REFERENCES

1. Bhadra Prasad Pokharel and Nava Raj Karki, "Electrical Engineering Materials"

2. SO KASAP, "Principles of Electronic Materials and Devices"

Chapter: 1

THEORY OF METAL

. Derive De- Broglie's wave equation.

ns: It was also observed that electromagnetic waves e.g. x-rays, amma rays, visible light, etc. sometimes exhibit properties imilar to properties of discrete particles of matter.

ouis De-Broglie in 1924 suggested his hypothesis that there is wave-particle dualism'. The wave associated with material articles is called "matter waves".

o show the wave particle dualism he made the use of Plancks heory of quantum radiation and Einstein's theory of relativity.

ccording the Plancks theory of quantum radiation energy of a hoton is given by

$$E = hf = \frac{hC}{\lambda} \text{ .... } (1)$$

here, C = Velocity of electromagnetic radiation

h = 6.62 × 10^{-34} Js is the Plancks constant

f = Frequency of electromagnetic radiation

gain from Einstein's mass energy relationship, energy of photon given by

$$E = Mc^2 \text{ .... } (2)$$

Comparing (1) and (2)

$$\frac{hc}{\lambda} = mc^2$$

$$\lambda = \frac{h}{mc} = \frac{h}{p} \text{ [where p = mc = momentum of photon]}$$

ccording to de-Broglie, the wave length 'λ' of the wave ssociated with a moving particle having momentum p = mc, is iven by

$$\lambda = \frac{h}{mv}$$

Experiment (1) : [Particle showing wave nature]

It is observed that a beam of electrons can get diffracts from a crystal in a manner similar to the diffraction of x-ray which are electromagnetic waves.

If an electron of mass 'm' is accelerated through t potential 'V' and velocity 'V' then

$$\frac{1}{2}mv^2 = ev$$

$$Q = \sqrt{\frac{2ev}{m}}$$

$$\therefore \quad \lambda = \frac{h}{mv} = \frac{h}{\sqrt{2mev}}$$

The wave associated with this wavelength is called De- Brogl electron wave.

Experiment : 2 [wave showing particle nature]

When a beam of electromagnetic radiation is incident on a sol or gas, it ejects electron from the material. For each material characteristic energy called the work function is required t remove electrons from the material. It is given by, $\phi = hf$.

Comparison of this equation with E = hf indicates th electromagnetic radiation behaves like particles

Quantum mechanics does not distinguish between wave an particles. Although it can predict the circumstances for which on type of behavior will dominate.

2. Discuss about wave function.

Ans: In quantum mechanics there exists an expression in th form of the equation that is used to represent wave - partic quality called wave function (Ψ).

A simple harmonic wave is represented by the equation y = a si (wt - kx). In such type of wave motion there is only the transfer energy, but in case of matter wave, there is a transfer o momentum (particle) in addition to the energy. The suitab function to represent wave function for matter wave is,

$$\Psi(x, t) = Ae^{-(wt-kx)} \text{.......................}(1)$$

Where A is called normalizing constant

$$\text{Since, } E = hf = \frac{h}{2\pi}.2\pi f = \hbar\omega$$

$$\omega = \frac{E}{\hbar}$$

and, $$P = \frac{h}{\lambda} = \frac{h}{2\pi}.\frac{2\pi}{\lambda} = \hbar k$$

$$K = \frac{P}{\hbar}$$

Equation (1) can also be represented as

$$\Psi(x,t) = Ae^{-i/\hbar(Et - px)} \text{....................}(2)$$

The wave function Ψ has no meaning in itself. When it is operated by the Schrodinger wave equation, it describes the motion of the particle associated with it as done by second law of motion in classical mechanics.

In quantum mechanics, energy, momentum and position of the particle are called observables and wave function Ψ is used to describe these observables.

The only quantity having the physical meaning is the square of its magnitude. The quantity $P = \Psi\Psi^* = |\Psi|^2$ evaluated at a particular point at a particular time was proportional to the probability of finding the particle at that time.

The probability of finding a particle in the volume element dx, dy dz is $|\Psi|^2$ dx dy dz or $|\Psi|^2 dV$.

Since the total probability of finding the particle in the entire space is unity.

$$f\,|\Psi|^2\,dV = 1$$

The wave function satisfying this condition is called normalized wave function. Every acceptable wave function can be normalized by multiplying it with an appropriate constant called normalizing constant.

3. What are the characteristics of the wave function?

Ans:

1. It must be normalized
2. It must be single valued and continuous
3. If$\varphi_1(x), \Psi_2(x)$ .... on Ψ_n must be solution of th Schrodinger wave equation.
4. The wave function $\Psi(x)$ must approaches zero as $x \to \pm \infty$

4. Derive the time independent Schrodinger wave equation Time independent Schrodinger wave equation.

Ans: Schrodinger wave equation describes the motion of a quantum mechanical particle as Newton's second law in classica mechanics. It has been observed that in many situations, potentia acting on the particle (v) does not depend upon time and varies only with its position only. For such conditions time independent form of Schrodinger's equation is applicable.

The wave function associated with quantum mechanical particle is given by–

$$\Psi = Ae^{-(wt-kx)} = Ae^{-i/\hbar(Et - px)}$$

Differentiating with respect to x,

$$\frac{d\Psi}{dx} = \left(\frac{-i}{\hbar}\right)(-p), Ae^{-i/\hbar(Et - px)}$$

$$= \frac{iP}{\hbar}\Psi$$

Again differentiating with respect to x

$$\frac{d^2\Psi}{dx^2} = \left(\frac{ip}{\hbar}\right)^2\Psi$$

$$\frac{d^2\Psi}{dx^2} = \frac{-p^2}{\hbar^2}\Psi$$

$$p^2\Psi = -\hbar^2\frac{d^2\Psi}{dx^2} \quad \text{..............................(1)}$$

The total energy of a particle is given by, E = KE + PE

$$E = \frac{1}{2}mu^2 + V = \frac{1}{2}\frac{(mu)^2}{m} + V = \frac{P^2}{2m} + V$$

$$E = \frac{P^2}{2m} + V$$

Multiplying both sides by Ψ

$$\left(\frac{p^2}{2m} + V\right)\Psi = E\Psi$$

$$-\frac{\hbar^2}{2m}\frac{d^2\Psi}{dx^2} + V\Psi = E\Psi \text{ [using equation (1)] } \text{.................(2)}$$

$$\frac{d^2\Psi}{dx^2} + \frac{2m(E - V)}{\hbar^2}\Psi = 0 \text{ ....(3)}$$

Which is the time independent Schrodinger wave equation. In three dimensional, it can be expressed as

$$\nabla^2\Psi + \frac{2m(E - V)}{\hbar^2}\Psi = 0 \text{ ....(4)}$$

Where, $\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$ is called a Laplacian operator

The equation (2) is

$$\frac{\hbar^2}{2m}\frac{d^2\Psi}{dx^2} + V\Psi = E\Psi$$

The Hamiltonian operator is given by, $\hat{H} = \left(\frac{-\hbar^2}{2m}\frac{d^2}{dx^2} + V\right)$

Therefore time independent Schrodinger wave equation takes the form,

$$\hat{H}\Psi = E\Psi \text{ ....(5)}$$

5. Derive time dependent Schrodinger wave equation.

Ans: The wave function associated with matter wave is

$$\Psi = Ae^{-i/\hbar(Et - px)}$$

Differentiating with respect to x, $\frac{d\Psi}{dx} = \left(\frac{-i}{\hbar}\right)(-P)\Psi = \frac{ip}{\hbar}\Psi$

Again differentiating with respect to x,

$$\frac{d^2\Psi}{dx^2} = \left(\frac{iP}{\hbar}\right)^2\Psi = \frac{-P^2}{\hbar^2}\Psi$$

$$P^2\Psi = -\hbar^2\frac{d^2\Psi}{dx^2} \quad \text{.......................(1)}$$

Now, differentiating Ψ with respect to t,

$$\frac{d\Psi}{dt} = \frac{-iE}{\hbar}\Psi \Rightarrow E\Psi = \frac{-\hbar}{i}\frac{d\Psi}{dt} = \frac{i^2\hbar}{i}\frac{d\Psi}{dt}$$

Hence, $E\Psi = i\hbar\frac{d\Psi}{dt}$ ....(2)

The total energy of a particle is given by, E = KE + PE

$$E = \frac{P^2}{2m} + V$$

Multiplying both sides by Ψ,

$$E\Psi = \frac{P^2\Psi}{2m} + V\Psi$$

Using equations (1) and (2)

$$i\hbar\frac{d\Psi}{dt} = \frac{-\hbar^2}{2m}\frac{d^2\Psi}{dx^2} + V\Psi \quad \text{....................(3)}$$

Which is the time dependent Schrodinger wave equation (3) can be re-written as

$$\frac{-\hbar^2}{2m}\frac{d^2\Psi}{dx^2} + V\Psi = i\hbar\frac{d\Psi}{dt}$$

The Homidtonial operator is given by $\hat{H} = \left(\frac{-\hbar^2}{2m}\frac{d^2}{dx^2} + V\right)$

Therefore time dependent Schrodinger wave equation takes the form

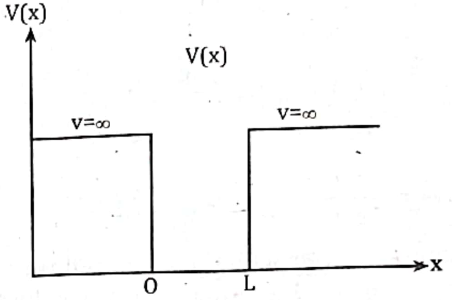
$$\hat{H}\Psi = i\hbar\frac{d\Psi}{dt} \quad \text{..............................(4)}$$

6. Derive the relation of energy level inside a potential well of width L.

Energy well model of metal.

Ans: (A particle confined in an one dimensional infinitely deep potential well)

Consider a particle (electron) restricted to move along the x-axis between x = 0 and x = L. The potential energy V of the particle is zero inside the box, but rises to infinity on the outside.



i.e. V = 0 for 0 < x < L

V = ∞ for x < 0 and x > L

The Schrodinger wave equation for the particle with in the box is

$$\frac{d^2\Psi}{dx^2} + \frac{2mE}{\hbar^2}\Psi = 0$$

$$\frac{d^2\Psi}{dx^2} + K^2\Psi = 0 \quad \text{..................(1)}$$

Where, $K^2 = \frac{2mE}{\hbar^2}$ ....(2)

The solution of equation (1) is,

$\Psi(x)$ = Asinkx + Bcoskx ....(3)

Where A and B are constants to be determined using boundary condition.

Since the particle cannot have infinite energy, it cannot exist outside the box. Therefore, the wave function Ψ most be zero outside the box, so Ψ must be zero at the walls i.e. at

x = O and x = L

$\therefore \quad \Psi(x) = 0$ at $x = 0$

From equation (3), $0 = 0 + B \Rightarrow B = 0$

and $\quad \Psi(x) = 0$ at $X = L$

$$0 = A\sin KL$$

$$\Rightarrow \quad \sin KL = 0$$

$$\sin KL = \sin n\pi, \; n = 0, 1, 2, 3 \ldots\ldots\ldots$$

$$KL = n\pi$$

$$K = \frac{n\pi}{L} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(4)$$

From equation (2) and (4), $\frac{2mE}{\hbar^2} = \frac{n^2\pi^2}{L^2}$

$$E = \frac{n^2\pi^2\hbar^2}{2mL^2} = \frac{n^2h^2}{8mL^2} \ldots\ldots\ldots\ldots\ldots\ldots\ldots(5)$$

This means the energy of particle in a potential well is quantized. Each value of energy given by equation (5) is called Eigen value and corresponding functions Ψ_n are called Eigen functions.

Now substituting, $B = 0$ and $K = \frac{n\pi}{L}$ in equation (3), the allowed solutions of Schrodinger equation are

$$\Psi_n(x) = A \sin \frac{n\pi x}{L}$$

The coefficient A is called normalizing constant and can be determined using normalizing condition,

$$\int_0^L \Psi\Psi^* \, dx = 1$$

$$A^2 \int_0^L \sin^2\left(\frac{n\pi x}{L}\right) dx = 1$$

$$A^2 \int_0^L \frac{1}{2}\left[1 - \cos 2\left(\frac{n\pi x}{L}\right)\right] dx = 1$$

$$A^2 . \frac{1}{2}\left[\int_0^L dx - \frac{1}{2}\int_0^L \cos \frac{2n\pi x}{L} dx\right] = 1$$

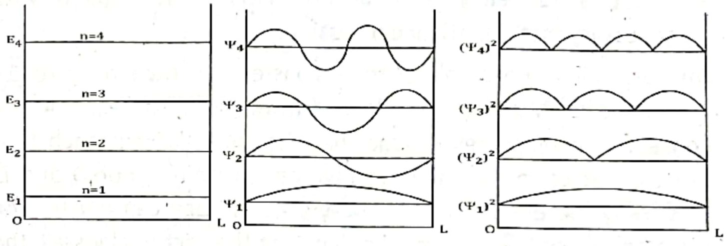
$$A^2 . \frac{L}{2} = 1$$

$$A = \sqrt{\frac{2}{L}}$$

Hence the normalized wave function of the electron is

$$\Psi_n = \sqrt{\frac{2}{L}} \sin \frac{n\pi x}{L}$$

This first four eigen functions Ψ_1, Ψ_2, Ψ_3&Ψ_4 together with the probability densities $(\Psi_1)^2$, $(\Psi_2)^2$,$(\Psi_3)^2$ and $(\Psi_4)^2$ are shown in the figure below. It is obvious that the quantum mechanical result is very different from the classical result. Classical mechanics predicts the same probability for the particle being anywhere in the box. But quantum mechanics predicts that the probability is different at different points.



For, particle infinite potential well the minimum energy that the electron possesses can be calculated by Heisenberg's uncertainty principle.

The particle is confined in the potential well of width L, this means uncertainty in position is L, i.e. $\Delta x = L$

And the particle has momentum +Px while moving in positive x-direction and -Px while moving in negative x direction. Therefore, uncertainty in momentum, $\Delta Px = Px - (-px) = 2Px = 2\hbar k$ for the ground state, $K = \frac{n\pi}{L} = \frac{\pi}{L}$ for $n = 1$

$$\therefore \quad \Delta Px. \Delta x = 2\hbar k.L = 2.\frac{h}{2\pi}.\frac{\pi}{L}.L = h$$

$$\therefore \quad \Delta Px.\Delta x = h \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(6)$$

Again, substituting the value of ΔPx and Δx in equation (6)

$$2Px.L = h \Rightarrow Px = \frac{h}{2L}$$

The potential energy of electron inside the well is zero. Hence the total energy is given by

$$E = \frac{1}{2}mu^2 = \frac{P^2}{2m}$$

$$E = \frac{1}{2m}\left(\frac{h}{2L}\right)^2 = \frac{h^2}{8mL^2}$$

$$\therefore \quad \left[E = \frac{h^2}{8mL^2}\right]$$

7. What is tunneling in quantum mechanics? Explain with necessary mathematical expression.

Ans: Consider an example of roller coaster as shown in figure (1). When the roller coaster is released from rest at a height A, the conservation of energy means that the carriage can reach B at most C but certainly not beyond C and definitely not D and E. Classically there is no possible way that carriage can reach. An extra energy of D – A is needed. Ignoring the frictional losses, the roller coaster will go back and forth between A and C.

Consider an analogous event on an atomic scale, when an electron of energy E is incident on the barrier of height 'V' which is greater than E classically region II and region III are forbidden to particle. However, quantum mechanics predicts and the solution of this equation is,

$$\Psi_1 = Ae^{ik1x} + Be^{-ik1x} \quad \text{...(1)}$$

For region II,

$$\frac{d^2\Psi_2}{dx^2} + \frac{2m(E-V)}{\hbar^2}\Psi_2 = 0$$

or, $$\frac{d^2\Psi_2}{dx^2} - \frac{2m(V-E)}{\hbar^2}\Psi_2 = 0$$

or, $$\frac{d^2\Psi_2}{dx^2} - K_2^2\Psi_2 = 0, \text{ where } K_2^2 = \frac{2m(V-E)}{\hbar^2}$$

The solution of this equation is

$$\Psi_2 = Ce^{k2x} + De^{-k2x} \quad \text{...................................(2)}$$

For region III,

$$\frac{d^2\Psi_3}{dx^2} + \frac{2mE}{\hbar^2}\Psi_3 = 0$$

or, $$\frac{d^2\Psi_3}{dx^2} + K1^2\Psi_3 = 0 \text{ where } K_1^2 = \frac{2mE}{\hbar^2}$$

The solution of this equation is,

$$\Psi_3 = Fe^{ik1x} + Ge^{-ik1x}$$

Since there is no reflected wave in the region (iii) so, G = 0

$$\therefore \quad \Psi_3 = Fe^{ik1x} \quad \text{..(3)}$$

Using boundary conditions at x = 0

$$\Psi_{1/x} = \Psi_{2/x} = 0$$

$$A + B = C + D \quad \text{...............................(4)}$$

and $$\Psi'/x = 0 = \Psi_2'/x = 0$$

$$iK_1A - iK_1B = K_2C - K_2D \quad \text{.........................(5)}$$

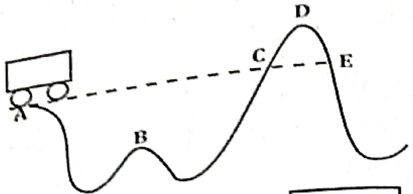
Again, using boundary conditions at x = l

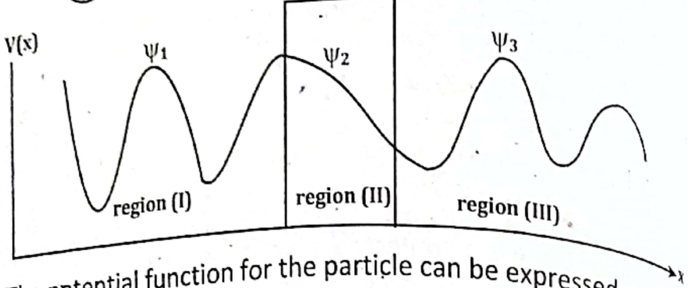
$$\Psi_2/x = l = \Psi_3/x = l$$

$$Ce^{k2l} + De^{-k2l} = Fe^{ik1l} \quad \text{....................................(6)}$$

and $$\Psi_2'/x = l = \Psi_3'x = l$$

Finite probability of finding the particle in region III. This effect of quantum mechanical particle is called "quantum leak" or "barrier tunneling" except in the region of infinite potential (where $\Psi = 0$), there will always be a solution $\Psi(x)$ and there always will be some probability of finding the electron.





The potential function for the particle can be expressed as

$V(x) = 0$ for $x < 0$ (for region I)

$V(x) = V$ for $0 < x < l$ (for region-II)

$V(x) = 0$ for $x > l$ (for region III)

The Schrodinger wave equations and their solutions for these regions are as follows:

For region I:

$$\frac{d^2\Psi_1}{dx^2} + \frac{2mE}{\hbar^2}\Psi_1 = 0$$

or, $$\frac{d^2\Psi_1}{dx^2} + K_1^2\Psi_1 = 0$$

Where, $K_1^2 = \frac{2mE}{\hbar^2}$

$$CK_2^{K2l} - DK_2e^{-k2l} = E\,(ik_1)e^{ik'l} \;\ldots\ldots\ldots\ldots\ldots\ldots\ldots(7$$

Solving equations (4), (5), (6) and (7) we get the transmission probability or transmission coefficient as

$$T = \left|\frac{F}{A}\right|^2 = \frac{4E(V-E)}{4E(V-E)+V^2 sinh^2 K_2 l} \;\ldots\ldots\ldots\ldots\ldots\ldots(8$$

If the width and height of potential are very large, the term $4E(V-E)$ in the denominator of equation (8) can be neglected in comparison to $V^2 \sinh^2 k_2 l$.

Also, $\text{Sinh}k_2l = \frac{e^{k2l} - e^{-k2l}}{2} = \frac{e^{k2l}}{2}$ for large l

Therefore, equation (8) can be written as,

$$T = \frac{4E(V-E)}{V^2\left(\frac{e^{k2l}}{2}\right)^2}$$

$$\therefore \quad T = \frac{16E(V-E)}{V^2} e^{-2k2l} \;\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(9)$$

Where, $K_2 = \sqrt{\frac{2m(V-E)}{\hbar^2}}$

Equation (9) gives the probability of tunneling of the particle of energy E through a potential barrier of width 'l' and height V.

This is the superiority of quantum mechanics over the classical mechanics which shows that there is a finite probability of emission of electron (β-emission) from the nucleus of atom even the electron has lower energy than the energy by which it is bounded.

The reflection coefficient or the probability of reflection of the particle at the barrier is given by–

$$R = \left|\frac{B}{A}\right|^2 = 1 - T$$

8. What is Heisenberg's uncertainty principle ?

Ans: Heisenberg uncertainty principle states that "It is impossible to determine precisely and simultaneously the value of both members of physical variables which describe the motion of an atomic system." Such pairs of variables are called canonically conjugate variables. For example the position coordinate and momentum coordinate, energy and time, the angular momentum and angular position, Let Δx and ΔPx are the uncertainty in measurement of position and momentum then,

$$\Delta x . \Delta Px \geq \hbar$$

Similarly, $\Delta E . \Delta t \geq \hbar$

and $\Delta J.\Delta\theta \geq \hbar$

Where, ΔE = uncertainty in energy

Δt = uncertainty in time

ΔJ = uncertainty in angular momentum

$\Delta\theta$ = uncertainty in angular position

If the position coordinate x of a particle in motion is accuratel determined at same instant so that $\Delta x = 0$, then at the sam instant the uncertainty ΔPx in the determination of th momentum becomes infinite and vice-versa.

Similarly, in all of the above cases, if one quantity is measure accurately, the measurement in the other quantity becomes les accurate.

9. What are the applications of uncertainty principle?

Ans: The uncertainty principle can explain a large number of facts

1. Nonexistence of electrons and existence of proton and neutro in nucleus.
2. Calculation of binding energy of an electron in an atom.
3. Determination of radius of hydrogen atom.
4. Determination of finite width of spectral lines.
5. To study the strength of nuclear forces and stability of th atom.

10. Discuss about energy operator $\overset{\Lambda}{E}$,momentum operator $\overset{\Lambda}{P}$ and position operator $\overset{\Lambda}{X}$.

Ans: An equation of the form, $A\Psi = a\Psi$ is called eigen value equation.

Here, 'A' is an operator

Ψ is called eigen function

and a is called eigen value

An operator is a function over a space of physical states to another space of physical states. The operators must yield real eigen values, since they are values which may come an as the result of the experiments.

We have the wave function associated with quantum mechanical particle as

$$\Psi = Ae^{-i/\hbar(Et - px)}$$

Differentiating with respect to 't'

$$\frac{d\Psi}{dt} = -\frac{i}{\hbar}E\Psi$$

$$E\Psi = \frac{-\hbar}{i}\frac{d\Psi}{dt}$$

$$E\Psi = i\hbar\frac{d\Psi}{dt}$$

$\overset{\Lambda}{E} \rightarrow i\hbar\frac{\partial}{\partial t}$ is called energy operator.

Again differentiating Ψ with respect to 'x'

$$\frac{\partial\Psi}{\partial x} = \left(\frac{-i}{\hbar}\right)(-P)\Psi = \frac{iP}{\hbar}\Psi$$

or, $P\Psi = \frac{\hbar}{i}\frac{\partial\Psi}{\partial x} = -\frac{i^2\hbar}{i}\frac{\partial\Psi}{\partial x}$

or, $P\Psi = -i\hbar\frac{\partial\Psi}{\partial x}$

or, $\overset{\Lambda}{P} \rightarrow -i\hbar\frac{\partial}{\partial x}$ is called momentum operator

Similarly, $\overset{\Lambda}{x} \rightarrow$ is called a position operator. All these operators are used to predict the respective parameters, i.e. energy, momentum and position of the electron.

11. What is the Pauli Exclusion Principle?

Ans: It states that no two electrons within a given system may have all four identical quantum numbers n, ℓ, m_ℓ and m_s. Each set of values for n, ℓ, m, m_ℓ, and ms represent a possible electronic state and correspondingly a wave function Ψn, l, ml, ms where, η

= principal quantum number, ℓ = orbital angular, quantu number, m_ℓ = magnetic quantum number, ms = spin magneti quantum number.

12. From free electron theory of metal, show that E - K diagra is parabolic.

Ans: According to this theory, the outermost electrons or valenc electrons of an atom in the metal are very loosely attached to th parent atom. These electrons are free to move throughout th metal. A free electron has the same potential energy everywher within the metal according to free electron theory of metal.

