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SOIL MECHANICS

Numerical Solution



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TUTORIAL-1

For a soil in natural state, given $e = 0.8$, $w = 24\%$ and $G_s = 2.68$, Determine the moist unit weight, dry unit weight and degree of saturation. If the soil is made completely saturated by adding water, what would its moisture content be at that time? Also find saturated unit weight.

Solution:-

Given that:-

$$e = 0.8$$

$$w = 24\%$$

$$G_s = 2.68$$

$$\text{Moist unit weight } (\gamma) = ?$$

$$\text{dry unit weight } (\gamma_d) = ?$$

$$\text{Degree of saturation } (S) = ?$$

We know that,

$$\gamma = \frac{(G + se)\gamma_w}{1+e} \quad \dots (1)$$

$$\therefore se = wG$$

$$\therefore S = \frac{wG}{e} = \frac{24}{100} \times 2.68 = 0.804$$

Now, eqⁿ (1) becomes,

$$\gamma = \frac{(2.68 + 0.804 \times 0.8)}{1 + 0.8} \times 9.81 \times 1000$$

$$= 18.11 \text{ kN/m}^3$$

Also,

$$\gamma_d = \frac{G\gamma_w}{1+e} = \frac{2.68 \times 9.81 \times 1000}{1 + 0.8} = 14.606 \text{ kN/m}^3$$

And,

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$$\text{Degree of saturation } (s) = \frac{wG}{e} \times 100\%$$

$$= 0.804 \times 100\%$$

$$= 80.4\%$$

Case II:-

If the soil is made completely saturated by adding water, then,

moisture content $(w) = ?$

Saturated unit weight $(\gamma_{sat}) = ?$

We have,

$$w = \frac{se}{G} = \frac{1 \times 0.8}{2.68} = 0.2985 = 29.85\%$$

and,

$\because s=1$ for saturated soil

$$\gamma_{sat} = \frac{(G+se)\gamma_w}{1+e}$$

$$= \frac{(2.68+0.8) \times 9.81 \times 1000}{1+0.8} \quad [\because s=1]$$

$$= 18.97 \text{ kN/m}^3$$

Hence, the moisture content is 29.85% at that time and saturated unit weight is 18.97 kN/m³.

8(F) PH.

2. A soil sample in its natural state has a mass of 2.29 kg and volume of $1.15 \times 10^{-3} \text{ m}^3$. Under an oven-dried state, the dry mass of the sample is 2.035 kg. The specific gravity of the solids is 2.68. Determine the total density, water content, void ratio, porosity, degree of saturation and air void ratio.

Solution:-

Given that,

Mass of soil sample $(m_s) = 2.29 \text{ kg}$.

Volume of soil sample $(V) = 1.15 \times 10^{-3} \text{ m}^3$

Dry mass of the sample $(m_d) = 2.035 \text{ kg}$.

Specific gravity of soil $(G_s) = 2.68$

Total density $(\rho) = ?$

Water content $(w) = ?$

void ratio $(e) = ?$

Porosity $(n) = ?$

Degree of saturation $(s) = ?$

Air void ratio $(n_a) = ?$

We know that,

$$\rho = \frac{m_s}{V} = \frac{2.29}{1.15 \times 10^{-3}} = 1991.30 \text{ kg/m}^3$$

$$w = \frac{W_w}{W_s} \times 100\%$$

$$= \frac{2.29 - 2.035}{2.035} \times 100\%$$

$$= 12.53\%$$

Now,

$$e = \frac{V_v}{V_s} \quad \dots (1)$$

$$\text{But } G = \frac{\gamma_s}{\gamma_w} \Rightarrow \gamma_s = G\gamma_w = 2.68 \times 1000 = 2680 \text{ kg/m}^3$$

$$\text{Also, } \gamma_s = \frac{W_s}{V_s}$$

$$\text{or, } V_s = \frac{W_s}{\gamma_s} = \frac{2.035}{2680} = 7.593 \times 10^{-4} \text{ m}^3$$

$$\therefore V_v = V - V_s = 1.15 \times 10^{-3} - 7.593 \times 10^{-4} = 3.907 \times 10^{-4} \text{ m}^3$$

from eq. (1),

$$e = \frac{3.907 \times 10^{-4}}{7.593 \times 10^{-4}} = 0.514$$

$$n = \frac{e}{1+e} = \frac{0.514}{1+0.514} = \frac{0.3398 \times 100\%}{33.98\%} \approx 34\%$$

Also,

$$S = \frac{wG}{e} = \frac{12.58 \times 2.68}{0.514} = 65.33\%$$

$$n_a = n_{ac} = n(1-S) = 34(1-0.6533) = 11.78\%$$

11(F) Additional practice

Q. A $5 \times 10^{-5} \text{ m}^3$ specimen of a moist soil weigh 0.932 N, its dry weight is 0.735 N and the sp. gravity of soil solid is 2.68. Compute water content, void ratio, porosity and degree of saturation.

