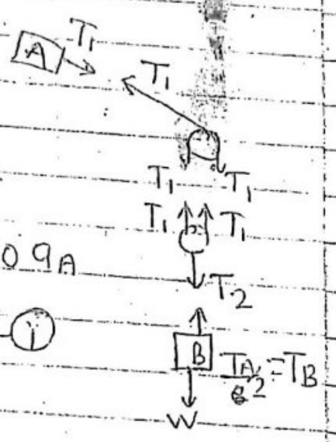
$(\uparrow +)\ \Sigma F_y = ma_y$

or, $N - W\cos 15° = m \times 0$

or, $N = W\cos 15° = 100 \times 9.81 \times \cos 15°$

$\therefore N = 947.58\,N$ Ans

$(+\downarrow)\ \Sigma F_x = ma_x$



or, $W \sin 15° - \mu N + T_A = ma_A$

or, $9.81 \sin 15° - 379.03 + T_A = 100\,a_A$

$\therefore T_A = 100\,a_A + 125.13$ — (i)

For block B,

$(\uparrow +)\ \Sigma F_y = ma_y$

or, $W - T_B = m a_B$

or, $400 \times 9.81 - T_B = 400\,a_B$

or, $T_B = 400 \times 9.81 - 400\,a_B$

$\therefore T_B = 3924 - 400\,a_B$ — (ii)

Also, we have from figure,

$T_2 = 2T_1$

From fig, if block A has velocity x m/sec along inclined direction, then block B should have velocity of $x/2$ m/sec downward.

i.e.,

$V_A = 2V_B$

which means

$a_A = 2a_B$

from eq^n (ii), (i)

$T_A = T_B/2$ i.e. $2T_A = T_B$

or, $100\,a_A + 125.13 = 200(9.81 - a_B)$

Putting $a_A = 2a_B$,

$100 \times 2a_B + 125.13 = 200(9.81 - a_B)$

or, $200a_B + 125.13 = 1962 - 200\,a_B$

or, $400\,a_B = 1962 - 125.13$

$\therefore a_B = \dfrac{1836.87}{400} = 4.592$ m/sec^2 #

So, $a_A = 2a_B = 4.59 \times 2 = 9.18$ m/sec^2 #

from eq^n (i),

$T_A = 100\,a_A + 125.13$

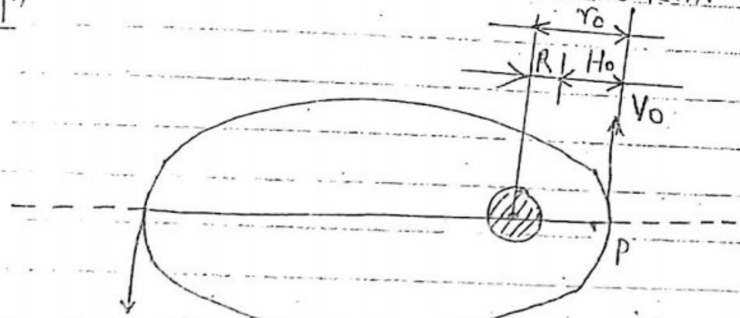
$= 100 \times 9.18 + 125.13$

$= 1043.13$ N #

$T_B = 2T_A$
$= 2086.26\ N$ #

(5) A satellite is launched in a direction parallel to the surface of the earth with a velocity of 36,900 km/hr. from an altitude of 500 km. Determine (a) the maximum altitude reached by the satellite (b) the periodic time of the satellite. Take radius of the earth $R = 6370$ km.

⇒ Sol^n



$V_0 = 36900\ km/h = \frac{36900 \times 1000}{60 \times 60}$

$= 10.25 \times 10^3\ m/sec.$

$R = 6370\ km = 6.37 \times 10^6\ m$

$H_0 = 500\ km = 5 \times 10^5\ m$

$r_0 = R + H_0 = 6.37 \times 10^6 + 5 \times 10^5$

$= 6.87 \times 10^6\ m$

we have,

$$\frac{1}{r} = \frac{GM}{h^2} + c\cos\theta \quad \text{—— (1)}$$

$\Rightarrow GM = gR^2 = 9.81 \times (6.37 \times 10^6)^2$
$= 3.98 \times 10^{14}$

$h^2 = r_0^2\, V_0^2 = (6.87 \times 10^6)^2 \times (10250)^2$
$= 4.96 \times 10^{21}$

For vertex A,

$\theta = 0,\ \cos 0° = 1,\ r = r_0$

hence,

$$\frac{1}{r_0} = \frac{GM}{h^2} + c$$

$\Rightarrow c = 6.53 \times 10^{-8}\ m^{-1}$

For maximum altitude attained by space vehicle.

$\theta = 180°,\ r = r_1$

From above equation (1),

$$\frac{1}{r_1} = \frac{GM}{h^2} + c\cos 180° = \frac{GM}{h^2} - c$$

or, $\frac{1}{r_1} = 8.03 \times 10^{-8} - 6.53 \times 10^{-8}$

or, $r_1 = 6.67 \times 10^7\ m$

$\therefore$ max altitude

$= (66700 - 6370)$

$= 60380$ km #

$\therefore$ Time period $(T) = \frac{2\pi ab}{h}$

$a = \frac{r_1 + r_0}{2} = \frac{6.87 + 66.7}{2} \times 10^6$

$= 36.8 \times 10^6$ m

$b = \sqrt{r_1 r_0} = \sqrt{6.87 \times 66.7} \times 10^6$

$= 21.4 \times 10^6$ m

$h = r_0 v_0 = 7.04 \times 10^{10}$ m

$T = \frac{2\pi ab}{h}$

$= \left[\frac{2\pi \times 36.8 \times 10^6 \times (21.4 \times 10^6)}{7.04 \times 10^{10}}\right]$

$\therefore T = 19$ h 31 min #

(6) The altitudes of the perigee P and apogee Q in the orbit of a space satellite are 250 mile and 500 mile respectively. For such an orbit, determine (a) the time period (b) the eccentricity (c) the maximum speed (d) the minimum speed. (see fig. 4.30)

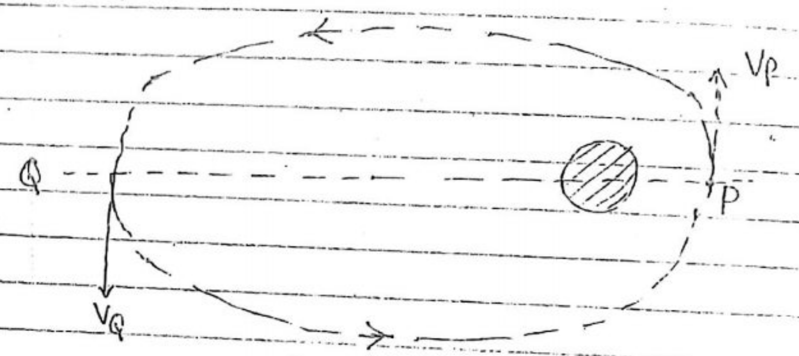


Fig. 4.30

Sol^n

$V_{max} = V_P$

$V_{min} = V_Q$

$r_0 = (250 \times 1.609 + 6370) \times 10^3$

$= 6.77 \times 10^6$ m

$r_1 = (500 \times 1.609 + 6370) \times 10^3$

$= 7.175 \times 10^6$ m

At vertex, P, $\theta = 0$, $\cos 0 = 1$

$r = r_0$, $V_0 = V_P$

$$\therefore \frac{1}{r_0} = \frac{GM}{r_0^2 V_p^2} + c \quad \text{--- (1)}$$

For vertex Q, $\theta = 180°$, $r = r_1$

$$\frac{1}{r_1} = \frac{GM}{r_0^2 V_p^2} + c\cos 180°$$

$$= \frac{GM}{r_0^2 V_p^2} - c \quad \text{--- (2)}$$

Solving equation (1) and (2)

$$\frac{1}{r_0} + \frac{1}{r_1} = \frac{2GM}{r_0^2 V_p^2}$$

$$\Rightarrow V_p^2 = 6.05 \times 10^7$$

$$V_p = 7778.50 \text{ m/sec.}$$

put V_p value in equation (1).

$$c = 4.15 \times 10^{-9}$$

$$V_p = 7778.5 \text{ m/sec} = 4.83 \text{ miles/sec} \quad \#$$

We know.

$$e = \frac{ch^2}{GM} = \frac{c r_0^2 V_p^2}{g R^2} = 0.028 \quad \#$$

$$\therefore T = \frac{2\pi ab}{r_0 V_p} = 1.611 \text{ hr} \quad \#$$

where,

$$a = \frac{r_0 + r_1}{2} = 6.99 \times 10^6 \text{ m}$$

$$b = \sqrt{r_0 r_1} = 6.96 \times 10^6 \text{ m}$$

from conservation of angular momentum.

$$r_0 m V_p = r_1 m V_Q$$

$$\text{or, } V_Q = \left(\frac{r_0}{r_1}\right) V_p = \frac{6.77 \times 10^6}{7.176 \times 10^6} \times 7778.50$$

$$= 7338 \text{ m/sec.}$$

$$= 4.56 \text{ miles/sec.} \quad \#$$

$\therefore$ Time period (T) = 1.611 hr #
eccentricity (e) = 0.028
maximum speed = 4.83 miles/sec
minimum speed = 4.56 miles/sec

7. The resultant external force acting on a 2 kg particle in space is $\vec{F} = (12t\hat{i} - 24t^2\hat{j} - 40t^3\hat{k})$ N, where t is the time measured in seconds. The particle is at rest at the origin.

when $t=0$. Determine the acceleration component $\vec{a_y}$, the velocity component $\vec{v_y}$ and the coordinate y of the particle at the instant of 4 sec.

⇒ Sol^n

$$\vec{F} = (12t\,\hat{i} - 24t^2\,\hat{j} - 40t^3\,\hat{k})$$

which is in the form of

$$\vec{F} = F_x\hat{i} + F_y\hat{j} + F_z\hat{k}$$

$$\therefore F_x = 12t,\ F_y = -24t^2,\ F_z = -40t^3$$

$$\text{or, } ma_x = 12t,\ ma_y = -24t^2,\ ma_z = -40t^3$$

$$\therefore a_y = -12t^2 \quad \text{— (x)}$$

Here,

$$ma_y = -24t^2$$

for $t = 4$ sec,

$$\text{or, } 2a_y = -24 \cdot 4^2 \Rightarrow a_y = -192\ \text{m/sec}^2$$

Hence,

$$\vec{a_y} = -192\,\hat{j}\ \text{m/sec}^2$$

from equation (x),

$$\text{or, } \frac{dv_y}{dt} = -12t^2$$

$$\text{or, } dv_y = -12t^2\,dt$$

Integrating,

$$\int dv_y = \int -12t^2\,dt$$

$$\text{or, } v_y = -12 \times t^3/3 + c_1$$

$$\therefore v_y = -4t^3 + c_1$$

To find c_1, at $t=0$, $v_y = 0$.

$$\therefore c_1 = 0$$

$$\therefore v_y = -4t^3 \quad \text{— (y)}$$

At $t = 4$ sec,

$$v_y = -256\ \text{m/sec.}$$

Hence,

$$\vec{v_y} = -256\,\hat{j}\ \text{m/sec} \ \#$$

from equation (y),

$$v_y = -4t^3$$

$$\text{or, } \frac{dy}{dt} = -4t^3$$

$$\text{or, } dy = -4t^3\,dt$$

Integrating,

$\int dy = \int -4t^3\, dt$

or, $y = -\frac{4t^4}{4} + C_2$

$\therefore y_{C_2} = -t^4 + C_2$

To find C_2,
Since at $t=0$, particle is at origin i.e. $y=0$.

$\therefore C_2 = 0$.

So,
$y = -t^4$
for, $t = 4$ sec.

$\therefore y = -256$ m ##

⑧ The motion of a 500 gm block B in a horizontal plane is defined by the relations $r = 2(1+\cos 2\pi t)$ and $\theta = 2\pi t$ where r is expressed in meters, t in seconds, and θ in radians. Determine the radial and transverse components of the force exerted on the block when (a) $t=0$, (b) $t=0.75$ sec.
(see fig. 4.31).

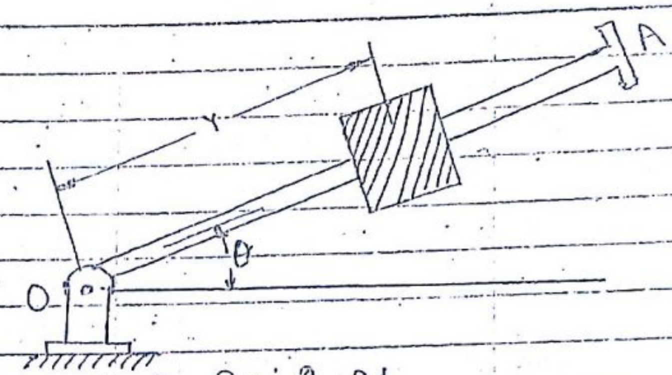


Fig: 4.31

⇒ Soln

$r = 2(1+\cos 2\pi t)$	$\theta = 2\pi t$
$\therefore \dot{r} = 2(2\pi)(-1)\sin 2\pi t$	$\dot{\theta} = 2\pi$
$= -4\pi \sin 2\pi t$	$\ddot{\theta} = 0$
&	
$\ddot{r} = -4\pi \cdot 2\pi \cos 2\pi t$	
$= -8\pi^2 \cos 2\pi t$	

Using,
$\Sigma F_r = ma_r = m(\ddot{r} - r\dot{\theta}^2)$

$\Sigma F_\theta = ma_\theta = m(r\ddot{\theta} + 2\dot{r}\dot{\theta})$

(a) At $t=0$, $r = 2(1+1) = 4$

$\dot{r} = -4\pi \sin 0 = 0$

$\ddot{r} = -8\pi^2 \cos 0° = -8\pi^2$

$\theta = 0$

$\dot{\theta} = 2\pi$

$\ddot{\theta} = 0$

$\therefore \Sigma F_r = 0.5\left(-8\pi^2 - 4\cdot(2\pi)^2\right)$

$= 0.5\left(-8\pi^2 - 16\pi^2\right)$

$= -118.428\ N$ #

$\Sigma F_\theta = m(r\ddot{\theta} + 2\dot{r}\dot{\theta})$

$= 0.5\,(4\times 0 + 2\times 0\times 2\pi)$

$= 0$

Hence,

$F_r = -118.428\ N$

$F_\theta = 0$ #

(b) At $t = 0.75$ sec,

$r = 2(1 + \cos 2\pi \times 0.75)$

$= 2(1+0) = 2$

$\dot{r} = -4\pi \sin(2\pi \times 0.75) = 4\pi$

$\ddot{r} = -8\pi^2 \cos 2\pi \times 0.75 = 0$

$\theta = 2\pi \times 0.75 = 4.71238^c$

$\dot{\theta} = 2\pi$

$\ddot{\theta} = 0$

$\Sigma F_r = m(\ddot{r} - r\dot{\theta}^2)$

$= 0.5\left[0 - 2\times(2\pi)^2\right]$

$\therefore F_r = -39.5\ N$ #

$\Sigma F_\theta = m(r\ddot{\theta} + 2\dot{r}\dot{\theta})$

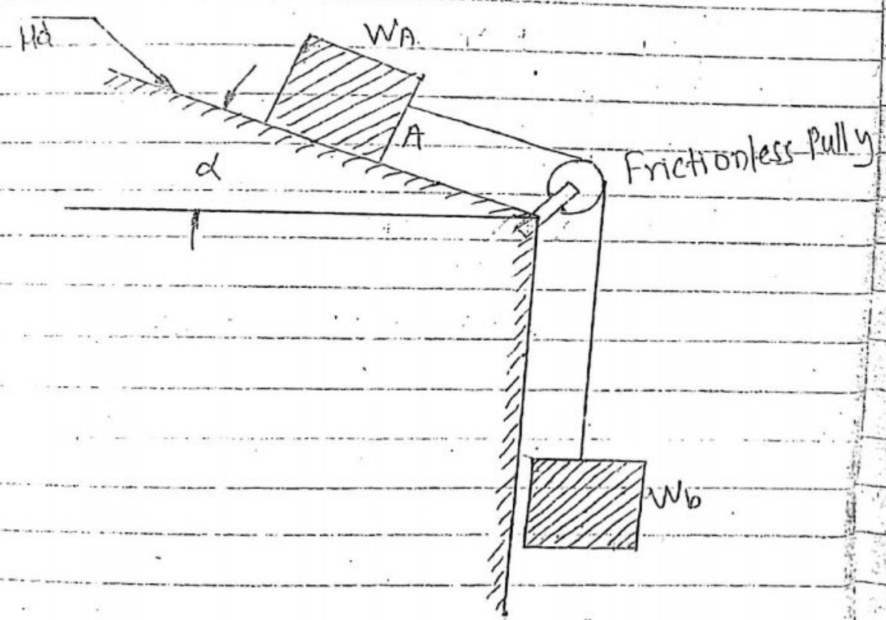
$= 0.5\,(2\times 0 + 2\times 4\pi \times 2\pi)$

$\therefore F_\theta = 79\ N$ #

~

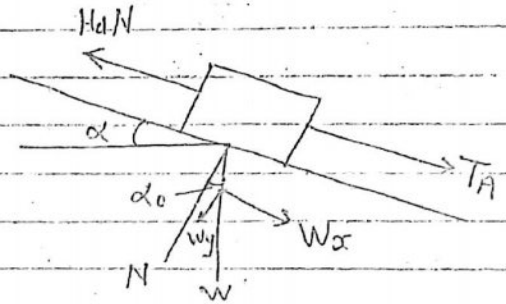
CHAPTER 5: Kinetics of Particles – Energy & Momentum.