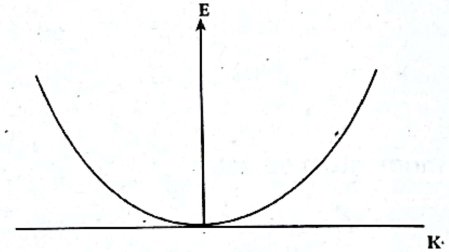
So, if the electron has kinetic energy only.

$$E = \frac{1}{2} mu^2 = \frac{1}{2m} m^2u^2 = \frac{P^2}{2m}$$

$$\therefore \quad E = \frac{\hbar^2 k^2}{2m} \quad \ldots\ldots\ldots\ldots\ldots\ldots(1)$$

This equation represents the kinetic energy of the free electro within the metal.

Equation (1) shows that the relationship between E and K i parabolic in nature as shown in the figure below.



13. Show the energy of a electron in a linear metal is quantized.

Ans: Consider a copper wave of length L. There are many free electrons within the copper rod. The wave length of electron in the wire depends upon kinetic energy.

For maximum wavelength the kinetic energy of electron will be smallest and K.E. will progressively increase as the wavelength decreases.

So for greatest wave length or minimum kinetic energy,

$$L = \frac{\lambda}{2} \Rightarrow \lambda = 2L$$

Similarly for smaller wavelength or larger value of kinetic energies, the relationship between length of wire and wave length of an electron is given by–

$$L = \frac{2\lambda}{2} \Rightarrow \lambda = \frac{2L}{2}$$

$$L = \frac{3\lambda}{2} \Rightarrow \lambda = \frac{2L}{3}$$

And so on upto n, we can write,

$$L = \frac{n\lambda}{2} \Rightarrow \lambda = \frac{2L}{n}$$

For wave length in terms of length of wire, we will get,

$$\lambda = 2L, \frac{2L}{2}, \frac{2L}{3}, \frac{2L}{n} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(1)$$

Here, Ψ is decreasing so, according to relation, $E = \frac{\hbar^2 K^2}{2m}$

$$= \frac{\hbar^2}{2m}\left(\frac{2\pi}{\lambda}\right)^2 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(2)$$

Substituting the value of wave length from the equation (1), we will get,

$$K = \frac{2\pi}{\left(\frac{2L}{1}\right)}, \frac{2\pi}{\left(\frac{2L}{2}\right)}, \frac{2\pi}{\left(\frac{2L}{3}\right)}, \ldots\ldots\ldots\ldots\ldots\ldots \frac{2\pi}{\left(\frac{2L}{n}\right)}$$

or, $$K = \frac{\pi}{L}, \frac{2\pi}{L}, \frac{3\pi}{L} \ldots\ldots\ldots\ldots\ldots\ldots \frac{n\pi}{L} \quad \ldots\ldots\ldots\ldots\ldots\ldots(3)$$

In normal condition, a solid is electrically neutral with not net flow of electrons in any direction. This means a free electron having a velocity in one direction must be matched by similar electron having some velocity in opposite direction. Since the velocity is related to momentum, and momentum related to wave number. So wave number must have both positive and negative values.

i.e. $K = \pm\frac{\pi}{L}, \pm\frac{2\pi}{L}, \pm\frac{3\pi}{L}$ .... $\pm\frac{n\pi}{L}$ ....(4)

In general, $K = \pm\frac{n\pi}{L}$, n = 1, 2, 3, 4, 5, 6 ....(5)

We have, $E = \frac{\hbar^2K^2}{2m}$ ....(6)

Using can (5) in (6) we will get

$$E = \frac{n^2\pi^2\hbar^2}{2mL^2} \quad ....(7)$$

14. What is a degenerate state?

Ans: Two or more different states of a quantum mechanical system are said to be degenerate states if they give some value of energy upon measurement.

In a single dimension solid and one dimensional potential well the energy of electron is expressed as,

$$E = \frac{n^2\pi^2\hbar^2}{2mL^2}$$

In three dimensions, $n^2 = nx^2 + ny^2 + nz^2$

$$\therefore \quad E_n = \frac{(nx^2 + ny^2 + nz^2)\pi^2\hbar^2}{2mL^2}$$

Where, as in the earlier cases, nx, ny and nz can only be integers i.e. 1, 2, 3, 4...................

The set of values of n along different axes in the three dimensional case will determine the energy associated with electron. Unlike a single dimension, the same n along x axis can represent different energies because this energy not only depends upon the principle quantum number along x-axis but also upon along y-axis and z-axis. These degenerate states are all equally probable of being filled. The number of such states gives the degeneracy of particular energy level. For example (nx, ny, nz) = (1, 1, 2), (1, 2, 1) and (2, 1, 1) represent the same electron energy. These all three states are degenerate states.

15. Write about Fermi Energy.

Ans: Fermi energy is defined as the energy of highest filled energy level at a temperature of absolute zero i.e. 0K. All the energy level up to the Fermi level are filled at 0k and empty above it.

A Fermi gas is an ensemble of a large number of Fermions. Fermions are the particles that obey Fermi-Dirac statistics. Free electrons inside a metallic conductor behave like an electron gas and obey Fermi-Dirac statistics.

If N is the total number of electrons to be accommodated on the line then for even N, because of the Pauli exclusion principle,

$$n_f = \frac{N}{2}$$

Where n_f represents the principle quantum number of the Fermi level i.e. number of topmost filled level.

As, electron energy in linear solid metal is

$$E = \frac{n^2\pi^2\hbar^2}{2mL^2}$$

For $n = n_f$ the Fermi energy is

$$E_f = \frac{{n_f}^2\pi^2\hbar^2}{2mL^2} = \frac{\hbar^2}{2m}\left(\frac{n_f\pi}{L}\right)^2$$

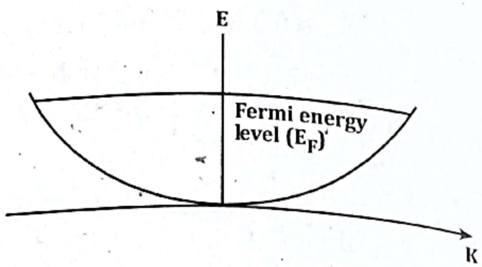
$$E_f = \frac{\hbar^2{K_f}^2}{2m} \quad ....(1)$$

Where, $K_f = \left(\frac{n_f\pi}{L}\right)^2$ is the Fermi wave vector. And the sphere of radius K_f in E-K space is called Fermi sphere.

Equation (1) shows that E–K curve is parabolic as shown in figure (1)

At T = OK, the electrons occupy the lowest quantum states to sa same energy. In other words, the quantum states $E \leq E_f$ a occupied while the states with $E > E_f$ are empty.

Since, $E_f = \frac{\hbar^2 K_F^2}{2m}$, this means the states with momentum $K \leq K_f$ a occupied and states with $K > K_f$ are empty. In other words, in th K-space the occupied states form a sphere with radius K_f. Th sphere is known as Fermi sphere (or the Fermi sea) as shown i figure (2)



The total volume of the Fermi sphere is $\frac{4\pi}{3} K_f^3$. Each quantum state occupies the volume, $\left(\frac{2\pi}{L}\right)^3$, Which comes from uncertainty relation.

$$\text{So the total number of quantum states} = \frac{\frac{4\pi}{3} K_f^3}{\left(\frac{2\pi}{L}\right)^3} = \frac{V K_f^3}{6\pi^2}$$

There are two electrons per state (because we have electrons with spin up and spin down). So the total number of occupied electrons in the sphere is,

$$N = 2 \times \frac{V K_f^3}{6\pi^2} = \frac{V K_f^3}{3\pi^2} \quad \text{.......(2)}$$

$$\Rightarrow \quad K_F = \left(3\pi^2 \frac{N}{V}\right)^{1/3} \quad \text{.......(3)}$$

Thus, K_f depends upon particle concentration, i.e. number of electrons per unit volume

$$\therefore \text{Fermi energy, } E_F = \frac{\hbar K_F^2}{2m} = \frac{\hbar^2}{2m}\left(3\pi^2 \frac{N}{V}\right)^{2/3} \quad \text{.......(4)}$$

This shows that E_f does not depend upon temperature and number of electrons but depends upon $\frac{N}{V}$ only. As a result the Fermi energy does not change when two identical metals are joined together (since $\frac{N}{V}$ is same)

If V_F is the velocity of the electron at the Fermi surface then,

$$m u_f = \hbar K_f$$

$$u_f = \frac{\hbar K_f}{m} = \frac{\hbar}{m}\left(3\pi^2 \frac{N}{V}\right)^{1/3} \quad \text{.......(5)}$$

16. What is the density of states?

Ans: The density of states function can be defined as the number of electronic energy states per unit energy range. A high value of density of states function at a specific energy level means that there are many states available for occupation. A zero value of density of state function means that no states can be occupied at that energy level.

$$\text{Density of states, } Z(E) = \frac{dM}{dE} \quad \text{.......(1)}$$

$$\text{Where, } E = \frac{n^2 \pi^2 \hbar^2}{2mL^2} = \frac{n^2 \pi^2}{2mL^2}\left(\frac{h}{2\pi}\right)^2 = \frac{n^2 h^2}{8mL^2}$$

$$n = \left(\frac{8mL^2 E}{h^2}\right)^{1/2} \quad \text{.......(2)}$$

In three dimensional quantum space, n^2 can be written as, $n^2 = nx^2 + ny^2 + nz^2$. A large number of states are associated with different value of nx, ny, nz. For each set of quantum numbers nx, ny and nz there exist a specific energy level called energy state. All these states lies within the sphere of radius n. The principle quantum number n can take positive integer values only so the energ states are defined only in positive octant ($\frac{1}{8}$ th equal part of sphere → octant) of the sphere made by the radius vector n. So the number of energy states within the sphere of radius n i given by,

$$N = \frac{1}{8}(4\pi/3)n^3 \text{..} (3)$$

Using equation (2) in (3)

$$N = \frac{1}{8}\left[\frac{8mL^2}{h^2}E\right]^{3/2}(4\pi/3)$$

$$N = \frac{\pi L^3}{6h^3}(8m)^{3/2}E^{3/2}$$

For the electron placed in three dimensional potential so x with each side equal to L, the volume of the box is V = L^3

$$N = \frac{\pi L^3}{6h^3}(2^2.2m)^{3/2}E^{3/2} = \frac{\pi L^3}{6h^3}.\ 2^3.\ (2m)^{3/2}E^{3/2}$$

$$N = \frac{4\pi V}{3h^3}(2m)^{3/2}E^{3/2}$$

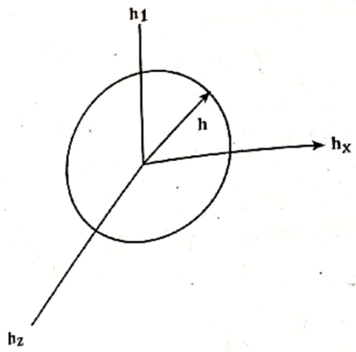
The density of state function is given by–

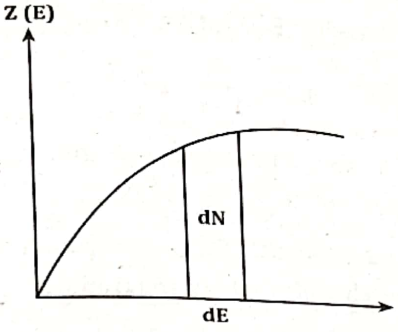
$$Z(E) = \frac{dN}{dE} = \frac{3}{2}\cdot\frac{4\pi V}{3h^3}(2m)^{3/2}E^{1/2}$$

$$Z(E) = \frac{2\pi V}{h^3}(2m)^{3/2}E^{1/2}$$

$$Z(E) = CE^{1/2} \text{..} (4)$$

Which is the equation of half parabola defined on first quadrant (since quantity under square root is only defined in positive space) as shown in figure (2)





17. Write about Fermi-Dirac distribution function.

Ans: The Fermi-Dirac distribution function applies to non interactive particles with half integer spin which must obey Pauli-exclusion principle. These particles which obey Fermi-Dirac statistics are called Fermians. It describes the probability of occupancy of particle in available energy levels in a given system.

Fermi-Dirac distribution function is written as.

$$F(E) = \frac{1}{1+e^{\frac{E-E_f}{KT}}}$$

Where, F(E) = occupation index or probability of occupation

E = Energy of electron

E_f = Fermi energy

T = Absolute temperature

K = 1.38×10^{-23} J/K called Boltzmassn constant.

Case-I, If E = E_f at 0°K, the Fermi-Dirac distribution function becomes indeterminable.

Case-II, if E < E_f at 0°K,

$$F(E) = \frac{1}{1+e^{-\infty}} = 1$$

i.e. There is absolute probability of finding the electrons below Fermi level at absolute zero.

Case-III, If E > E_f at T = 0°K,

$$F(E) = \frac{1}{1+e^{\infty}} = 0$$

This shows that electrons cannot occupy a state higher than Ferm level at 0°K.

Thus at T = 0°K, Fermi distribution function becomes step functio which switches between 0 and 1.

Casse-IV, If E ≫ E_f, T ≠ 0°K

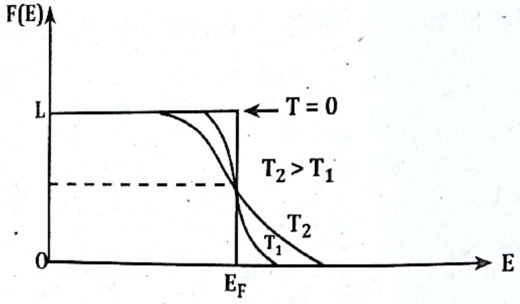
$$F(E) = \frac{1}{e^{\frac{E-E_f}{KT}}} = e^{\frac{-(E-E_f)}{KT}}$$

Hence at higher temperature, the Fermi-Dirac distributio function approximates to Maxwell-Boltzmann classica distribution function.

The higher the temperature, the higher is the probability of higher energy states being occupied.

One interesting property of Fermi energy is that for any temperature the occupation index is 0.5, if the energy of electron is equal to the Fermi energy.

$$F(E) = \frac{1}{1+e^{\frac{E-E_F}{KT}}} = \frac{1}{1+e^{0}} = \frac{1}{1+1} = \frac{1}{2} = 0.5 \text{ for } E = E_f$$



18. Find population density N(E) and Fermi energy E_f from Fermi-Dirac distribution function.

Ans: By using Fermi-Dirac distribution function we can calculate the number of energy states actually occupied by the electrons.

The number of energy states N(E) in an energy interval dE is given by the product of density of state function N(E) and Fermi-Dirac distribution function F(E).

i.e. $dN(E) = Z(E)\,F(E)\,dE$

The total number of electrons up to Fermi level is given by

$$N(E) = \int_0^{E_F} 2z(E)F(E)dE$$

Here, N(E) is also called "population density" or "population density function"

Since the probability of occupancy of electron from ground state to Fermi level is 100% i.e. F(E) = 1.

Therefore, $N(E) = \int_0^{E_F} 2z(E)dE = 2\int_0^{E_f} CE^{\frac{1}{2}}dE$

$$= \frac{2CE_F^{\frac{3}{2}}}{\frac{3}{2}} = \frac{4}{3}CE_F^{\frac{3}{2}}$$

Since, $C = \frac{2\pi V}{h^3}(2m)^{3/2}$

$$\therefore \quad N(E) = \quad (2m)^{3/2}(E_f)^{3/2}$$

$$N(E) = \frac{8\pi V}{3h^3}(2mE_f)^{3/2} \quad \text{..........(1)}$$

Which is the expression for population density.

Let, 'n' be the number of electrons per unit volume

$$n = \frac{M}{V} = \frac{8\pi V}{3h^3}(2mE_f)^{3/2} \quad \text{..........(2)}$$

or, $(2mE_f)^{3/2} = \frac{3nh^3}{8\pi} \Rightarrow 2mE_f\left(\frac{3nh^3}{8\pi}\right)^{2/3}$

$$E_f = \frac{h^2}{2m}\left(\frac{3n}{8\pi}\right)^{2/3} \quad \text{.............................} (3)$$

$$= \frac{h^2}{2m}\left(\frac{1}{8\pi}\right)^{2/3} . (3n)^{2/3}$$

$$= \frac{h^2}{2m}\left(\frac{1}{2^3\pi}\right)^{2/3} . \left(\frac{3M}{V}\right)^{2/3}$$

$$= \frac{h^2}{2m}\left(\frac{1}{4.\pi^{2/3}}\right)^{2/3} . \left(\frac{3M}{V}\right)^{2/3}$$

$$= \frac{h^2}{8m\pi^{2/3}} . \left(\frac{3M}{V}\right)^{2/3}$$

$$= \left(\frac{h}{2\pi}\right)^2 . \frac{4\pi^2}{.8m\pi^{2/3}} . \left(\frac{3M}{V}\right)^{2/3}$$

$$= \frac{\hbar^2\pi^{4/3}}{2m}\left(\frac{3M}{V}\right)^{2/3}$$

$$= \frac{\hbar^2(\pi^2)^{2/3}}{2m}\left(\frac{3M}{V}\right)^{2/3}$$

$$\text{Or, } E_f = \frac{\hbar^2}{2m}\left(\frac{3\pi^2 M}{V}\right)^{2/3} \quad \text{....................................} (4)$$

Hence, by using equation (4), Fermi energy for any metal can be calculated if number of electrons per unit volume for that metal is known.

19. What is Thermionic emission and work function?

Ans: When a metal is heated the free electrons becomes more energetic as the Fermi- Dirac statistics extends to higher temperature. Some of the electrons have sufficiently large energies to leave the metal and become free. This phenomenon is called Thermionic emission.

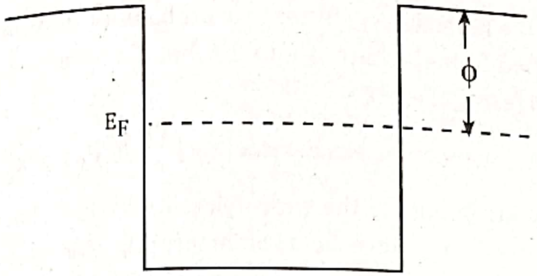


Fig: Fermi level and work function in metal

This situation is self limiting because as the electrons accumulate outside the metal, they prevent electrons from leaving the metal. Also the emitted electrons leave a net positive charge behind. Which pulls the electron in only those electrons with energy greater than $E_F + \phi$(Fermi energy + work function) can leave the metal. The electrons at Fermi level must given a threshold value of energy to leave the metal surface. This threshold value of energy is called work function (ϕ) of metal. The number of emitted electrons depends on the temperature by virtue of Fermi-Dirac statistics.

The conduction electrons behave as if they are free with in metal. We can therefore take PE to be zero with in the metal. The total energy E of the electron with in the metal is then purely kinetic i.e.,

$$E = \frac{1}{2}mu_x^2 + \frac{1}{2}mu_y^2 + \frac{1}{2}mu_z^2$$

Suppose that the surface of the metal is perpendicular to the direction of emission, say along x. For an electron to be emitted from the surface of metal, its K.E. $= \frac{1}{2}mu_x^2$ along x-direction must be greater than the potential energy barrier $E_F + \phi$

i.e., $\frac{1}{2}mu_x^2 > E_F + \phi$

The number of electrons per unit volume having momentum between Px and Px + dPx is given by N(Px) dPx

The number of electrons among at the surface of metal per unit time volume = velocity × N(Px) dPx.

$$= \frac{Px}{m} N(Px)\, dPx$$

Let r be the reflection coefficient i.e. probability that the electron will be reflected from the barrier into the metal. Therefore, probability of emission (escape) = 1-r

$$\text{Number of escaping electrons} = [1-r]\frac{Px}{m} N(Px)\, dPx$$

Adding contribution by the entire electrons, which have momentum greater than Pxo (threshold momentum), the emission current density can be written as.

$$J = e \int_{Pxo}^{\infty} (1-r)\frac{Px}{m} N(Px)\, dPx$$

$$J = \frac{e}{m}\int_{Pxo}^{\infty} (1-r)\frac{Px}{m} N(Px)\, dPx \;\dots\dots\dots\dots\dots\dots (1)$$

On calculation we get the number of electrons in the momentum range Px to Px + dPx as,

$$N(Px)\, dPx = \frac{4\pi mKT}{h^3} e^{\frac{E_f}{KT}} e^{\frac{-P_x{}^2}{2mKT}} dPx \;\dots\dots\dots\dots (2)$$

Solving equations (1) and (2) using integral table, we get

$$J = Ao\,(1-r)\,T^2\, e^{-\frac{\phi}{KT}} \;\dots\dots\dots\dots (3)$$

Where, (3) is called Richardson's equation which shows that the emission current density is heavily dependent upon both work function of material and it's temperature. As temperature increases emission current density increases. As work function increases emission current density decreases.

20. Derive the Richardson's expression for the thermionic emission for Schottky effect.

Ans: When an electric field applied to metal is increased, the work function is decreased and hence thermionic emission from the metal surface increases. This effect is called Schottky effect.

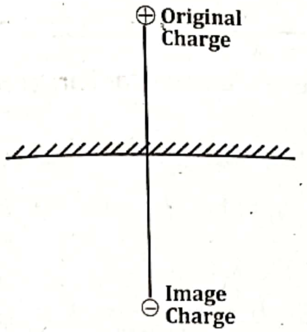


Fig (1): Metal surface with image charge

To study the lowering of barrier with increase in electric field we make the use of image charge method in electrostatics. In thermionic emission an emitted electron from the metal surface leave an equivalent positive charge behind. In this way an image charge builds up in the metal.

Image charge builds up in the metal electrode reduces the effective barrier height. This barrier reduction depends on the applied voltage. The electrostatic force between the real and image charge can be calculated by Coloumb's law.

$$F = \frac{(-e).e}{4\pi\epsilon o(2x)^2} = \frac{-e^2}{16\pi\epsilon o X^2}$$

The potential energy can be found by integrating F from x to ∞

$$V(x) = \int_{X}^{\infty} f(x)\, dPx = \frac{e^2}{16\pi\epsilon o}\int_{X}^{\infty}\frac{dx}{x^2} = \frac{-e^2}{16\pi\epsilon o}\left[-\frac{1}{X}\right]_{X}^{\infty}$$

$$V(x) = \frac{-e^2}{16\pi\epsilon o X} \;\dots\dots\dots\dots\dots (2)$$

The application of image field does not make effective difference in the work function and potential energy barrier for electron at metal-vacuum interface. The applied external electric field make clear difference.

If E is the applied electric field, then the potential energy due to this field will be

$$V(x) = -eEx \;\dots\dots\dots\dots\dots (3)$$

Now by adding contributions from image field and applied external field, the potential energy of te barrier will be

$$PE = E_F + \phi - \frac{e^2}{4\pi\epsilon_o X} - e\epsilon x \text{ ....} (4)$$

The reduction in work function (or barrier height) will be

maximum if, $\frac{d}{dx}$

$$\left(\frac{-e^2}{16\pi\epsilon_o x} - e \in x\right) = 0$$

$$\Rightarrow \frac{e^2}{16\pi\epsilon_o x^2} - eE = 0 \Rightarrow \frac{e^2}{16\pi\epsilon_o x^2} = eE \Rightarrow E = \frac{e}{16\pi\epsilon_o x^2}$$

$$x = \left(\frac{e}{16\pi\epsilon_o E}\right)^{\frac{1}{2}} \text{ ....} (5)$$

$$\text{Hence, } V_{max} = \frac{-e^2}{-16\pi\epsilon_o x^2} - eE_x = \frac{-e^2}{16\pi\epsilon_o}\left(\frac{16\pi\epsilon_o E}{e}\right)^{\frac{1}{2}} - eE\left(\frac{e}{16\pi\epsilon_o Ee}\right)^{\frac{1}{2}}$$

$$V_{max} = \frac{-e^{\frac{3}{2}}E^{\frac{1}{2}}}{(16\pi\epsilon_0)^{\frac{1}{2}}} - \frac{-e^{\frac{3}{2}}E^{\frac{1}{2}}}{(16\pi\epsilon_0)^{\frac{1}{2}}} = \frac{-2e^{\frac{3}{2}}E^{\frac{1}{2}}}{(16\pi\epsilon_0)^{\frac{1}{2}}}$$

$$V_{max} = -\left(\frac{4e^3E}{16\pi\epsilon_0}\right)^{1/2} = -\left(\frac{e^3E}{4\pi\epsilon_0}\right)^{1/2}$$

$$V_{max} = -\left(\frac{e^3E}{4\pi\epsilon_o}\right)^{1/2} \text{ ....} (6)$$

Here, V_{max} is the maximum reduction in potential energy barrier after taking into account at the effect of image field and applied external field.

The effective work function can now be written as

$$\phi_{eff} = \phi + V_{max} = \phi - \left(\frac{e^3E}{4\pi\epsilon_o}\right)^{\frac{1}{2}} \text{ ....} (7)$$

The reduced work function means the increase in emission current density. Which is given as.

$$J = A_o(1 - r)\, T^2 e^{-\frac{\phi_{eff}}{KT}} \text{ ....} (8)$$

This equation is called Richardson's equation for Schottky effect. For Simplicity we can omit the reflection coefficient considering negligible reflection into the metal. Then the equation (8) can be written as

$$J = A_o T^2 e^{-\frac{\phi_{eff}}{KT}},$$

$$\text{Where, } A_o = \frac{4\pi mk^2}{h^3} = 1.2 \times 10^6 \text{ Am}^{-2}\text{K}^{-2}$$

21. Write short note on contact potential : Fermi level at equilibrium.

Ans: When two metals with different Fermi energy and work function are brought in contact, electron from metal with higher Fermi level will start crossing over to metal with lower Fermi level metal having lost electrons become positively charged where as metal having received electrons becomes negatively charged. consequently a potential difference is developed all the Junction called contact potential.