→ Solution:-

Given that,

Volume of specimen (V) = $5 \times 10^{-5} \text{ m}^3$.

Weight of moist soil (W_m) = 0.932 N

Weight of dry soil (W_d) = 0.735 N

Sp. gravity of soil (G_s) = 2.68

Water content (w) = ?

Void ratio (e) = ?

Porosity (n) = ?

Degree of saturation (S) = ?

We know that,

$$w = \frac{W_w}{W_s} \times 100\%$$

$$= \frac{(0.932 - 0.735)}{(0.932 + 0.735) \cdot 0.735} \times 100\%$$

=

$$S = \frac{(G_s e) \rho_w}{1 + e}$$

$$S_e = G_w$$

$$S_d = \frac{G_w}{1 + e}$$

$$S_d = \frac{M}{V} = ?$$

$$S_d = \frac{0.935}{5 \times 10^{-5}} \text{ N/m}^3$$

$$S_d = \frac{\rho}{1 + w}$$

3. The wet density of soil is 2 gm/cc . The specific gravity of solids is 2.7 and the moisture content of the soil is 15% . Calculate a) Dry density. b) Porosity c) void ratio d) Degree of saturation e) % of air voids.

→ Solution:-

Given that,

Wet density of soil (ρ) = 2 gm/cm^3

Specific gravity of solids (G_s) = 2.7

Moisture content of the soil (w) = 15%

a) Dry density (ρ_d) = ?

b) Porosity (n) = ?

c) void ratio (e) = ?

d) Degree of saturation (s) = ?

e) % of air voids (n_a) = ?

We know that,

$$a) \rho_d = \frac{\rho}{1+w} = \frac{2}{1+0.15} = 1.74 \text{ gm/cm}^3$$

We have,

$$\rho_d = \frac{G_s \rho_w}{1+e}$$

$$\text{or, } 1.74 = \frac{2.7 \times 1}{1+e}$$

$$\text{or, } 1+e = \frac{2.7}{1.74} \Rightarrow e = 0.5517$$

$$b) n = \frac{e}{1+e} = \frac{0.5517}{1+0.5517} = 0.3555 = 35.55\%$$

$$c) e = 0.5517$$

$$d) s = wG_s$$

$$\text{or, } s = \frac{wG_s}{e} = \frac{0.15 \times 2.7}{0.5517} = 0.734$$

$$e) n_a = n s$$

$$= n(1-s)$$

$$= 0.3555(1-0.734)$$

$$= 0.0945$$

$$\approx 9.45\%$$

A pycnometer weighing 620 gm was used in the following measurements on samples 'A' and 'B' of the same soil. Sample A was oven dried and 'B' was completely saturated. Weight of pycnometer when filled with water only is 1495 gm . Determine:

sample	A	B
Weight of sample only (gm)	980	1020
Weight of Pycnometer full of sample & water	2112	2030

→ The specific gravity of the soil.
→ The water content & void ratio of sample B.

→ Solution:-

Given that,

Mass of pycnometer (m_1) = 620 gm

$m_2 = (980 + 620) \text{ gm} = 1600 \text{ gm}$

$m_3 = 2112 \text{ gm}$

$m_4 = 1495 \text{ gm}$

→ Sp. gravity of soil (G_s) = ?

→ $w = ?$ & $e = ?$

We know that,

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$$\begin{aligned} \text{Mass of dry soil } (m_s) &= m_2 - m_1 \\ &= 1600 - 620 \\ &= 980 \text{ gm.} \end{aligned}$$

And,

$$\therefore G = \frac{m_s}{m_s - (m_3 - m_4)} = \frac{980}{980 - (2112 - 1495)} = 2.7$$

ii) For sample B,
 $se = wG$

$$\text{or, } e = \frac{wG}{s}$$

$$w = \frac{m_w}{m_s} =$$

5. A partially saturated soil sample from earth fill has a natural moisture content of 20% and bulk unit weight of 1.8 gm/cm^3 . The specific gravity of solid is 2.7. Determine the degree of saturation and void ratio. If the sample gets saturated subsequently, determine its unit weight.