① Two blocks A & B are connected by means of an inextensible and weightless cord as shown in fig 5.21. Initially the bodies are at rest. If the dynamic coefficient of friction is 'μ_d' for block A on the surface inclined at an angle α. Compute the velocity of the bodies at any time t, before body A has reached the end of the incline.



⇒ Sol^n

FBD of block A,



$W_y = W_A \cos\alpha$, $W_x = W_A \sin\alpha$

For block A,

$(\uparrow +) \sum F_y = ma_y = 0$

or, $N - W_y = 0$

or, $N - W_A \cos\alpha = 0$

$\therefore N = W_A \cos\alpha$ —— ①

Again using linear-impulse momentum principle,

$$\int_0^t F_a\, dt = m_A (V_2 - V_1)$$

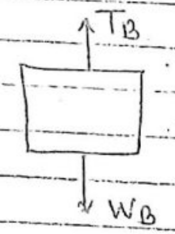
or, $$\int_0^t (W_A \sin\alpha + T_A - \mu_d N)\, dt = \frac{W_A}{g}(V_2 - 0)$$

Let $V_2 = V$

$$\therefore W_A \sin\alpha t + T_A t - \mu_d N t = \frac{W_A}{g} V$$

or, $\left(w_A \sin\alpha + T_A - \mu_d w_A \cos\alpha\right) t = \frac{w_A}{g} V$ —— (i)

For block B,



Using linear momentum principle,

$$\int_0^t F_y\,dt = m_B\left(V_2 - \cancel{V_1}^{\,0}\right)$$

or, $\int_0^t (w_B - T_B)\,dt = m_B V$ (let $V_2 = V$)

or, $(w_B - T_B)\,t = m_B V$

$\therefore (w_B - T_A)\,t = \frac{w_B}{g} V$ —— (ii) ($\because T_B = T_A$)

Adding equation (i) & (ii), we have,

$$w_B t - \cancel{T_A t} + w_A \sin\alpha \cdot t + \cancel{T_A t} - \mu_d w_A \cos\alpha \cdot t = \left(\frac{w_A}{g} + \frac{w_B}{g}\right) V$$

or, $\left(w_B + w_A \sin\alpha - \mu_d w_A \cos\alpha\right) t = (w_A + w_B)\, V/g$

$$\therefore V = \frac{gt}{w_A + w_B}\left[w_B + w_A \sin\alpha - \mu_d w_A \cos\alpha\right]$$

which is the required expression.

(2) A 5 kg sphere travelling at 20 m/sec collides with a 10 kg sphere travelling at 10 m/sec in the same direction as shown in fig 5.22. If the impact is direct and central, what will the final velocities of the spheres be if the impact is (a) elastic (b) perfectly inelastic? (c) How much K.E. is lost in each case?

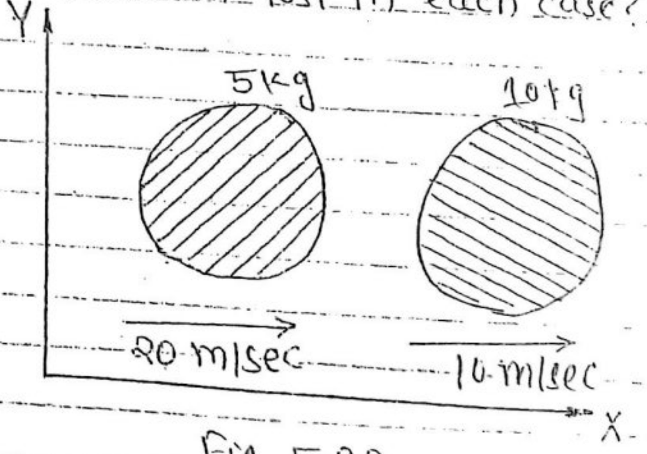


Fig 5.22

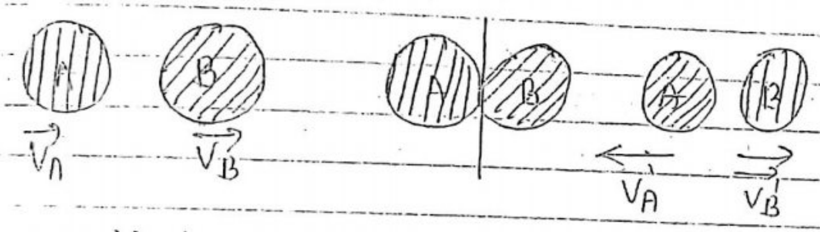
⇒ Sol^n

$m_A = 5\,kg$ $\quad V_A = 20\,m/sec$

$m_B = 10\,kg$ $\quad V_B = 10\,m/sec.$

F.B.D. is

(i) Perfectly elastic collision.



$V_A > V_B$

For perfectly elastic collision, coeff. of restitution is $e = 1$ and is given as

$$e = \frac{\vec{V_B'} - \vec{V_A'}}{\vec{V_A} - \vec{V_B}} \Rightarrow e = \frac{V_B' - V_A'}{V_A - V_B}$$

$$\text{or, } 1 = \frac{V_B' - V_A'}{V_A - V_B}$$

or, $V_B' - V_A' = V_A - V_B = 20 - 10 = 10$ — (i)

Again, applying conservation of momentum principle, $\longrightarrow$ (+ve) & $\longleftarrow$ (−ve) (say)

$$m_A V_A + m_B V_B = m_A V_A' + m_B V_B'$$

or, $5 \times 20 + 10 \times 10 = 5 \times V_A' + 10 V_B'$

or, $200 = 10 V_B' + 5 V_A'$

or, $2 V_B' + V_A' = 40$ — (ii)

from equation (i),

$2 V_B' + (V_B' - 10) = 40$

or, $3 V_B' = 50$

$\therefore V_B' = +16.67\,m/sec = 16.67\,m/sec\,(\longrightarrow)$

&

$V_A' = V_B' - 10 = 16.67 - 10 = 6.67\,m/sec\,(\longrightarrow)$

⇒ For perfectly elastic collision, there is no loss of K.E.

(ii) For perfectly plastic collision.

→ coefficient of restitution, $e = 0$ and both particles move with common velocity as,

$$e = \frac{V_B' - V_A'}{V_A - V_B} \Rightarrow 0 = \frac{V_B' - V_A'}{V_A - V_B}$$

or, $V_B' = V_A' = v$ (say)

Using conservation of momentum principle,

$m_A v_A + m_B v_B = (m_A + m_B) v$

$\therefore v = \dfrac{m_A v_A' + m_B v_B}{m_A + m_B}$

$= \dfrac{5 \times 20 + 10 \times 10}{5 + 10}$

$= \dfrac{200}{15} = 13.33$ m/sec ($\longrightarrow$)

Loss of K.E.

$= \left[\frac{1}{2} m_A v_A^2 + \frac{1}{2} m_B v_B^2\right] - \left[\frac{1}{2} m_A v^2 + \frac{1}{2} m_B v^2\right]$

$= \left[\frac{1}{2} \times 5 \times 20^2 + \frac{1}{2} \times 10 \times 10^2\right] - \left[\frac{1}{2} \times 5 \times 13.33^2 + \frac{1}{2} \times 10 \times 13.33^2\right]$

$= 167.3$ J #.

③ Two balls (of the same size and mass) collide with the velocities of approach as shown in Fig 5.23. For a coefficient of restitution of 0.90, what are the final velocities of the balls directly after they part? What is the loss of K.E. and where does this energy go?

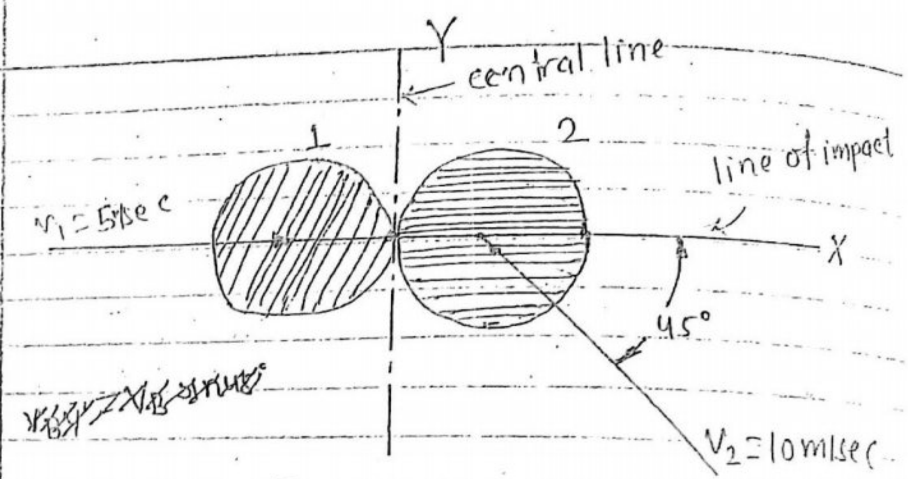


Fig. 5.23

⇒ Sol^n

V_A, V_A' & V_B, V_B' be the initial and final velocities or velocity before impact and after impact of block A and B respectively.

$V_A = 5$ m/sec, $V_B = 10$ m/sec.

momentum of each particle along the contact line will be conserved.

$m_A (\vec{V_A})_t + m_A (\vec{V_A'})_t = 0$

or, $m_A (V_A)_t + m_A (V_A')_t = 0$ [scalar only]

but,

$(V_A)_t = 0$; $(V_A')_t = 0$

for block B

$m_A (\vec{V_B})_t = m_B (\vec{V_B'})_t$

or, $V_B \sin 45 = V_B'$

$\therefore (V_B')_t = V_B \sin 45^\circ = 7.07$ — (I)

total momentum along the impact will be conserved.

or, $m_A (\vec{V_A})_n + m_B (\vec{V_B})_n = m_A (\vec{V_A'})_n + m_B$

$\Rightarrow 5 + (-10 \cos 45^\circ) = -v_A' + (\vec{V_B'})_n$

$\therefore (V_B')_n - (V_A')_n = -2.07$ — (II)

By the relation of coeff. of restitution

$$e = \frac{(V_B')_n - (V_A')_n}{(V_A)_n - (V_B)_n}$$

$$\text{or, } 0.9 = \frac{(V_B')_n + V_A'}{V_A + V_B \cos 45^\circ}$$

$$\text{or, } 0.9 = \frac{(V_B')_n + V_A'}{5 + 10 \cdot (1/\sqrt{2})}$$

$\therefore (V_B')_n + V_A' = 10.864$ — (III)

from equation (II) and (III)

$(V_B')_n = 4.4$ m/sec.

$V_A' = 6.47$ m/sec.

$(V_B')_t = 7.07$ m/sec.

$\therefore \vec{V_A'} = 6.47 \hat{i}$ m/sec #

$\therefore$ At last velocity of particle A

$= 6.47 \hat{i}$ m/sec #

and velocity of B $= (4.4 \hat{i} + 7.07 \hat{j})$ m/sec #

$\therefore$ loss of K.E. = Initial K.E. − final K.E.

$= \frac{1}{2} m_A V_A^2 + \frac{1}{2} m_B V_B^2 - \frac{1}{2} m V_A'^2 - \frac{1}{2} m_B V_B'^2$

$= \frac{1}{2} m [25 + 100 - (6.47)^2 - (8.33)^2]$

$= 6.89$ N m (Joule) #

(4) A 10 kg-collar slides without friction along a vertical rod as shown in fig 5.24. The spring attached to the collar has an undeformed length of 100 mm and a constant of 500 N/m. If the collar is released from rest in position 1, determine its after it has moved 150 mm to position 2.

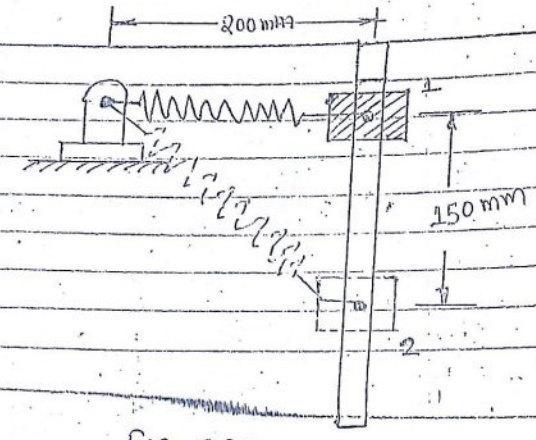


Fig. 5.24

$\Rightarrow$ Sol^n

$k = 500\ N/m$

Undeformed length of spring $= 100\ mm$
$= 0.1\ m$

We have from conservation of energy,

$T_1 + V_1 = T_2 + V_2$ —— ①

$\therefore T_1 = 0 \quad [\because u_1 = 0]$

$\therefore V_1 = V_g + V_e$ [V_e = P.E. due to elastic force produced by spring]

$= 0 + \frac{1}{2} kx^2$ [V_g = P.E. due to gravity]

$= \frac{1}{2} \times 500 \times (0.2 - 0.1)^2$

$T_2 = \frac{1}{2} \times 10 \times v_2^2 = 5v_2^2$

$\therefore V_2 = V_e + V_g$
$= \frac{1}{2} kx_2^2 + W.y \quad [\because W = mg]$
$= \frac{1}{2} \times 500 \times (0.25 - 0.1)^2 + 98.1(-0.15)$
$= -9.09$

putting all values in equation ①,

or, $0 + 2.5 = 5v_2^2 - 9.09$

$\therefore v_2 = 1.52\ m/sec\ (\downarrow)$ #

⑤ A ball is thrown against a frictionless vertical wall. Immediately before the ball strikes the wall, its velocity has a magnitude v and forms an angle of 30° with the horizontal. Knowing that $e = 0.90$, determine the magnitude and direction of the velocity of the ball as it rebounds from the wall. (see fig 5.25)

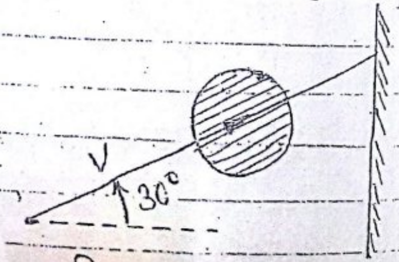
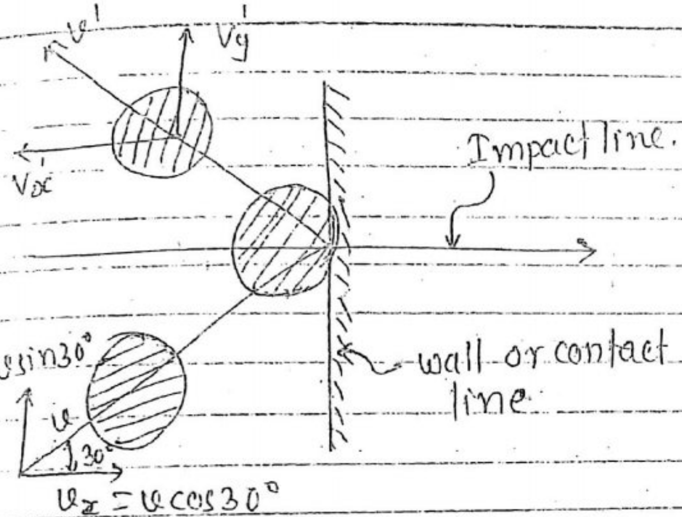


Fig: 5.25

⇒ Sol^n.



momentum of ball is conserved along y-axis.

$\therefore v_y' = v_y = v\sin 30° = 0.5v\ (\uparrow)$

$\therefore e = \dfrac{V_B' - V_A'}{V_A - V_B}$ | B = wall, A = Ball

or. $0.9 = \dfrac{0 - (-V_A')}{V_x - 0}$ (wall is in rest)

or, $0.9v\cos 30° = V_x'$

or, $V_x' = 0.779v\ (\leftarrow)$

$\therefore v' = \sqrt{(v_x')^2 + (v_y')^2}$

$= \sqrt{(0.5v)^2 + (0.779v)^2}$

$= 0.926v$ #.