This electron transfer from one metal to another reduces the total energy of electrons in metal-metal system. This process continues till the contace potential is large enough to prevent further transfer of electrons so the system reaches equilibrium. At equilibrium the Fermi level of both atom will be same.

The contact potential (ΔV is due to the difference in work functions of metal is metal in contact.

i.e. $e\Delta V = \phi_2 - \phi_1$

$$\Delta V = \frac{\phi_2 - \phi_1}{e}$$

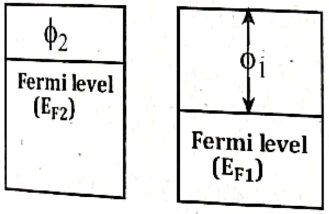


Fig 1: When metals are separate

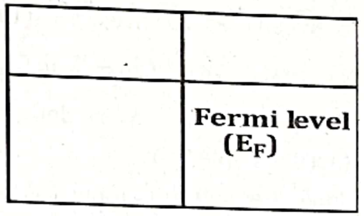


Fig 2: When metals are in contact

22. Discuss about Seeback Effect.

Ans: The Seeback effect is phenomenon in which temperature difference between two dissimilar metal produces a voltage difference.

When two metals one hot and other cold are brought together, more energetic electrons in the hot metal will diffuse to the total metal. The electrons diffusing to cold from hot side leave being an equivalent positive charge. So there is net potential difference between hot metal and cold metal due to difference in temperature.

The ratio of the potential difference ΔV across metal-metal Junction to temperature difference ΔT is called Seeback coefficient.

i.e. $S = \frac{\Delta V}{\Delta T}$ .... (1)

The Seeback coefficient for many metal is given by molt and Jones equation.

$S = \frac{-\pi^2 K^2 T}{3eE_{Fo}}$ .... (2)

Where E_{Fo} is Fermi energy at OK

This effect is used in Thermo couple. A thermocouple is a sensor used to measure temperature. Thermo couple consists of two wire legs made from different metals. The wire legs are weilded together at one end, creating a Junction. When the Junction experiences a change in temperature, a voltage is created. This change in voltage is calibrated to find the temperature.

If ΔV_H and ΔV_c are the potential of hot and cold junction and S_H and S_c are the respective Seebeck coefficient, the potential difference between the two wires called thermo-emf can be found as

ΔV_H and $\Delta V_c = \int_{T_C}^{T_H} (S_H - S_C)dT$ .... (3)

Here, $S_{Hc} = S_H - S_c$ is defined as thermo electric power for the thermocouple pair.

Solving equation (2) and (3) and then integrating, leads to familiar thermocouple equation.

$V_{Hc} = a\,\Delta T + b(\Delta T)^2$

Where a and b are called thermo couple coefficients and

$\Delta T = T_H - T_C$

T_H = Temperature of hot Junction

T_c = Temperature of cold Junction

According to equation (4) The variation of thermo emf (V_{HC}) with change in temperature is parabolic as shown in figure.

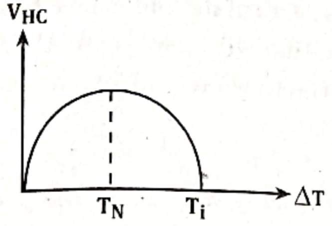


Fig: Variation in thermo-emf with change in temperature

Here T_N is called neutral temperature defines as the temperature at which thermo-emf is maximum.

T_1 is called inversion temperature defined as the temperature at which thermo-emf again falls to zero.

NUMERICALS

1. Find the wavelength of an electron accelerated by 100V.

Solution:

The whole potential energy will be converted into kinetic.

K.E. = P.E

or, $\frac{P^2}{2m} = ev$

$\therefore P = (2mev)^{\frac{1}{2}}$

Now, the wavelength of the electron will be

$\lambda = \frac{h}{p} = \frac{h}{(2mev)^{\frac{1}{2}}}$

$= \frac{6.624 \times 10^{-34}\ Js}{[(9.11\times10^{-31}kg)\ (1.6\times10^{-19}C)\ (100V)]^{\frac{1}{2}}} = 1.22 \times 10^{-10}m = 0.123\ nm$

2. In the photoelectric experiment, green light, with a wavelength of 522nm is the longest wavelength radiation that can cause photoemission of electrons from a clean sodium surface. Calculate the work function of sodium. If ultraviolet radiation with a wavelength 250 nm is incident to the sodium surface, what will be the K.F of photo-emitted electrons?

Solution:

At threshold, photon energy just causes the photo emission, that is, electrons just overcome the potential barrier ϕ. Thus,

The work function of sodium

$hf = \phi$

$$\text{or } \phi = \frac{hc}{\lambda} = \frac{(6.624\times10^{-34}\text{Js})\,(3\times10^{8}\text{ms}^{-1})}{522\times10^{-9}\text{m}} = 2.30\text{ ev}$$

The energy of incoming photon is

$$E_1 = \frac{hc}{\lambda_1} = \frac{(6.624\times10^{-34}\text{Js})\,(3\times10^{8}\text{ms}^{-1})}{250\times10^{-9}\text{m}} = 4.96\text{ev}$$

Only 2.3rev is required to eject electrons from the surface of sodium. So, the exces energy i.e., 4.96 ev – 2.30ev

= 2.58 ev

This will go to electron is form of K.E.

3. An electron is confined to an infinite potential well of size 0.1 nm. Calculate the ground energy of the electron and radian frequency. How this electron can be put to the third energy level?

Solution:

The energy of electron confined to an infinite potential well is given by:

$$E_n = \frac{n^2h^2}{8mL^2}$$

n = 1 for ground state

1 = 0.1 nm.

$$\text{So, } E_1 = \frac{1^2\times(6.624\times10^{-34}\text{Js})^2}{8\times(9.11\times10^{-31}\text{kg})\,(0.1\times10^{-9})^2} = 37.6\text{ ev}$$

Radian frequency of electron associated with this energy is

$$\omega = \frac{E}{\lambda} = \frac{37.6\times1.6\times10^{-19}\text{J}}{1.055\times10^{-34}\text{Js}} = 5.71\times10^{16}\text{ rad. s}^{-1}$$

Energy of electron in third energy level is

$$E_3 = n^2E_1 = 3^2\times37.6\text{ ev} = 338.4\text{ ev}$$

The short fall in energy required to put electron into the third level is

$$E_3 - E_1 = 338.4\text{ev} - 37.6\text{ ev} = 300.8\text{ ev}$$

This difference or short fall can be provided by a photon having exactly the same energy, no less, no more. The wavelength of such photon is given by

$$\lambda = \frac{hc}{E} = \frac{(6.624\times10^{-34}\text{Js})}{(3\times10^{8}\text{ms}^{-1})300.8\times1.6\times10^{-19}\text{J}} = 4.12\text{nm}$$

Hence,

Photon with a wavelength of 4.12 nm is X-ray photon.

4. For an electron confined to an infinite potential well of width 0.1 nm, determine the uncertainly in momentum and K.E.

Solution:

Since, Heisenberg's uncertainty principle states that,

$\Delta P_x \Delta x \geq h$

$$\Delta P_x \approx \frac{h}{\Delta x} = \frac{1.054\times10^{-34}\text{Js}}{0.1\times10^{-9}\text{m}} = 1.054\times10^{-24}\text{ kg. m.s}^{-1}$$

Now,

The uncertainty in K.E. associated with this uncertainty in momentum

$$\Delta m\,(KE) = \frac{\Delta P_x^2}{2m} = \frac{(1.054\times10^{-24}\text{kg m.s}^{-1})^2}{2\times9.11\times10^{-31}\text{kg}} = 6.1\times10^{-19}\text{J} = 3.81\text{ev}$$

5. A transmitter type vacuum tube has a cylindrical Th-coated W cathode, which is is 4cm long and 2mm in diameter. Estimate the saturation current if the tube is operated at 1600°C. The emission constant Ao = 3×10⁴ Am⁻²K⁻² for thorium on tungsten. Work function for Th or W is 2.6ev

Solution

Temperature (T) = 1600+273K = 1873K

Richardson's equation is

$$J = A_oT^2\exp\left[\frac{-\phi}{KT}\right]$$

$$= (3\times10^{4}\text{Am}^{-2}\text{k}^{-2})\,(1873\text{k})^2\exp\left[\frac{-2.6\times1.6\times10^{-19}}{(1.38\times10^{-33}\text{Jk}^{-1})\,(1873\text{k})}\right]$$

$$= 1.08\times10^{4}\text{Am}^{-2}$$

The emission surface area is

$$A = \pi dL = \pi(2(2\times10^{4}\text{m})\times(4\times10^{-2}\text{m}) = 2.5\times10^{-4}\text{m}^2$$

The saturation current or the maximum current obtainable is, I = JA

$= (1.08\times10^4 Am^{-2})\ (2.5\times10^{-4} m^2) = 2.7A$

6. **Consider a thermocouple pair from Al and Cu which have Fermi energes and x as shown in table. Estimate the emf available from this thermocouple if one junction is held at 0°C and the other at 100°C**

Metal	S at 0°C (μVK^{-1})	S at 27°C (μVK^{-1})	Ef(ev)	x
Al	−1.6	−1.8	11.6	2.78
Cu	+1.70	+1.84	7.0	−1.79

Solution:

For each metal there will be a voltage a across its given by integrating the seeback coefficient from To (at low temperature end) to T.

$$\Delta V = \int_{To}^{T} SdT = \int_{To}^{T} \frac{-x\pi^2K^2T}{3eE_{Fo}}dT = \frac{-x\pi^2K^2}{6eE_{Fo}}(T^2 - To^2)$$

The available emf (VAB) is the difference is ΔV for the two metals (A and B), so

$$VAB = \Delta V_A - \Delta V_B = \frac{-\pi^2K^2}{6e}\left[\frac{X_A}{E_{FAO}} - \frac{X_B}{E_{FBO}}\right](T^2 - To^2)$$

Where, $T = 373K$

$To = 273K$

For Al i.e., $E_{FAO = 11.6\,V}$, $x_A = 2.78$

For Cu i.e., $E_{FBO} = 7.0\ eV$, $x_B = -1.79$

$\therefore V_{AB} = -189\ \mu V - (+201\ \mu V) = -390\ \mu V$

We have, taking Pt as reference at junction 0°C, $V_{Al} - pt = 0.42\ mV$

and, $V_{cu} - pt = 0.76\ mV$

then,

$V_{Al} - Cu = V_{Al} - pt - V_{cu} - pt$

$= 0.42\ mV - 0.76 mV = -0.34 mV$

Chapter 2

FREE ELECTRON THEORY OF CONDUCTION IN METALS

1. **Describe different crystalline structure of cubic type.**

Ans:

a. Simple cubic structure

Atoms are placed at the corners of the unit cell geometry and by repeating this in all directions crystal is formed. The length of a side is equal to the distance between two atoms. Each touches along the cube edges only. The structure is closely packed.

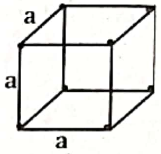


Fig: Unit cell of SCC

Only 1/8th of each of the atoms belongs to a unit cell. So, each unit cell has $8\times\frac{1}{8} = 1$ atom. The packing density is defined as the ratio of the volume of the unit cell.

$$\text{Packing density} = \frac{V_{atom}}{U_{unitcell}}$$

So, for SCC we have, volume of cube = a^3

$$\therefore \text{Packing density} = \frac{\frac{4}{3}\pi R^3}{a^3} = \frac{\frac{4}{3}\pi R^3}{(2R)^3} = 0.52 = 52\%$$

Thus, shows that 52% of simple cubic structure is occupied by atoms.

b. Face centered cubic (FCC) structure

Here, the atoms are placed as in the case of SCC at all the corners of the unit cell and additional atoms are placed at the intersection of face diagonals in all faces. Half of the atom at each of the face diagonals belongs to the unit cell. So the total number of atoms in a unit cell equals to

$$\left(\left(1 + 6 * \frac{1}{2}\right) = 4\right)$$

four. Each FCC unit cell has four atoms in it. The number of neighbouring atoms is termed as coordination number. In FCC structure the coordination number is 12.

In the structure, each atom touches each other along the face diagonal.

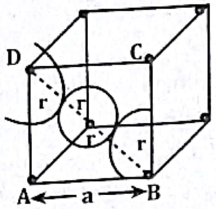


Fig: Unit Cell of FCC

Now,

Total no. of atom = $N_i + \frac{N_f}{2} + \frac{N_c}{8} = 0 + \frac{6}{2} + \frac{1}{8} \times 8 = 4$

If 'a' be the side of the cube of lattice constant and 'r' be the radius of atom. From figure:

$$BD = \sqrt{AB^2 + AD^2}$$

$$\text{or, } 4r = \frac{\sqrt{2}}{4}a$$

$$\therefore r = \frac{a}{2\sqrt{2}}$$

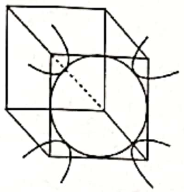
Then, Total volume occupied by atoms = no. of atom volume of atom

$$= 4 \times \frac{4}{3}\pi r^3 = 4 \times \frac{4}{3} \times \pi \times \frac{a^3}{(2\sqrt{2})^3}$$

$$\therefore \text{Packing density} = \frac{\text{volume of atom in cube}}{\text{volume of cube}} = \frac{\frac{\pi a^3}{3\sqrt{2}}}{a^3} = 0.74 = 74\%$$

C. Body centred cubic structure (BCC):

In this type of unit cell structure in addition to atoms at the corner of the cell geometry, there is an atom occupying the central place within the unit cell. All the corner atoms are touching the central atom. The coordination number is 8.



Total no. of atoms $N = Ni + \frac{N_f}{2} + \frac{N_c}{8} = 1 + 0 + \frac{8}{8} = 2$

If 'a' be the side of cube and 'R' be the radius of atom. Then,

Body diagonal = R + 2R + R = 4R

$$\text{So, } R = \frac{\sqrt{3}}{4}a$$

So, Total volume of atoms = no. of atoms × volume of an atom

$$= 2 \times \frac{4}{3}\pi r^3 = 2 \times \frac{4}{3} \times \pi\left(\frac{\sqrt{3}}{4a}\right)^3 = \frac{\sqrt{3}}{8}\pi a^3$$

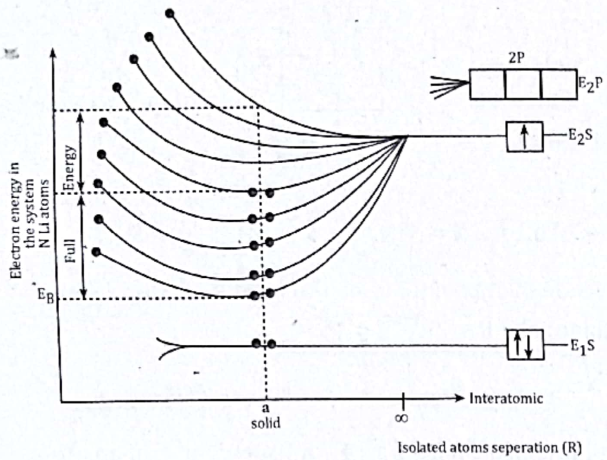
Hence,

$$\text{Packing density} = \frac{\frac{\sqrt{3}}{8}\pi a^3}{a^3} = \frac{\frac{\sqrt{3}}{8}\pi a^3}{a^3} = 0.68 = 68\%$$

So, the atom of body centered cubic structure is more compact than that of SCS.

2. **Explain, how energy bands are formed in solids, taking the example of N number of Lithium atoms for the explanation.**

Ans: There are number of orbitals in atoms of different elements (except hydrogen atom) which at normal temperature give rise to a solid when they come together in large number when identical orbitals, like Ψ_{1s} and Ψ_{2s} of of different atoms overlap with respective orbitals, although the energy levels formed by overlapping of identical orbitals are finely separated from each other. The separation is so small that for very large number of atoms required to form a solid, it can be assumed continuous. So that continuous energy level is regarded as energy band.



Fig(a) : Formation of 2s energy band from 2s orbitals when Li atoms come together to form Li solid

Fig (a) Formation of 2S energy band from 2S orbital. When N Li atoms come together to form Li solid When N Li atoms are brought together to form Li solid. We have N electrons in N Ψ_2, orbitals. Since its orbital is full and close to nucleus it is not affected band E_2 will be formed from $N\Psi_2$ orbitals. The atoms separated by distance greater than R = a will cause lesser amount of energy splitting. The inter atomic interactions between NY_2 orbitals spread the N energy levels from bottom level E_B to top E_T respectively which is determined by closest inter atomic distance a.

The single 2s energy level E_2, splits into N finely separated energy levels forming an energy bond. Consequently there are N separate energy levels each of which can take two electrons with opposite spins. The N electrons fill all level of N/2. So, the bond is half filled. The energy band formed by atomic energy levels are termed as their respective bands such as 2S band, 3d band etc.

As atoms approach each other to form solids. At first, outermost energy levels are affected. These close atoms molecular orbitals interact resulting in new orbital of which only where the energy of the system is lower will be occupied by electrons. The splitting of energy level in very large finely separated energies that can be considered continuous gives rise to energy band.

In solids, valence band is either completely or partially full. This is the highest occupied energy band with valence band. Valence band is the highest filled band. The next higher energy band is called conduction band which is completely or partially empty. The energy gap between valence band and conduction band is called forbidden gap where no. states are available and so no electrons.

The covalent forces of a crystal lattice are due to the valence band. If a valence electron absorbs enough energy, it can jump the energy gap and become a conduction electron after which it can freely move entirely through the material.

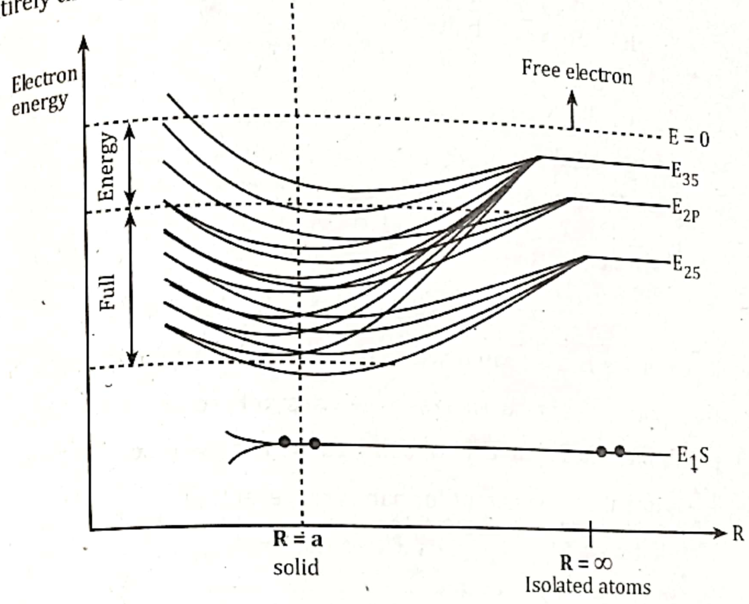


Fig: Formation of different energy bands in Li solid.

3. Explain the effective mass of electron within a crystal. How do you understand negative and infinite mass of electron?

Ans: For a free particle the relationship between energy and momentum and correspondingly the wave number is

$$E = \frac{p^2}{2m} = \frac{h^2k^2}{2m} \quad \text{..........................} \quad (1)$$

For a particle in the crystal, energy in terms of kapplies

Differentiating equation (1) w. r. to K, we have,

$\frac{dE}{dK} = \frac{h^2k}{m}$ .... (2)

But velocity, $v = \frac{P}{m} = \frac{hk}{m}$

And, comparing this with equation (2), we will get

$\frac{1}{h}\frac{dE}{dK} = v$ .... (3)

In equation (3), v is the velocity of particle in the crystal. Now, acceleration is given by

$a = \frac{dv}{dt} = \frac{d}{dt}\left(\frac{1}{h}\frac{dE}{dx}\right) = \frac{1}{h}\frac{d}{dt}\frac{dE}{dK}$

or, $\frac{d}{dt} = \frac{dk}{dt}\frac{d}{dk}$

Also, $a = \frac{h}{m}\frac{dk}{dt}$

So, $\frac{h}{m}\frac{dk}{dt} = \frac{1}{h}\frac{dk}{dt}\frac{d^2E}{d^2K}$

or, $m = \frac{h^2}{(d^2E/dK^2)}$.................... (4)

Thus, mass is associated with curvature of energy in k-space. For a free particle this curvature $\frac{d^2E}{d^2K}$ is constant, so the E-K diagram is parabolic. But in the crystal solid this is not the case. The curvature $\frac{d^2E}{d^2K}$ is not always constant for bands representing

F-K diagram for electrons within the solids

For solids we can write

$m^* = \frac{h^2}{(d^2E/dK^2)}$......................... (5)

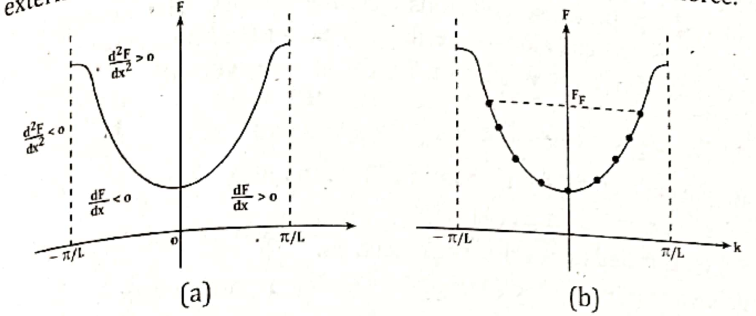
Where,

m* represents the effective mass of the electron in crystal structure.

The value $\frac{d^2E}{d^2K}$ is positive at concave upward of the E-k curve and is negative at concave downward of the E-K curve. Such that the effective mass is positive at K=0 is negative at vicinity of $K = \pm\frac{\pi}{a}$

The mass will be infinite if $\frac{d^2F}{dk^2} = 0$

The positive mass behaves normally to the applied force i.e., ordinary mass. The negative mass implies that is dis-accelerated by applied force, it is due to the applied force is over matched by internal lattice force. The infinite mass implies that it can't be accelerated by external force. The applied force is canceled by internal lattice force.



(a) (b)

Fig: Electrun motion in the conduction band of a conductor for a simplied 1-dimentional structure (a) Empty band (b) At Equilibrium.

4. Write about electron mobility, conductivity and resistivity.

Ans:

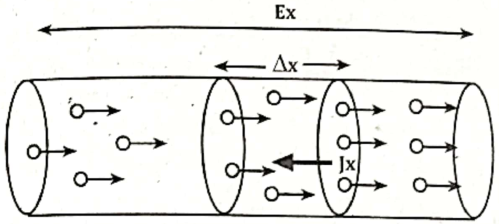


Fig: Drift of electrons in a conductor in the presence of an applied electric field

Although electrons are in thermal random motion, no net flow of electron in a particular direction persists in the absence of external electric field. So to maintain the flow of electron we need to apply an external electric field. V is the applied potential across the length L of the conduction. Then an electric field is given by:

$E = \frac{V}{L}$ .... (1)

The electric field is present at every point of the solid. The application of this field will result in acceleration of electron given by

$F = m^*a$

$F = -eE$

So, $a = \left(\frac{-e}{m^*}\right) . E$ .... (2)

Thus, in addition to random velocity, electron acquires a velocity in direction of the applied field. This directed velocity is lost after each collision with lattice atoms because of the fact that electron is very light compared to the atom. Therefore, only the part of velocity that is picked up between collisions actually counts. The average time between two collisions is termed as mean free time, then the final velocity of electron will be a τ. So, the average velocity is:

$v_{av} = a\frac{\tau}{2}$ .... (3)

Replacing factor 2 from equation (3) with unity, we get,

$\therefore v_{av} = a\tau$ .... (4)

So, v_{av} is called drift velocity of electron.

From equation (2) and (4), and Neglecting negative sign,

$V_D=(e\tau/m)E$

Or, $V_D=\mu_D E$ .... (5)

Where, μ_D = drift mobility of electron

So, $\mu_D = \frac{e}{m}\tau \ cm^2V^{-1}S^{-1}$ .... (6)

Assuming that all the electrons drift with V_D, the number of electrons crossing a plane of unit area per second can be obtained by multiplying drift velocity by free electron dencity N_e .