→ Solution:-

Given that,

$$\text{Natural moisture content } (w) = 20\%$$

$$\text{Bulk unit weight } (\gamma_t) = 1.8 \text{ gm/cm}^3$$

$$\text{Specific gravity of solid } (G) = 2.7$$

$$\text{Degree of saturation } (S) = ?$$

$$\text{Void ratio } (e) = ?$$

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We know that,

$$\gamma_t = G \gamma_d (1+w)$$

$$\text{or, } 1.8 = \gamma_d (1+0.2)$$

$$\text{or, } \gamma_d = 1.5$$

And,

$$\gamma_d = \frac{G \gamma_w}{1+e}$$

$$\text{or, } 1+e = \frac{2.7 \times 1}{1.5}$$

$$\text{or, } e = 0.8.$$

Now,

$$se = wG$$

$$\text{or, } s = \frac{20 \times 2.7}{80} = 0.675$$

$$= 67.5\%$$

Case II:-

When the sample gets saturated subsequently, $S=1$.

$$\text{or, } 1 \times e = 0.2 \times 2.7$$

$$\text{or, } e = 0.54$$

$$\text{Also, } \gamma_{\text{sat}} = \frac{(G+e) \gamma_w}{(1+e)}$$

$$= \frac{(2.7+0.8) \times 1}{(1+0.8)}$$

$$= 1.94 \text{ gms/cc.}$$

The unit weight is 1.94 gms/cc when the sample gets saturated subsequently.

6) A 50cc volume of moist soil weights 95gm. Its dry weight is 75gm and specific gravity of solids is 2.68. Compute the water content, void ratio, porosity and unit weight.

→ Solution:-

Given that,

$$\text{Volume (V)} = 50 \text{cc}$$

$$\text{Weight of moist solid (m)} = 95 \text{gm}$$

$$\text{Weight of dry solid (m}_s) = 75 \text{gm}$$

$$\text{Specific gravity (G}_s) = 2.68$$

$$\text{Water content (w)} = ?$$

$$\text{Void ratio (e)} = ?$$

$$\text{Porosity (n)} = ?$$

$$\text{Unit weight (}\gamma) = ?$$

We know that,

$$w = \frac{m_w}{m_s} = \frac{m - m_s}{m_s} = \frac{95 - 75}{75} \times 100\% = 26.67\%$$

$$\gamma = \frac{m}{V} = \frac{95}{50} = 1.9 \text{ gm/cc.}$$

We have,

$$\gamma_d = \frac{\gamma}{1+w} = \frac{1.9}{1+0.2667} = 1.5 \text{ gm/cc.}$$

We have,

$$\gamma_d = G_s \gamma_w / (1+e)$$

$$\text{or, } 1.5 = \frac{2.68 \times 1}{1+e} \Rightarrow e = 0.786$$

$$\text{and, } n = \frac{e}{1+e} = \frac{0.786}{1+0.786} = 0.44 \%$$

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Rakshan Sab

1.8. A cube of dried clay having sides 4cm long has a mass of 110gm. The same cube of soil, when saturated at unchanged volume has a mass of 135gm. Draw a phase diagram showing the constituents of volumes and weights and then determine the specific gravity of soil solids and the void ratio.

→ Solution:-

Given that-

$$\text{Mass of dry clay (m}_s) = 110 \text{gm.}$$

$$\text{Mass of saturated clay (m}_{sat}) = 135 \text{gm.}$$

$$\text{Specific gravity of soil (G}_s) = ?$$

$$\text{Void ratio (e)} = ?$$

$$\text{Side of cube (l)} = 4 \text{cm}$$

$$\therefore \text{Volume of cube (V)} = 4^3 = 64 \text{cm}^3$$

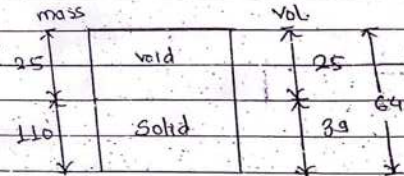


fig. Phase diagram.