$\therefore \theta = \tan^{-1}\left(\dfrac{v'_y}{v'_x}\right)$

$= \tan^{-1}\left(\dfrac{0.5v}{0.779v}\right)$

$\therefore \theta = 32.7°$ ( ) #.

⑥ A 30kg block is dropped from a height of 2m onto the 10kg pan of a spring scale. Assuming the impact to be perfectly plastic, determine the maximum deflection of the pan. The constant of the spring is K=20kN/m. (see fig. 5.26.)

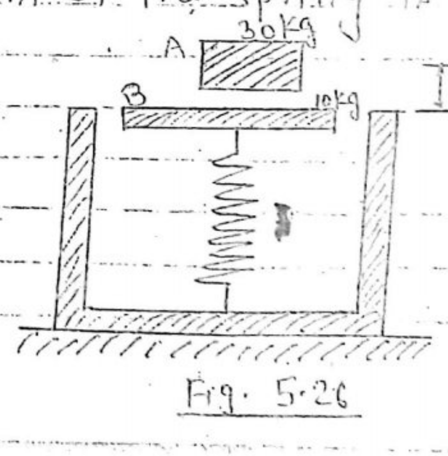
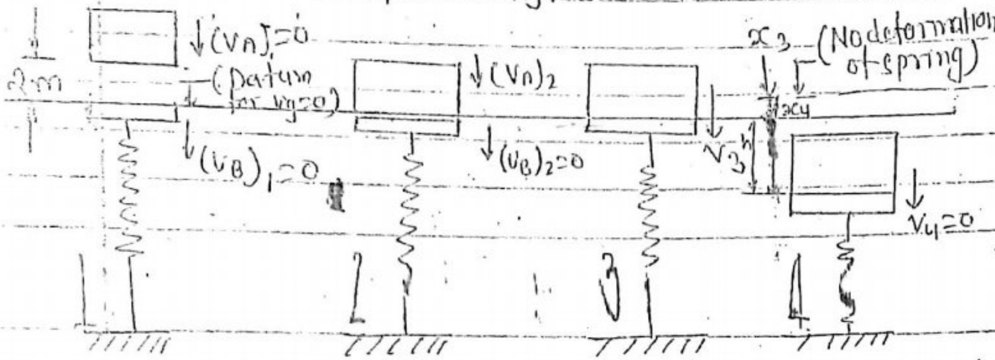


Fig. 5.26

⇒ Solⁿ

The impact between block and pan considered separately.



Conservation of energy

$w_A = mg = 30 \times 9.81 = 294N.$

$T_1 = \frac{1}{2} m_A (v_A)_1^2 = 0$

$V_1 = w_A y = 294 \times 2 = 588$

$T_2 = \frac{1}{2} \times m_A (V_A)_2^2 = \frac{1}{2} \times 30 \times (V_A)_2^2$

$V_2 = 0$

$\therefore T_1 + V_1 = T_2 + V_2$

or, $0 + 588 = \frac{1}{2} \times 30 \times (V_A)_2^2 + 0$

$\therefore (V_A)_2 = 6.26$ m/sec

Conservation of momentum.

for perfectly plastic, $e = 0$.

$m_A (V_A)_2 + m_B (V_B)_2 = (m_A + m_B) v_3$

Conservation of energy:-

$x_3 = \frac{W_B}{k} = \frac{10 \times 9.81}{20 \times 10^3} = 4.91 \times 10^{-3}$ m

$T_3 = \frac{1}{2}(m_A + m_B) V_3^2 = \frac{1}{2}(30+10)(4.7)^2$

$= 442J$

$T_4 = 0$

$V_4 = V_g + V_e = (W_A + W_B)(-h) + \frac{1}{2} k x_4^2$

$= -(392)h + \frac{1}{2}(20 \times 10^3) x_4^2$

$\therefore T_3 + V_3 = T_4 + V_4$

or, $442 + 0.241 = 0 + -(392)h + \frac{1}{2} \times (20 \times 10^3) \times x_4^2$

Insert

or, $442 + 0.241 = -(392)(x_4 - 4.91 \times 10^{-3}) + \frac{1}{2} \times (20 \times 10^3) x_4^2$

$[\because h = x_4 - x_3]$

$\therefore x_4 = 0.230$ m

$h = x_4 - x_3$
$= 0.230 - 4.91 \times 10^{-3}$
$= 0.225\, m$
$\therefore h = 225\, mm$ //

(7) The magnitude and direction of the velocities of two identical smooth balls before they strike each are as shown in fig 5.27. Assuming e = 0.80, determine the magnitude and direction of the velocity of each ball after the impact. How much K.E. will be lost due to the impact?

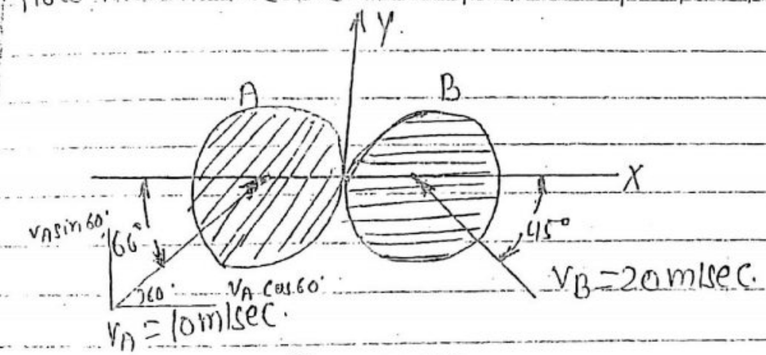


Fig. 5.27

⇒ Sol^n

$V_{1t} = V_A \sin 60° = 8.66\, m/sec$
$V_{1n} = V_A \cos 60° = 5\, m/sec$
$V_{2t} = V_B \sin 45° = 14.14\, m/sec$
$V_{2n} = V_B \cos 45° = 14.14\, m/sec$

Momentum along the contact axis of each particle is conserved i.e.
$m_A V_{1t} = m_A V'_{1t}$ $\therefore V_{1t} = V'_{1t} = 8.66\, m/sec$

Again,
$m_B V_{2t} = m_B V'_{2t}$ $\therefore V'_{2t} = V_{2t} = 14.14\, m/sec$

Total momentum along the impact axis will be conserved i.e.
$m_A \vec{V_{1n}} - m_B \vec{V_{2n}} = -m_A \vec{V'_{1n}} + m_B \vec{V'_{2n}}$

or, $V_{1n} (-V_{2n}) = -V'_{1n} + V'_{21}$
or, $V_{1n} - V_{2n} = V'_{2n} - V'_{1n}$
or, $V'_{2n} - V'_{1n} = 5 - 14.14 = -9.14$ — (I)

Again,
$$e = \frac{\vec{V'_{21}} - \vec{V'_{1n}}}{\vec{V_{1n}} + \vec{V_{2n}}}$$

or, $V'_{2n} + V'_{1n} = e(V_{1n} + V_{2n})$
or, $V'_{2n} + V'_{1n} = 0.8(5 + 14.19)$
or, $V'_{2n} + V'_{1n} = 15.352$ — (II)
From equation (I) and (II),

$V_{2n}' = 3.1$ m/sec.

$V_{1n}' = 12.25$ m/sec.

$\therefore \vec{V_A'} = (-12.25\hat{i} + 8.66\hat{j})$ m/sec

$\vec{V_B'} = (3.1\hat{i} + 14.14\hat{j})$ m/sec #

* ~ *

CHAPTER 6: SYSTEM OF PARTICLES

(1) A system of particles at time t has the following velocities and masses:

$v_1 = 20$ ft/sec, $m_1 = 1$ lbm

$v_2 = 18$ ft/sec, $m_2 = 3$ lbm

$v_3 = 15$ ft/sec, $m_3 = 2$ lbm

$v_4 = 5$ ft/sec, $m_4 = 1$ lbm

Determine (a) the linear momentum of the system, (b) the angular momentum of the system about the origin, and (c) the angular momentum of the system about point a. (see Fig 6.18).

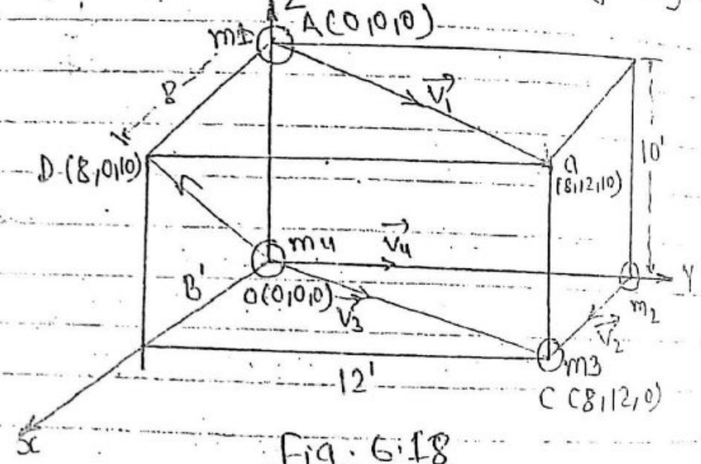


Fig. 6.18

⇒ Sol^n

$m_1 = 1$ lbm $= \frac{1}{32.2}$ slug [$\because g = 32.2$ ft/sec²]

$m_2 = 3$ lbm $= \frac{3}{32.2}$ slug

$m_3 = 2\ lbm = \frac{2}{32.2}$ slug

$m_4 = 1\ lbm = \frac{1}{32.2}$ slug

$\vec{v_1} = 20 ft/sec \Rightarrow \vec{v_1} = \frac{20(8\hat{i}+12\hat{j})}{\sqrt{64+144}} = 11.09\hat{i}+16.64\hat{j}$

$v_2 = 18 ft/sec \Rightarrow \vec{v_2} = 18\hat{i}$

$v_3 = 15 ft/sec \Rightarrow \vec{v_3} = \frac{15(-12\hat{j}+10\hat{k})}{\sqrt{144+100}}$

$= -11.52\hat{j} + 9.60\hat{k}$

$V_A = 5 ft/sec \Rightarrow \vec{v_4} = 5\hat{j}$

ⓐ The linear momentum of the system.

$\sum_{i=1}^{4} \vec{P_i} = \vec{P_1} + \vec{P_2} + \vec{P_3} + \vec{P_4}$

$= m_1\vec{v_1} + m_2\vec{v_2} + m_3\vec{v_3} + m_4\vec{v_4}$

$= \frac{1}{32.2}\left[1\times(11.09\hat{i}+16.64\hat{j}) + 3\times(18\hat{i}) + 2\times(-11.52\hat{j}+9.60\hat{k}) + 1\times 5\hat{j}\right]$

$= \frac{1}{32.2}\left[65.09\hat{i} - 1.40\hat{j} + 19.20\hat{k}\right]$

$\therefore \sum_{i=1}^{4} \vec{P_i} = 2.02\hat{i} - 0.04\hat{j} + 0.60\hat{k}$ slug-ft/sec #

ⓑ Angular momentum of the system about the origin.

$\sum_{i=1}^{4} (\vec{H_o})_i = (\vec{H_o})_1 + (\vec{H_o})_2 + (\vec{H_o})_3 + (\vec{H_o})_4$

$= \vec{r_1}\times m_1\vec{v_1} + (\vec{r_2}\times m_2\vec{v_2}) + (\vec{r_3}\times m_3\vec{v_3}) + (\vec{r_4}\times m_4\vec{v_4})$

where,

$\vec{r_1} = \vec{r_{OA}} = 10\hat{k}$

$\vec{r_2} = \vec{r_{OB}} = 12\hat{j}$

$\vec{r_3} = \vec{r_{OC}} = 8\hat{i} + 12\hat{j}$

$\vec{r_4} = \vec{r_{OD}} = 0$

$= \frac{1}{32.2}\left[10\hat{k}\times 1(11.09\hat{i}+16.64\hat{j}) + (12\hat{j})\times 3(18\hat{i}) + (8\hat{i}+12\hat{j})\times 2(-11.52\hat{j}+9.60\hat{k}) + 0\right]$

$= \frac{1}{32.2}\left[110.9\hat{j} + 166.4(-\hat{i}) - 648\hat{k} - 184.32\hat{k} - 153.60\hat{j} + 230.40\hat{i}\right]$

$= \frac{1}{32.2}\left[64\hat{i} - 42.70\hat{j} - 832.32\hat{k}\right]$

$= 1.99\hat{i} - 1.33\hat{j} - 25.85\hat{k}$ slug-ft/sec #

ⓒ Angular momentum of the system about point a:

$\sum_{i=1}^{4} (\vec{H_o})_i = (\vec{H_a})_1 + (\vec{H_a})_2 + (\vec{H_a})_3 + (\vec{H_a})_4$

$$= (\vec{r}_{GA} \times m_1 \vec{v}_1) + (\vec{r}_{AB} \times m_2 \vec{v}_2) + (\vec{r}_{AC} \times m_3 \vec{v}_3) + (\vec{r}_{AD} \times m_4 \vec{v}_4)$$

where,

$\vec{r}_{GA} = -8\hat{i} - 12\hat{j}$

$\vec{r}_{AB} = -8\hat{i} - 10\hat{k}$

$\vec{r}_{AC} = -10\hat{k}$

$\vec{r}_{AD} = -8\hat{i} - 12\hat{j} - 10\hat{k}$

$$= \frac{1}{32.2}\Big[(-8\hat{i} - 12\hat{j}) \times (11.09\hat{i} + 16.64\hat{j}) + (-8\hat{i} - 10\hat{k}) \times 3(18\hat{i}) - 10\hat{k} \times 2(-11.52\hat{j} + 9.60\hat{k}) + (-8\hat{i} - 12\hat{j} - 10\hat{k}) \times -5\hat{j}\Big]$$

$$= \frac{1}{32.2}\Big[-133.12\hat{k} + 133.03\hat{k} - 540\hat{j} - 230.40\hat{i} - 40\hat{k} + 50\hat{i}\Big]$$

$$= -5.60\hat{i} - 16.77\hat{i} - 1.24\hat{k} \text{ slug-ft}^2/\text{sec}^2$$

(d) Angular momentum of the system about centre of mass.

Let G be the centre of mass and $\vec{r}_G$ be position vector and $\vec{v}_G$ be the velocity of centre of mass.