Electron flow = $N_e V_D S^{-1}m^{-2}$

Current density = charge flow

or, $J = N_e V_D e$

But, $V_D = \mu_D E = \frac{e}{m}\tau E$

So, $J = eN_e\frac{e}{m}\tau E = \frac{e^2N_e\tau}{m}E$ .... (7)

$\therefore J = \sigma E$ .... (8)

Where,

σ = Electrical conductivity

i.e., $\sigma = \frac{e\tau}{m}eN_e$

$\therefore \sigma = \mu P_e N_e$ Smm^{-1} .... (9)

So, it is clear from the equation (9) that conductivity of any metal is product of charge density and drift mobility of electron.

NUMERICALS

1. **Calculate the drift mobility of electrons in copper at room temperature. Conductivity, mass and density of copper are 5.9×10^5 sm/cm, 63.5 g/mol, and 8.93 g/cm^3 respectively.**

Solution:

Drift mobility = μd = J/en

Where, n = electron concentration

From Avogadro's number

1 mole of copper = 6.022×10^{23} atoms

Since, each cu atom contributes one electron to the conduction band.

The mass of 1 mole of Cu = 63.5 g

So, the concentration of electrons in copper is given as;

$$n = \frac{\rho N_A}{M_{at}} = \frac{(98.93\ g/cm^3)\ (6.022\times10^{23}/mol)}{(63.5\ g/mol)} = 8.5 \times 10^{22}/\ cm^3$$

$$\text{Now, drift mobility } (\mu d) = \frac{\sigma}{en} = \frac{(5.9 \times10^5\ 5m/cm)}{(1.6\times10^{-19}C)(5.5\times10^{22}/cm^3)}$$

$\therefore \mu d = 43.4\ cm^2v^{-1}s^{-1}$

Which is more than the experimental value of 32 $cm^2v^{-1}s^{-1}$ and due to fact that more than one electron is contributed to the conduction band by each atom.

2. **For silver with E_{Fo} = 5.5 ev and ϕ = 4.5 ev, calculate the total number of states per unit volume and compare this with atomic concentration of silver. Density and atomic mass of silver are 10.5 g/cm^3 and 107.3 g/mol respectively.**

Solution:

For silver,

Fermi energy at absolute zero = 5.5 ev

work function (ϕ) = 4.5 ev

Width of energy band (E) = 5.5 + 4.5 = 10ev

The density of states function reaches maximum at the center of band. Integrating from bottom of the band to center of band, we will find number of states per unit volume up to center of the band which is half the total number of states.

$$\frac{1}{2}N(E) = \int_0^5 Z\,(F)\,dE = \int_0^5 8\pi\sqrt{2}\,\frac{me^{\frac{3}{2}}}{n^3}$$

$$\text{or, } \frac{1}{2}N(F) = \frac{16\sqrt{2}\pi me^3_2}{3n^3}\,[5-o]^{\frac{5}{2}}$$

$\therefore N(E) = 2 \times 5.08\times10^{128}m^{-3} = 10.16 \times 10^{28}$ states /m^3

Now, density (ρ) = $\frac{nMat}{N_A}$

$$\text{or } n = \frac{\rho N_A}{Mat} = \frac{\left(\frac{10.5\times10^{-3}kg}{106m^3}\right)(6.022\times10^{23}/mol)}{107.9\times10^{-3}kg/mol}$$

$\therefore n = 5.85 \times 10^{28}$ atoms/ m^3

So, even with a crude approximation, as expected, the number of states is double the atomic concentration.

3. Calculate the Fermi energy at ox for copper given its density is 8.96 g/cm³ and atomic mass 63.5 g/mole. $N_a = 6.022 \times 10^{23}$/ mole.

Solution

Assuming each atom contributes one electron to the conduction band, the electron concentration of copper is

$$n = \frac{\rho NA}{Mat} = \frac{(8.36\ g/cm^{-3})\ (6.022\times10^{23}/\ mol)}{63.5\ g/mol} = 8.5 \times 10^{22}\ cm^{-3}$$

The Fermi energy at ox is given by

$$E_{FO} = (h^2/8me)\ (3n/2\pi)^{2/3} = \frac{(6.624\times10^{-34}JS)^2}{8\times9.1\times10^{-31}kg}\left(\frac{3\times8.5\times10^{28}m^{-3}}{2\pi}\right)^{\frac{2}{3}}$$

$\therefore E_{Fo} = 1.1 \times 10^{-18}J = 7ev$

4. Drift mobility of conduction electron is 43 $cm^2v^{-1}s^{-1}$ and mean speed is 1.2 × 10^6 ms^{-1}. Calculate the mean free path of electrons between collisions.

Solution:

The mean free path between collisions is given by

$Le = Ve\tau$

Where, ve is the velocity of electron τ is the mean free time between collision

$$\text{But, } \mu d = \frac{e\tau}{me} \text{ i.e., } \tau = \frac{\mu dme}{e}$$

$$\text{So, } Le = v_e\frac{\mu dme}{e} = 1.2\times10^6ms^{-1}\frac{(43cm^2v^{-1}s^{-1})(9:110^{-31}\mu g)}{1.6\times10^{-19}C}$$

$\therefore Le = 2.99 \times 10^{-8}m = 29.9$ nm.

Chapter 3

DIELECTRIC MATERIALS

1. Write a short note an Macroscopic view of polarization.

Ans: According to macroscopic view, polarization is the increase in electric flux density.

Mathematically,

$P = D - Do$

But, we have,

$D = \frac{Q}{A} = \epsilon o \epsilon oE$ (i.e., electric flux density when the dielectric medium is used).

$\therefore P = \epsilon o\ \epsilon rE - \epsilon oE = \epsilon o(\epsilon r - 1)E = \epsilon o x_e E$

Where xe = dielectric susceptibility.

Also, polarization is induced charge within dielectric per unit area

$P = \frac{Q'}{A}\ [Q' = Q - Qo]$

2. Describe microscopic/ electronic view of polarization.

Ans:

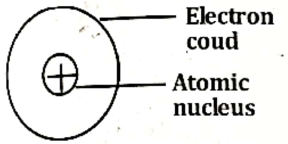


Fig: Neutral atom in E = 0

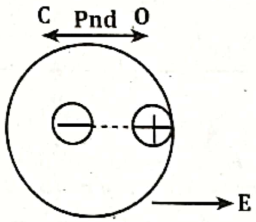


Fig: Atom placed in electric field

Electronic dipolar moment,

$P = Qa$

$Pind \propto E$

$Pind = \propto_e E$

Where $\propto_e$ = electric polarizability

The net charge within a neutral atom is zero. Furthermore, the centre of -ve charge of electrons coincide with the positive nuclear charge which means that the atom has no net dipole moment. When this atom is placed in an external field, the electrons being lighter in weight than the positive nucleus become easily displaced by the field. This results in separation of the –ve charge centre from the positive charge centre. This separation of positive charge and –ve charge centre. This separation of positive and –ve charges and the resulting induced dipole moment are termed as polarization.

An atom is said to be polarized if it possesses an effective dipole moment.

Induced dipole moment depends on electric field causing it.

$P_{induced} = \propto_e E$

$\propto_e = \frac{Ze^2}{m_e w_o^2}$ Where, z = atomic weight

ω_o = oscillation frequency of the center of electron cloud about nucleus

m_e = mass of electron

3. Derive the relation between microscopic and macroscopic view of polarization.

Ans:

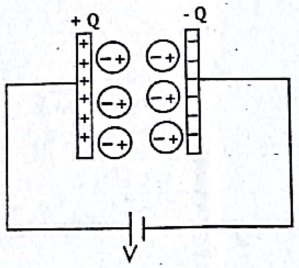


Fig: polarization of dielectric medium due to applied field

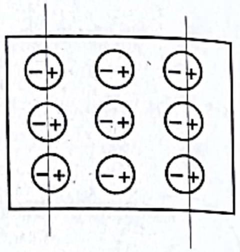


Fig: Bound molecular charges

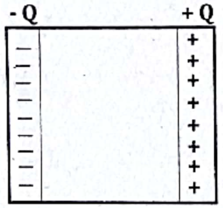


Fig: Surface polarization charge

Total induced dipole moments per unit volume,

$P = \frac{1}{V}[P_1 + P_2 + \dots\dots\dots + P_n]$

If Pav is average induced dipole moment,

$P = NP_{av}$ .... (i)

Where, N = no. of atoms per unit volume.

If -QP and +QP are surface polarization charges and 'd' is the seperatiuon then total dipole moment P per unit volume

$P = \frac{Qpd}{Ad} = \frac{Qp}{A}$

P = 6 = surface charge density

Average dipole moment of an atom $P_{av} = P_{ind} = \propto_e E$

From equation (i)

$P = N\propto_e E$

Also, $P = \epsilon_0 X_e E$, from macroscopic view

So, $\epsilon_0 X_e E = N\propto_e E$

$X_e = \frac{N\propto_e}{\epsilon o}$ and $\boxed{\epsilon r = 1 + \frac{N\propto e}{\epsilon o}}$

This equation relates macroscopic property of material (Er) to microscopic property (∝e).

4. Define local field in relation to polarization. Derive the Clausius - Massotti equation relating polarizability with permittivity.

Ans: The actual field experienced by a molecule in dielectric is defined as the local field, E_{local}. It depends not only on the free charges on the plates but also on the arrangement of all pólarized molecule around this point. The greater the polarization the greater is the local field.

In a material with cubic structure or liquid, the field is given by

$E_{local} = E + \frac{P}{3\epsilon_o}$

The induced polarization of the molecule (dipole moment) depends on this local field rather than applied field E.

$P_{ind} = \propto_e E_{local} = \propto_e\left(E + \frac{P}{3\epsilon_o}\right)$

$P = N\,P_{ind} = N\propto_e E + \frac{N\propto_e P}{3\epsilon_o} = \frac{N\propto_e E}{1 - \frac{N\propto_e}{3\epsilon_o}}$

So, $x_e\epsilon_o E = \frac{N\propto eE}{1 - \frac{N\propto e}{3\epsilon o}}$ or, $x_e\epsilon_o\left(1 - \frac{N\propto e}{3\epsilon o}\right) = N\propto e$

$$\text{or, } x_e\frac{(3\epsilon_o - N\alpha_e)}{3} = N\alpha_e \text{or, } x_e\epsilon_o - \frac{x_e N\alpha_e}{3} = N\alpha_e$$

$$\text{or, } x_e\epsilon_o = N\alpha_e\left[1+\frac{x_e}{3}\right] \text{or, } x_e\epsilon_o = N\alpha_e\left(\frac{3+x_e}{3}\right)$$

$$\text{or, } \frac{N\alpha_e}{3\epsilon_o} = \frac{x_e}{3+x_e} \therefore \frac{N\alpha_e}{3\epsilon_o} = \frac{x_e}{3+x_e} = \frac{\epsilon_r - 1}{\epsilon_r + 2}$$

This is the Clausius – Massotti equation. It gives the accurate relation between α_e and ϵ_r.

5. What are the different types of polarization mechanism in dielectric Medium ?

a. Electronic polarization medium
b. Ionic polarization
c. Oriental polarization
d. Inter-facial polarization

Ans:

a. Electronic polarization

- (same from the microscopic view of polarization)
- It is exhibited by covalent solids, like silicon, Germassium etc.

b. Ionic polarization

- It occurs in ionic crystals like NaCl, KCl, LiBr etc.

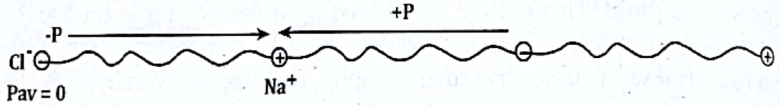


Fig (a)

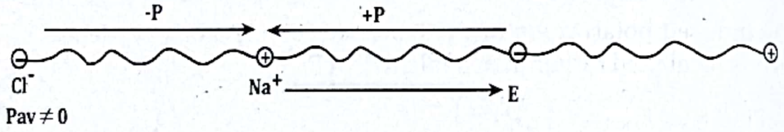


Fig (b)

In the absence of applied field, there is not net polarization of the specimen as the dipole moments of equal magnitude are lined up head to head and tail to tail as shown in fig (a)

When external electric field is applied as shown in fig (b), Cl^- ions are pushed opposite to the direction of applied field and Na^{++} ions in the direction of field about their equilibrium positions. Thus, dipole moment P+ increases to P'+ and P– decreases to P'– increases to P'+ and P– decreases to P'– . And hence net dipole moment is no longer zero.

i.e., $P_{av} = N\alpha_i E_{local}$

Where, α_i = ionic polarizability

C. Oriental/ dipolar polarization

It is exhibited by polar liquids such as water, alcohol, acetone etc and dipolar gases such as gaseous HCl,. steam etc.

Molecular like HCl possess permanent dipole moments. In the absence of external field, the dipole moments are randomly oriented and average field E is applied, both the ions experience force in opposite directions. But the nearly rigid bond between these ions holds them together. Thus, the molecule experiences a torque τ that tries to rotate dipole P_o and align with the applied field

$P = NP_o$

Where, P = Polarization vector

N = Number of molecules per unit volume

D. Interfacial polarization

It is the polarization due to accumulation of charge at interface between two materials or regions.

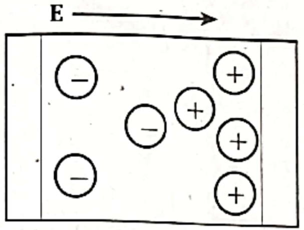


Fig: Accumulation of charge

There are always some defects within the dielectric material, which contribute, free charge carriers. In a dielectric material like ceramic, the positive and negative ions are equal, but positive ions are more mobile. After application of a field, positive ions migrate near to negative electrode. These positive charge at interface attracts electrons and thus bulk of positive and –ve ions remain together constituting dipole moment.

6. What is ferroelectricity?

Ans: Certain crystals like Barium Titanate ($BaTiO_3$) which have permanent polarization even in the absence of an applied field are called Ferro electric crystals and the phenomenon is called ferro electricity. At above 130°C the crystal structure of $BaTiO_3$ has cubic structure and the center of mass of positive and negative charges coincide and the net polarization is zero. Below 130°C, $BaTiO_3$ has tetragonal structure and center of mass of positive and negative charges do not coincide, the net polarization vector P is not zero. The temperature above which the ferro electricity is lost is called curie Temperature T_c. Below this, the whole specimen becomes spontaneously polarized and above T_c, disorderness of dipole increases which is called pyroelectric state. By applying temporary field and letting the crystal cool below T_c, we can induce spontaneous polarization to develop along the field direction.

Above T_c, the dielectric constant behaves as according to $E_r = \frac{C}{T-T_o}$

Where C is curie constant and To is Curie-Weiss temperature

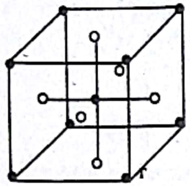


Fig (a):
$BaTiO_3$ cubic
crystal structure
above 130°C

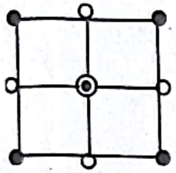


Fig (b): $BaTiO_3$
cubic structure above
130°C

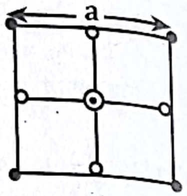


Fig (c): $BaTiO_3$
tetragonal structure
below 130°C

7. What is piezoelectricity?

Ans: Certain crystals become polarized under mechanical stress. Also, when an electric field is applied to these crystals, they will be mechanically deformed. These two effects are complimentary and termed as piezoelectricity.

The phenomenon requires a special crystal structure which has no center of symmetry. The NaCl unit cell has cubic symmetry and there is no net polarization when there is no force applied and the specimen is still not polarized after it has been applied by external force. In absence of external force, the centre of masses of both the charges coincide. But when an external force is applied, the centre of masses of both the charges are shifted and there is now net polarization of the specimen. The direction of the induced polarization depends upon the direction of applied stress.

8. Show that the dielectric loss per unit volume is a function of frequency of the applied field and the loss tangent.

Ans: When the applied field is sinusoidal ac, polarization of material under these conditions lead to the dielectric constant different from dc. Let us consider the oriental polarization in dipolar liquids. If the instantaneously induced dipole moment per molecule can readily follow the field variations then at any instant the dipole is

$P = \alpha_d E$ .... (i)

Where, $\alpha_d = \frac{Po^2}{3KT}$

At low frequencies, the dipoles can respond with the change in applied field, so dipolar polarizability is maximum in this region. In high frequency range, the dipoles cannot respond to the change in applied field, consequently the high frequency field cannot polarized the material i.e., the orientational polarizability is zero.

In the figure above, the dielectric is applied by a dc field Eo for a long time before the field is reduced to E. α_d (o) corresponds to polarizability at zero frequency. The induced dipole moments are also reduced as shown in fig. The decrease in dipole moments is achieved by random collision between molecules. Assume τ as average relaxation time between molecular collision, P is the induced dipole moment, then $P-\alpha_d(o)$ Eo is the excess moment which must disappear as t tends to infinity. τ is the average time to eliminate the access dipole moment. Rate of change of dipole moment is

$$\frac{dP}{dt} = \frac{P-\alpha_d(o)E}{\tau} \text{ .... (ii)}$$

Electric field in ac condition is

$E = E_o \exp(jwt)$ .... (iii)

From equation (ii) and (iii), we can write,

$$\frac{dP}{dt} = \frac{-P}{\tau} + \frac{\alpha_d(o)}{\tau} E_o \exp(jwt)$$

$P = \alpha_o E_o \exp(jwt)$ .... (iv)

Where, $\alpha_o(w) = \frac{\alpha(o)}{1+jw\tau}$

$\alpha_d(\omega)$ represents dipolar orientational polarizability under ac conditions.

From the complex dipolar polarizability, we can obtain the complex dielectric constant as polarizability is related to dielectric constant. So, the dielectric constant in ac is

$Er = Er' - jEr''$ .... (v)

Where,

$Er' = \frac{Er_{\infty} + Er'(o) - E'r_{\infty}}{1+\omega^2\tau^2}$ and, $Er'' = \frac{Er'(o) - E'r_{\infty}}{1+\omega^2\tau^2}$

Here, Er' is the real part and Er" is the imaginary part. Both are frequency dependent.

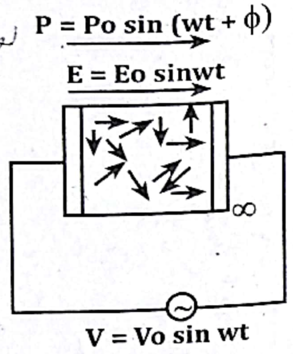


Fig: (a) AC field applied to dipolar medium

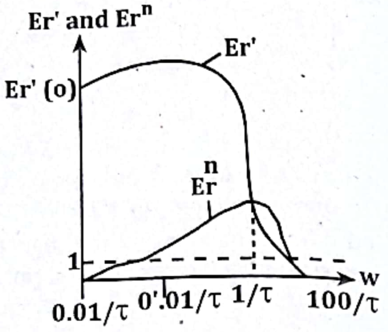


Fig: (b) Relative permittivity in ac

For a capacitor shown in fig (a), the admittance can be calculated as

$$Y = \hat{j}\,\frac{\omega A E_0 E_r(\omega)}{d} = \hat{j}\,\frac{\omega A E_0 E_r'(\omega)}{d} + \hat{j}\,\frac{\omega A E_0 E_r''(\omega)}{d}$$

or, $Y = J\omega C + Gp$

Where, $C = \frac{AE_0E_r'(\omega)}{d}$

$Gp = \frac{\omega A E_0 E_r''(\omega)}{d}$ .... (vi)

Since, there is no real power dissipation in C but there is real power dissipation in Gp (Rp). The input power is

$$Pin = VI = V(VY) = \hat{j}\,\omega CV^2 + V^2Gp$$

Thus, the power dissipated in the dielectric medium depends only on the imaginary part of the dielectric constant and is maximum when Er" is maximum as Gp is directly proportional to Er" from equation (vi)

The relative magnitude of Er" with respect to Er' is defined as tan

$\delta = \frac{\epsilon_r''}{\epsilon_r'}$

It is frequency dependent and peaks just beyond $\omega = \frac{1}{\tau}$

Again,

Power loss per unit volume is expressed as

$$P_{vol} = \frac{V^2Gp}{V_{volume}} = \frac{V^2\omega A\,\epsilon_0\epsilon_r''}{d}\,\frac{1}{dA}$$

$$P_{vol} = \frac{V^2}{d^2}\,\omega\epsilon_0\epsilon_r'' = \epsilon^2\omega\epsilon_0\epsilon_r'' = \epsilon^2\omega\epsilon_0\epsilon_r'\tan\delta$$

From above the equation, it is clear that power loss per unit volume is a function of frequency, applied field and loss tangent.

9. Describe how a thermal breakdown and electro mechanical breakdown results in dielectric breakdown results in dielectric breakdown in solids.

Ans: In solids, thermal breakdown results in dielectric breakdown as: There is a Joule heat σE^2 being released within the solid in case of finite conductivity of the insulation. At high frequencies, dielectric loss $V^2\omega\tan\delta$ is high. If the heat generated in the solid cannot be removed quickly by thermal conduction or other means, the temperature of dielectric increases and leads to higher electrical conductivity, which will cause more joule heating. Certain local area can become hot spots and if there are many such hot spots in the sample there can be a permanent conducting channel connecting two electrodes and hence a thermal breakdown of dielectric occurs. At the same temperature the breakdown field can be drastically reduced if the duration is increased.

Electromechanical breakdown results in dielectric breakdown in solids as:

The basis of the electromechanical breakdown is the force experienced by dielectric due to oppositely charged plates between which it is placed. The compressive forced to increase with the increase in voltage, hence the electric field. The thickness of the dielectric is reduced, which will increase the charge on the electrode as given by

$$Q = CV = \epsilon_0\epsilon_r\frac{AV}{d}$$

This will lead to further increase in compressive force and decrease in thickness. This process will continue till the dielectric deforms plastically causing the insulation to break down by increase in the field. The increase in E and decrease in d produces more joule heat and dielectric loss, heat, causing more problems for the mechanical stability of material. It is possible to reach thermal breakdown due to mechanical deformation. There is a possibility of internal cracks being developed and these cracks experience shear forces, which combined with large electric field, can cause crack propagation and thus mechanical and dielectric failure.

Numericals

1. **The electronic polarizability of Ar atom is 1.7×10^{-40} Fm^2. Calculate the static dielectric constant of solid Ar given its density is 1.8 g cm^{-3}. M_{at} = 39.95 gm $mole^{-1}$ and NA = 6.02×10^{23} $mole^{-1}$.**

Solution

Number of Ar atoms per unit volume

$$N = \frac{\rho N_a}{M_{at}} = \frac{(6.02\times10^{23}\ mol^{-1})\ (1.8gcm^{-3})}{39.952gmol} = 2.71 \times 10^{22}\ cm^{-3}$$

Using Clausius – Massotti equation,

$$\in_r = \frac{1+\frac{2N\propto_e}{3\in_o}}{1-\frac{N\propto_e}{3\in_o}} = 1.63$$

2. **The number of electrons per unit volume of Si is 5×10^{28} m^{-3} and its dielectric constant is 11.9. Calculate**

a. Electronic polarizability due to valence electrons per Si atom (if one could portion the observed crystal polarization to individual atoms.

b. If the Si crystal sample is electrode on opposite faces, by how many times the local field is greater than the applied field?

Solution

a. Using Clausius - Massotti equation,

$$\propto_e = \frac{3\in_o(\in_r-1)}{N(\in_r+2)} = \frac{3(8.854\times10^{-12}Fm^{-1})(11.-9-1)}{5\times10^{28}m^{-3}\ (11.9+2)} = 4.17\times10^{-40}Fm^2$$

b. The local field, $E_{10c} = E + \frac{P}{3\in_o}$

And, $P = x_e\in_o\in = (\in_r-1)\cdot\in_o\in$

$$\therefore \in_{10c} = E + \frac{(\in_r-1)\in_o E}{3\in_o} = E\left(1+\frac{\in_o(\in_r-1)}{3\in_o}\right)$$

$$\frac{\in_{10c}}{\in} = 1 + \frac{11.9-1}{3} = 4.63$$

Thus, the local field is 4.63 times greater than applied field.