Since the volume remains constant before and after saturation,

$$\text{mass of water} = 135 - 110 = 25 \text{ gm.}$$

$$\text{Volume of water in void} = \frac{25 \text{ gm}}{1 \text{ gm/cc}} = 25 \text{cc.}$$

$$\therefore \text{Volume of void} = 25 \text{cc.}$$

$$G = \frac{\gamma_s}{\gamma_w} = \frac{2.82}{1} = 2.82$$

$$\gamma_s = \frac{m_s}{V_s} = \frac{110}{39} = 2.82 \text{ gm/cc.}$$

and,

$$e = \frac{V_v}{V_s} = \frac{25}{39} = 0.64 \%$$

8. A sample of sand has porosity of 40% and specific gravity of its solid is 2.75. Find out the dry unit weight, saturated unit weight, when fully saturated, submerged unit weight and saturated unit weight, when the degree of saturation is 50%.

→ Solution:-

Given that,

Porosity of sand (n) = 40%

Specific gravity (G_s) = 2.75

Dry unit weight (w_d) = ?

Saturated unit weight (w_{sat}) = ?

Submerged unit weight (w_{sub}) = ?

Saturated unit weight (w_{sat}) = ?

} $S = 100\%$

} $S = 50\%$

We know that,

$$\gamma_d =$$

$$n = \frac{V_v}{V} = \frac{V_w}{V} = \frac{w}{G_s + w}$$

$$0.40 = \frac{w}{2.75 + w}$$

$$0.40(2.75 + w) = w$$

$$1.1 + 0.40w = w$$

$$1.1 = 0.60w$$

$$w = \frac{1.1}{0.60} = 1.83 \text{ gm/cc}$$

$$\gamma_{sat} = \frac{G_s + w}{1 + e}$$

γ_d

$$n = \frac{e}{1+e}$$

$$0.40 = \frac{e}{1+e} \quad \text{or, } 0.40 + 0.40e = e \quad \text{or, } e = 0.67$$

$$n = \frac{e}{1+e}$$

$$\text{or, } 0.40 = \frac{e}{1+e} \Rightarrow e = 0.67$$

Now,

$$w_d = \frac{G_s \gamma_w}{1+e} = \frac{2.75 \times 1}{1+0.67} = 1.65 \text{ gm/cc}$$

Also,

$$\gamma_{sat} = \frac{(G_s + e) \gamma_w}{1+e} = \frac{(2.75 + 0.67) \times 1}{1+0.67} = 2.05 \text{ gm/cc}$$

Case II

$$\gamma_{sat} = \frac{(G_s + Se) \gamma_w}{1+e} = \frac{(2.75 + 0.5 \times 0.67) \times 1}{1+0.67} = 1.85 \text{ gm/cc}$$

and

$$\gamma_{sub} = \gamma_{sat} - \gamma_w$$

$$= 1.85 - 1$$

$$= 0.85 \text{ gm/cc}$$

g. Prove that:-

$$S = \frac{w}{\gamma_w(1+w)} \cdot \frac{\gamma_d}{G_s}$$

> Solution:-

We have,

$$\gamma_d = \frac{\gamma}{1+w} \quad \dots (1)$$

$$\gamma_d = \frac{G_s \gamma_w}{1+e} \quad \dots (2)$$

Equating (1) & (2), we get,

$$\frac{\gamma}{1+w} = \frac{G\gamma_w}{1+e}$$

$$\text{or, } \gamma(1+e) = G\gamma_w(1+w) \quad \left[\because e = \frac{Gw}{S} \right]$$

$$\text{or, } \frac{1+wG}{S} = \frac{G\gamma_w(1+w)}{\gamma}$$

$$\text{or, } \frac{wG}{S} = \frac{G\gamma_w(1+w)}{\gamma} - 1$$

$$\text{or, } \frac{w}{S} = \frac{\gamma_w(1+w)}{\gamma} - \frac{1}{G}$$

$$\text{or, } S = \frac{w}{\frac{\gamma_w(1+w)}{\gamma} - \frac{1}{G}} \quad \text{proved.}$$