Then,

$$\sum_{i=1}^{4} (\vec{H}_G)_i = (\vec{H}_G)_1 + (\vec{H}_G)_2 + (\vec{H}_G)_3 + (\vec{H}_G)_4$$

$$= (\vec{r}_{G1} \times m_1 \vec{v}_{G1}) + (\vec{r}_{GB} \times m_2 \vec{v}_{G2}) + (\vec{r}_{GC} \times m_3 \vec{v}_{G3}) + (\vec{r}_{GD} \times m_4 \vec{v}_{G4})$$

where,

$$\vec{r}_4 = \vec{r}_{OG} = \frac{\sum_{i=1}^{4} m_i \vec{r}_i}{\sum_{i=1}^{4} m} = \frac{m_1\vec{r}_1 + m_2\vec{r}_2 + m_3\vec{r}_3 + m_4\vec{r}_4}{m_1 + m_2 + m_3 + m_4}$$

$$= \frac{[1 \times 10\hat{k} + 3 \times 12\hat{j} + 2 \times (3\hat{i} + 12\hat{j}) + 1 \times 0]}{\frac{32.2 \times 1}{32.2}[1 + 3 + 2 + 1]}$$

$$= \frac{10\hat{k} + 60\hat{j} + 16\hat{i}}{7} = 2.29\hat{i} + 8.57\hat{j} + 1.43\hat{k}$$

$$\vec{v}_G = \frac{\sum_{i=1}^{4} m_i \vec{v}_i}{\sum_{i=1}^{4} m_i} = \frac{m_1\vec{v}_1 + m_2\vec{v}_2 + m_3\vec{v}_3 + m_4\vec{v}_4}{m_1 + m_2 + m_3 + m_4}$$

$$= \frac{1(11.09\hat{i} + 16.64\hat{j}) + 3(18\hat{i}) + 2(-11.52\hat{j} + 9.60\hat{k}) + 1(5\hat{j})}{1 + 3 + 2 + 1}$$

$$= \frac{1}{7}(65.09\hat{i} + 1.40\hat{j} + 19.20\hat{k})$$

$$= 9.30\hat{i} - 0.20\hat{j} + 2.74\hat{k}$$

$$\therefore \vec{r}_{GA} = \vec{r}_1 - \vec{r}_G = 10\hat{k} - (2.29\hat{i} + 8.57\hat{j} + 1.43\hat{k})$$

$$= -2.29\hat{i} - 8.57\hat{j} + 8.57\hat{k}$$

$$\therefore \vec{r}_{GB} = \vec{r}_2 - \vec{r}_G = 12\hat{i} - (2.29\hat{i} + 8.57\hat{j} + 1.43\hat{k})$$

$$= -2.29\hat{i} + 3.43\hat{j} - 1.43\hat{k}$$

$\therefore \vec{r}_{G4} = \vec{r}_4 - \vec{r}_G = 0 - (2.29\hat{i} + 8.57\hat{j} + 1.43\hat{k})$

$= -(2.29\hat{i} + 8.57\hat{j} + 1.43\hat{k})$

$\vec{V}_{G1} = \vec{V}_1 - \vec{V}_G = (11.09\hat{i} + 16.64\hat{j}) - (9.30\hat{i} - 0.20\hat{j} + 2.74\hat{k})$

$= 1.79\hat{i} + 16.84\hat{j} - 2.74\hat{k}$

$\therefore \vec{V}_{G3} = \vec{V}_3 - \vec{V}_G = 18\hat{i} - (9.30\hat{i} - 0.20\hat{j} + 2.74\hat{k})$

$= 8.70\hat{i} + 0.20\hat{j} - 2.701\hat{k}$

$\therefore \vec{V}_{G3} = \vec{V}_3 - \vec{V}_G = (-11.52\hat{j} + 9.60\hat{k}) - (9.30\hat{i} - 0.20\hat{j} + 2.74\hat{k})$

$= -9.30\hat{i} - 11.32\hat{j} + 6.306\hat{k}$

$\therefore \vec{V}_{G4} = \vec{V}_4 - \vec{V}_G = 5\hat{j} - (9.30\hat{i} - 0.20\hat{j} + 2.74\hat{k})$

$= -9.30\hat{i} + 5.2\hat{j} - 2.74\hat{k}$

Now,

$\sum_{i=1}^{4} (\vec{H}_G)_i = \frac{1}{32.2}\left[(-2.29\hat{i} - 8.57\hat{j} + 8.57\hat{k}) \times (1.79\hat{i} + 16.84\hat{j} - 2.74\hat{k})\right.$

$+ 3(-2.29\hat{i} + 3.43\hat{j} - 1.43\hat{k}) \times (8.70\hat{i} + 0.20\hat{j} - 2.74\hat{k})$

$+ 2(5.71\hat{i} + 3.43\hat{j} - 1.43\hat{k}) \times (-9.30\hat{i} - 11.32\hat{j} + 6.86\hat{k})$

$\left. + (-2.29\hat{i} + 8.57\hat{j} + 1.43\hat{k}) \times (-4.30\hat{i} + 5.2\hat{j} - 2.74\hat{k})\right]$

$= \frac{1}{32.2}\left[-120.84\hat{i} + 9.07\hat{j} - 23.22\hat{k} + 3(-9.11\hat{i} - 18.72\hat{j} - 30.30\hat{k}) + 2(-30.92\hat{i} - 19.57\hat{j} - 32.74\hat{k})\right.$

$\left. + (-30.92\hat{i} - 19.57\hat{j} + 67.79\hat{k})\right]$

$= \frac{1}{32.2}(-240.93\hat{i} - 105.80\hat{j} - 111.8\hat{k})$

$= -7.48\hat{i} - 3.29\hat{j} - 3.47\hat{k}$ slug-ft^2/sec.

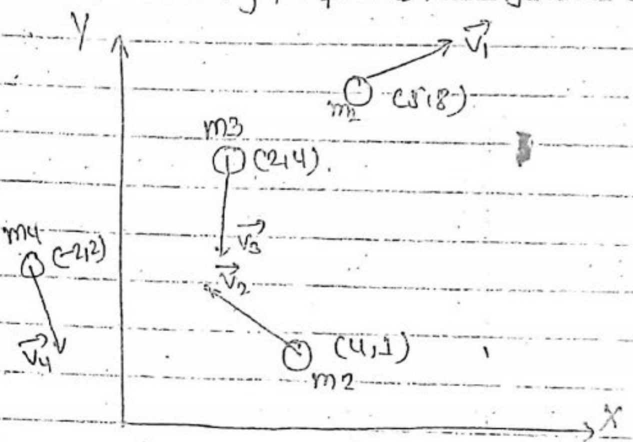
(2) A system of particles is shown in fig 6.19 at time t moving in the xy-plane. The following data apply.

$m_1 = 1$ kg, $\vec{v}_1 = 5i + 5j$ m/sec

$m_2 = 0.7$ kg, $\vec{v}_2 = -4i + 3j$ m/sec

$m_3 = 2$ kg; $\vec{v}_3 = -4j$ m/sec

$m_4 = 1.5$ kg, $\vec{v}_4 = 3i - 4j$ m/sec



(a) What is the total linear momentum of the system?

(b) What is the linear momentum of the centre of mass?

c) What is the total moment of momentum of the system about the origin and about point (2,6)?

$\Rightarrow$ Sol^n

$m_1 = 1\,kg$, $\vec{v_1} = 5\hat{i} + 5\hat{j}$ m/sec.
$m_2 = 0.7\,kg$, $\vec{v_2} = -4\hat{i} + 3\hat{j}$ m/sec.
$m_3 = 2\,kg$, $\vec{v_3} = -4\hat{j}$ m/sec.
$m_4 = 1.5\,kg$, $\vec{v_4} = 3\hat{i} - 4\hat{j}$ m/sec.

(a) Total linear momentum of the system.

$$\sum_{i=1}^{4} (\vec{P_i}) = \vec{P_1} + \vec{P_2} + \vec{P_3} + \vec{P_4}$$
$$= m_1\vec{v_1} + m_2\vec{v_2} + m_3\vec{v_3} + m_4\vec{v_4}$$
$$= 1(5\hat{i}+5\hat{j}) + 0.7(-4\hat{i}+3\hat{j}) + 2(-4\hat{j}) + 1.5(3\hat{i}-4\hat{j}).$$
$$\therefore \sum_{i=1}^{4} \vec{P_i} = 6.70\hat{i} - 6.90\hat{j}\ kg\text{-}m^2/sec.$$

(b) the linear momentum of the centre of mass

$$m\vec{v_c} = \sum_{i=1}^{4} m_i\vec{v_i} = \sum_{i=1}^{4} \vec{P_i} = 6.70\hat{i} - 6.90\hat{j}\ kg\text{-}m^2/sec$$

(c) Total moment of momentum of the system.

(i) about origin.

$$\sum_{i=1}^{4} (\vec{H_o})_i = (\vec{H_o})_1 + (\vec{H_o})_2 + (\vec{H_o})_3 + (\vec{H_o})_4$$
$$= \vec{r_1} \times m_1\vec{v_1} + \vec{r_2} \times m_2\vec{v_2} + \vec{r_3} \times m_3\vec{v_3} + \vec{r_4} \times m_4\vec{v_4}$$
$$= (6\hat{i}+8\hat{j}) \times 1(5\hat{i}+5\hat{j}) + (4\hat{i}+\hat{j}) \times 0.7(-4\hat{i}+3\hat{j}) + (2\hat{i}+4\hat{j}) \times 2(-4\hat{j}) + (-2\hat{i}+2\hat{j}) \times 1.5(3\hat{i}-4\hat{j})$$

$$= 25\hat{k} - 10\hat{k} + 8.40\hat{k} + 2.80\hat{k} - 16\hat{k} + 8\hat{k} - 9\hat{k}$$
$$= -20.80\hat{k}$$

(ii) about (2, 6) i.e. say point is P(2, 6)

$$\sum_{i=1}^{4} (\vec{H_P})_i = (\vec{H_P})_1 + (\vec{H_P})_2 + (\vec{H_P})_3 + (\vec{H_P})_4$$
$$= \vec{r_{P1}} \times m_1\vec{v_1} + \vec{r_{P2}} \times m_2\vec{v_2} + \vec{r_{P3}} \times m_3\vec{v_3} + \vec{r_{P4}} \times m_4\vec{v_4}$$
$$= (3\hat{i}+2\hat{j}) \times 1(5\hat{i}+5\hat{j}) + (2\hat{i}-5\hat{j}) \times 0.7(-4\hat{i}+3\hat{j}) + (-2\hat{j}) \times 2(-4\hat{j}) + (-4\hat{i}-4\hat{j}) \times 1.5(3\hat{i}-4\hat{j})$$
$$= (15-10)\hat{k} + 0.7(6+30)\hat{k} + 2(0) + 1.5(16+12)\hat{k}$$
$$= 65.20\hat{k}\ \#$$

(3) A system of particles at time t_1 has masses $m_1 = 2\,lbm$, $m_2 = 1\,lbm$, $m_3 = 3\,lbm$ and locations and velocities as shown in fig 6.20a. The same system of masses is shown in fig. 6.20 b at time t_2. What is the total linear impulse on the system during this time interval?

What is the total angular impulse $\int \vec{M}\,dt$ during this time interval about the origin?

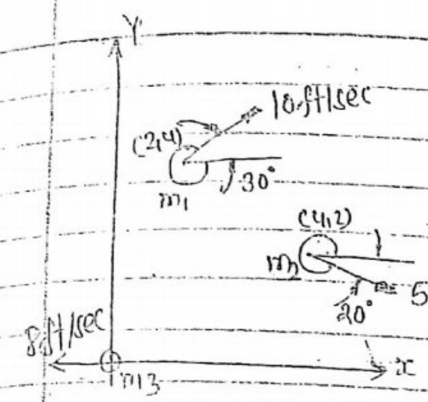
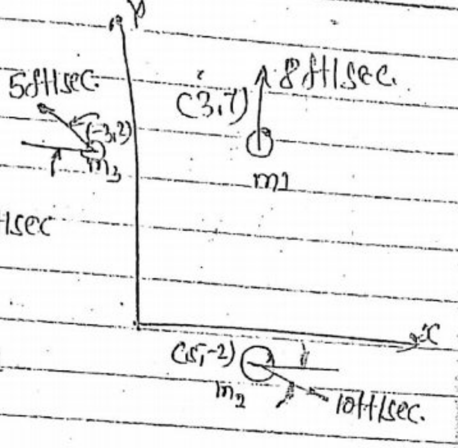


Fig: 6.209: Time t_1



⇒ Sol^n

$m_1 = 2\,lbm = \frac{2}{32.2}$ slug ;

$m_2 = 1\,lbm = \frac{1}{32.2}$ slug

$m_3 = 3\,lbm = \frac{3}{32.2}$ slug.

$\vec{v_1} = \frac{10(2\hat{i} + 4\hat{j})}{\sqrt{4+16}} = 4.47\hat{i} + 8.94\hat{j}$

$\vec{v_2} = \frac{5(4\hat{i} + 2\hat{j})}{\sqrt{16+4}} = 4.47\hat{i} + 2.24\hat{j}$

$\vec{v_3} = -8\hat{i}$

$\vec{v_1}' = \frac{3(3\hat{i} + 7\hat{j})}{\sqrt{9+49}} = 3.15\hat{i} + 7.35\hat{j}$

$\vec{v_2}' = \frac{10(5\hat{i} - 2\hat{j})}{\sqrt{25+4}} = 9.28\hat{i} - 3.71\hat{j}$

$\vec{v_3}' = \frac{5(-3\hat{i} + 2\hat{j})}{9+4} = -4.16\hat{i} + 2.77\hat{j}$

We know, from impulse momentum principle,

i.e. $\sum \int_{t_1}^{t_2} (\vec{F_{ext}})\,dt = (\sum m\vec{v})_{final} - (\sum m\vec{v})_{initial}$

$= \frac{1}{32.2}\left[2(3.15\hat{i} + 7.35\hat{j}) + 1(9.28\hat{i} - 3.71\hat{j}) + 3(-4.16\hat{i} + 2.1\hat{j})\right]$

$- \frac{1}{32.2}\left[2(4.47\hat{i} + 8.94\hat{j}) + (4.47\hat{i} + 1.24\hat{j}) + 3(-8\hat{i})\right]$

$= \frac{1}{32.2}\left[13.69\hat{i} - 0.82\hat{j}\right]$

$\sum \int_{t_1}^{t_2} (\vec{F_{ext}})\,dt = 0.43\hat{i} - 0.03\hat{j}$ slug-ft/sec.

Further, total angular Impulse = change in angular momentum of system of particles.

i.e. $\sum \int_{t_1}^{t_2} (\vec{m_o})_{ext}\,dt = (\sum \vec{H_o})_{final} - (\sum \vec{H_o})_{Initial}$

$= \{(\vec{r_1}' \times m_1\vec{v_1}') + (\vec{r_2}' \times m_2\vec{v_2}') + (\vec{r_3}' \times m_3\vec{v_3}')\}_{final}$

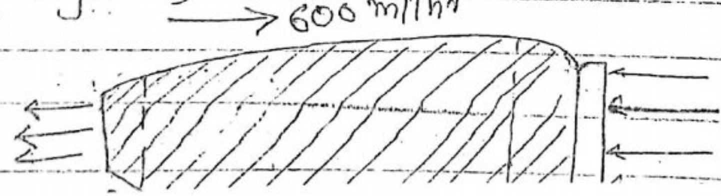
$- \{(\vec{r_1} \times m_1\vec{v_1}) + (\vec{r_2} \times m_2\vec{v_2}) + (\vec{r_3} \times m_3\vec{v_3})\}_{initial}$

$= \{(3\hat{i}+7\hat{j}) \times 1(3.15\hat{i}+7.35\hat{j}) + (5\hat{i}-2\hat{j}) \times 1(9.28\hat{i}-3.71\hat{j})$
$+ (-3\hat{i}+2\hat{j}) \times 3(-4.16\hat{i}+2.77\hat{j})\}_{final}$
$- \{(2\hat{i}+4\hat{j}) \times 2(4.47\hat{i}+8.94\hat{j}) + (4\hat{i}+\hat{j}) \times 1(4.47\hat{i} - 2.44\hat{j}+0\}_{initial}$
$= \{2(22.05-22.05)\hat{k} + (-18.55-18.56)\hat{k} + 3(-8.31 + 8.32)\hat{k}\}$
$\cancel{= 0.01\hat{k} + 0.03\hat{k} - 17.90}$
$- \{2(17.88-17.88)\hat{k} + (-8.96-8.94)\hat{k}\}$

$= 0.01\hat{k} + 0.03\hat{k} - 17.90\hat{k}$
$= -17.86\hat{k}$ slug-ft²/sec.

④ A jet airplane is crushing at speed of 600 mile/hour in level flight. Each of its jet engines scoops in air at the rate of 180 lb/sec and ejects with a speed of 2000 ft/sec relative to the airplane. Neglecting effect of fuel consumption, determine for each jet engine (a) the magnitude of the propulsive force generated (b) the total horsepower generated relative to the ground (see fig. 6.21).

→ 600 m/hr



⇒ Soln

According to the question, this problem related to steady stream of particles,

ⓐ the magnitude of the propulsive force generated,

i.e. $\Sigma F = \frac{dm}{dt}(v_c - v_B)$

$= \frac{180}{32.2}(2000 - 879.59)$

$= 6263$ lb $= 6.26$ kips (kilopower)

$\frac{dm}{dt} = \frac{180}{32.2}$ slug/sec.

$v_B = \frac{600 \times 1609 \times 3.28}{60 \times 60}$

$= 879.59$ ft/sec. #

ⓑ total horsepower generated relative to the ground; i.e. power generated $(P) = P_1 + P_2$

where,

P_1 = Power generated to propel the airplane

$= \frac{F}{550} \times v_B$ hp $\quad [\because 550 \frac{lb\,ft}{sec} = 1hp]$

$= \frac{6763}{550}$ hp $= 11.39$ hp.

P_2 = power generated for ejection

$= \frac{KE}{time} = \frac{1}{2}\frac{mv_e^2}{t} = \frac{1}{2}\frac{dm}{dt}V_e^2$

[Here, V_e = velocity of ejection w.r.t. earth
$= 2000 - 879$
$= 1121$ ft/sec]

$= \frac{1}{2} \times \frac{180}{32.2} \times (1121)^2$

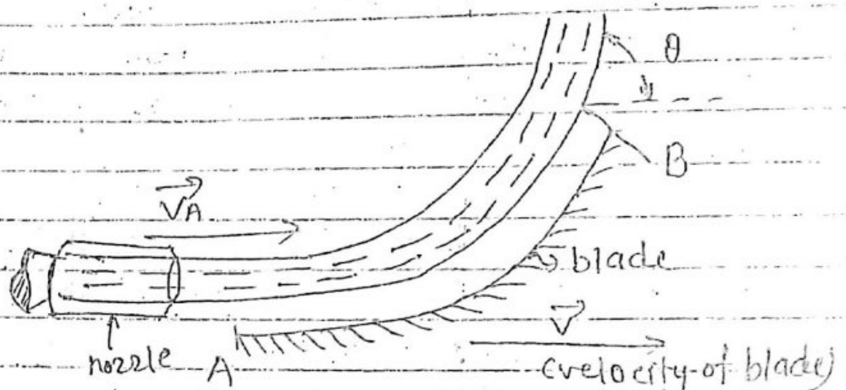
$= 3512350.62$ lbft/sec.