3. **The optical index of refraction and the dielectric constant for glass are 1.45 and 6.5 respectively. Calculate the percentage of ionic polarizability.**

Solution

From Clausius – Massotti equation,

$$\frac{\in_r-1}{\in_r+2} = \frac{N(\propto_e+\propto_i)}{3\in_o}$$

At optical frequencies,

We know that,

$$\frac{n^2-1}{n^2+2} = \frac{N\propto_e}{3\in_o}$$

Now, dividing the second relation by the first, we get,

$$\frac{n^2-1}{n^2+2} \times \frac{\in_r+2}{\in_r-1} = \frac{\propto_e}{\propto_o+\propto_i}$$

$$\therefore \%\ \text{of ionic polarizability} = \frac{\propto_i}{\propto_e+\propto_i}\times100$$

$$= \left[1-\frac{\propto_e}{\propto_e+\propto_i}\right]\times100 = \left[1-\left(\frac{1.1025}{4.1025}\right)\left(\frac{8.5}{5.5}\right)\right]*100 = 57.98\%$$

Chapter 4

MAGNETIC MATERIALS

Some definitions

a. Magnetic dipole moment

- The strength of a magnetic dipole is called magnetic dipole moment. It may be thought of as a measure of dipole's ability to turn itself into alignment is presence of external magnetic field.
- A current carrying loop with A as enclosed area has current I circulating through it , μ_n is a unit vector coming out of the area A.
- The magnetic dipole moment is given by

  $\mu_m = \mu_n IA$
- If a coil has N number of turns,

$\mu_m = NIA$

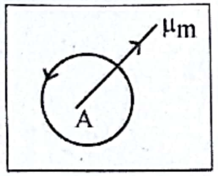


Fig: Current loop

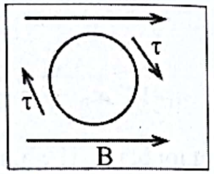


Fig: Dipole moment in a magnetic field

b. Magnetization vector 'M'

- It describes the extent of magnetization of the medium. 'M' is defined as magnetic dipole moment per unit volume.

c. Magnetic permeability 'μ'

- Magnetic field per unit magnetizing field is magnetic permeability

i.e., $\mu = \frac{B}{H}$

d. Relative permeability 'μ_r'

- It represents fractional increase in the magnetic field with respect to the field in free space when a material medium is introduced.

i.e., $\mu_r = \frac{B}{B_o} = (B/H\mu_o)$

or, $\mu_r\mu_o = \frac{B}{H} \Rightarrow \mu = \mu_r\mu_o$

1. Based on magnetization vector, classify the magnetic material.

a. Diamagnetic materials

Ans: When a diamagnetic material is placed in magnetic field, the magnetization field M is in opposite direction to the applied field. This causes the magnetic field within the material to be less than the applied magnetic field.

Their magnetic susceptibility is negative and relative permeability is less than unity.

Note: Magnetic susceptibility

$X_m = \frac{M}{H}\left(\frac{\textit{Magnetisation}}{\textit{Magnetising field}}\right)$

Superconductor are perfect diamagnetic with X_m −1. Other examples are Bi, Sb, Au, Cu, Water, Air, Diamond etc.

b. Paramagnetic materials

Ans: In the absence of external magnetic field, the molecular moments of paramagnetic materials are randomly oriented. So, the average dipole moment and net magnetization both are zero. When an external magnetic field is applied, the molecular magnetic moments take various alignments with the field as shown in fig (b).

i.e., $M_{av} \neq o$, $M = X_m H$

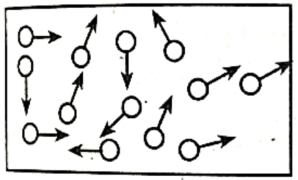


Fig (a): Randomly oriented molecular moments in the absence of exernal field. i.e., $M_{av} = o$, $M = o$

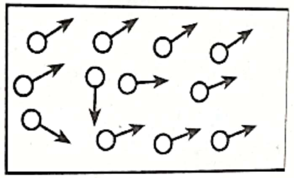


Fig (b): Alignments of molecular magnetic moments in presence of external field.

i.e., $M_{av} \neq o$, $M = X_m H$

The degree of alignment of molecular moment increases with the applied and hence magnetization M increases with the strength of applied field. Paramagnetic materials have small positive susceptibility. eg. Pt, Al, Cr, Mn and dilute solution of ferromagnetic materials.

c. Ferromagnetic materials

Ans: It possesses large permanent magnetization even in the absence of an applied field. The ferromagnetic crystal has magnetic moments of all crystal aligned in an orderly manner so as to give rise to the net magnetization vector M.

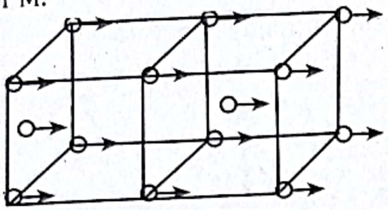


Fig: Magnetized region of ferromagnetic material

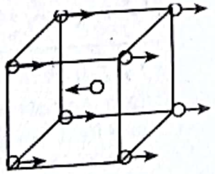
The relation between magnetization M and M_oH is highly non-linear. At sufficiently high fields, magnetization M saturates.

Magnetic susceptibility X_m is positive and large depends on applied field. Eg: Fe, Ni, Co

d. Anti-ferro magnetic material

Ans: They do not posses only magnetization in the absence of external magnetic field. They possesses magnetic order in which the magnetic moments of alternating atoms in the crystal align in the opposite direction as shown in fig below. The net result is that in the absence of an applied field, there is no net magnetization.

Anti-ferro magnetization occurs below a critical temperature called Neel temperature.

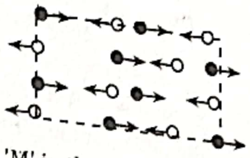


Xm is small and positive.

Eg. CoO, NiO, MnO, FeO, $FeCl_3$, $FeCl_3$, MnO_4 etc.

e. Ferrimagnetic material

Ans: It exhibits magnetic behavior similar to ferro-magnetism below a critical temperature called curie temperature. Above Tc, there is no net magnetization i.e., they become paramagnetic. They magnetic ordering of ferri-magnetic material is shown below.



All 'A' atoms have their spins aligned in one direction and all 'B' atoms have their spins aligned in opposite direction. As the magnetic moment of a 'B' atom is greater than that of 'A' atom, there is net magnetization 'M' in the crystal.

- X_m is small and positive

For eg: Generally ferrites are ferri-magnetic materials.

XFe_3O_4

Where, X = Mg, Cu, Mn, Ni, Zn, Ca.

2. What is magnetic domain and domain wall.

a. Magnetic domain

Ans: A magnetic domain is a region of the crystal in which all the spin magnetic moments are aligned to produce a magnetic moment in one direction only.

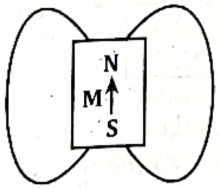
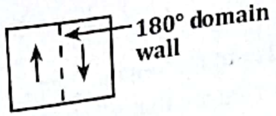


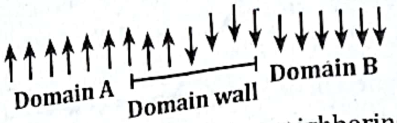
Fig: a single crystal with only one domain

Fig: Crystal with two domains with opposite magnetization

b. Domain wall/ Block wall

Ans: It is a boundary between two domains where the magnetization changes from one direction to another direction.





In a block wall, the neighboring spin magnetic moments rot gradually and it takes several hundred atomic spacing to rotate t magnetic moment by 180°.

3. Explain domain structure of ferromagnetic material.

Ans: A single crystal of iron does not necessarily possess a r permanent magnetization in the absence of an applied field. T reason for the absence of net magnetization is due to the formation magnetic domains that effectively cancel each other. A magne domain is a region of the crystal in which all the spin magne moments are aligned to produce a magnetic moment in one directi only.

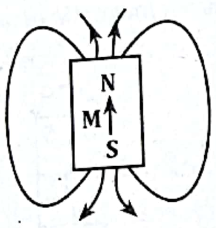


Fig (a): Crystal with only one domain, external magnetic field present

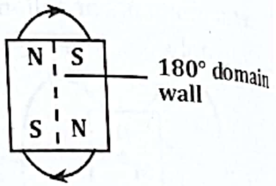


Fig (b): Two domains, redu magnetic field

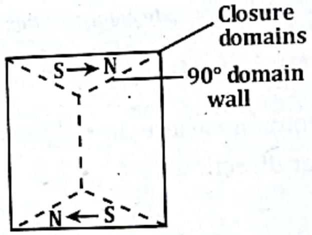


Fig (c): External fields at the end eliminated

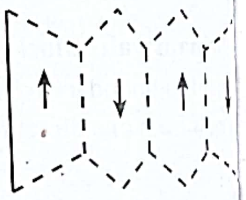


Fig (d): A specimen with sever domains and closure domains. N external magnetic field

Fig (a) shows a single crystal of iron that has a permanent magnetization. The crystal is like a bar magnet with magnetic field lines around it. There is a potential energy stored in these of magnetic field.

This energy in the external magnetic field is reduced if the crystal is divided into two domains as shown in fig (b). The two domains are separated by 180° domain wall. Here, the magnetization is in the opposite direction. The external magnetic field lines are reduced and there is now less potential energy stored in the magnetic field. There are field lines only at the end. The energy associated with these field lines can further be reduced by closing the ends with side way domains with magnetization at 90° as shown in fig (c). These end domains are called closure domains and have 90° walls. The magnetization is rotated through 90° walls. The magnetization is rotated through 90° through the wall.

When the magnetization changes form one direction to another, it requires energy i.e., energy is required to change atomic spin. The domain wall region where the neighbouring atomic spins change their relative direction from one domain to the neighbouring one has higher P.E than the bulk of the domain, where all the atomic spins are aligned. When the magneto static energy associated with the external field lines are reduced by the formation of domains, the potential energy in te wall is increased by adding additional walls. The creation of magnetic domains continue until the PE reduction in creating an additional domain is same as the increase in creating an additional wall. The specimen then possess minimum PE and is in equilibrium with no net magnetization. (Fig (d) shows a specimen with several domains and do net magnetization. The sizes, shapes and distribution of domains depend upon no. of factors including size and shape of the whole specimen.

4. Explain Domain Theory of Magnetization in details.

Ans:

The magnetization of each domain is normally along one of the preferred direction in which the atomic spin alignments are easiest. These directions are called easy direction. The domain have magnetization along these easy directions. When the crystal is placed in a magnetic field, the magnetization occurs such that the domain with magnetization along applied field will grow.

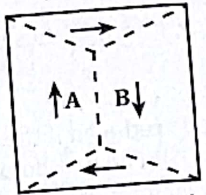


Fig (a) Un magnetized iron crystal in the absence of applied field

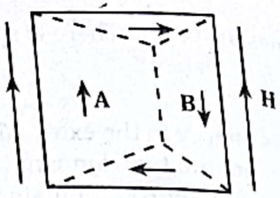
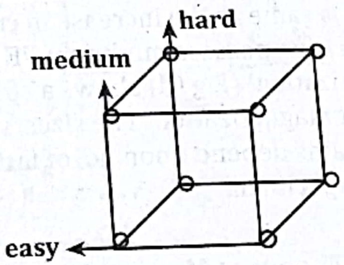


Fig (b) Magnetized crystal in presence of H

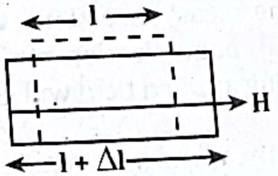
For simplicity, the magnetizing field is taken along an easy direction. The block wall between the domains A and B migrates toward the right, which enlarges domain A and shrinks domain B. The net result is that the crystal has an effective magnetization M along H. The migration of the block wall is caused by the spins in the wall and also spins in section B adjacent to the wall, being gradually rotated by the applied field. The magnetization process therefore involve the motion of block walls in the crystals.

5. What do you mean by magnetostriction?

Ans: The magnetization of the crystal generates strains or changes in the physical dimension of the crystal. For eg. When an iron crystal is magnetized along the hard direction by a strong field, the atomic spins within domains are rotated away from their easy direction towards the hard direction.

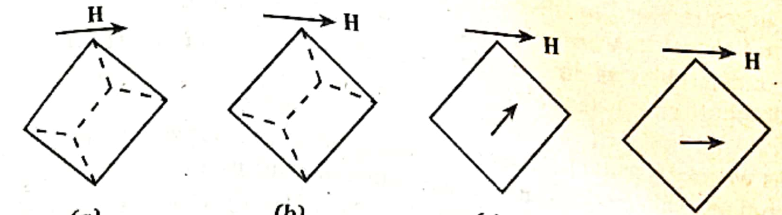


These e⁻s spin rotations involve changes in the e⁻ charge distribution around the atoms and therefore effect the inter-atomic bonding and hence the inter-atomic spacing. When an iron crystal is placed in magnetic field along an easy direction, it gets longer along the direction but contracts in transverse direction.



The longitudinal strain $\frac{\Delta l}{l}$ along direction of magnetization is called the magnetostrictive constant, l.

6. Explain Hysteresis loop, Eddy current losses with necessary diagrams.



(a) Reversible boundary motion

(b) Irreversible boundary motion

(c) Rotation of Motion

(d) Saturation of Motion

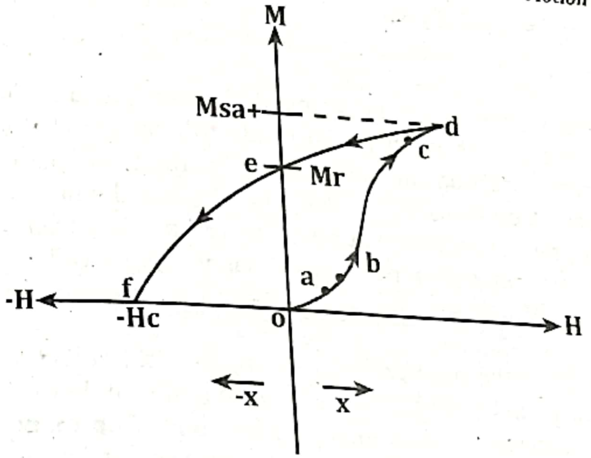


Fig: M vs H behavior

Ans: Let us take a small piece of iron and apply the very small magnetic field along x-direction. The domain walls within various grains begin to move small distances. The favourably oriented domains grow a little larger at the expense of those pointing away from the field as indicated by point 'a' as shown in fig. There is a very small but net magnetization along the field as indicated by 'oa' region in M vs H curve.

As we increase the magnetizing field, the domain motion extent larger distances as shown for point 'b'. Here, walls encounter various obstacles such as crystal imperfections, impurities etc which tend to hinder the motion. A domain wall that is stuck at an imperfection to a given field cannot move until the field increases sufficiently to provide necessary force to snap the wall free which then suddenly surges forward to the next obstacle. As the wall suddenly snap free and shoots forward to the next obstacle, it leads to heat generation. Sudden changes in the magnetization induce eddy current that dissipate energy via heating.

As we increase the field, magnetization continues to increase by jerky domain wall motions. Eventually, domain wall motions leave each crystal grain with a single domain and magnetization in one direction as indicated by point 'C'. The magnetization in massy grains will be pointing at some angle to H. From then until point 'd', the increase in applied field forces the magnetization to rotate towards the direction of H. When the applied field is sufficiently strong to align M along H, the specimen reaches saturation magnetization M_{sat}.

If we were to decrease and remove the magnetizing field, the magnetization in each grain would rotate to align parallel with the nearest easy direction in that grain. Further, in some grains additional small domains may develop that reduce the magnetization within that grain as indicated at point 'c'. The process from 'd' to 'e' leaves the specimen with a permanent magnetization called residual magnetization, Mr.

If we apply 'H' in the negative x-direction, 'M' still decrease and at sufficiently large field -]He],]M] will be zero and sample would have been totally demagnetized as indicated by point. **The magnetizing field 'Mc' required to totally demagnetize the sample is called the coercivity or coercive field.**

If we increase 'H' along 'x, the process from point 'f' onwards, similar to that described for magnetization from point 'a' to 'd'. The full M vs H behavior as the magnetizing field is cycled between +x to -x has closed loop said **hysteresis loop.**

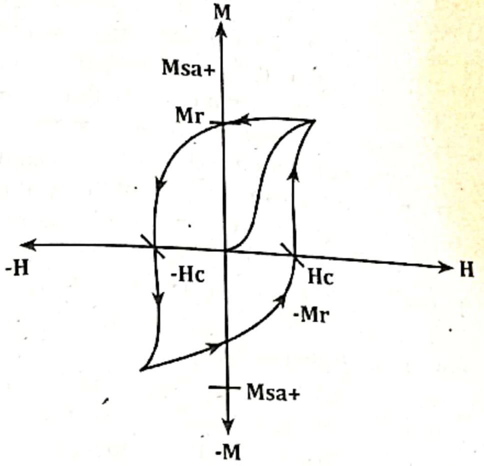
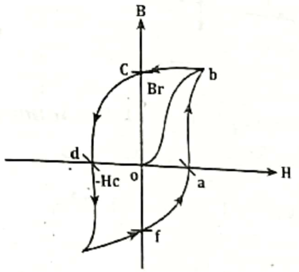


Fig: Hysteresis loop

The area enclosed within the loop is the energy dissipated per unit volume per cycle of the applied field. This is called **hysteresis loss.**

7. Explain Deperming method of Demagnetization.

Ans:



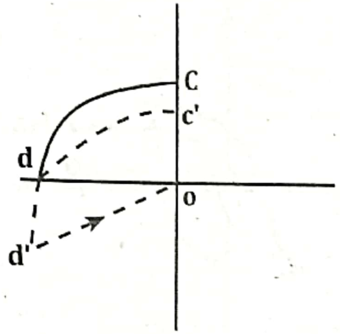


Fig (a) shows a B-H curve of a specimen.

Br is residual magnetism. If we try to demagnetize the material we need to apply reverse field, the magnetization would move from point C to point D as shown in figure. Now, if we switch off the applied field, 'B' does not actually remain zero. But recovers along 'd' to point C and attains some value Br. If we remove the field at point d as shown, the same recovers along d'O and the magnetization ends up being zero. However, to remove the field intensity at d', we need to know both the B-H curve behavior and the location of d'. This is not a easy task.

The simplest method to demagnetize the sample is first to reach saturation and then to continue cycling H but with gradually decreasing magnitude as shown in fig (c).

As H is cycled, with decreasing magnitude, the B-H curve will be traced smaller and smaller and end of at the origin where magnetization is zero. This process is called deperming.

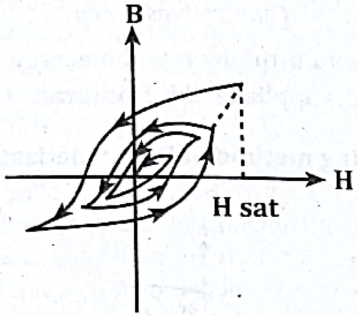
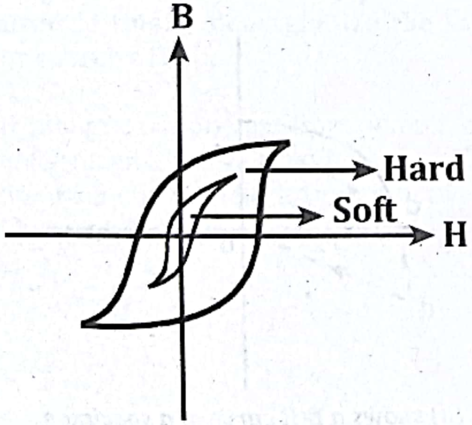


Fig (c) Deperming method of demagnetization.

8. Differentiate between soft and hard magnetic material.



Ans:

Soft magnetic materials	Hard magnetic materials
Soft magnetic materials are easy to magnetize and demagnetize. And hence require relatively low magnetic field intensities.	Hard magnetic materials are difficult to magnetize and demagnetize so require relatively high magnetic field intensities.
Their B-H loops are narrower and thus the hysteresis loop has smaller area. So, the hysteresis power loss per cycle is small.	Their B-H loops are broater and thus the hysteresis loop has larger area. So, the hysteresis power loss per cycle is large
They posses low coercivity.	They posses high coercivity which means that they need large applied field to be demagnetized.
They are suitable for high frequency application where repeated cycles of magnetization and demagnetization are involved eg: electric motor, transformer and inductor	They are useful as permassent magnets in variety of applications. eg: Alnico (Fe-Al-Ni-Co-Cu).

9. What type of magnetic material would you choose for electromagnetic relays?

Ans:

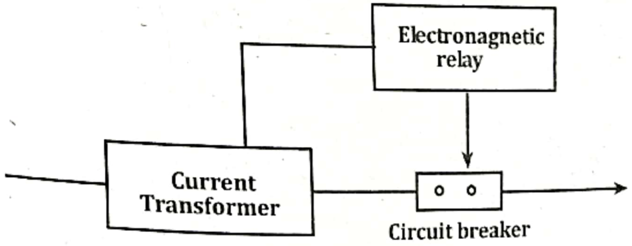


Fig: Block diagram of relay connection

Electromagnetic relay breaks/ opens a circuit during a fault. It is a protective device that is a protective device that is used to operate a circuit during fault. The relay detects the fault and sends signal to circuit breaker to break the circuit.

During fault, a large current flows through current transformer whose secondary coil is connected to electromagnetic relay. The emf induced in the secondary coil will force a current to flow through electromagnetic relay. The coil in the relay thus gets magnetized and pulls the circuit. breaker arm, thus opening the circuit. The magnetization process should be easy for the protection of a device. Relay requires frequent on and off i.e. it requires the iron core to be magnetized and demagnetized correspondingly. Thus, soft magnetic materials are used in electromagnetic relay.

Chapter 5

SUPER CONDUCTIVITY

1. What is Meissner Effect?

Ans:

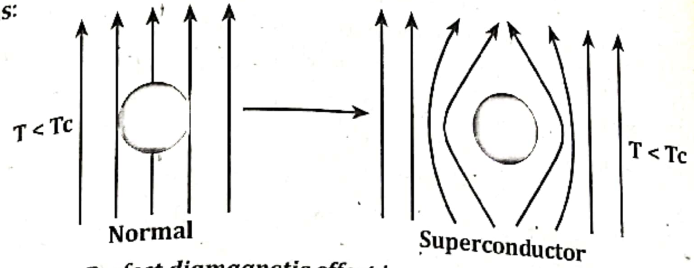


Fig: Perfect diamagnetic effect in a superconductivity state

If a superconductor is cooled in a magnetic field from above the transition temperature then at the transition temperature and below, the lines of induction of 3 are pushed out.

This phenomenon was first discovered by Meissner and Ochsenfeld in 1933, and is called Meissner effect. It shows that a bulk superconductor behaves in an applied external field as it inside the specimen B = 0 i.e., perfect diamagnetic effect.

Inside the superconductor B = 0

Thus,

$$B = \mu_o (H+M) = 0$$

$$\text{i.e., } H = -M \text{ and } x = \frac{M}{H} = -1$$

Which is true for a perfect diamagnet. The perfect diamagnetic behavior of superconductor cannot be explained simply by considering zero resistivity. Now let an electric field E would accelerate superconducting electrons then

$m = \left(\frac{dv}{dt}\right) = eE$ and super current

density, $J_s = n_s e v$

$$\left(\frac{dJ_s}{\delta t}\right) = \left(\frac{n_s e^2}{m}\right) E \text{ ....} \quad (1)$$

i.e., if there is zero resistance, relaxation time would accelerate infinitely, which is not possible, so E must be zero inside the metal. From Maxwell's equation.

$$\Delta\times E = -\frac{dB}{dt} = 0$$

i.e. B = constant .... (2)

Thus, B is constant in perfect conductor, which is contradiction to the Messiner effect. The behavior of superconductor is different from that of a perfect conductor. Hence the superconducting state has mutually independent properties.

i.e., zero resistivity and perfect diamagnetism i.e, $E = 0$ and $B = 0$ Materials showing such behavior are known as soft or type I super conductor.

2. Explain how strong magnetic field effects superconductor.

Ans: A sufficiently strong magnetic field will destroy the superconductor. The threshold value of the applied magnetic for the distribution of superconductivity is represented by H_c (T) and is a function of temperature. At critical temperature the critical field is zero is $H_c(T) = 0$

This fig depicts the variation of critical magnetic field with temperature where s is superconducting state and N is the normal state. The nature of the curve is approximately parabolic and statisfies the parabolic relation.

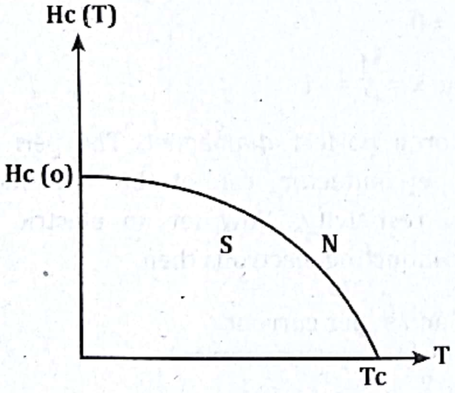


Fig: Variation of critical field with temperature

$$H_c(T) = H_c(o)\left[1-\left(\frac{T}{T_c}\right)^2\right]$$

Where $H_c(o)$ is the critical field at 0k. Thus we find that the superconducting state is stable only in some definite range of magnetic fields and temperature. For higher fields and temperatures, normal state is more stable.

3. Derive the relation of critical current in superconductor with necessary diagram.

Ans: In addition to $H_c(T)$ and T_c a third critical parameter is often specified to a conductor. A given superconductor cannot support an electric current density greater than a critical value J_c which is temperature dependent and reaches a maximum at T = 0k. The critical current density and critical magnetic field are closely related because on passing current necessarily produces a magnetic field.