11) $\gamma_d = (1-n)G\gamma_w$

→ Solution:-

We have,

$$V = V_a + V_w + V_s$$

$$\text{or, } V = V_w + V_s \quad \left[\because V_a = V_w + V_s \right]$$

$$\text{or, } 1 = \frac{V_w}{V} + \frac{V_s}{V}$$

$$\text{or, } 1 = n + \frac{V_s}{V}$$

$$\text{or, } \frac{V_s}{V} = 1-n$$

$$\text{or, } \frac{M_s}{\gamma_s} \cdot \frac{\gamma}{V} = 1-n \quad \left[\because \gamma_d = \frac{M_s}{V} \right]$$

$$\text{or, } \gamma_d \times \frac{1}{G\gamma_w} = 1-n$$

$$\text{or, } \gamma_d = (1-n)G\gamma_w \quad \text{proved.}$$

10. In a field density, the mass of soil removed from a small hole was 1.59 kg. The volume of the hole is 900 cm³. The field water content was found to be 25%. If the density of the soil is 2700 kg/m³, determine the dry density and degree of saturation of the fill.

→ Solution:-

Given that,

$$\text{mass of soil (} m_s \text{)} = 1.59 \text{ kg}$$

$$\text{Volume of hole (} V \text{)} = 900 \text{ cm}^3 = 900 \times 100^{-3} = 9 \times 10^{-4} \text{ m}^3$$

$$\text{water content (} w \text{)} = 25\% = 0.25$$

$$\text{Density of soil (} \rho \text{)} = 2700 \text{ kg/m}^3$$

$$\text{Dry density (} \rho_d \text{)} = ?$$

We have,

$$M_s = M = \frac{1.59}{1+0.25} = 1.272$$

And,

$$\rho_d = \frac{M_s}{V} = \frac{1.272}{9 \times 10^{-4}} = 1413.33 \text{ kg/m}^3$$

$$\text{Degree of saturation (} s \text{)} = ?$$

$$V_w = \frac{W_w}{\gamma_w} = \frac{M - M_s}{1000} = \frac{1.59 - 1.272}{1000} = 3.18 \times 10^{-4}$$

$$\neq V_w = V - V_s$$

$$= 9 \times 10^{-4} - \frac{1.272}{2700} = 4.288 \times 10^{-4}$$

$$\therefore S = \frac{V_w}{V_s} \times 100 = \frac{3.18 \times 10^{-4}}{4.288 \times 10^{-4}} \times 100\% = 74.16\%$$

11 A sample of wet soil has a volume of 0.0192 m^3 and a mass of 32 kg . When the sample is dried out in an oven, its mass reduces to 28.5 kg . Determine i) dry density ii) void ratio iii) porosity iv) Degree of saturation. Take specific gravity of solids as 2.65 .

> Solution:-

Given that:-

mass of wet soil (M) = 32 kg .

Volume of wet soil (V) = 0.0192 m^3 .

Mass of dry soil (M_s) = 28.5 kg .

i) Dry density (ρ_d) = ?

ii) Void ratio (e) = ?

iii) Porosity (n) = ?

iv) Degree of saturation (s) = ?

We have,

$$\left. \begin{aligned} M_s + M_w &= M \\ \text{or, } M_s + wM_s &= M \\ \text{or, } M &= M_s(1+w) \dots (1) \end{aligned} \right\}$$

$\therefore e$
 $M_w = M - M_s = 32 - 28.5 = 3.5 \text{ kg}$.

Now,

$$V_w = \frac{wM_s}{\rho_w} = \frac{3.5}{1000} = 3.5 \times 10^{-3} \text{ m}^3$$

and, $V_s = \frac{M_s}{G_s \rho_w} = \frac{28.5}{2.65 \times 1000} = 0.01075$

$$\begin{aligned} \therefore V_v &= V - V_s \\ &= 0.0192 - 0.01075 \\ &= 8.445 \times 10^{-3} \text{ m}^3 \end{aligned}$$

Now,

i) $\rho_d = \frac{M_s}{V} = \frac{28.5}{0.0192} = 1484.375 \text{ kg/m}^3$

ii) $e = \frac{V_v}{V_s} = \frac{8.445 \times 10^{-3}}{0.01075} = 0.78$

iii) $n = \frac{V_v}{V} \times 100 = \frac{8.445 \times 10^{-3}}{0.0192} \times 100 = 43.98\%$

iv) $s = \frac{V_w}{V_v} \times 100 = \frac{3.5 \times 10^{-3}}{8.445 \times 10^{-3}} \times 100 = 41.44\%$

12. A soil sample with specific gravity of solids 2.65 has a bulk (mass) specific gravity of 1.8 . Determine the void ratio assuming the soil to be perfectly dry.