$= \frac{3512350.62}{550}$ hp

$= 6386$ hp

$\therefore P = (11.39 + 6386)$ hp
$= 6397.39$ hp
$\simeq 6.39 \times K^3$ hp ##

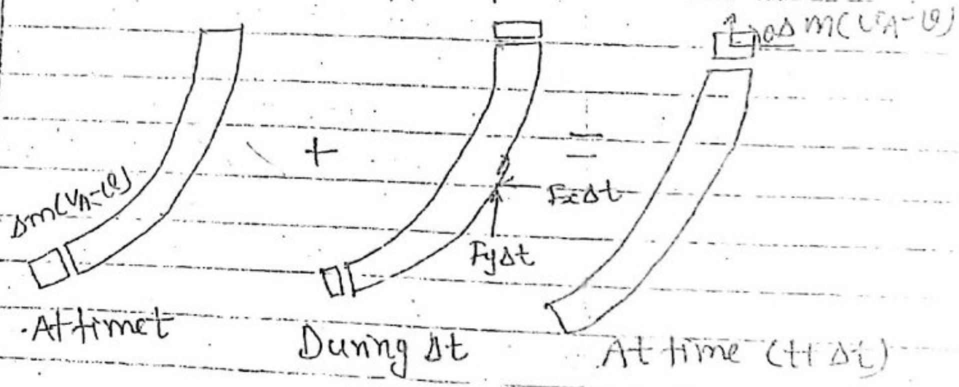
(5) A nozzle discharges a stream of cross sectional area A with a velocity $\vec{V_A}$. The stream is deflected by a single blade which moves to the right with a constant velocity $\vec{V}$. Assuming that the water moves along the blade at constant speed determine (a) the components of the force $\vec{P}$ exerted by the blade on the stream. (b) the velocity $\vec{V}$ for which maximum power is developed? (see fig 6.22)



$(v < v_A)$ Fig 6.22

⇒ Sol^n

This numerical can be analysed using the Impulse momentum principle as below.



From x-component of Impulse & momentum principle.

($\xrightarrow{+}$) cancelled out $\Sigma m\vec{v}$ i.e. momentum of system at t & (t+Δt) are equal.

$$\Delta m\,(v_A - v) - F_x \Delta t = \Delta m\,(v_A - v)\cos\theta$$

or, $\rho A\,(v_A - v)^2 \Delta t - F_x \Delta t = \rho A\,(v_A - v)^2 \Delta t \cos\theta$

$\left[\because \rho = \frac{m}{V},\ \Delta m = \rho Q_r \Delta t = \rho \times \text{Area} \times \text{velocity} \times \Delta t = \rho A (v_A - v)\Delta t\right]$

or, $F_x = \rho A\,(v_A - v)^2 (1-\cos\theta)$ ($\leftarrow$)

Y-component ($+\uparrow$)

$$0 + F_y \Delta t = \Delta m\,(v_A - v)\sin\theta$$

or, $F_y \Delta t = \rho A\,(v_A - v)^2 \sin\theta\,\Delta t$

$\therefore F_y = \rho A\,(v_A - v)^2 \sin\theta$ ($\uparrow$)

Power (P) = $F_x \times$ velocity of blade (v)

$$= \rho A\,(v_A - v)^2 (1-\cos\theta)\,v$$

$$= \rho A\,(1-\cos\theta)\,(v_A^2 v - 2v_A v^2 + v^3)$$

Diff. w.r.t. velocity

$$\frac{dP}{dv} = \rho A\,(1-\cos\theta)\,(v_A^2 - 4v_A v + 3v^2)$$

For maximum and minimum power,

$$\frac{dP}{dv} = 0$$

i.e. $\rho A\,(1-\cos\theta)\,(v_A^2 - 4v_A v + 3v^2) = 0$

Since, $\rho A\,(1-\cos\theta) \neq 0$

$\therefore v_A^2 - 3v_A v - v_A v + 3v^2 = 0$

or, $v_A(v_A - 3v) - v(v_A - 3v) = 0$

or, $(v_A - v)(v_A - 3v) = 0$

$\therefore v = \frac{v_A}{3},\ v_A$

If $v = v_A$, No power is developed.

$\therefore v = \frac{v_A}{3}$ #

(6) A double pendulum shown in fig 6.23, oscillates in the xy plane. At the instant shown $\omega_1 = 2$ rad/sec ccw and $\omega_2 = 3$ rad/sec ccw. What is H_o at this instant if $m_1 = m_2 = 1$ kg? Note that the lower pendulum is connected to mass m_1 by a pin joint and is free rotate about this point.

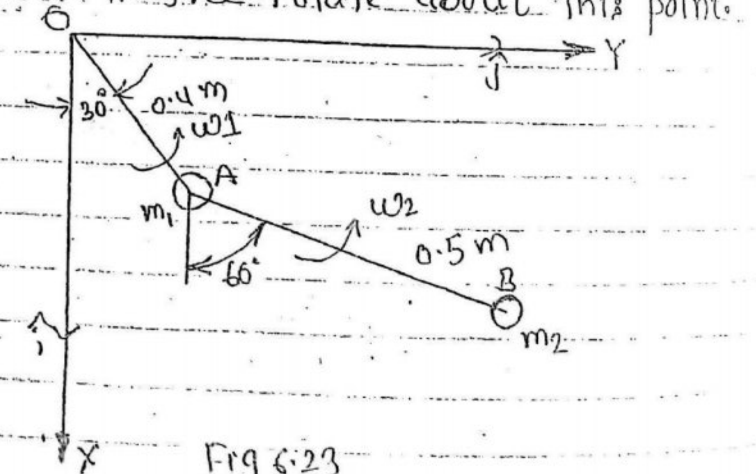


Fig 6.23

⇒ Sol^n

$m_1 = m_2 = 1\,kg$ | $\Sigma \vec{H_0} = ?$

$\omega_1 = 2$ rad/sec.

$r_{OA} = r_1 = 0.4\,m$

$\omega_2 = 3$ rad/sec

$r_{AB} = r_2 = 0.5\,m$

Angular momentum of the system about O is,

i.e. $\Sigma \vec{H_0} = (\vec{H_0})_1 + (\vec{H_0})_2$

$= (\vec{r_1} \times m_1 \vec{v_1}) + (\vec{r_{OB}} \times m_2 \vec{v_{2/0}})$

where,

$\vec{r_{OA}} = \vec{r_1} = 0.4(\cos30\,\hat{i} + \sin30\,\hat{j}) = 0.35\hat{i} + 0.20\hat{j}$

$= 6.4(0.87\,\hat{i} + 0.5\,\hat{j})$

$\vec{r_{OB}} = \vec{r_{OA}} + \vec{r_{AB}}$

$= 6.4(0.87\hat{i} + 0.5\hat{j}) + 0.5(\cos60\hat{i} + \sin60\hat{j})$

$= 0.60\,\hat{i} + 0.63\,\hat{j}$

$\vec{v_1} = \vec{\omega_1} \times \vec{r_1}$

$= 2\hat{k} \times (0.35\hat{j} + 0.20\hat{j})$

$= 0.7\hat{j} - 0.4\hat{i}$

$\vec{v_{2/0}} = \vec{v_1} + \vec{v_{2/1}}$

$= 6.7\hat{j} - 0.4\hat{i} + (\vec{\omega_2} \times \vec{r_{AB}})$

$= 0.7\hat{j} - 0.4\hat{i} + \{3\hat{k} \times (0.25\hat{i} + 0.43\hat{j})\}$

$= 0.7\hat{j} - 0.4\hat{i} + 0.75\hat{j} - 1.29\hat{i}$

$= 1.45\hat{j} - 1.69\hat{i}$

Now,

$\Sigma \vec{H_0} = (0.35\hat{i} + 0.20\hat{j}) \times (0.7\hat{j} - 0.4\hat{i}) +$

$(0.60\hat{i} + 0.63\hat{j}) \times (1.45\hat{j} - 1.69\hat{i})$

$= 0.33\hat{k} + 1.93\hat{k}$

$= 2.26\hat{k}$ kg m²/sec ##

(7) Two particles shown in fig 6.24 oscillate on the smooth plane in the x-direction. (a) write the differential equation of motion for each mass (b) Find the equation of motion for the centre of mass (c) Write the expression for kinetic and potential energy of the system of particles.

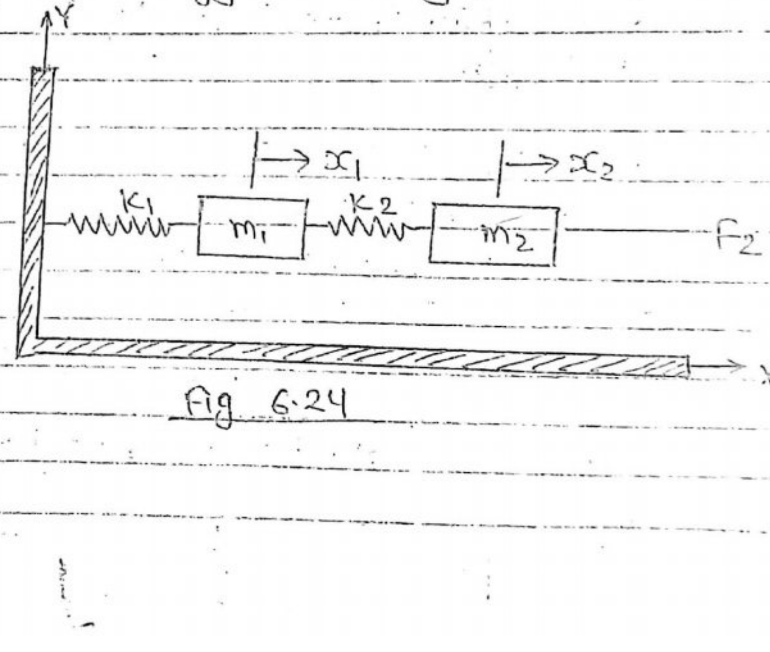
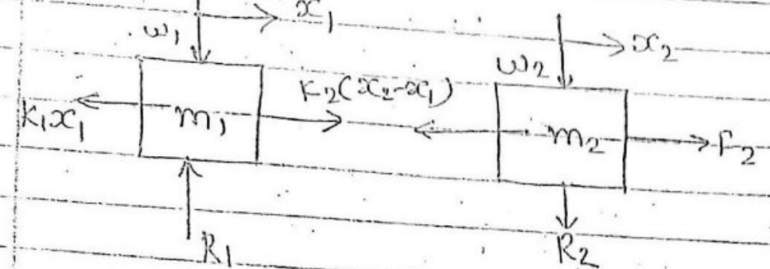


Fig 6.24

⇒ Sol^n

Free body diagram for m_1 and m_2 are shown below;



a) Differential equation of motion for each mass:

we know, from Newton's 2nd law,

For mass m_1

$(\xrightarrow{+})$ $-k_1x_1 + k_2(x_2-x_1) = m_1\ddot{x}_1$

For mass m_2

$(\xrightarrow{+})$ $-k_2(x_2-x_1) + F_2 = m_2\ddot{x}_2$

b) The equation of motion for centre of mass;

We know, from Newton's second law;

$(\xrightarrow{+})$ $\Sigma(F_{ext}) = \Sigma m a_c$

i.e. $-k_1x_1 + F_2 = (m_1+m_2)\ddot{x}_c$

c) Expression kinetic and potential energy of the system.

Total kinetic Energy (KE) $= \frac{1}{2}m_1\dot{x}_1^2 + \frac{1}{2}m_2\dot{x}_2^2$

Total Potential Energy (PE) = (PE of 1) + (PE of 2)

$= \{(V_e)_1 + (V_w)_1\} + \{(V_e)_2 + (V_w)_2\}$

$= \{\frac{1}{2}k_1x_1^2 + mgh_1\} + \{\frac{1}{2}k_2(x_2-x_1)^2 + mgh_2\}$

$\therefore \boxed{PE = \frac{1}{2}k_1x_1^2 + \frac{1}{2}k_2(x_2-x_1)^2}$

* ~ *

① In the engine system shown in fig 7.27, the crank AB has a constant clockwise angular velocity of 2000 rpm. For the crank position indicated, determine (a) the angular velocity of the connecting rod BD, (b) the velocity of the position P.

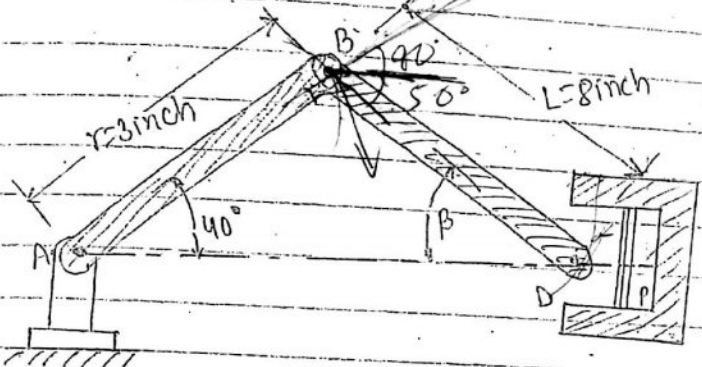
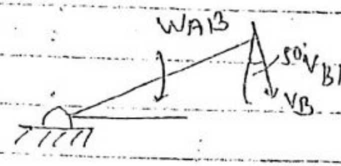


Fig. 7.27

Motion of crank shaft (rotation above)

a)



FBD of crank.

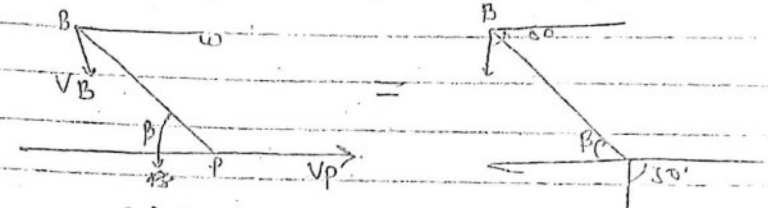
We have,

$V_B = r_{AB}\,\omega_{AB}$ ($V = r\omega$)

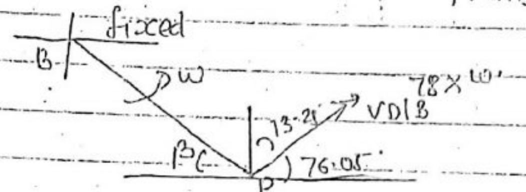
$= 3 \times \frac{2000 \times 2\pi}{60} = 200 \times 22/7$

$= 628.57$ inch/sec.

Motion of piston rod BD (GPM)



G.P.M. = Translation with B

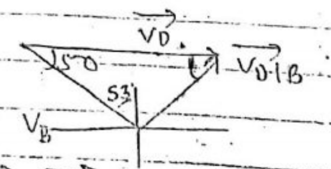


Rotation about B.

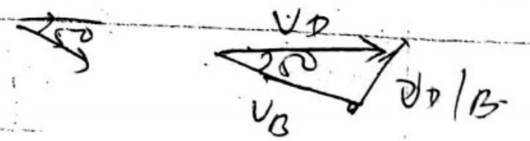
From sine rule from fig.

$\frac{\sin 40}{8} = \frac{\sin \beta}{3}$

$\Rightarrow \beta = \sin^{-1}(3/8 \sin 40) = 13.95$



$\vec{V_D} = \vec{V_B} + \vec{V_{D/B}}$



From vector Δ.

$$\frac{\sin 50}{V_{D/B}} = \frac{\sin 53.95}{V_D} = \frac{\sin 76.05}{V_B}$$

we have,

$$V_D = \frac{\sin 53.95}{\sin 76.05} \times V_B$$

$$= 525.64 \text{ inch/s.}$$

∴ velocity of piston $(V_P) = V_D = 323.$

Further,

$$V_{D/B} = \frac{\sin 50}{\sin 76.05} \times V_B$$

$$= 436.1456 = 43.6 \text{ ft/sec.}$$

$\Rightarrow \omega_{BD} = 62.01$ rad/s (↑).

\#..

② The centre of the double gear has a velocity of 1.2 m/sec to the right and an acceleration of 3 m/sec² to the right as shown in fig 7.28. Recalling that the lower rack is stationary determine (a) the angular acceleration of the gear, (b) the acceleration of point B, C and D of the gear.