If a wire of radius a of type I superconductor carries a current 'I' a surface magnetic field 'H' is set up. In a addition if a transverse magnetic field Ha is applied to the wire, then the condition for transition to normal state is

$$H_c = H_I + 2Ha$$

$$\text{i.e. } H_I = H_c - 2Ha$$

Where $H_I = \frac{1}{2\pi a}$ and at critical value $I = I_c$

$$\therefore \frac{I_c}{2\pi a} = H_c - 2H_a$$

i.e., $I_c = 2\pi a\,(H_c - 2H_a)$ .... (4)

Hence critical current decreases linearly with increase of the applied force until $I_c = 0$ and $H_a = \frac{H_c}{2}$. This is also called Silsbee's rule.

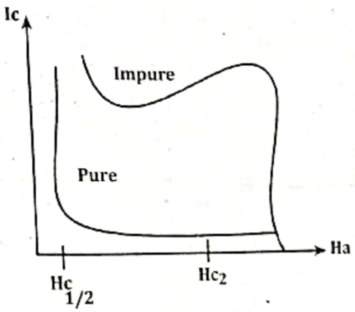


Fig: Variation of critical current of pure and impure materials with transverse applied field.

4. What are the differences between Type I and Type II superconductor.

Ans:

Type I Superconductor		Type II Superconductor	
1.	The superconductor in which the meissner effect is complete up to the critical field H_c is called type I superconductor.	1.	Those in which Meissner effect is incomplete, i.e., between lower, upper critical field H_c, and H_{c2}, B ≠0.
2.	The mean free path of type I superconductor is greater than penetration depth.	2.	It is observed that the penetration depth ($>_2$) of these superconductors is larger than coherence length (ξa).
3.	Pure specimen of various metals exhibits this type of behavior.	3.	Alloys, ceramic or hard transition metals with high value of electrical resistivity exhibits this type of behavior.
4.	These also called soft or pure superconductor.	4.	These are also called hard or impure superconductors.

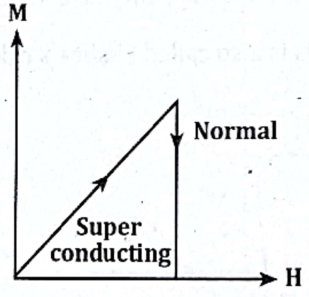


Fig: Magnetization curve

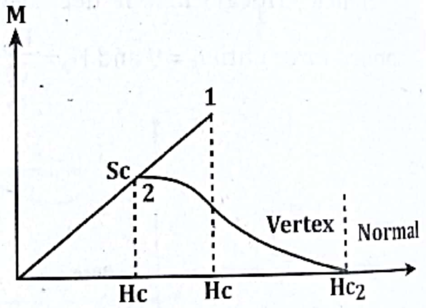


Fig: Magnetization curve

Numerical

1. The superconducting state of a lead specimen has critical temperature 6.2K at zero magnetic field and the critical field is 0.064 M $Am^{-1}A$ at 0k. Estimate the critical field at 4k.

Solution

We have,

$h_c(0) = 0.064\ MAm^{-1}$

$T = 4k$

$T_c = 6.2\ K$

We have formula as:

$$H_c = H_c(o)\left[1-\left(\frac{T}{T_c}\right)^2\right] = \left[1-\left(\frac{4}{6.2}\right)^2\right] = 0.037\ MAm^{-1}.$$

Hence, Critical field of lead (Pb) at 4k is 0.037 MAm^{-1}.

Chapter 6

SEMI-CONDUCTOR

1. Give a brief introduction of semiconductor

Ans: Semiconductors are materials whose electrical conductivity lies between that of good conductors and insulators. Elemental semi-conductors include antimony, arsenic, boron, carbon, germanium, selenium, sulphur and tellurium. Silicon is the best-known of these forming the basis of most integrated circuits. Common semiconductor compounds include gallium Arsenide, indium-antimonide and oxides of most metals of these gallium Arsenide (has) are widely used in low-noise, weak-signal amplifying devices.

At absolute zero, a semiconductor acts as an insulator. When temperature increases, Some of the balance electrons are able to cross the small forbidden gap and reach to the conduction band. Hence conductivity increases as temperature increases. The forbidden band gap does not exist in metal, it is narrow in semiconductor and wide in insulator.

2. Write short note on Electrons and Holes in semiconductor.

Ans: In semi-conductor, as absolute zero the valence band is fully occupied and the conduction band is empty. For a valence electron to cross the band gap, the electron must gain an energy at least equal to the band gap eg. The newly entered electron to the conduction band can move freely throughout the solid is termed as free electron. The space left behind in the valence band by electron excited to the conduction band is called hole. So in instrinsic semiconductor both the conduction electron and hole are generated together called electron hole pair.

Electron hole pairs can be generated not only by incident photon, but also by thermal energy. Thermal energy causes lattice band, and in a certain region, the atoms may be moving in such a way that the band is overstretched and rupturing of the over stretched band takes place. As the bond ruptures, the electron corresponding to that bond becomes free and is excited to the conduction band. When an external field is applied, the valence an electron move from the end at negative potential to the succeeding hole by creating a hole behind. Again the later electron Jump to succeeding hole by creating a hole behind. In this way there occurs a movement of a hole in the direction opposite to that of electrons. Thus the current in semiconductor is due to both movement of electron in conduction band called electron current and movement of holes in the valence band constitutes a current called hole current. Though the movement of electrons in the conduction band and the movement of holes in the valence band is in the opposite direction, the electron current and hole current have the same direction.

When an electrons fall from the conduction band, it fills the hole in the valence band. This process is called re-combination.

3. How does electrical conduction occur in semi-conductor?

Ans: When an electric field is applied to semiconductor, valance holes drift in the direction of applied field where as the conduction electron drift in the direction opposite to the applied field. The drifting of both electrons in the conduction band and holes in the valence band contribute to current in one direction.

Therefore the total current density in semi-conductor can be written as.

$J = n\,eV_{de} + peV_{dh}$

Where; n_e = electrons concentration in the conduction band

P = holes concentration in valence band

V_{de} = drift velocity of an electron

V_{dh} = drift velocity of holes

Since $\mu = \frac{V_d}{E} \Rightarrow V_d = \mu E$

$\therefore J = n\,e\mu_c E + Pe\mu_h E$

we substitute effective mass m* instead if mi

$\sigma E = (n\,e\mu_e + pe\mu_h)E$

$\sigma = n\,e\mu_e + pe\mu_h$

Where μ_e and μ_n are electron and hole mobility

$\mu_e = \frac{e\tau_e}{m_e^*}$, $\mu_n = \frac{e\tau_h}{m_h^*}$

In case of metal we used the mass of free electron to calculate mobility, but here we have to use effective mass of electron (m_e^*) in crystal to account for the internal forces that hinders its movement. For hole also we will use its effective mass (m_h^*), where is different from the effective mass of electron in conduction band.

4. Derive the equation for Electron concentrations in intrinsic semiconductor.

Ans: In an intrinsic semi-conductor every electron transferred to the conduction band leaves behind a hole in the valence band. Therefore, the total number of electrons in the conduction band is equal to the total number of holes in the valence band.

If n is the density of electrons i.e. the numbers of electrons per unit volume in the conduction of electrons, Z(E) be the density of states in the conduction band, F(E) is the probability, then in the equilibrium conduction,

$$n_e = \int_{E_c}^{\infty} Z(E)\, F(E)\, dE \quad \text{.......................(i)}$$

Where, $Z(E) = \frac{4\pi}{h^3}(2m^*)^{3/2}(E-E_e)^{1/2}$

is the density of states in the conduction band. As the bottom of the conduction band has an energy E_e and not zero we substitute $(E-E_e)$ for E.

The Fermi-Dirac distribution function is given by

$$F(E) = \frac{1}{1+\frac{E-E_F}{KT}} = e^{\frac{-(E-E_F)}{KT}} \quad \text{for } (E - E_F) >> KT$$

$$\therefore n = \frac{4\pi}{h^3}(2m^*)^{3/2}\int_{E_c}^{\infty}(E-E_c)^{1/2}\, e^{\frac{-(E-E_F)}{KT}}\, dE$$

$$n_1 = \frac{4\pi}{h^3}(2m^*)^{3/2}\int_{E_c}^{\infty}(E-E_c)^{1/2} e^{\frac{-(E_F-E_c)}{KT}}\, e^{\frac{-(E-E_c)}{KT}}\, dE$$

$$n = \frac{4\pi}{h^3}(2m^*)^{3/2}\, e^{\frac{(E_F-E_c)}{KT}}\int_{E_c}^{\infty}(E-E_c)^{1/2}\, e^{\frac{-(E-E_c)}{KT}}\, dE$$

Let, $\frac{E-E_c}{KT} = x \Rightarrow E - E_C = KTx$

$dE = (KT)\, dx$

When, $E = E_c$, $x = o$

$E = \infty$, $x = \infty$

and $(E-E_C)^{1/2} = (KT)^{1/2}x^{1/2}$

$$n = \frac{4\pi}{h^3}(2m^*)^{3/2}\, e^{\frac{(E_F-E_c)}{KT}}\int_0^{\infty}(KT)^{1/2}x^{1/2}e^{-x}KT\,dx$$

$$n = \frac{4\pi}{h^3}(2m^*)^{3/2}\, e^{\frac{(E_F-E_c)}{KT}}\int_0^{\infty}x^{1/2}e^{-x}dx$$

Using Integration table,

$$n = \frac{4\pi}{h^3}(2m^*KT)^{3/2}\, e^{\frac{(E_F-E_c)}{KT}}\,\frac{\sqrt{\pi}}{2}$$

$$n = 2\left(\frac{2m^*KT}{h^2}\right)^{\frac{3}{2}} e^{\frac{(E_F-E_c)}{KT}}$$

$$n_e = N_C\, e^{\frac{(E_F-E_c)}{KT}} \quad \text{..................... (3)}$$

Where $N_C = 2\left(\frac{2\pi m^*KT}{h^2}\right)^{\frac{3}{2}}$

is called effective density of state at the conduction band edge. So by multiplying the effective density of states at conduction band edge Boltzmassn's statistics, we can calculate the electron n at E_c.

5. Derive the equation for Hole concentration in intrinsic Semiconductor.

Ans: A 'hole' is a state of energy in the valence hand unoccupied by an electron. As F(E) gives the probability of occupation for an energy state. As F(E) gives the probability of occupation for an energy state. The probability that it may be unoccupied by an electron is [1−F(E)] which is something as that it may be occupied by a hole.

Therefore the concentration of holes in the valence band is given by

$$P = \int_{-\infty}^{E_V} Z(E)\,[1-f(E)]\,dE \quad \text{.................... (1)}$$

Here, $[1-f(E)] = 1 - \frac{1}{1+e^{\frac{E-E_F}{KT}}} = \frac{e^{\frac{E-E_F}{KT}}}{1+e^{\frac{E-E_F}{KT}}}$

As $E << E_F$ being in the valence band, so the term $e^{\frac{E-E_F}{KT}}$ in the denominator can be neglected in comparison to L.

$$\therefore [1-f(E)] = e^{\frac{(E-E_F)}{KT}} \quad \text{............... (2)}$$

From equation (2) we find that the value of [1-F(E)] i.e., the probability that the finding a hole decreases exponentially as we move from top to bottom of valence band. The value of Z(E) near the top of valence band is given by

$$Z(E) = \frac{4\pi}{h^3}(2m^*)^{3/2}(E_V - E)^{1/2}$$

Where, m* is the effective mass of hole.

$$P = \frac{4\pi}{h^3}(2m^*)^{3/2}\int_{-\infty}^{E_V}(E_V - E)^{1/2}\, e^{\frac{E-E_F}{KT}}\, dE$$

$$= \frac{4\pi}{h^3}(2m^*)^{3/2}\int_{-\infty}^{E_V}(E_V - E)^{1/2}\, e^{-\frac{(E_V-E)}{KT}}\, e^{\frac{(E_V-E_F)}{KT}}\, dE$$

$$= \frac{4\pi}{h^3}(2m^*)^{3/2} e^{\frac{(E_V-E_F)}{KT}}\int_{-\infty}^{E_V}(E_V - E)^{1/2}\, e^{-\frac{(E_V-E)}{KT}}\, dE$$

Let $\frac{E_V - E}{KT} = x \Rightarrow E_V - E = KTx$

$\Rightarrow dE = -KT\,dx$

When, $E = -\infty$, $x = \infty$

When $E = E_V$, $x = 0$

and $(E_V - E)^{1/2} = (KT)^{1/2}x^{1/2}$

$$P=\frac{4\pi}{h^3}(2m^*)^{3/2} e^{\frac{E_V - E_F}{KT}}\int_{\infty}^{0}(KT)^{1/2}x^{1/2}e^{-x}(-KT)dx$$

$$\therefore P = \frac{4\pi}{h^3}(KT.2m^*)^{3/2}\, e^{\frac{(E_V-E_F)}{KT}}\int_0^\infty x^{1/2}e^{-x}dx$$

$$= \frac{4\pi}{h^3}(2m^*KT)^{3/2}\, e^{\frac{(E_V-E_F)}{KT}}.\frac{\sqrt{\pi}}{2}$$

$$P = 2\left(\frac{2\pi m^*KT}{h^2}\right)^{\frac{3}{2}} e^{\frac{(E_V-E_F)}{KT}}$$

$$P = N_V = e^{\frac{-(E_F-E_V)}{KT}} \;\ldots\ldots\ldots\ldots\ldots\ldots\; (3)$$

Where $N_V = 2\left(\frac{2\pi m^*KT}{h^2}\right)^{\frac{3}{2}}$ is called effective density of states at the valence band edge.

6. Show Fermi level in intrinsic Semiconductor lies between the conduction band and valence band.

Ans: We have an electron concentration in conduction band,

$$n = N_C\, e^{-\frac{(E_C - E_F)}{KT}}$$

And the hole concentration in valence band

$$P = N_V\, e^{-\frac{(E_F - E_V)}{KT}}$$

Multiplying,

$$n.P = N_C . N_V\, e^{-\frac{(E_C - E_F)}{KT}}$$

This equation is commonly termed as "mass action law"

$$n\,P = N_c N_V\, e^{-\frac{E_g}{KT}} \;\ldots\ldots\ldots\ldots\ldots\ldots\; (7)$$

Where, $E_g = E_e - E_V$

∴ The product of electron and hole concentration is constant for a given temperature.

For intrinsic semiconductor, the electrons and holes are generated simultaneously

i.e. $n = P = ni$ (say)

From equation (7)

$$\therefore n_i^2 = (NcN_V)^{1/2}\, e^{-\frac{E_g}{2KT}}$$

Since $P = ni$

$$\therefore N_V\, e^{-\frac{E_F - E_V}{KT}} = (NcN_V)^{1/2}\, e^{-\frac{E_g}{KT}}$$

For an intrinsic semiconductor, the Fermi energy is denoted by E_{Fi}

$$e^{-\frac{E_F - E_V}{KT}} = \left(\frac{N_C}{N_V}\right)^{\frac{1}{2}} {}^{-\frac{E_g}{2KT}}$$

Talking ln on both sides

$$-\left(\frac{E_F - E_V}{KT}\right) = Ln\left(\frac{N_C}{N_V}\right)^{\frac{1}{2}} e^{-\frac{E_g}{2KT}}$$

$$-E_{Fi} + E_V = \frac{KT}{2}\ln\left(\frac{N_C}{N_V}\right) - \frac{E_g}{2}$$

$$E_{Fi} = E_V + \frac{E_g}{2} - \frac{KT}{2}\ln\left(\frac{N_C}{N_V}\right)$$

$$E_{Fi} = E_V + \frac{E_g}{2} - \frac{KT}{2} \ln\left(\frac{n^{*}e}{m_n^{*}}\right)^{\frac{3}{2}}$$

If effective masses of electrons and holes are equal ,

$$Efi=Ev+\frac{Eg}{2}$$

$$Or, Efi=Ec-\frac{Eg}{2}$$

In silicon the effective mass of hole is slightly greater than that of electron, so the intrinsic Fermi level is slightly above the mid-gap.

7. Discuss about n-type semiconductor.

Ans: When a pentavalent impurity like Arsenic(As) ,Phosphorus(P), Antimony(Sb) is added to pure, semi-conductor, a n-type semi-conductor is formed. The four electrons out of five valence electrons of the impurity make covalent bond with four valence electron of silicon. So one electron remains free on every odd of impurity. This extra electron remains continuously revolves around the impurity ion core as similar to the electron in the hydrogen atom as shown in the figure.

The energy required to free this extra electron from inner site calculated in analogy to ionize hydrogen atom, to ionize hydrogen atom, which means to free an electron from the ground state of the hydrogen atom.

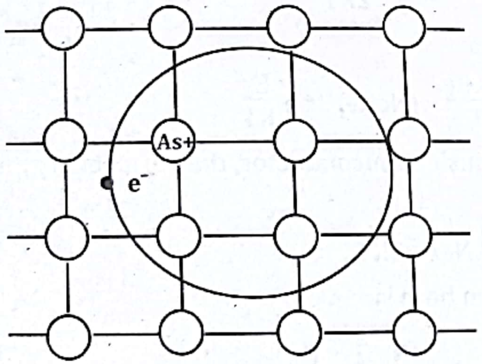


Fig: Arsenic doped Si-crystal.

The binding energy of the electron in a hydrogen atom is

$$E_b = \frac{e^4 m_c}{8\epsilon_0{}^2 h^2} = 13.6 \text{ ev}$$

To free the extra electron from the inner ion core, the energy required is

$$E_b{}^{si} = \frac{e^4 m_e{}^{*}}{8\epsilon^2 h^2} = \frac{e^4 m_e{}^{*}}{8\epsilon^2 \epsilon_r{}^2 h^2} = 0.032 \text{ ev}$$

Where, effective mass of electron m_e^{*} and $\epsilon_r = 11.9$

This energy is comparable to the average thermal energy of atomic vibration at room temperature ≈KT (≈0.07ev). Thus the fifth valence electron can be readily freed by thermal vibration of Si lattice.

In this type of semiconductor the concentration of electron increases with every addition of impurity. So it is called negative type (n-type) semiconductor. Since the pentavalent atom donate electron for conduction. So it is called doner atom. In n-type semiconductor electrons are majority charge comers and holes are minority charge carrier.

If n and p represent extrinsic electron and hole concentration of semiconductor, Nd be doner concentration, then supposing all doner sites are ionized (i.e., all donor atom contribute an electron to conduction band).

$$n = p + Nd$$

$$\text{but, } n.p = n_i^2 \Rightarrow p = \frac{n_i^2}{n}$$

$$n = \frac{n_i^2}{n} + Nd = \frac{n_i^2 + n.Nd}{n}$$

$$n^2 - n.Nd - n_i^2 = 0$$

$$n = \frac{-(-Nd) \pm \sqrt{(-Nd)^2 - 4.1\,(-n_i^2)}}{2}$$

$$n = \frac{Nd}{2} \pm \sqrt{\frac{Nd^2}{2} + n_i^2}$$

Ignoring the –ve sign and for $\frac{Nd^2}{4} >> n_i^2$

$$n = \frac{Nd}{2} + \frac{Nd}{2}$$

$$n = Nd$$

Therefore in n-type Semiconductor the electron concentration is nearly equal to donor concentration.

Here, $n.p = n_i^2$

$$\therefore p = \frac{n_i^2}{n} = \frac{n_i^2}{Nd}$$

Therefore hole concentration in n-type semiconductor is: $p = \frac{n_i^2}{Nd}$

The hole concentration in n-type semiconductor is less than the intrinsic hole concentration. This is because some of the electrons in conduction band recombine with holes in valence band so as to maintain $n.P \approx n_i^2$. As we go on increasing concentration of donor atoms there by increasing majority carrier concentration, the minority carrier concentration will be decreased simultaneously. This is called minority carrier suppression.

The conductivity of a semiconductor is then

$$\sigma = n\, e\, \mu_e + P\, e\, \mu_n = Nde\, \mu_e + \left(\frac{n_i^2}{Nd}\right) e\, \mu_n$$

or, $\sigma \approx Nd\, e\, \mu_e$

or, $\sigma = e\, Nd\, \mu_e$

So the conductivity is mainly due to mobility of electrons in n-type semiconductor.

8. Discuss about P-type Semiconductor.

Ans: When a trivalaent (group-III elements in periodic table) impurity like Boron (B), Aluminium (Al), helium (He) or indium (In) is added to pure silicon, a p-type semiconductor is formed.

The three valence electron of these impurities can form only three complete covalent bonds with silicon which has valence electrons. There is deficiency of one electron to form forth bond. This means an electron vacancy is left in the fourth bond. This give rise to a hole.

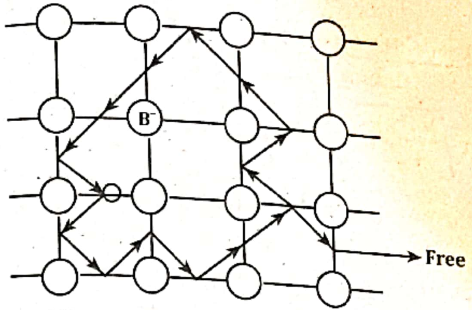


Fig: Borom doped Si crystal. The hole orbitals around the B⁻ site. Thermally vibrating Si atoms provide enough energy to free the hole from the e⁻ site into VB.

A nearby electron can tunnel to this hole and displace the hole further away from the Boron atom. As the hole moves away, it gets attracted by the negative charge left behind on the Boron atom and therefore taken into orbit around the B^- ion as shown in figure. The binding energy of this hole to the B^- ion can be calculated using the hydrogenic atom analogy. This binding energy turn out to be small ~ 0.05 eV, so at room temperature the thermal vibration of lattice can free the hole away from the B^- site.

In this type of semiconductor the concentration of holes increases with every addition of impurity. So it is called positive type (p-type) semiconductor. Since the trivalent atom creates a hole and this has tendency to accept an electron, so trivalent impurity is called acceptor atom.

Let n and p are the electron and hole concentration in the semiconductor and Na is the acceptor concentration. The doping of trivalent impurities will increase the hole concentration in valence band but but does not increase the electron concentration in conduction band (or cannot put the valence electron to conduction band to create a hole in valence band) so,

$p = Na + n$

Since, $n.p = n_i^2$

$$n = \frac{n_i^2}{P}$$

Therefore, $P = Na + \frac{n_i^2}{P} = \frac{P\,Na + n_i^2}{P}$

$P^2 - P\,Na - n_i^2 = 0$

$$P = \frac{-(Na) \pm \sqrt{(-Na)^2 - 4.1\,(-n_i^2)}}{2}$$

$$P = \frac{Na}{2} \pm \sqrt{\frac{Na^2}{4} + n_i^2}$$

Ignoring the negative sign and for $\frac{Na^2}{4} >> n_i^2$

$$P \approx \frac{Na}{2} + \frac{Na}{2} = Na$$

Therefore, in p-type Semiconductor hole concentration is nearly equal to acceptor concentration.

Here, $n.p = n_i^2$

$$n = \frac{n_i^2}{P} = \frac{n_i^2}{Na}$$

Therefore, electron concentration in p-type Semiconductor is: $n = \frac{n_i^2}{Na}$

In trivalent doping of semiconductor, the hole concentration increases and electron concentration decreases. This means the minority charge carries are supressed by majority charge carriers. This phenomena is called minority carrier suppression.

9. What is Compensation Doping? Explain in detail.

Ans: Compensation doping is a term used to describe the doping of semiconductor with both donors and acceptors to control the properties. The effect of donors compensate for the effect of acceptors and vice versa..

For example; a p-type semiconductor doped with Na acceptor can be converted to an n-type semiconductor by simply adding donor until the concentration Nd exceeds Na. The holes due to acceptor doping will recombine with the electrons due to donor doping.

So the electron concentration is: $n = Nd - Na$

from mass-action law, $n.p = n_i^2$

$$\therefore \text{Hole concentration } p = \frac{n_i^2}{n} = \frac{n_i^2}{Nd - Na}$$

Similarly, a n-type semiconductor doped with Nd donor can be converted to a P-type semiconductor by simply adding acceptor until the concentration Na exceed Nd. The electrons due to donor doping will recombine with the holes due to acceptor doping,

So the hole concentration is: $p = Na - Nd$

From mass action law, $n.p = n_i^2$

Where n_i is the intrinsic concentration in semiconductor.

10. Explain how carrier concentration of a semi-conductor depends on temperature.

Ans: The temperature dependence of electron concentration has three regions

a. Low - temperature range ($T < T_s$)

As the temperature is increased starting from very low temperature, some of donors become ionized and donate the electrons to CB as shown in figure (1). The Si - Si bond breaking i.e. thermal excitation from VB to CB is difficult because it takes too much excitation from VB to CB is difficult as it takes too much energy. Sice the donor ionization energy $\Delta E = E_C - E_d$ is very small (i.e. $\Delta E << Eg$), thermal generation involves exciting electrons from Ed to Ec.