> Solution:-

Given that,

Sp. gravity of solids (G_s) = 2.65

Bulk sp. gravity (γ) = 1.8

void ratio (e) = ?

$$\gamma_{dry} = \left(\frac{G_s}{1+e} \right) \gamma_w$$

or, $\gamma_{dry} = \frac{2.65}{1+e}$
 $1.8 = \frac{2.65}{1+e} \times 1$

or, $1.8 + 1.8e = 2.65$

or, $e = \frac{2.65 - 1.8}{1.8}$

$= 0.4722$

Hence, the void ratio was found to be 0.47 .

13) Two soil C and D are mixed dry in proportion of 35%:65% by mass. The specific gravity of soil C and D are 2.65:2.75 respectively. If the bulk density of mixed soil is 1.7 gm/cc at 15% water content, determine void ratio and degree of saturation.

→ Solution:-

Given that,

$$\frac{\text{Mass of C}}{\text{Mass of D}} = \frac{35}{65}$$

$$\frac{G_C}{G_D} = \frac{2.65}{2.75}$$

Bulk density of mixed soil (ρ_{mix}) = 1.7 gm/cc

water content (w) = 15%

void ratio (e) = ?

Degree of saturation (s) = ?

We know that,

$$\rho_{\text{mix}} = \frac{m_C G_C + m_D G_D}{m_C + m_D}$$

$$= \frac{0.35 \times 2.65 + 0.65 \times 2.75}{0.35 + 0.65} = 2.715$$

Also,

$$(\rho_{\text{mix}})_d = \frac{\rho_{\text{mix}}}{1+w} = \frac{1.7}{1+0.15} = 1.478$$

$$\text{And, } (\rho_{\text{mix}})_d = \frac{G_s \rho_w}{1+e}$$

$$\text{or, } 1.478 = \frac{2.715 \times 1}{1+e}$$

$$\therefore e = 0.83$$

And, $s_e = G_w$

$$\text{or, } s \times 0.83 = 2.715 \times 0.15$$

$$\therefore s = 0.4906 \approx 49.06\%$$

14. A natural soil deposit has a bulk unit weight of 16 kN/m^3 and water content of 11%. Determine the amount of water required to be added to 1 m^3 of soil to raise the water content to 18%. Assume void ratio to remain constant.

→ Solution:-

Given that,

$$\text{Bulk unit weight } (\gamma) = 16 \text{ kN/m}^3$$

$$\text{water content } (w) = 11\%$$

$$\text{Volume of soil } (V) = 1 \text{ m}^3$$

$$\text{final water content } (w_f) = 18\%$$

Amount of water required to be added (V_w) = ?

We have,

$$\gamma_d = \frac{\gamma}{1+w} = \frac{16}{1+0.11} = 14.41 \text{ kN/m}^3$$

$$\therefore \text{Weight of dry soil } (W_s)_1 = \gamma_d \times V = 14.41 \times 1 = 14.41 \text{ kN}$$

And,

$$W_w = w W_s = 0.11 \times 14.41 = 1.58 \text{ kN}$$

But,

$$\gamma_w = \frac{W_w}{V_w} \Rightarrow (W_w)_2 = \frac{W_w}{\gamma_w} = \frac{1.58}{9.81} = 0.161 \text{ m}^3$$

Similarly,

for $w=18\%$,

$$\gamma_d = \frac{\gamma}{1+w} = \frac{16}{1+0.18} = 13.56 \text{ kN/m}^3$$

$$\therefore (W_s)_2 = 13.56 \text{ kN}$$

$$W_w = W W_s = 0.18 \times 13.56 = 2.44 \text{ kN}$$

and,

$$(W_w)_2 = \frac{W_w}{W_w} = \frac{2.44}{9.81} = 0.248 \text{ m}^3$$

Now,

$$\begin{aligned} \text{amount of water to be added } (V_w) &= (W_w)_2 - (W_w)_1 \\ &= 0.248 - 0.161 \\ &= 0.087 \text{ m}^3 \end{aligned}$$

Hence, 0.087 m^3 of water is required to be added to 1 m^3 of soil to raise the water content to 18%.

15. A soil has a liquid limit 25%, flow index 12.5% and the plastic limit 15%. Determine the plasticity index and toughness index if water content of the soil in the natural condition in the field is 20%. Find the liquidity index and relative consistency.