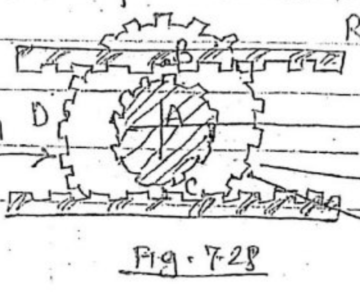


Fig. 7.28

$\Rightarrow$ Solⁿ

(a) Angular acceleration of Gear:-

$$V_A = -r_1\omega$$

or, $1.2 = -(0.150)\omega$

$\therefore \omega = -8$ rad/sec

$$a_A = -r_1 \alpha$$

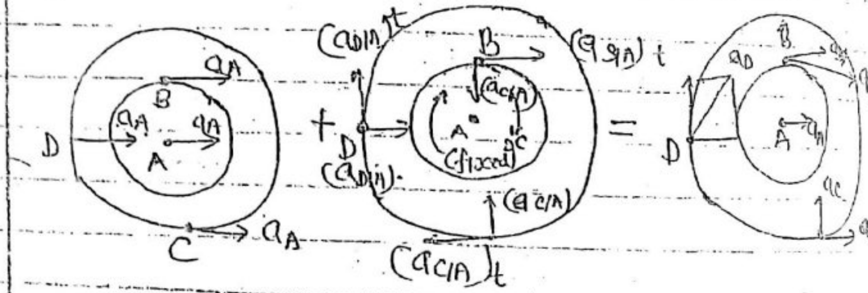
or, $3 = -(0.150)\alpha$

$\therefore \alpha = -20$ rad/sec²

i.e. $\vec{\alpha} = \alpha k = -20k$ rad/sec²

(b) Accelerations:-

The rolling motion is resolved into to components motion viz, a translation with the centre A and a rotation about the centre A as fig. below.

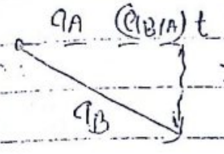


Translation + Rotation = Rolling motion.

Acceleration of points:

Adding vectorically the acceleration corresponding to the translation and the rotation.

$\vec{a_B} = \vec{a_A} + \vec{a_{B/A}} = \vec{a_A} + (\vec{a_{B/A}})_t + (\vec{a_{B/A}})_n$



$= \vec{a_A} + \alpha k \times \vec{r_{B/A}} - \omega^2 \vec{r_B}$

$= 3\vec{i} - 20\vec{k} \times \vec{r_{B/A}} - \omega^2 \vec{r_B}$

$= 3\vec{i} - 20\vec{k} \times (0.1)\vec{j} - (8)^2$

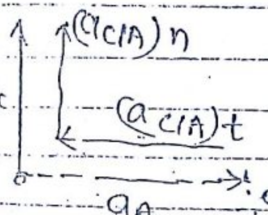
$= 3\vec{i} + 2\vec{i} - 6.4\vec{j}$

$= 5\vec{i} - 6.4\vec{j}$

$\therefore a_B = 8.12 \ m/sec^2$ (52°) #

Acceleration of point C:

$\vec{a_C} = \vec{a_A} + \vec{a_{C/A}} = \vec{a_A} + \alpha k \times \vec{r_{C/A}} - \omega^2 \vec{r_{C/A}}$



$= 3\vec{i} - 20\vec{k} \times (-0.15)\vec{j}$

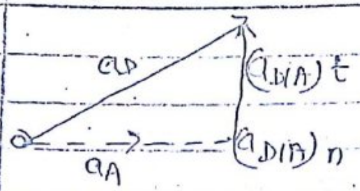
$- (8)^2 (-0.15)\vec{j}$

$= 3\vec{i} - 3\vec{i} + 9.6\vec{j}$

$\therefore a_C = 9.60 \ m/sec^2 (\uparrow)$ #

Acceleration of point D.

$\vec{a_D} = \vec{a_A} + \vec{a_{D/A}} = \vec{a_A} + \alpha k \times \vec{r_{D/A}} - \omega^2$



$= \vec{a_A} + \alpha k \times \vec{r_{D/A}} - \omega^2$

$= 3\vec{i} - 20\vec{k} \times (0.1)$

$- (8)^2 (-0.15)\vec{i}$

$= 3\vec{i}$.

$\vec{a_D} = 12.15 \ m/sec^2$ (13.4°)

#

(3) Disk D, of radius R, is pinned to end A of the arm OA of length L located in the plane of the disk. The arm rotates about a vertical axis through O at the constant rate ω_1, and the disk rotates about A at the constant rate ω_2. Determine (a) the velocity of point P located directly above A, (b) the acceleration of P, (c) the angular velocity and angular acceleration of the disk. (see fig. 7.29)

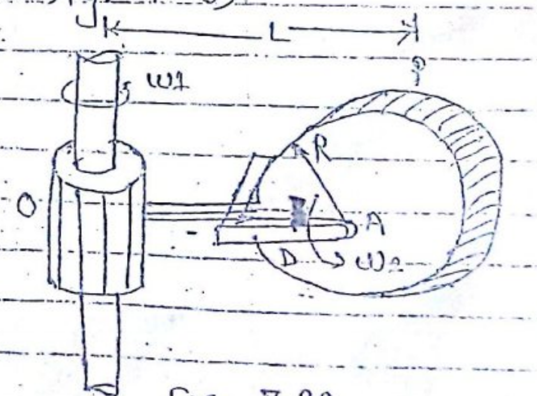


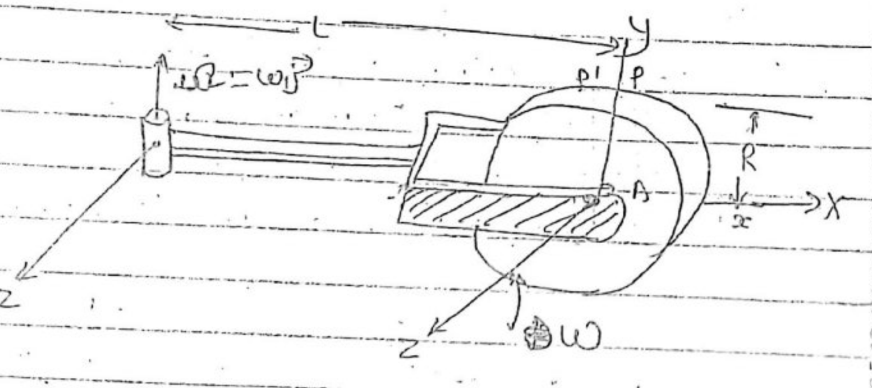
fig. 7.29

Scanned by CamScanner

⇒ Sol^n

Here, the frame OXYZ is fixed. As fig: the moving frame Axyz is attached to the arm OA. Its angular velocity w.r.t. the frame OXYZ is $\vec{\Omega} = \omega_1\vec{j}$. The angular velocity of disk D relative to the moving frame Axyz is $\omega_{D/F} = \omega_2\vec{k}$. The position vector of P relative to O is $\vec{r} = L\vec{i} + R\vec{j}$ and its position vector relative to A is,

$\vec{r}_{P/A} = R\vec{j}$



Velocity

Denoting by P' the point of the moving frame which coincides with P

$$V_P = V_{P'} + V_{P/F} \quad \text{—— (i)}$$

Where $V_{P'} = \vec{\Omega} \times \vec{r} = \omega_1\vec{j} \times (L\vec{i} + R\vec{j})$

$\vec{V}_{P/F} = \vec{\omega}_{D/F} \times \vec{r}_{P/A} = \omega_2\vec{k} \times R\vec{j} = -\omega_2 R\vec{i}$

from eq – (i)

$$\boxed{\therefore V_P = -\omega_2 R\vec{i} - \omega_1 L\vec{k}}$$

#

Acceleration:

$$\therefore a_P = a_{P'} + a_{P/F} + a_c \quad \text{—— (2)}$$

$\vec{a}_{P'} = \vec{\Omega} \times (\vec{\Omega} \times \vec{r}) = \omega_1\vec{j} \times (-\omega_1 L\vec{k})$
$= -\omega_1^2 L$

$\vec{a}_{P/F} = \vec{\omega}_{D/F} \times (\vec{\omega}_{D/F} \times \vec{r}_{P/A})$
$= \omega_2\vec{k} \times (-\omega_2 R\vec{i})$
$= -\omega_2^2 R\vec{j}$

$\vec{a}_c = 2\vec{\Omega} \times \vec{V}_{P/F} = 2\omega_1\vec{j} \times (-\omega_2 R\vec{i})$
$= 2\omega_1\omega_2 R\vec{k}$

From eq – (ii)

$$\boxed{\therefore a_P = -\omega_1^2 L\vec{i} - \omega_2^2 R\vec{j} + 2\omega_1\omega_2 R\vec{k}}$$

#

Angular velocity & Angular Acceleration of Disk.

$\vec{\omega} = \vec{\Omega} + \vec{\omega}_{D/F} = \omega_1\vec{j} + \omega_2\vec{k}$

we have,

$$(\vec{\dot{Q}})_{oxyz} = (\vec{\dot{Q}})_{oxyz} + \vec{\Omega} \times \vec{Q}$$

with $\vec{q} = \omega$, then.

$$\gamma = (\dot{\omega})_{oxyz} = (\dot{\omega})_{Axyz} + \vec{\Omega} \times \omega$$

$$= 0 + \omega_1 \vec{j} \times (\omega_1 \vec{j} \, g + \omega_1)$$

$$\boxed{\therefore \alpha = \omega_1 \omega_2 \vec{i}}$$ #

④ A drum of radius 120 mm is mounted on a wheel of radius 180 mm as shown in fig. 7.30, where a rope is wound around the drum. the end E of the rope is pulled with a constant velocity $V_E = 0.6$ m/sec (⟶) and the wheels rolls without slipping. Using velocity center, determine (a) V_D of the point D, (b) the rate at which the rope is being wound or unwound.

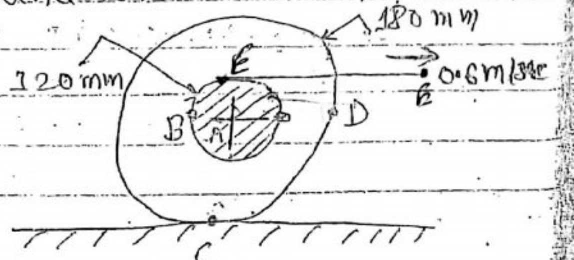


Fig. 7.30

⇒ Sol^n

Let C be the instantaneous centre of rotation.

$\therefore V = \omega r$ ——— ①

$V_E = V_A + V_{E/A}$

$V_A = \omega r$

$= \omega (0.18)$

$V_E = 0.18\,\omega + \omega (0.12)$

$= \omega \times 0.30$

$\Rightarrow 0.6 = \omega \times 0.30$

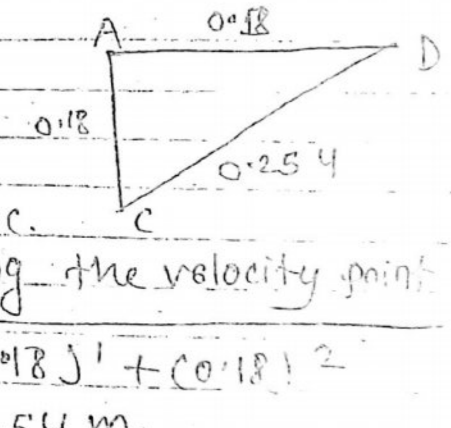
$\omega = 2$ rad/s.

$r = (0.18 + 0.12)$ m

$= 0.3$ m

from eq. — ①

$V = \omega r$



or, $0.6 = \omega \times 0.3$

$\therefore \omega = 2$ rad/sec.

for determining the velocity point D.

$r = CD = \sqrt{(0.18)^2 + (0.18)^2}$

$= 0.254$ m.

$\therefore V_D = \omega r = 2 \times 0.354$

$= 0.509$ m/s

(ω_e) #

$V_A = \omega r$

$= 2 \times 0.18$

$= 0.36$ m/sec.

$V_{E/A} = V_E - V_A = 0.60 - 0.36$ m/sec

$= 0.24$ m/sec.

$\therefore \omega_{AA} = \omega r$

$= 2 \times 0.18 = 0.36$ m sec^{-1}

$\therefore V_{E/A} = V_E - V_A$

$= 0.6 - 0.36$

$= 0.24$ m/sec (unwound)

#

(5) The gear A of the system shown in fig. 7.31 rotates with $\omega_A = 180$ rpm (↻) and the connecting arm CD rotates with $\omega_{CD} = 60$ rpm (↺). Determine ω_B of the gear B.

60

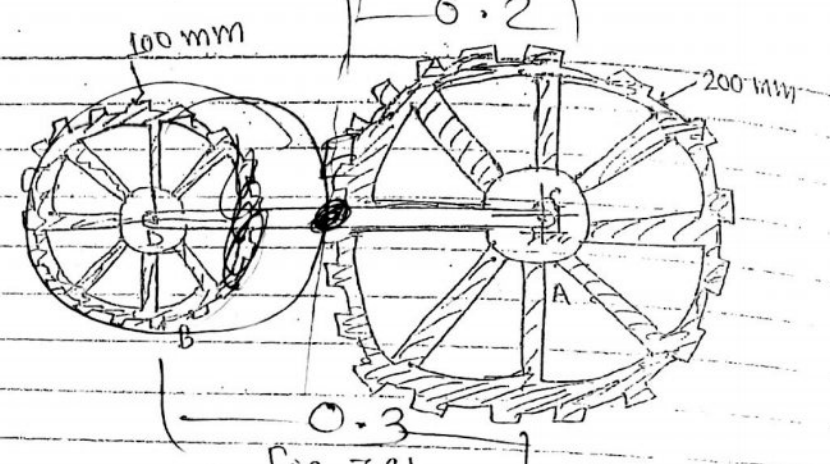


Fig 7.31

⟹ Sol^n

$\omega_A = 180$ rpm (↻) $= 6\pi$ rad/sec.

$\omega_{CD} = 60$ rpm (↺) $= 2\pi$ rad/sec.

we have,

$V_D = V_A + V_{D/A}$

$= 0 + 2\pi \times 0.3$

$= 0.6\pi$ m/sec (↑)

$V_E = V_A + V_{E/A}$ (from gear A)

$= 0 + 6\pi \times 0.2$

$= 1.2\pi$ m/sec (↓) — (I)

$V_E = V_D + V_{D/E}$ (from gear B)

$= 0.6\pi\,(\uparrow) + \omega r$

$= 0.6\pi + \omega_B \times 0.1$ — (II)

Equality equation (I) & (II).

or $-1.2\pi = 0.6\pi + \omega_B \times 0.1$

or, $\omega_B = 18\pi$

$\therefore \omega_B = 18\pi \times \frac{60}{2\pi} = 540$ rpm #.

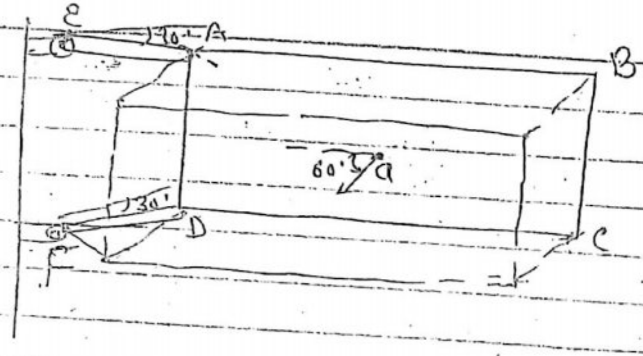
* ~ *

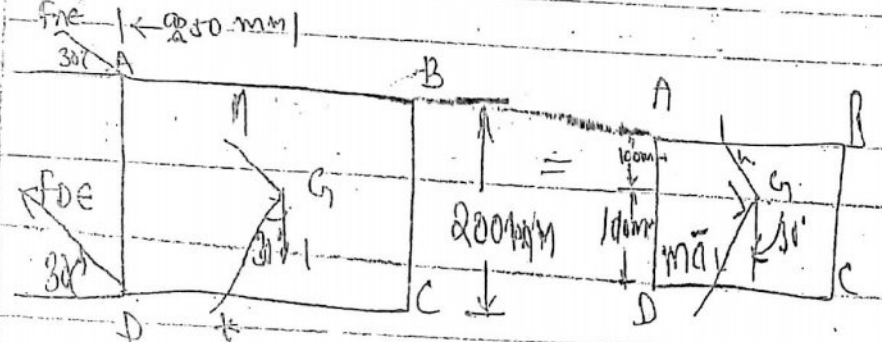
CHAPTER 8: Plane motion of rigid bodies: forces, moments and acceleration.