The electron concentration at low temperature is given by an expression as similar to intrinsic case.

$$n = (N_C N_V)^{1/2}\, e^{-\frac{Eg}{2KT}} \quad \text{.......................} \ (1)$$

In our case it is given by,

$$n = \left(\frac{1}{2} N_c N_d\right)^{\frac{1}{2}} e^{-\frac{\Delta E}{2KT}} \quad \text{.......................} \ (2)$$

The numerical factor $\frac{1}{2}$ in equation (2) arises because donor occupation statistics is different by this factor from the usual Fermi-Dirac function.

The donor ionization continues until we reach a temperature called saturation temperature (TS) where all donor atoms have been ionized. This temperature range up to T_s is called ionization range.

b. Medium - temperature range ($T_S < T < T_i$)

Nearly all donors have been ionized in this range, $n_i = N_d$. This condition remains unchanged until $T = T_i$ as shown in figure (2). This is the temperature when n_i = Nd. It is that temperature range $T_S < T < T_i$ that utilizes that n-type doping in pn Junction device applications. This temperature range is often referred to as the extrinsic range.

c. High-temperature range ($T > T_i$)

At very high temperature, thermal vibrations of the atoms will be so strong that many Si-Si bonds will be broken. The electron concentration in the CB will then be mainly due to thermal excitation from VB to CB as shown in figure (3). So ni is now much larger than Nd. But this process also generates an equal concentration of holes in the VB. Accordingly the semiconductor behaves as if it were intrinsic. The electron concentration at this temperature will therefore be equal to intrinsic concentration ni as given by equation (1). This temperature range is often referred to as intrinsic range.

Figure (4) shows the change in electron concentration with temperature in n-type semiconductor.

Figure (5) shows the dependence of intrinsic carrier concentration n Ge, Si and Ga As with temperature.

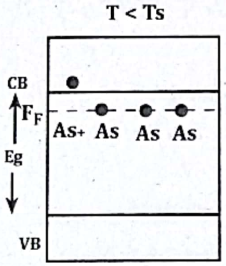


Figure (1)

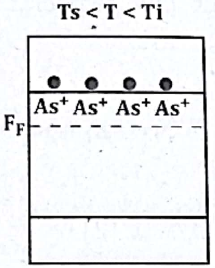


Figure (2)

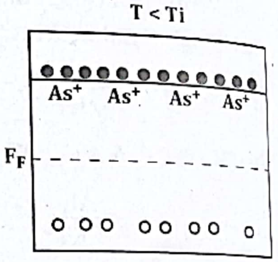


Figure (3)

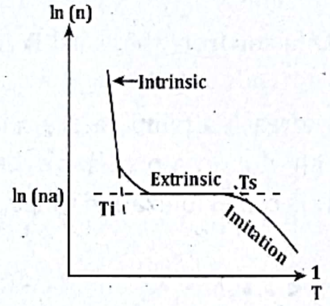


Figure (4): the temperature dependence of the electron concentration in an n-type semiconductor

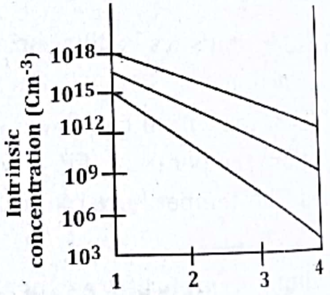


Figure (6): The temperature dependence of in intrinsic concentration

11. Discuss about Drift mobility: Temperature and Impurity dependence.

Ans: The electron drift mobility is given by,

$\mu = \frac{e\tau}{m_e^*}$, τ = mean free time between scattering

where, $\tau = \frac{1}{AV_{th}N}$

A = crosssectional area of scatter
V_{th} = mean speed of electron called thermal velocity
N = number of scatters per unit volume

Here, $A = \pi a^2$, a = amplitude of atomic vibration following $a^2 \propto T$ (Temperature)

Since $\frac{1}{2} m_e^* V_{th}^2 = 3/2\ KT$

$V_{th} \propto T^{1/2}$

$$\tau = \frac{1}{\pi a^2 V_{th} N} \propto \frac{1}{T.T^{1/2}N} \propto \frac{T^{-3/2}}{N}$$

$$\therefore \mu_L \propto \frac{T^{-3/2}}{N_S} \quad \text{.....................} \quad (1)$$

He re μ_L is called "Lattice Vibration Scattering Mobility". Therefore at high temperature, the drift mobility is limited by scattering from lattice vibrations. As the magnitude of atomic vibration increases with temperature, the drift mobility decreases in the fashion $\mu \propto T^{-3/2}$.

At low temperature, scattering of electrons by thermal vibration of the lattice will not be so strong as the electron scattering by ionized donor impurities. The electron scattering by ionized donor impurities limits the drift mobility at low temperature.

At low temperature, as an electron passes by an ionized donor As^+, it is attracted and thus deflected from it's straight path as shown in figure (6). The PE of electron at a distance 'r' rom an As^+ ion is due to the Coloumbic attraction, and it's magnitude is given by,

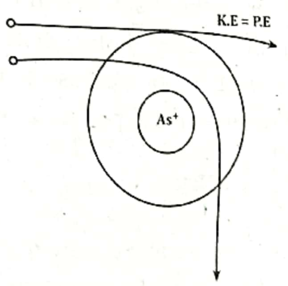


Fig (6): Scattering of electrons by on ionized impurity

$$(PE) = \frac{e^2}{4\pi\epsilon_r}$$

When the electron is Just scattered, i.e. PE=KE

$$(PE) \approx \frac{3}{2} KT$$

$$r = \frac{e^2}{6\pi \in KT}$$

$$S = \pi r^2 = \frac{\pi e^4}{(6\pi \in KT)^2} \propto T^{-2}$$

Also from, $\frac{1}{2}$ $mu_{th}^2 = 3/2KT$

$$V_{th} \propto T^{1/2}$$

Therefore mean scattering time, $\tau = \frac{1}{AV_{th}N} \propto \frac{1}{T^{-2}T^{1/2}N_I}$

$\tau \propto \frac{T^{\frac{3}{2}}}{N_I}$, Here N_I is the concentration of ionized impurities.

$$\therefore \mu_I \propto \frac{T^{\frac{3}{2}}}{N_I}$$

Here μ_L is called "Ionized Impurity Scattering Limited Mobility".
Note: As temperature increases, N_I increases and μ_I decreases.
The overall temperature dependence of the drift mobility is then simply the reciprocal addition of μ_I and μ_L ,

i.e., $\frac{1}{\mu_e} = \frac{1}{\mu_I} + \frac{1}{\mu_L}$

The conductivity of semiconductor as given by $\sigma = n_e\mu_e + pe\mu_n$ depends on carrier concentration and drift mobility. For n-type semiconductor (extrinsic range), it will depend on doner concentration and in p-type semi-conductor it will depend on acceptor concentration and hole drift mobility. In n-type semiconductor, at lowest temperatures in the ionization range, the electron concentration depends exponentially on the temperature.

$$n = \left(\frac{1}{2} N_C N_V\right)^{\frac{1}{2}} \exp\left[\frac{-(E_C - E_d)}{2KT}\right]$$

which dominates the temperature dependence of and activity. In intrinsic range at the highest temperatures, the conductivity is dominanted by temperature dependence of ni since, $\sigma = n_i e (\mu e + \mu n)$ and n_i is exponential function of temperature in contrast to $\mu \propto T^{-1/2}$. In the extrinsic range (high temperature range) n = Nd and is constant, so that the conductivity follows the temperature dependence of the drift mobility.

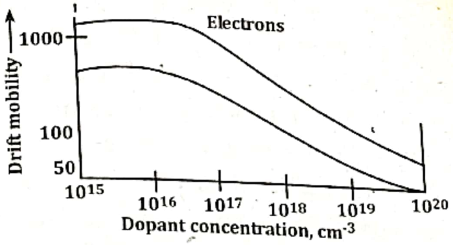


Figure (7): The variation of the drift mobility with dopant concentration in Si for electron and holes at 300 K.

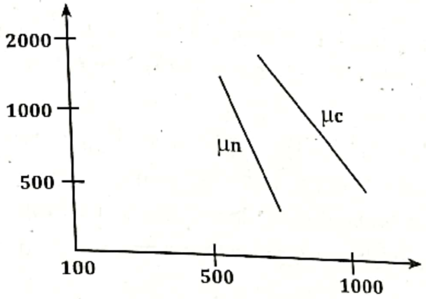


Figure (8):L Temperature dependence of mobility in extrinsic semiconductor.

12. Write short note on: Degenerate and Non-degenerate Semiconductor.

Ans: For an intrinsic semiconductor, the electron concentration is given as.

$$n = N_e e^{-\frac{-(E_c - E_F)}{KT}} \quad \text{..................} \text{ (i)}$$

which is based on replacing Fermi-Dirac statistics with Boltzmassn statistics which is only valid when E_c is several KT above E_F. This means there are very large number of energy states than the number of electrons there. So the probability of occupation of the same state by two electrons is almost nil. This means the Pauli-exclusion Principle has no role and the electron statistics can be described by the Boltzmassn statistics. Here the equation (i) is valid only when n << N_c. Here semiconductor for which n << Nc and $n_n < N_v$ are termed as non degenerate semiconductors. They essentially follow all the normal semiconductor properties.

When the semiconductor has been excessively doped with donors then n may be so large, typically 10^{-19} -10^{20}cm^{-3}, that if may be comparable to or greater than N_c. In that case the Pauli-exclusion Principle becomes important and we have to use the Fermi-Dirac statistics. Equation (i) for n is no longer valid. Such a semiconductor exhibits metal-like properties than semiconductor like. For example resistivity follows $\rho \propto T$. Such semiconductors that have $n > N_c$ or $P > N_v$ are called degenerate semiconductors.

Due to the heavy doping in semiconductor, the donor atoms become so close to each other that their orbital overlap to form a narrow energy band and become a part of the conduction band. The mass action law $nP = n_i^2$ is not valid for degenerate semiconductor. Degenerate semiconductor has many important uses. For example, they are used in laser diode, Zenor diodes, Ohmic contracts in IC's and as metal gates in many micro electronics MOS devices.

Degenerate Semiconductor	Non-degenerate
• Semiconductors with n<<Nc and p<<Nv. • Pauli's exclusion principle can be neglected. • Boltzmassn Stastistics have to be used.	• Semiconductors with n>>Nc and p>>Nv. • Pauli's exclusion principle cannot be neglected. • Fermi-Dirac Stastistics have to be used.

13. Discuss about direct and indirect recombination with necessary diagram.

Ans: Above absolute zero of temperature, the thermal excitation of electrons from the VB to the CB continuously generates free electron - hole pairs. It should be apparent that in equilibrium there should be same annihilation mechanism that returns the electron from the CB down to an empty state (a hole) in the VB. When a free electron, wandering around in the CB of a crystal, meets a hole, it falls into this as shown in figure (1) . This process is called recombination. Figure (i) shows a direct recombination mechanism for example it occurs in GaAS. The excess energy of electron is lost as a photon of energy hf = Eg. In fact, it is this type of recombination that results in the emitted light from light emitting diodes (LEDs).

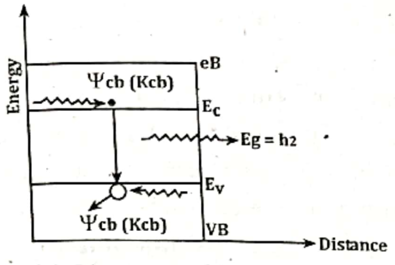


Fig (1): Direct recombination in GaAs

The recombination process between an electron and a hole, like every other process in nature, must obey the momentum conservation law. The wave function of an electron in the CB, Ψ_{cb} (K_{cb}), will have vector K_{cb} and, similarly, the electron wavefunction $\Psi_{vb}(K_{vb})$ in the VB will have a momentum $\hbar K_{vb}$ associated with the with the wave vector K_{vb}. Conservation of linear momentum during recombination requires that when the electrons drop from the CB to the VB, it's wave vector should remain the same, $K_{vb}a = K_{eb}$. For the elemental semiconductors, Si and Ge, the electronic states Ψ_{vb} (K_{vb}) with $K_{vb} = K_{cb}$ are right in the middle of the VB and are therefore fully occupied. Consequently there are no empty states in the VB that can statisfy $K_{vb} = K_{cb}$ and so direct recombination in Si and Ge is next to impossible. For some compound semiconductors, such as GaAS and InSb, for example, the states with $K_{vb} = K_{cb}$ are at the top of the valence band, so they are essentially empty (contain holes). Consequently an electron in the CB of GaAS can drop down to an empty electronic state at the top of the VB and maintain $K_{vb} = K_{cb}$. Thus, direct recombination is highly probable in GaAS, and this is the reason that makes GaAS an LED material.

In elemental semiconductor crystal, for example in Si and Ge, electrons and holes usually recombine through recombination centers. A recombination center increases the probability of recombination because it can 'take up' only momentum difference between a hole and electron. This process essentially involves a third body, which may be an impurity atom or a crystal defect. The electron is captured by the recombination center and thus becomes localized at this site. It is held at the center until some hole arrives and recombines with it as shown in figure (2). This type of recombination is called indirect recombination.

In this recombination process, the energy of the electron is lost as lattice vibrations called photons. A photon is a quantum of energy associated with atomic vibrations in the crystal analogous to the photon in electromagnetic radiations. Typical recombination centers, other than the donor and acceptor impurities, might be metallic impurities and crystal defects such as dislocations, vacancies etc.

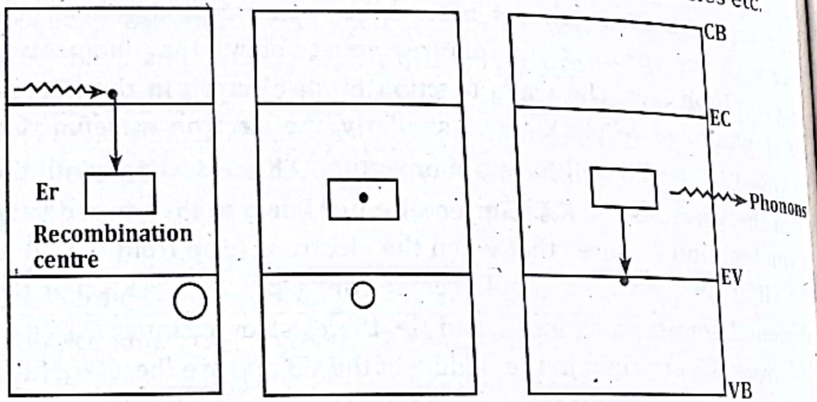


Figure (2): Recombination in Si vies a recombination centre that has a localized energy level at E_r in the band gap, usually near the middle

14. What is diffusion? Derive Einstein relationship for an n-type semiconductor.

Ans: Diffusion is the process in which particles tend to spread out and redistribute themselves as a result of their random thermal vibration migrating from region of high concentration to region of low concentration.

In a semiconductor when doping is done then there is more concentration of one type of charge so it gets diffused. Since, the

particles are charged, the diffusion of charge cause rise in electric current is directly proportional to carrier concentration gradient.

$$J_{e/diff} = eD_e \frac{dn}{dx} \text{......................} (1)$$

$$J_{h/diff} = eD_h \frac{dp}{dx} \text{......................} (2)$$

Where,

D_e = electron diffusion coefficient

D_h = holes diffusion coefficient

Also,

$$J_{e/diff} = en\,\mu_e E \text{ ....} (3)$$

Under equilibrium condition, the current on the semiconductor must be zero.

i.e., $J_e = 0$

or, $J_{e/diff} + J_{e/drift} = 0$

$$\text{or, } en\,\mu_e E + eD_e \frac{dn}{dx} = 0 \text{ ....} (4)$$

The potential energy in ev is related to electrostatic potential at any point by:

$$P.E = -ev \text{ ....} (5)$$

P.F with respect to some reference is given by $P.F = E_C - E_{ref}$ .... (6)

From equation (5) and (6)

$$V = -\frac{1}{e}(E_C - E_{ref})$$

The electric field, $\vec{E} = -\frac{dv}{dx}$

$$\text{or, } E = \frac{1}{e}\left[\frac{dE_c}{dx} - \frac{dE_{ref}}{dx}\right]$$

$$= \frac{1}{e}\frac{dE_c}{dx} \qquad \therefore \left(\frac{dE_{ref}}{dx} = 0\right)$$

$$= \frac{1}{e}\frac{dE_V}{dx} = \frac{1}{e}\frac{dE_{Fi}}{dx}$$

Electron concentration in n-type

$$n = N_c \exp\left[-\frac{E_C - E_F}{KT}\right]$$

Now, Intrinsic concentration

$$n_i = N_c \exp\left[-\frac{E_C - E_{Fi}}{KT}\right]$$

or, $\frac{n}{ni} = \exp\left[-\frac{E_C - E_F}{KT} + \frac{E_C - E_{FI}}{KT}\right]$

$\therefore n = ni \exp\left[-\frac{E_F - E_{Fi}}{KT}\right]$

And, $\frac{dn}{dx} = ni \exp\left[-\frac{E_F - E_{Fi}}{KT}\right]\frac{1}{KT}\left(-\frac{dE_F}{dx} - \frac{dE_{Fi}}{dx}\right)$

$\frac{dE_F}{dx} - \frac{dE_{Fi}}{dx}$

Under equilibrium conditions, Fermi level inside a material cannot vary with respect to position i.e., $\frac{dF_{Fi}}{dx} = 0$

So, $\frac{dn}{dx} = \frac{n}{KT}\frac{dF_{Fi}}{dx} = -\frac{n}{KT}$

Putting in equation (4), we get

on en μ_e E – e $D_e\frac{n}{KT}$ eE = 0

on en μ_e E = e^2 $D_e\frac{n}{KT}$ F

$\therefore \frac{D_e}{\mu_e} = \frac{KT}{e}$ .... (7)

Equation (7) is the required Einstein relation.

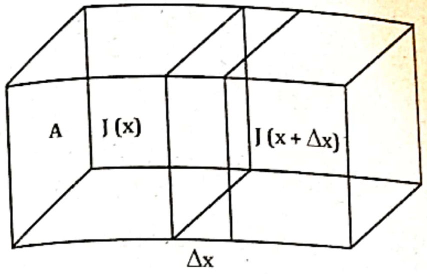
Also, for holes,

$\frac{D_h}{\mu_h} = \frac{KT}{e}\frac{D_h}{\mu_h}$

15. Derive the continuity equation for holes:

Ans: Consider small portion of a volume of n-type semiconductor the hole current density J(x + Δx) leaving volume can be larger or smaller than the current density J(x) entering depending upon the generation and recombination in the sample.

The net increase in the hole concentration per unit time is the difference between the hole flux per unit volume entering and leaving minus the recombination rate. The hole current density can be converted to hole particle flux density by dividing J by 'e'.



$$\frac{\partial P}{\partial t} = \frac{1}{e}\;\frac{\underline{J(x) - J(x+\Delta x)}}{\Delta x} - \frac{\Delta P}{\tau}$$

Rate of hole build up .... (i)

increse of hole concentraion involve AΔx perunit tim

Recombination rate

Using Taylor series expansion,

$J(x+\Delta x) = J(x) + \frac{\partial J(x)}{\partial x}\Delta x$

Again ΔP = P-Po

$\frac{\partial(\Delta P)}{\partial t} = \frac{\partial P}{\partial t}$

Therefore, equation (i) can be written as,

$\frac{\partial(\Delta P)}{\partial t} = -\frac{1}{e}\frac{\partial T}{\partial x} - \frac{\Delta P}{\tau}$ .... (2)

Equation (2) is called the continuity equation for holes. The total current density is due to the diffusion and drifting

i.e., $J = J_{drift} + J_{diffusion} = Pe\,\mu E - eD\frac{\partial P}{\partial x}$

Now substituting J in equation (2),

$\frac{\partial(\Delta P)}{\partial t} = -\frac{\Delta P}{\tau} - \frac{1}{e}\frac{\partial}{\partial x}\left(Pe\mu E - eD\frac{\partial P}{ox}\right)$

For uniformly doped Semiconductor $\vec{E}$ is small, so neglecting the term containing $\vec{E}$,

$\frac{\partial(\Delta P)}{\partial t} = \frac{-\Delta P}{\tau_h} + D\frac{\partial^2 P}{\partial x^2}$ .... (3)

which is called the diffusion equation for holes.

Under steady state condition, $\frac{\partial(\Delta P)}{ot} = 0$

From (3),

$$D\frac{\partial^2 \Delta P}{\partial x^2} = \frac{\Delta P}{\tau}$$

$$\frac{\partial^2 \Delta P}{\partial x^2} - \frac{\Delta P}{D\tau} = 0$$

$$\frac{\partial^2 \Delta P}{\partial x^2} - \frac{\Delta P}{L^2} = 0 \text{ .... } (4)$$

Where $L = \sqrt{D\tau}$ is called the diffusion length for the holes in n-type semiconductor. 'L' is the length a hole diffuses in average before it recombines with an electron in n-type semiconductor. 'L' is also called minority carrier diffusion length.

The solution of equation (4) is

$$\Delta P = Ae^{\frac{x}{L}} + Be^{-\frac{x}{L}} \text{ .... } (5)$$

where 'A' and 'B' are constants that have to be found from the boundary conditions.

For an infinite bar, at $x = \infty$, $\Delta P(\infty) = 0$

From (5), $0 = A + 0 \Rightarrow A = 0$

At $x = 0$, $\Delta P(x) = \Delta P(0)$

Thus, the excess hole concentration apposition x is

$$\Delta P(x) = \Delta P(0)\, e^{-\frac{X}{L}} \text{ .... } (6)$$

16. Derive the continuity equation for electrons.

Ans: Consider small portion of a volume of p-type semiconductor the electron current density J(x+Δx) leaving the portion can be larger or smaller than the current density J(x) entering, depends upon the generation and recombination in the sample. The net increase in electron concentration $\left(\frac{\partial n}{\partial t}\right)$ is the difference between the electron concentration entering and leaving minus the recombination rate.

$$\frac{\partial n}{\partial t} = \frac{1}{(-e)}\left[\frac{J(x)-J(x+\Delta x)}{\Delta x}\right] - \frac{\Delta n}{\tau} \text{ .... } (1)$$

Using Taylor series expansion,

$$J(x+\Delta x) = J(x) + \frac{\partial J(x)}{\partial x}\Delta x$$

Again, $\Delta n = n - no \Rightarrow \frac{\partial(\Delta n)}{\partial t} = \frac{\partial n}{\partial t}$

Therefore, equation (1) can be written as,

$$\frac{\partial(\Delta n)}{\partial t} = \frac{1}{e}\left[\frac{-J(x) + J(x) + \frac{\partial J(x)}{\partial x}\Delta x}{\Delta x}\right] - \frac{\partial n}{\partial t}$$

$$\frac{\partial(\Delta n)}{\partial t} = \frac{1}{e}\frac{\partial J(x)}{\partial x} - \frac{\partial n}{\tau} \text{ .... } (2)$$

Equation (2) is called Continuity Equation for Electrons.

The total current density is due to both diffusion and drifting of electrons.

i.e., $J = J_{drift} + J_{diffusion} = ne\mu E + eD\frac{\partial n}{\partial x}$

Now substituting 'J' in equation (2),

$$\frac{\partial(\Delta n)}{\partial t} = \frac{1}{e}\frac{\partial}{\partial x}\left[ne\mu E + eD\frac{\partial n}{\partial x}\right] - \frac{\Delta n}{\tau}$$

For uniformly doped semiconductor, $\vec{E}$ is small, so neglecting the term containing $\vec{E}$,we have,

$$\frac{\partial(\Delta n)}{\partial t} = -\frac{\Delta n}{\tau} + D\frac{\partial^2 n}{\partial x^2} \text{ .... } (3)$$

which is the diffusion equation for electrons.

Under steady state condition, $\frac{\partial(\Delta n)}{\partial t} = 0$

From (3), $D\frac{\partial^2(\Delta n)}{\partial x^2} = \frac{\Delta n}{\tau} \Rightarrow \frac{\partial^2(\Delta n)}{\partial x^2} = \frac{\Delta n}{D\tau}$

$$\frac{\partial^2(\Delta n)}{\partial x^2} - \frac{\Delta n}{L^2} = 0 \text{ .... } (4)$$

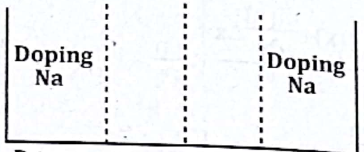
where $L = \sqrt{D\tau}$ is called diffusion length for the electrons. The solution of equation (4) is:

$$\Delta n = Ae^{\frac{x}{L}} + Be^{-\frac{x}{L}} \text{ .... } (5)$$

Here, 'L' is defined as the length an electron diffuses in average before it recombines in p-type semiconductor. It is also called minority carrier diffusion length.

17. What is PN junction? Derive the relation for built in potential and depletion layer of a PN junction.

Ans:



Fig(1): Geometry of on abrunt pn Junction

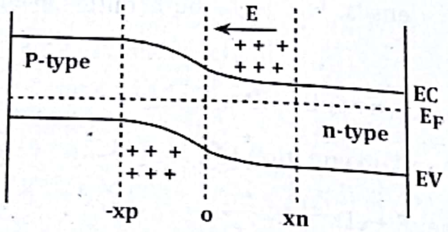


Fig: Bond diagram for an abrung pn function

An n-type and p-type semi-conductor are brought together to form a junction called metallurgical junction n-region has excess in condition band and immobile +ue ions whereas p-region has excess holes and immobile negative ions.