→ Solution:-

Given that,

$$\text{Liquid limit (LL)} = 25\%$$

$$\text{Flow index (I}_f) = 12.5\%$$

$$\text{Plastic limit (P.L)} = 15\%$$

$$\text{Plasticity Index (P.I)} = ?$$

$$\text{Toughness Index (I}_t) = ?$$

$$\text{water content } (w) = 20\%$$

$$\text{Liquidity index (L.I)} = ?$$

$$\text{Relative consistency (I}_c) = ?$$

We have,

$$P.I = LL - P.L$$

$$= 25\% - 15\%$$

$$= 10\%$$

$$I_t = \frac{P.I}{I_f} = \frac{10}{12.5} = 0.8$$

$$I_c = \frac{LL - w}{P.I} = \frac{25\% - 20\%}{10\%} = 0.5$$

$$\text{and, } L.I = \frac{w - P.L}{P.I} = \frac{20\% - 15\%}{10\%} = 0.5$$

16. Soil A and B were saturated in their natural condition and they have the same specific gravity. The water content and index properties of the soils are in the following table:

Soil	A	B
Liquid limit (%) (L.L)	62	62
Plastic limit (%) (P.L)	26	40
Natural water content (%)	27	60

Which of these soils:

- a) Has greater liquidity index?

→ Solution:- $P.I = LL - P.L =$

$$\text{For soil A, } (P.I)_A = 36\% \quad (P.I)_B = 22$$

$$(L.I)_A = \frac{w - P.L}{P.I} = \frac{27 - 26}{36} = 0.0278$$

$$(L.I)_B = \frac{60 - 40}{22} = 0.90$$

$$\therefore (L.I)_B > (L.I)_A$$

- b) Is softer in its natural condition. (L.I) (L.I)

Here,

$$(I_t)_A = 36$$

Since,

$$(L.I)_B > (L.I)_A$$

So, soil B is softer in its natural condition.

iii) Has greater plasticity index?

→ Here,

$$(P.I)_A = 36\%$$

$$(P.I)_B = 22\%$$

$$\therefore (P.I)_A > (P.I)_B$$

iv) Is more cohesive? (P.I)

$$(P.I)_A = 36\%$$

$$(P.I)_B = 22\%$$

$$\therefore (P.I)_A > (P.I)_B$$

So, soil A is more cohesive than soil B

(c)

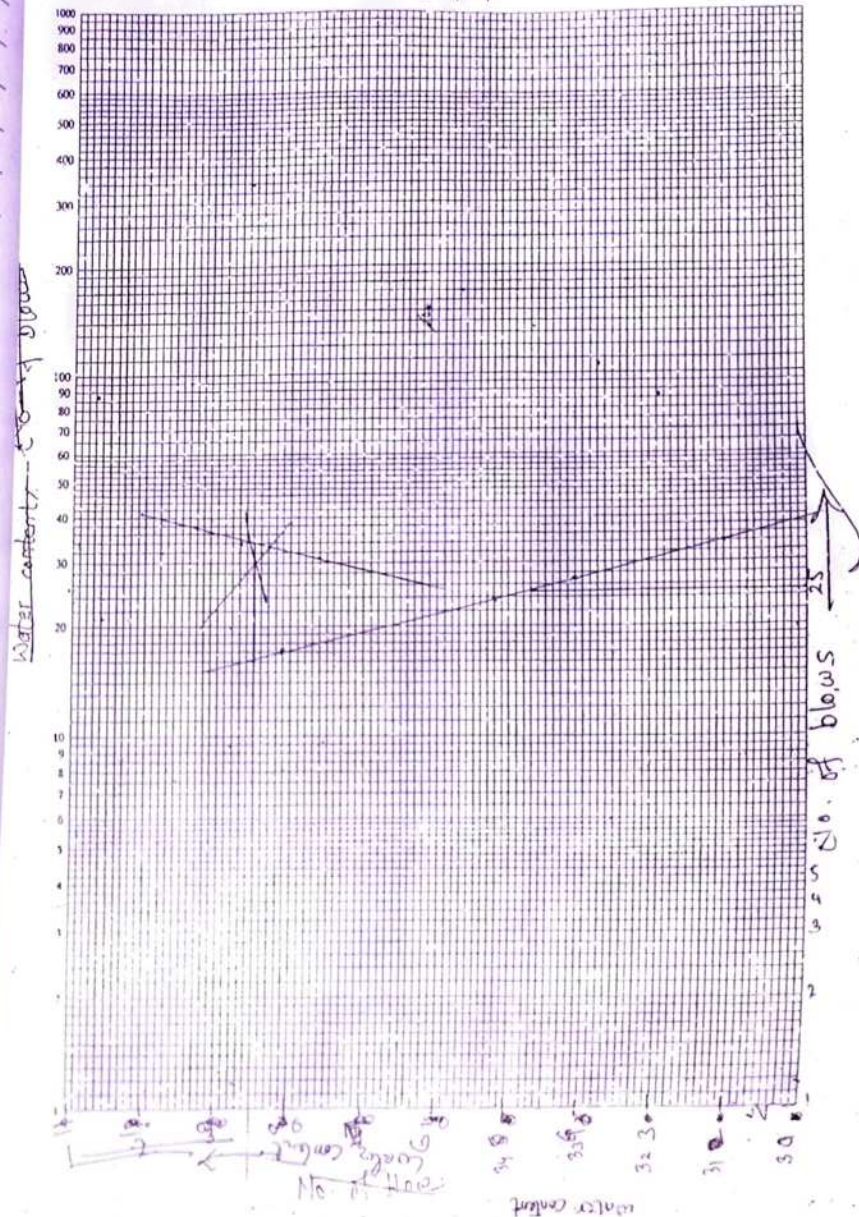
v) Has greater void ratio in its natural condition? Give reasons to your answers