(1) The thin plate ABCD has a mass of 0.8 kg and is held in the position shown by the wire BH and two links AE and DF. Neglecting the mass of the links determine immediately after wire BH has been cut the acceleration of the plate. (b) the force in each link (see fig 8.14)

Fig 8.14

After wire BH is cut, then corners A and D move along parallel circles of radius 150 mm centered at E and F. The motion of plate is curvilinear translation, then the particles forming the plate move along parallel circles of radius 150 mm. If the wire BH is cut, the velocity of plate is zero.





Here, the external forces consists of the weight w and the force F_{AE} and F_{DF} exert by the links.
Then the plate is in translation,

$\uparrow \sum F_t = [\sum F_t]_{eff}$.

$w \cos 30^\circ = m\bar{a}$ $[\because \sum F_t = \sum (F_t)_{eff}]$

or, $mg \cos 30^\circ = m\bar{a}$

or, $\bar{a} = g \cos 30^\circ = (9.81) \cos 30^\circ$

$\therefore \bar{a} = 8.50$ m/sec² # ($\measuredangle 60^\circ$)

(2) $\sum F_n = \sum (F_n)_{eff}$.

or, $F_{AE} + F_{DF} - w \sin 30 = 0$ —— (1)

(3) $(+\circlearrowright) \sum M_G = \sum (M_G)_{eff}$.

or, $(F_{AE} \sin 30^\circ)(250\text{ mm}) - (F_{AE} \cos 30^\circ)(100\text{ mm})$
$+ (F_{DF} \sin 30^\circ)(250\text{ mm}) + (F_{DF} \cos 30^\circ)(100\text{ mm})$

or, $38.4 F_{AE} + 211.6 F_{DF} = 0$

$\therefore F_{DF} = -0.1815 F_{AE}$ —— (2)

from equation (1) & (2),

$F_{AE} - 0.1815 F_{AE} - w \sin 30^\circ = 0$

$F_{AE} = 0.6109 w$

$F_{DF} = -0.1815 F_{AE} = -0.1815 (0.6109)$
$= -0.1109 w$.

$\therefore F_{AE} = 0.6109 w = 0.6109 (mg)$
$= 47.9$ N (T) ($\measuredangle 60^\circ$)

$F_{DE} = -0.1109 mg = 0.1109 \times 8 \times 9.8$
$= 8.70$ N (C) #

(2) A uniform sphere of mass m and radius r is projected along a rough horizontal surface with

ling by μ_k the coefficient of kinetic friction between the sphere and the floor, determine (a) the time t_1 at which the sphere will start rolling without sliding (b) the linear velocity and angular velocity of the sphere at time t_1. (see fig 8.15)

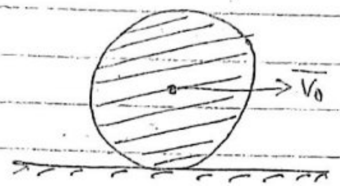


fig 8.15

⇒ Sol^n

The external forces acting on the sphere consist of the weight W, the normal reaction N, and the friction force F as fig (b). The direction of sliding is opposite to that of frictional force F. In case of sliding, the frictional force F is given by $F = \mu_k N$. The effective force consist of the vector $m\bar{a}$ attached at G and the couple $\bar{I}\alpha$.

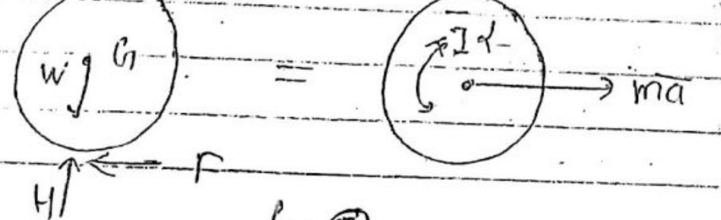


fig (b)

Expressing that the systems of the external forces is equivalent to the systems of the effective forces, we write.

$(+\uparrow)\ \Sigma F_y = \Sigma (F_y)_{eff}$.

or, $N - W = 0$

or, $N = W = mg$

$(\xrightarrow{+})\ \Sigma F_x = \Sigma (F_x)_{eff}$:

or, $-F = m\bar{a}$

or, $-\mu_k N = m\bar{a}$

or, $-\mu_k mg = m\bar{a}$ $\therefore \bar{a} = -\mu_k g$

$(+\circlearrowleft)\ \Sigma M_G = \Sigma (M_G)_{eff}$

$\therefore Fr = \bar{I}\alpha$ $\quad (\because \bar{I} = \frac{2}{5} mr^2)$

or, $(\mu_k N) r = (\frac{2}{5} mr^2)\alpha$

or, $(\mu_k mg) r = (\frac{2}{5} mr^2)\alpha$

$\therefore \alpha = \frac{5}{2} \frac{\mu_k g}{r}$

The sphere both rotates and slides, linear and angular motions are accelerated.

$t=0, \bar{v} = \bar{v_0}$ $\qquad \bar{v} = \bar{v_0} + \bar{a}t$

$\qquad\qquad\qquad = \bar{v_0} - \mu_k g t$

$t=0, \omega_0 = 0$ $\qquad \omega = \omega_0 + \alpha t$

$\qquad\qquad\qquad = 0 + \left(\frac{5}{2} \frac{\mu_k g}{r} t\right)$

$\Rightarrow \omega = \frac{5}{2} \frac{\mu_k g}{r} \times t$ —— (2)

The sphere will start rolling without sliding when the velocity v_c of the of contact c is zero. At that time point c becomes the instantaneous centre of rotation. and we have,

$\vec{v_1} = r\omega_1$ —— (3)

Substituting in (3) the values obtained for $\bar{v_1}$ and ω_1 by making $t = t_1$ in (1) and (2), respectively, we write.

$v_0 - \mu_k g t_1 = r\left(\frac{5}{2} \frac{\mu_k g}{r} t_1\right)$.

$\therefore t_1 = \frac{2}{7}\left(\frac{v_0}{\mu_k g}\right)$ #

Substituting for t_1 into (2), we have,

$\omega_1 = \frac{5}{2} \frac{\mu_k g}{r} t_1 = \frac{5}{2} \frac{\mu_k g}{r} \left(\frac{2}{7} \frac{\bar{v_0}}{\mu_k g}\right)$

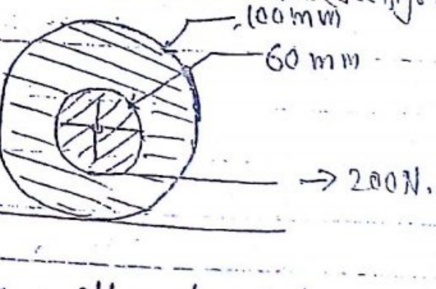
$= \frac{5}{7} \frac{\bar{v_0}}{r}$

$\therefore \omega_1 = \frac{5}{7} \frac{\bar{v_0}}{r}$ ($\circlearrowright$) #

$v_1 = r\omega_1 = r\left(\frac{5}{7} \frac{\bar{v_0}}{r}\right) = \frac{5}{7} \bar{v_0}$

$\therefore v_1 = \left(\frac{5}{7} \bar{v_0}\right) (\rightarrow)$ #

(3) A cord is wrapped around the inner drum of a wheel and pulled horizontally with a force of 200N. The wheel has a mass of 50 kg and a radius of gyration of 70 mm. knowing that $\mu_s = 0.20$ and $\mu_k = 0.15$. determine the acceleration of G and and angular acceleration of the wheel. (see fig 8.16)



$\Rightarrow$ Sol^n

<u>Assuming Rolling without sliding:</u>

$\bar{a} = r\alpha = (0.100 \text{ m})\alpha$

$\bar{I} = m\bar{k}^2 = (50)(0.070\,m)^2$
$= 0.245\ kg\,m^2$

$(\circlearrowleft +)\ \Sigma M_c = \Sigma (M_c)_{eff}$

or, $(200\,N)(0.040\,m) = m\bar{a}(0.100\,m) + \bar{I}\alpha$

or, $(8\,Nm) = (50\,kg)(0.100\,m)\alpha(0.100\,m) + 0.245\alpha$

$\therefore \alpha = 10.59\ rad/sec^2$

$\bar{a} = r\alpha = (0.100\,m)(10.74\ rad/sec^2)$
$= 1.074\ m/sec^2$

$(\rightarrow +)\ \Sigma F_x = \Sigma (F_x)_{eff}$

$F + 200 = m\bar{a}$

or, $F + 200 = 50 \times 1.074$

$F = -146.3\,N = 146.3\,N$ 192.37

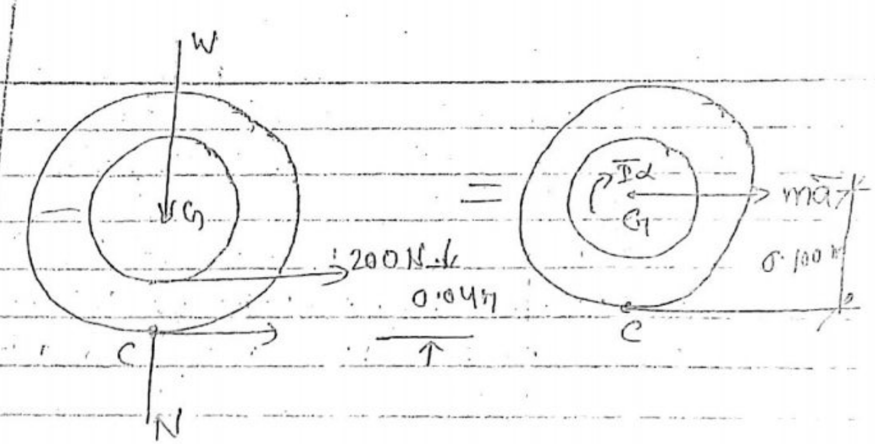
$(\uparrow +)\ \Sigma F_y = \Sigma (F_y)_{eff}$

or $N - W = 0$

or $N = W = mg = 50 \times 9.2 = 490.5\,N$.

$F_{max} = \mu_s N = 0.2\,(490.5) = 98.1\,N$

Since, $F > F_{max}$, the assumed motion is impossible.



ⓑ Rotating and sliding:

Since the wheel must rotate and slide at the same time, we draw a new diagram where $\bar{a}$ and α are independent and where

$F = F_k = \mu_k N = 0.15\,(490.5\,N)$
$= 73.6\,N$

$(\rightarrow +)\ \Sigma F_x = \Sigma (F_x)_{eff}$

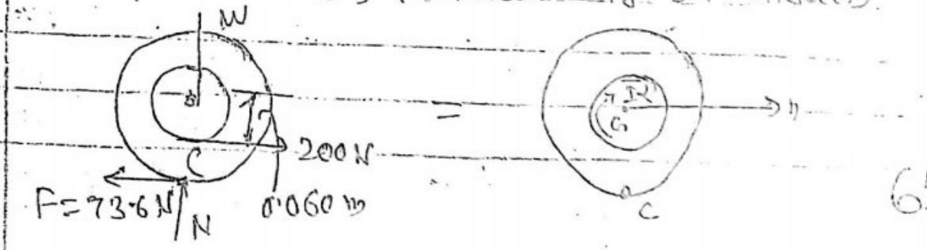
$200 - 73.6 = (50\,kg)\bar{a}$

$\therefore \bar{a} = 2.53\ m/sec^2\ (\rightarrow)$

$(+\circlearrowright)\ \Sigma M_G = \Sigma (M_G)_{eff}$

or, $(73.6)(0.100\,m) - (200)(0.060) = (0.245)\alpha$

$\alpha = -18.94\ rad/sec^2 = 18.94\ rad/sec^2\ (\circlearrowleft)$



(4) The extremities of a 1.2 m rod of mass 25 kg may move freely and with no friction along two straight tracks as shown in fig 8.17. If the rod is released with no velocity from the position shown, determine (a) the angular acceleration of the rod (b) the reactions at A & B.

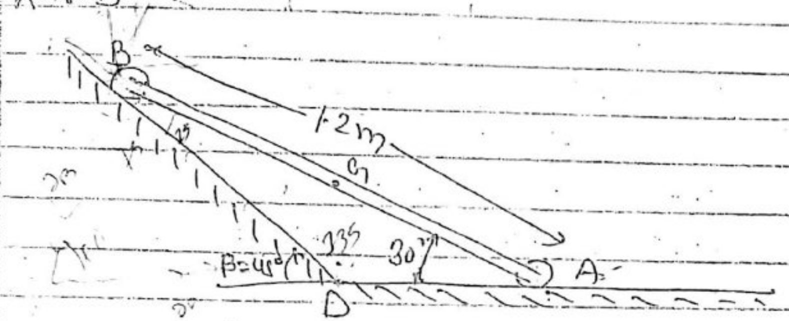
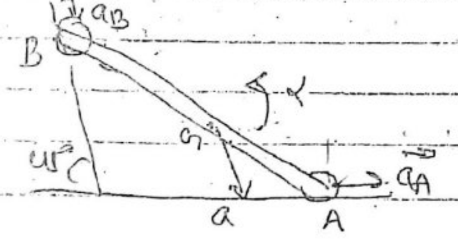
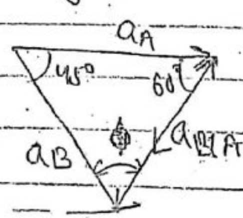


Fig 8.17

⇒ Sol^n

Here the acceleration of G must be to the angular acceleration α because constrained motion. Assuming α direction counterclockwise and noting that $a_{B/A}$.





We have from vector diagram.

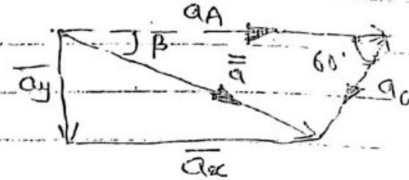
$$a_B = a_A + a_{B/A}$$

$$[a_B \measuredangle 45°] = [a_A \rightarrow] + [1.2\alpha \measuredangle 60°]$$

Nothing $\phi = 75°$, using sine laws.

$$a_A = 1.639\alpha$$

$$a_B = 1.47\alpha.$$



The acceleration of G is now obtained by writing.

$$\bar{a} = a_G = a_A + a_{G/A}$$

$$\bar{a} = [1.639\alpha \rightarrow] + [0.6\alpha \measuredangle 60°]$$

Resolving $\bar{a}$ into x and y components.

$$\bar{a}_x = 1.639\alpha - 0.6\alpha\cos 60° = 1.339\alpha$$

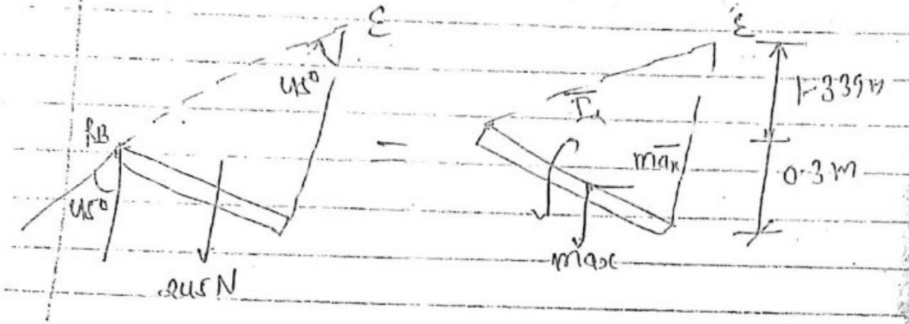
$$\therefore \bar{a}_x = 1.339\alpha \rightarrow$$

$$\vec{a_y} = -0.6\alpha\sin 60° = -0.520\alpha$$

$$\therefore \bar{a}_y = 0.520\alpha \downarrow$$

Here the system of external forces is equivalent to the system of effective forces represented by the vector of components

$m\bar{a}_x$ and $m\bar{a}_y$ attached at G and the couple $\bar{I}\alpha$. (See the below sketches).



$$\bar{I} = \frac{1}{12} ml^2 = \frac{25}{12}(1.2)^2 = 3\,kg\,m^2$$

$$\therefore \bar{I}\alpha = 3\alpha$$

$$m\bar{a}_x = 25 \times 1.339\alpha = 33.5\alpha.$$

$$m\bar{a}_y = -(25)\times(0.52\alpha) = -13\alpha.$$

we have,

$$W = mg = 25 \times 9.81 = 245N$$

① $(+\circlearrowleft)\ \Sigma M_e = \Sigma (M_e)_{eff.}$

or, $245 \times 0.52 = (33.5\alpha)(1.339) + (13\alpha)(0.52) + \text{--}$

or, $\alpha = 2.33\ rad/sec^2$ (↻)

② $(\rightarrow +)\ \Sigma F_x = \Sigma (F_x)_{eff.}$

or, $R_B \sin 45° = 33.5 \times 2.33$

$\therefore R_B = 110.5N = 110.5N$ ($\measuredangle$ 45°)

③ $(\uparrow +)\ \Sigma F_y = \Sigma (F_y)_{eff.}$

$$R_A + R_B \cos 45° - 245 = -(13)(2.33)$$

$$B_A = 136.6N\ (\uparrow)$$

\# .