Electron concentration in n-region=n_{no}

Hole concentration in n-region= n_{po}

Electron concentration in p-region=p_{po}

Due to concentration gradient of holes from p-region to n-region, holes will diffuse towards n-region. Due to concentrationgradient of electrons from n-region to p-region, electrons will diffuse towards p-region. The electrons and holes diffusing towards each other meet and recombine near the junction. The junction region becomes free of charge carrier and is called depletion region.

Electrons leaving the n-side near the junction leave behind positively charged donor atoms of concentration N_d. Holes leaving the p-region near expose negatively charged acceptor atoms of concentration N_a. Therefore, there is a space charge layer around. Through this region, the concentration of electrons and holes change in such a way that they transform from majority to minority carrier and vice versa. So, this region is also called transition region.

There is an internal field. Eo directed from positive ions to negative ions that tries to drift holes back into p-region and electrons back into n-region. It is seen that as more and more holes diffuse towards right and electrons towards left, the internal field increase until an equilibrium is reached when the rate of charge carriers diffusing is just balanced by the drifting of charge carrier due to field Eo. For uniformly doped p and n-regions, the net space charge density net ρ_{net}.

Electric field E and net space charge ρ_o are related to space charge density as:

$$\frac{dE}{dx} = \frac{\rho_{net}}{\in}$$

or, $$E = \int_{xp}^{x} \frac{\rho_{net}}{\in}$$

Where, $\in$ = Permittivity of medium

ρ_{net} = Space charge density

Maximum electric field, $E_o = \frac{eN_a x_p}{\in}$

$$= -\frac{eN_d x_n}{\in} \quad \text{..................}(i)$$

For overall charge neutrality,

$$N_a x_p = N_d x_n$$

Also,

$$E = \frac{-dV}{dx}$$

At equilibrium the total conduction electron current is zero.

$$J_{e/net} = 0$$

or, $$J_{e/diff} + J_{e/drift} = 0$$

or, $$en\mu_e E + eD_e \frac{dn}{dx} = 0$$

$\Rightarrow$ $$E = \frac{-D_e dn}{n\mu_e dx}$$

From Einstein relation

$$\therefore \frac{D_e}{\mu_e} = \frac{KT}{e}$$

or, $$\frac{-dV}{dx} = \frac{-KT}{en}\frac{dn}{dx}$$

or, $\int_{x_p}^{x_n} dV = \frac{-KT}{e}\int_{n(-x_p)}^{n(x_n)} \frac{dn}{n}$

or, $V_o = \frac{KT}{e}[\log(n)]_{n(-x_p)}^{n(x_n)}$ ....(ii)

At $x = -x_p$, $n[-x_p] = n_{po} = \frac{n_i^2}{N_a}$

At $x = x_n$, $n[x_n] = n_{no} = N_d$

So, $V_o = \frac{KT}{e}\log\frac{N_a N_d}{n_i^2}$ ....(iii)

Equation (iii) is the required equation for build -in potential.

Form figure,

$V_o = -\frac{E_o x_o}{2}$

Where $x_o = x_n + x_p$

$\therefore V_o = e\frac{N_d x_n}{2\epsilon} x_o$

We have.

$N_a x_p = N_d x_n$

$\Rightarrow$ $x_n = \frac{N_a}{N_a+N_d} x_o$

So, $V_o = \frac{e N_d N_a}{2\epsilon(N_a+N_d)} x_o^2$

$\therefore$ $x_o = \left[\frac{2\epsilon V_o}{e}\,\frac{N_a+N_d}{N_a N_d}\right]^{\frac{1}{2}}$

$= \sqrt{\frac{2\epsilon V_o}{e}\left(\frac{1}{N_a}+\frac{1}{N_d}\right)}$ ....(iv)

Equation (iv) is the required equation for depletion width (width of depletion layer).

18. Explain the forward biased P-N junction and derive the mathematical expressions for the same with necessary diagrams.

Ans: When an external voltage is applied to the Pn junction in such a massner that the positive terminal of the battery is attached to the P-side and negative terminal to the n-side, the P-n Junction is said to be forward biased.

When a voltage 'V' is applied, it reduces built in potential 'V_o' to $V_o - V$. Built in potential is one which act against diffusion, consequently massy holes can now diffuse across the depletion region and enter n-side. This results in the injection of excess minority carriers (holes) into the n-region. Similarly, excess electrons can now diffuse towards the P-side and enter the region and there by become injected minority carriers.

Now, the probability that a hole will surmount the potential barrier $(V_o - V)$ and diffuse to n-region becomes proportional to :

$\exp\left[\frac{-e(V_o-V)}{KT}\right]$

i.e., $P_n(o) = P_p e^{\frac{-e(V_o - V)}{KT}}$ ....(1)

Here, $P_n(o)$ is the hole concentration just outside the depletion region at $x = X_p$. And P_p is the total concentration of hole in p-region.

We have, $\frac{P_P}{P_n} = e^{\frac{eV_o}{KT}}$ ....(2)

Here P_n = concentration of holes in n-region

Using P_p in equation (1),we get,

$P_n(o) = P_n e^{\frac{eV_o}{KT}} \cdot e^{\frac{eV_o + Ev}{KT}} = P_n e^{\frac{eV_o}{KT} - \frac{eV_o}{KT} + \frac{eV}{KT}}$

$P_n(o) = P_n e^{\frac{eV_o}{KT}}$ ....(3)

which is called Law of Junction. It describes the effects of the applied voltage 'V' on the injected minority carrier concentration just outside the depletion region $P_n(o)$. For no applied voltage, V=o, $P_n(o) = P_n$ which is exactly what we expect.

Similarly, electrons are injected from the n-side to the p-side. The electron concentration $n_p(o)$ just outside the depletion region towards p-side at $x = -x_p$ given by

$$n_p(o) = n_p\, e^{\frac{ev}{KT}}$$

Here, n_p = electron concentration in p-region.

By forward biasing a p-n junction, a current through the junction can be maintained. Hole diffusion in n-side and electrons diffusion in p-side will give rise to a current in the same direction. The current flow due to the diffusion of minority charge carriers, there is, however, current due to drift of majority carriers as well.

Now,

$$\Delta P_n = P_n(o) - P_n = P_n e^{\frac{ev}{KT}} - P_n$$

$$\Delta P_n = P_n \left(e^{\frac{ev}{KT}} - 1\right) \text{ .... (4)}$$

The current density for hole is calculate as,

$$J = Pe\mu E - eD\frac{dP}{dx}$$

The drift contribution can be neglected because the diffusion current is dominant.

$$\text{Therefore, } J = -eD\frac{dP}{dx} = -eD\frac{d(\Delta P)}{dx}$$

If the length of p-side and n-side are larger than minority carrier diffusion length, then the excess hole concentration $\Delta P_n(x)$ in n-region fall exponentially form equilibrium value $\Delta P(o)$ as:

$$\Delta P_n = \Delta P(o)\, e^{\frac{-x}{L}}$$

$$\Rightarrow \frac{d(\Delta P)}{dx} = \frac{-1}{L}\Delta P(o)\, e^{\frac{-x}{L}} = \frac{-1}{L}\Delta P_n$$

$$\text{Therefore, current density, } J = \frac{eD}{L}\Delta P_n$$

Substituting ΔP_n from equation (4),

$$J = \frac{eD}{L} P_n \left(e^{\frac{ev}{KT}} - 1\right)$$

But, $P_n = \frac{n_i^2}{n_n} = \frac{n_i^2}{N_d}$ [If all donors are ionized $n_n = N_d$]

$$\text{Therefore, } J = \frac{eDn_i^2}{LNd}\left(e^{\frac{ev}{KT}} - 1\right)$$

Putting subscript 'h' for holes on J, D and L

$$J_h = \frac{eDn_i^2}{L_h Nd}\left(e^{\frac{ev}{KT}} - 1\right) \text{ .... (5)}$$

Similarly the diffusion current density of electron is,

$$J_e = \frac{eDn_i^2}{L_e Na}\left(e^{\frac{ev}{KT}} - 1\right) \text{ .... (6)}$$

The total diffusion current density is,

$$J = J_e + N_h = en_i^2\left(\frac{D_e}{L_e N_a} + \frac{D_h}{L_h N_d}\right)\left[e^{\frac{ev}{KT}} - 1\right]$$

$$J = J_s\left[e^{\frac{ev}{KT}} - 1\right] \text{ .... (7)}$$

Equation (7) is called the ideal diode equation.

In term of current, this can be expressed as,

$$I = JA = A\, J_s\left[e^{\frac{ev}{KT}} - 1\right]$$

$$I = I_s\left[e^{\frac{ev}{KT}} - 1\right] \text{ .... (8)}$$

Where, Is = A Js is called saturation current

And $J_s = \left(\frac{D_e}{L_e N_a} + \frac{D_h}{L_h N_d}\right) en_i^2$ is called reverse saturation current density

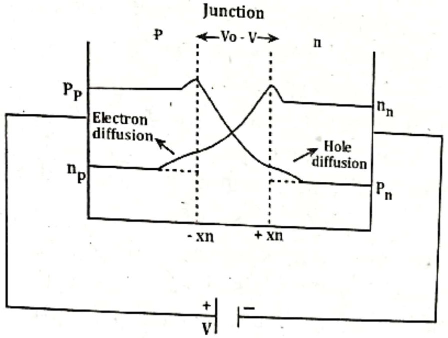


Fig: Carrier concentration profides across the device under forword bias

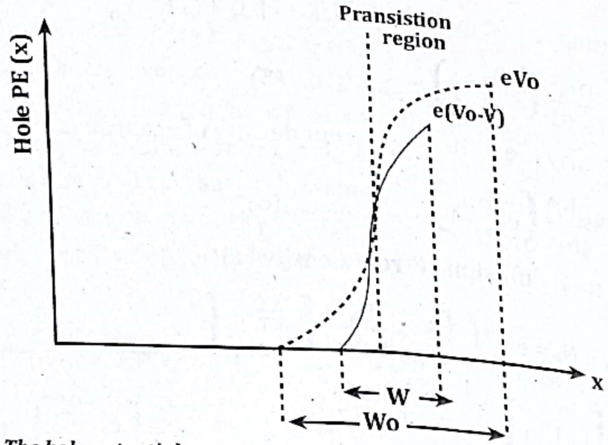


Fig: The hole potential energy with and without an applied bias. W is the width of Junction with forward bias. Wo is the width of junction with no bias.

19. Explain reverse biased p-n junction and derive the mathematical expressions with necessary diagrams.

Ans:

Whan a pn junction is connected to a voltage source with p-region connected to negative terminal of voltage source and n-region to positive terminal, then the junction is called to be reverse biased.

The negative terminal of the battery will attract holes in p-region to move away from depletion layer. It results in more exposed acceptor ions and creates a wide space charge layer..

Similarly, the positive terminal will attract electrons away form the depletion layer exposes more positively charged donors and results in the widening the depletion region in this side also.

The movement of electrons in the n-region toward the positive battery terminal cannot be sustained because there is no electron supply to this n-side. The p-side also cannot supply electrons to the n-side because it has almost no electron.

The electrons from the battery cannot enter the p-side and cannot move to n-side because before they got a chance to do so. So a current cannot be sustained through p-n junction in reverse biased condition. The applied voltage adds to built in potential as shown in figure (2).

Similarly, the electric field in the depletion layer is the sum of internal field and applied field.

The holes in the n-side near the depletion layer can be extracted by the field and driven across depletion layer. The holes exit p-region and enter to battery and again give rise to a small current. There will be small diffusion current through the junction due to diffusion from depletion layer to p-region. Therefore, there is a small reverse current due to these causes.

The reverse current is given by,

$$J = -Js\left[e^{\frac{ev}{KT}} -1\right] \text{ .... } (1)$$

$$\text{where, } Js = \left(\frac{D_e}{L_e N_a} + \frac{D_h}{L_h N_d}\right) e n_i^2 \text{ .... } (2)$$

It is called reverse saturation current density. Here equation,(2) is called Shockley Equation.

The equation for reverse current can be expressed as:

$$J_{rev} = \left(\frac{D_e}{L_e N_a} + \frac{D_h}{L_h N_d}\right) e n_i^2 + \frac{e n_i W}{\tau g} \text{ .... } (3)$$

Here, current density due to thermal generation, $J_{gen} = \frac{e n_i W}{\tau g}$

and τ_g is the mean thermal generation time to generate electron hole pair.

The width of depletion region with no bias is

$$W = \left[\frac{2\in V_o}{e}\left(\frac{1}{N_a} + \frac{1}{N_d}\right)\right]^{\frac{1}{2}}$$

When the reverse voltage V is applied, the total potential of the junction becomes $(V_o + V)$, so the total width of depletion region for reverse biased is given by:

$$\left[\frac{2\in}{e}\left(\frac{1}{N_a} + \frac{1}{N_d}\right)(V_o + V)\right]^{\frac{1}{2}} \text{ .... } (4)$$

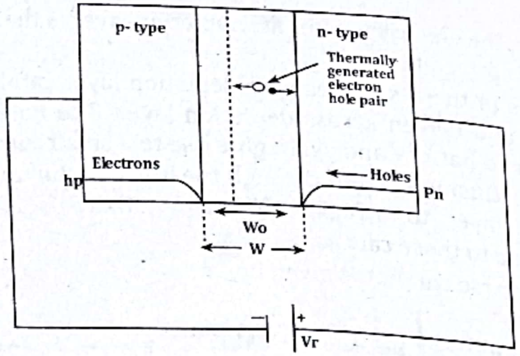
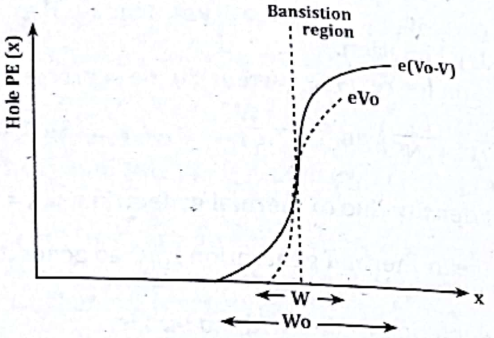


Fig (1): Reversed - biased pn junction: minority carrier profiles and the origin of the reverse current



Fig(2): Hole PE across the junction under reverse bras

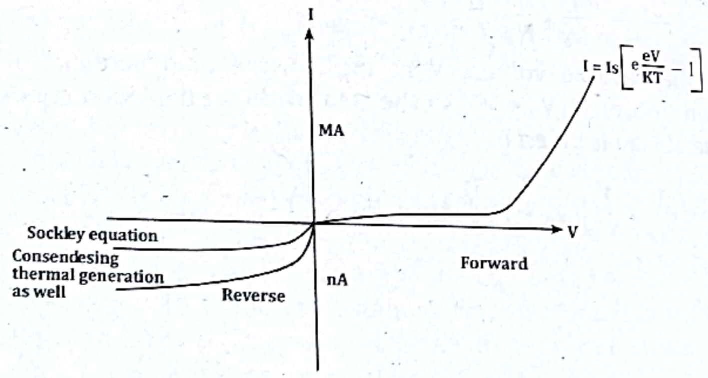


Figure (3): Forward and reverse I-V characteristics of a pn junction

Numerical

1. The density of states related effective masses of electrons and holes in silicon are approximately 1.08 me and 0.56 me, respectively. The electron and hole drift mobilities at room temperature are 1350 and 450 cm²γ²S⁻¹ respectively. Calculate intrinsic concentration and intrinsic resistivity of silicon. The energy band gap for silicon is 1.1 ev.

Solution:

Effective densities of states at conduction band edge and valence band edge are given by

$$N_c = \left[\frac{8\pi m_e^* KT}{h^2}\right]^{\frac{3}{2}} \text{ and } N_V = \left[\frac{8\pi m_h^* KT}{h^2}\right]^{\frac{3}{2}}$$

$$\text{So, } N_c = \left[\frac{8\pi(1.08\times9.1\times10^{-31}\text{kg})\,(1.38\times10^{-23}\text{J/K})\,1300\text{K}_2}{16.624\times10^{-34}\text{Js}}\right]^{\frac{3}{2}}$$

$$= 2.8 \times 10^{25}\text{m}^{-3}$$

$$= 2.8 \times 10^{19}\ \text{cm}^{-3}$$

Similarly,

$$N_V = \left[\frac{8\pi(10.56\times9.1\times10^{-31}\text{kg})\,(1.38\times10^{-3}\text{J/K})\,1300\text{K}_2}{16.624\times10^{-34}\text{Js}}\right]^{\frac{3}{2}}$$

$$= 1.05 \times 10^{25}\text{m}^{-3}$$

$$\therefore N_V = 1.05 \times 10^{19}\text{cm}^{-3}$$

The intrinsic concentration is given by

$$n_i = \sqrt{N_c N_v}\,\exp\left[\frac{-Eg}{2KT}\right]$$

$$\text{or} \quad n_i = \sqrt{(2.8\times10^{25}\text{m}^{-3})\,(1.05\times10^{25}\text{m}^{-3})}\ \exp\left[\frac{-1.1\text{ev}\times1.6+10^{-19}\text{J/ev}}{2(1.38\times10^{-23}\text{J/ev})(300\text{K})}\right]$$

$$\therefore n_i = 1.0 \times 10^{16}\ \text{m}^{-3}$$

The resistivity is given by

$$\rho = \frac{1}{en_i(\mu_e + \mu_n)} = \frac{1}{(1.6\times10^{-19}\text{C})\,(10^{16}\text{m}^{-3})\,(1350 + 450)\ \text{cm}^2\text{V}^{-1}\text{S}^{-1}}$$

$$= 3.5 \times 10^{3}\Omega\text{m}$$

$$\therefore \rho = 3.5 \times 10^{5}\Omega\text{cm}$$

2. Calculate the resistance of pure silicon cubic crystal of 1cm³ at room temperature. What will be the resistance of the cube when it is doped with 1 arsenic in 10^3 silicon atoms and 1 boron atom per billion silicon atoms? Atomic concentration of silicon is $5\times10^{22}cm^{-3}$, $n_i = 1.45\times10^{10}cm^{-3}$. Assume the mobilities as 1350 and 450 $cm^2V^{-1}s^{-1}$ of electron and hole respectively.

Solution

Resistance of cube is given by

$$R = \frac{\rho l}{A}$$

$$\text{or } \rho = \frac{-l}{\sigma}\frac{1}{en_i(\mu_e + \mu_n)}$$

$$\text{or, } R = \frac{1}{(1.6\times10^{-19}C)(1.45\times10^{10}cm^{-3})(1350+450)cm^2V^{-1}S^{-1}} = 2.39\times10^5\Omega$$

When silicon is doped with 1 arsenic atom per 10^9 silicon atoms, then doping concentration is calculated as:

$$Nd = \frac{Nsi}{10^9} = \frac{5\times10^{22}cm^{-3}}{10^9} = 5\times10^{13}cm^{-3}$$

At room temperature, all donors are ionized, so the electron concentration is given by donor concentration

i.e $n \simeq Nd = 5\times10^{23}cm^{-3}$

The hole concentration then becomes

$$P = \frac{n_i^2}{Nd} = \frac{(1.45\times10^{10}cm^{-3})^2}{5\times10^{13}cm^{-3}} = 4.2\times10^6\ cm^{-3}$$

Since P << n, we can neglect hole concentration for calculation of resistance, which is given by

$$R = \frac{1}{(1.6\times10^{-19})(5\times10^{23}cm^{-3})(1350cm^2V^{-1}S^{-1})} = 92.6\,\Omega$$

So, there is drastic fall in resistance of cubic crystal when it is doped with 1 arsenic atom in 10^9 silicon atoms.

Similarly, when sample is doped with 1 baron atom per billion silicon atoms, the acceptor concentration is again $5\times10^3cm^{-3}$ and resistance of sample is calculated as, by neglecting contribution from electrons.

$$R = \frac{1}{(1.6\times10^{-19})(5\times10^{23}cm^{-3})(450cm^2V^{-1}S^{-1})}$$

$$\therefore R = 278\,\Omega$$

3. An ntype semiconductor doped with 10^{16} cm^{-3} phosphorous atoms has been doped with 10^{17} boron atom calculate electron and hole concentration in the semi-conductor.

Solution:

A semiconductor doped with both the donors and acceptors is called compensated semiconductor. Here pentavalent phosphorous is donot and trivalent boron is acceptor. The resultant acceptor concentration is $Na-Nd = 10^{17} - 10^{16} = 9\times10^{16}cm^{-3}$. The hole concentration as the intrinsic concentration is very small compared to accepter concentration.

$$P = \frac{n_i^2}{P} = \frac{(1.45\times10^{10}cm^{-3})^2}{9\times10^{16}cm^{-3}} = 2.34\times10^3cm^{-3}$$

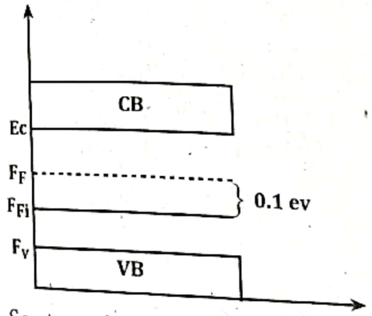
This semiconductor, through having much higher acceptor concentration than donors, is still called compensation doped semiconductor.

4. If it is derived that the Fermi - level is to be raised to 0.1 ev above intrinsic Fermi-level at room temperature, what type of dopant is to be used? Determine its doping level.

Solution:

Here, Fermi-level is to be raised i.e.

$F_F - F_{Fi} = 0.1$ ev



So, to raise the Fermi level above the intrinsic Fermi-level, we need to dope pentavalent atom. The doping level is given is:

$$\frac{n}{n_i} = \exp\left[\frac{E_F - E_{Fi}}{KT}\right]$$

$$\text{or, } n = n_i \times \exp\left[\frac{E_F - E_{Fi}}{KT}\right] = 1.45\times10^{10}cm^{-3}) \times \exp\left[\frac{0.1ev}{0.0259\ ev}\right]$$

$$\therefore n = 6.88\times10^{11}cm^{-3}$$

5. A silicon sample has been doped with 10^{17} Arsenic atoms/cm^3. Calculate the conductivity of the sample at 300K and 400K. Use the mobility vs impurity concentration graph to determine mobility.

Solution:

To calculate conductivity we can use the donor concentration only since $N_a << n_i$ and p>>n. The electron mobility at $N_d = 10^{17} cm^{-3}$ is 800cm^{2}Y^{-1}S^{-1} at 300 K. So, the required conductivity at 300K

$$\sigma = en\mu e = (1.6\times10^{-19}C)\ (10^7 cm^{-3})\ (800 cm^2 Y^{-1} S^{-1}) = 12.85\ m/cm$$

Similarly at 400K, drift mobility of electron is 420cm^{2}Y^{-1}S^{-1} at given donor concentration. So, the conductivity of sample is

$$\sigma = en\mu e = (1.6\times10^{-19}C)\ (10^{17} cm^{-3})\ (420 cm^2 Y^{-1} S^{-1}) = 6.72\ m/cm$$

6. A heavily doped p-side with acceptor concentration of $10^{18}cm^{-3}$ is connected to n-side with donor concentration of $10^{16}cm^{-3}$. Calculate the built in potential depletion width in n-side and p-side and overall depletion width. The intrinsic concentration is $1.45\times10^{10}cm^{-3}$ and T = 300K.

Solution:

The built - in potential in the junction is calculated as

$$Vo = \frac{KT}{e} \ln \frac{N_a N_d}{n_i^2} = (0.0259V) \ln \frac{(10^{16}\times10^{18})}{(1.45\times10^{10})^2} = 0.816\ v$$

The depletion width can be calculated by using the following expression

$$Wo = \sqrt{\frac{2\epsilon Vo}{e}\left(\frac{1}{N_a} + \frac{1}{N_a}\right)}$$

But here since $N_a >> N_d$, we simply write the formula as

$$Wo = \sqrt{\frac{2\epsilon Vo}{e}\frac{1}{N_d}} = \sqrt{\frac{2\times(11.9\times8.85\times10^{12}\ Fm^{-1})\ (0.816\ v)}{(1.6\times10^{-19}E)(10^{22}, m^{-3})}} =$$

$3.3\times10^{-7}m$

Now,

Depletion width in n-region is

$$Wn = \frac{WoNa}{Na + Nd} = \frac{(3.3\times10^{-7}m)(10^{24}m^{-3})}{(10^{22}m^{-3} + 10^{24}m^{-3})} = 3.3\times10^{-9}m$$

Similarly, the depletion width in p-region is

$Wp = Wo - Wn = 3.3\times10^{-7}m - 3.267\times10^{-7}m = 3.3\times10^{-9}m$

So, almost 99% of depletion layer falls in n-region due to nearly doped p-region where only around 1% depletion layer falls.

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