* ~ *

CHAPTER 9: Plane motion of rigid bodies : energy and momentum methods.

① Gear A has a mass of 10 kg and a radius of gyration of 200 mm, while gear B has a mass of 3 kg and a radius of gyration of 80 mm. The system is at rest when a couple $\vec{m}$ of magnitude 6 Nm is applied to gear B. Neglecting friction, determine ⓐ the number of revolutions executed by gear B before its angular velocity reaches 600 r/min. ⓑ the tangential force which gear B exerts on gear A. (see fig 9.13)

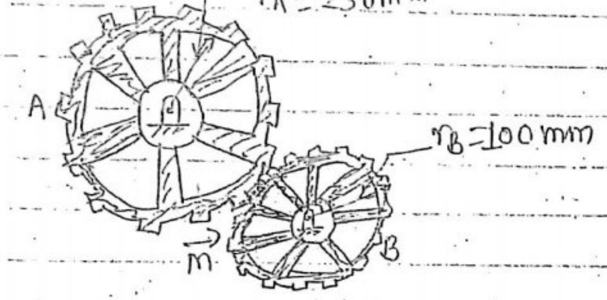


Fig 9.13

⇒ Solⁿ

Considering the peripheral speeds of gears be equal,

$$r_A \omega_A = r_B \omega_B$$

$$\therefore \omega_A = \omega_B \frac{r_B}{r_A} = \omega_B \frac{100}{250} = 0.4\omega_B.$$

We have,

$\omega_B = 600$ r/min.

$\omega_B = 62.8$ rad/sec, $\omega_A = 0.4\omega_B = 25.1$ rad/sec.

$\bar{I}_A = m_A \bar{k}_A^2 = 10(0.2)^2 = 0.4$ kg m²

$\bar{I}_B = m_B \bar{k}_B^2 = 3(0.08)^2 = 0.0192$ kg m²

Since, system is initially at rest

$\therefore T_1 = 0$

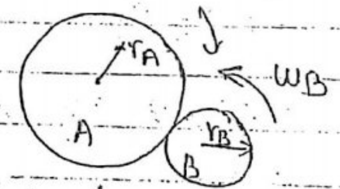
$\therefore T_2 = \frac{1}{2} \bar{I}_A \omega_A^2 + \frac{1}{2} \bar{I}_B \omega_B^2$

$= \frac{1}{2}(0.4)(25.1)^2 + \frac{1}{2}(0.0192)(62.8)^2$

$= 163.9$ J

Let θ_B be the angular displacement of gear B,

work $(U_{1-2}) = m\theta_B = 6 \times \theta_B = (6\theta_B)$ J



Principle of work and energy,

$$T_1 + U_{1-2} = T_2$$

$$0 + (6\theta_B) J = 163.9 J$$

$$\therefore \theta_B = 27.32 \text{ rad}$$

$$= 4.35 \text{ rev}$$ #

Motion of Gear A:

K.E. of gear A at rest, $T_1 = 0$.

When $\omega_B = 600$ r/min, $\omega_A = 25.1$ rad/sec

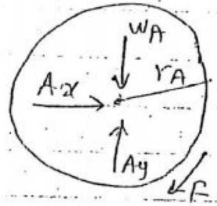
K.E. of gear A, $T_2 = \frac{1}{2} \bar{I}_A \omega_A^2$

$= \frac{1}{2} (0.4)(25.1)^2$

$= 126 J$

Work:

The force acting on gear A are as fig.



Since, peripheral distance are,

$\theta_A r_A = \theta_B r_B$

we have,

$U_{1-2} = F(\theta_B r_B)$ (force × distance)

$= F(27.32)(0.1 m)$

$= F(2.73)$.

principle of work and energy:

$T_1 + U_{1-2} = T_2$

or $0 + F.(2.73) = 126 J$

$F = 46.2 N$ #

Q) A 15-kg slender rod AB is 1.5 m long and is pivoted about a point O which is 0.3m from end B. The other end is pressed against a spring of constant k = 300 kN/m until the spring is compressed 25 mm. The rod is then in a horizontal position. As the rod is released from this position, determine its angular velocity and the reaction at the pivot O as the rod passes through a vertical position. (see fig 9.14)

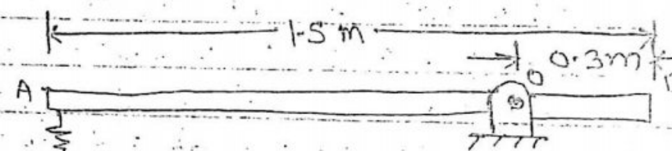


Fig. 9.14

⇒ Sol^n

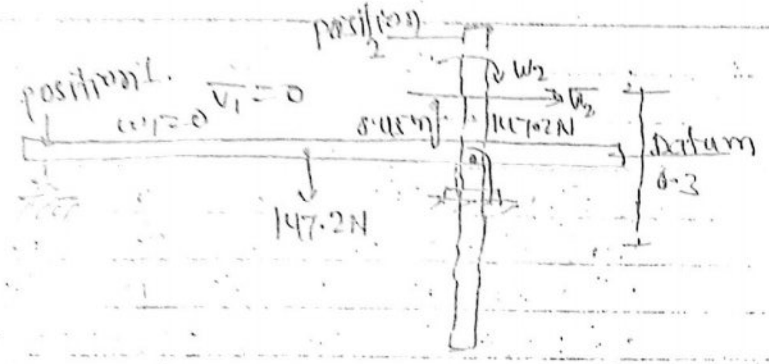
Here since the spring is compressed 25mm

$d_1 = 0.025 m$

$V_e = \frac{1}{2} k x_1^2 = \frac{1}{2} (300 kN/m)(0.025)^2 = 93.8 J$

Choosing the datum as shown, $V_g = 0$,

$V_1 = V_e + V_g = 93.8 J$



K.E.

Since the velocity in position 1 is zero, $T_1 = 0$.

P.E. (position 2)

The wt. of rod AB is,

$W = mg = 15 \times 9.81 = 147.2N$

The elongation of spring is zero, and we have $V_e = 0$ since the centre of gravity of the rod is now 0.45 m above the datum

$V_g = (147.2N)\lambda(+0.45\,m) = 66.2J$

$V_2 = V_e + V_g = 66.2J$

K.E.

Let ω_2 be the angular velocity of rod in position 2.

$\bar{v}_2 = \bar{r}\omega_2 = 0.45\,\omega_2$

$\bar{I} = \frac{1}{12} ml^2 = \frac{1}{12}(15)(1.5)^2 = 2.81\ kg\,m^2$

$T_2 = \frac{1}{2} m\bar{v}_2^2 + \frac{1}{2}\bar{I}\omega_2^2$

$= \frac{1}{2}(15)(0.45\omega_2)^2 + \frac{1}{2}(2.81)\omega_2^2$

$= 2.92\,\omega_2^2$

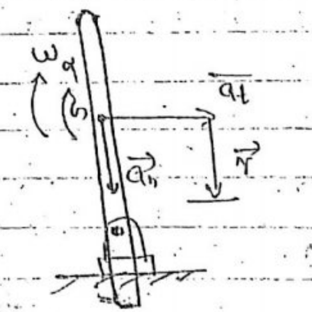
concentration of energy.

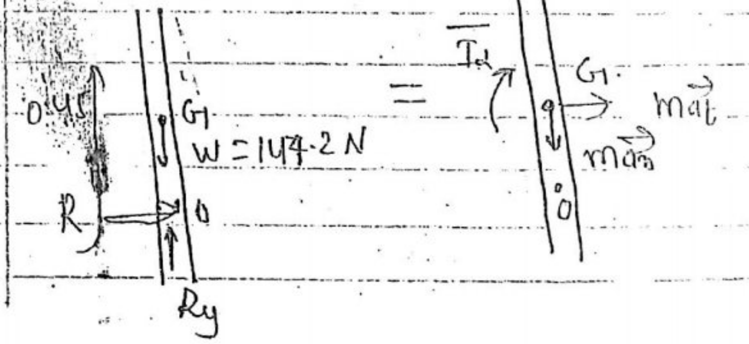
$T_1 + V_1 = T_2 + V_2$

or, $0 + 93.8 = 2.92\,\omega_2^2 + 66.2$

$\therefore \omega_2 = 3.07\ rad/sec\ (\circlearrowleft)$ #

Reaction in position 2:





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Since, $\omega_2 = 3.07$ rad/sec, the components of the acceleration of G as the rod passes through position 2 are:

$$\vec{a}_n = \bar{r}\,\omega_2^2 = (0.45)(3.07)^2 = 4.24\ m/sec^2$$

$$\bar{a}_n = 4.24\ m/sec^2 (\downarrow)$$

$$\bar{a}_t = \bar{r}\alpha\ (\rightarrow)$$

As above fig (*), system of external force is equivalent to the system of effective forces represented by the vector of components $m\bar{a}_t$ and $m\bar{a}_n$ attached at G and the couple $\bar{I}\alpha$.

① $(\circlearrowleft)\ \Sigma M_o = \Sigma (M_o)_{eff}$

$$0 = \bar{I}\alpha + m(\bar{r}\alpha)\bar{r}$$

$$\Rightarrow \alpha(\bar{I} + m\bar{r}^2) = 0$$

$$\therefore \alpha = 0$$

② $(\rightarrow +)\ \Sigma F_x = \Sigma (F_x)_{eff}$

$$R_x = m(\bar{r}\alpha)$$

$$\text{or, } R_x = m\bar{r} \times 0 = 0$$

$$\therefore R_x = 0$$

③ $(\uparrow +)\ \Sigma F_y = \Sigma (F_y)_{eff}$

$$R_y - (147.2\,N) = -m\bar{a}_n$$

$$R_y - (147.2\,N) = -(15)(4.24)$$

$$R_y = 183.6\,N$$

$$\therefore R = 83.6\ N\ (\uparrow)\ \#$$

③ A 20-gm bullet B is fired with a horizontal velocity of 450 m/sec into the side of a 10-kg square panel suspended from a hinge at A. Knowing that the panel is initially at rest, determine (a) the angular velocity of the panel immediately after the bullet becomes embedded, (b) the impulsive reaction at A, assuming the bullet becomes embedded in 0.0006 sec. (see fig. 9.15).

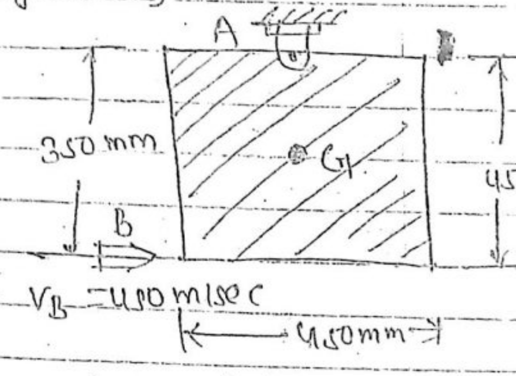
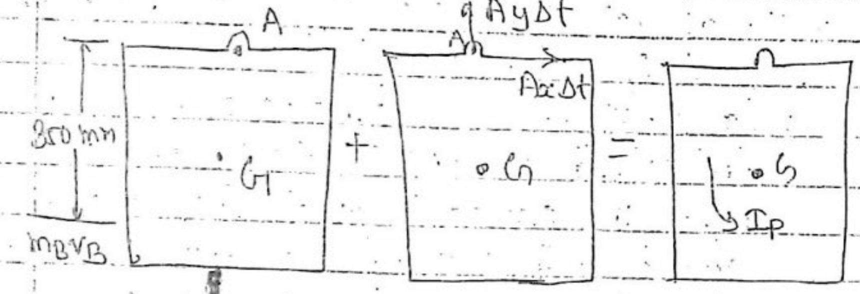


Fig 9.15

Here the initial momentum of the bullet and panel and impulses of the external force are together equivalent to the final momentum of the system. Being time interval very short, we neglect all nonimpulsive forces consider only external impulses $A_x\Delta t$ and $A_y\Delta t$:



Syst Momentum + Syst Ext Imp$_{1-2}$

= Syst moment.

Moment of momentum (+↺) moments about A:

$m_Bv_B(0.35) = m_p\bar{v}_2(0.225) + \bar{I}_p\omega_2$

(→+) x components:

momentum only

$m_Bv_B + A_x\Delta t = m_p\bar{v}_2$ — ①

(moments of momentum taken)

(↑+) y components:

$0 + A_y\Delta t = 0$ — ③

The centroidal mass moment of inertia of the square panel is

$\bar{I} = \frac{1}{6}m_pb^2 = \frac{1}{6}(10)(0.450\ m)^2 = 0.3375\ kgm^2$

substituting this value as well as the given data into ① and noting that

$\bar{v}_2 = (0.225\ m)\omega_2$

we write,

$m_Bv_B \times 0.35 = m_pv_2 \times 0.225 + I_p\omega_2$

or, $(0.020)(450)(0.350) = (10)(0.225\omega_2)(0.225) + 0.3375\omega_2$

or, $\omega_2 = 3.73$ rad/sec.

$\therefore \omega_2 = 3.73$ rad/sec #

$\bar{v}_2 = (0.225\ m)\omega_2 = (0.225\ m)(3.75\ rad/sec)$

$= 0.839$ m/sec.

Substituting $\bar{v}_2 = 0.839$ m/sec and $\Delta t = 0.0006$ sec into equations ② or, $(0.020)(450) + A_x(0.0006) = (10)(0.839)$

$A_x = -1017\,N$

$\therefore A_x = 1017\,N\ (\leftarrow)$ #

from eq^n (5)

$A_y = 0$ #

* ~ *

CHAPTER 10: Mechanical vibrations

(1) A 50 kg block moves between vertical guides as shown in fig 10.9. The block is pulled 40 mm down from its equillibrium position and released. Determine the period of vibration, the maximum velocity and maximum acceleration of the block.

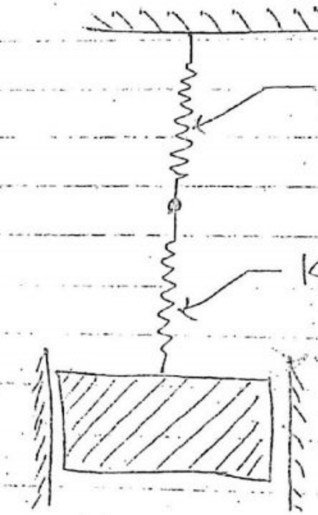


Fig. 10.9

⟹ Sol^n

$$d = d_1 + d_2 = \frac{P}{K_1} + \frac{P}{K_2} = \frac{12KN}{4KN/m} + \frac{12KN}{6KN/m}$$

$$= 5m.$$

To facilitate the computation a static load of magnitude $P = 12KN$ is used.

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$K = P/\delta = \frac{12\,kN}{5\,m} = 2400\,N/m$

$p^2 = k/m = \frac{2400\ N/m}{50\,kg}$

$\therefore P = 6.93$ rad/sec.

Time period $(T) = \frac{2\pi}{p} = 0.907$ sec #

$\therefore V_m = x_m\, p$
$= 0.277$ m/sec (↓)

$\therefore a_m = x_m p^2 = 1.920$ m/sec² (↓) #

Q A cylinder of mass m and radius r is suspended from a looped cord as shown in fig 10.10. One end of the cord is attached directly to a rigid support, while the other end is attached to a spring of constant K. Determine the period and frequency of vibration of the cylinder.

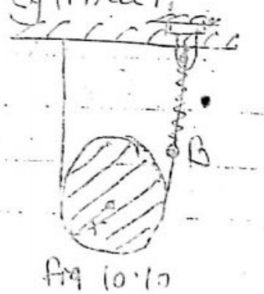
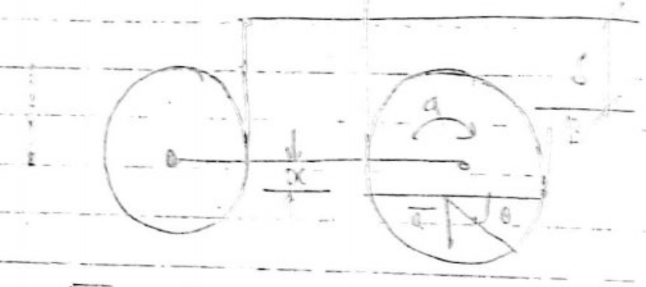


fig 10.10

$= \ldots$

Taking positive sense as clockwise ... meaning the displacements from the equilibrium position.



$\bar{x} = r\theta \quad \therefore \delta = 2\bar{x} = 2r\theta$

$\alpha = \ddot{\theta}$ $\therefore \bar{a} = r\alpha = r\ddot{\theta}$

The system of external forces acting on the cylinder consists of weight W and of forces T_1 and T_2 exerted by the cords.