$$\rightarrow e_A = \frac{WG}{S_r} = \frac{27 \times 2.72}{1} = 0.739$$

$$e_B = \frac{WG}{S_r} = \frac{60 \times 2.72}{1} = 1.63$$

So, the soil B has greater void ratio in its natural condition.

Semi-log Graph paper



17) The following data has been obtained from liquid and plastic limit tests on the soil in the laboratory.

Details	Liquid limit test data				Plastic limit test data	
Mass of container with wet soil	43.39	47.62	45.24	45.58	43.62	40.38
" " " " dry "	40.18	43.61	41.87	41.5	41.94	38.85
Mass of container	29.86	31.5	32.01	30.51	33.1	30.9
No. of blows (N)	34	27	23	17		

Determine:-

- Liquid limit, plastic limit and flow index.
- Classify the soil as per USCS
- Liquidity index if the natural water content of soil is 28.9%.

⇒ Solution:-

Details	L.L. test limit				P.L. test limit.	
Mass of wet soil (gm)	43.39	47.62	45.24	45.58	43.62	40.38
dry	40.18	43.61	41.87	41.5	41.94	38.85
Mass of container	29.86	31.5	32.01	30.51	33.1	30.9
No. of blows (N)	34	27	23	17		
Mass of water (mw)	3.21	4.01	3.37	4.08	1.68	1.53
Dry mass of soil (ms)	10.32	12.11	9.86	10.99	8.84	7.95
water content, $w = \frac{m_w}{m_s} \times 100\%$	31.1%	33.1	34.2%	37.1	19%	19.2

Now,

$$L.L. = \frac{33.9}{34} \text{ (from Graph) (water content corresponding to 25 blows)}$$

Now,

S.No	Mass of container (g)	Mass of container with wet soil (g)	Mass of container with dry soil (g)	m_w (g)	m_s (g)	w (%)
1	33.1	43.62	41.94	1.68	8.84	19
2	30.9	40.38	38.85	1.53	7.95	19.25

And,

$$\text{Plastic limit} = \frac{19 + 19.25}{2} = 19.13\%$$

$$\text{Plasticity index} = (34 - 19.13)\% = 14.87$$

$$\rightarrow \text{Flow index (I}_f) = \frac{w_2 - w_1}{\log(w_2/25)} = \frac{33.1 - 31.1}{\log(33.1/25)} = \dots$$

For liquid limit of 34%, the plasticity index from the A line equation is

$$PI_{34} = 0.73(34 - 20)$$

$$PI_{34} = 10.22\%$$

Since $w_L < 50\%$ and $I_p = 14.87\%$, the point lies above A line. From USCS classification, the soil is classified as clay of low plasticity (CL).

$$\text{Liquidity index} = \frac{w_n - PI}{PI} \times 100\% = \frac{0.289 - 0.1913}{0.148} \times 100\% = 66.01\%$$

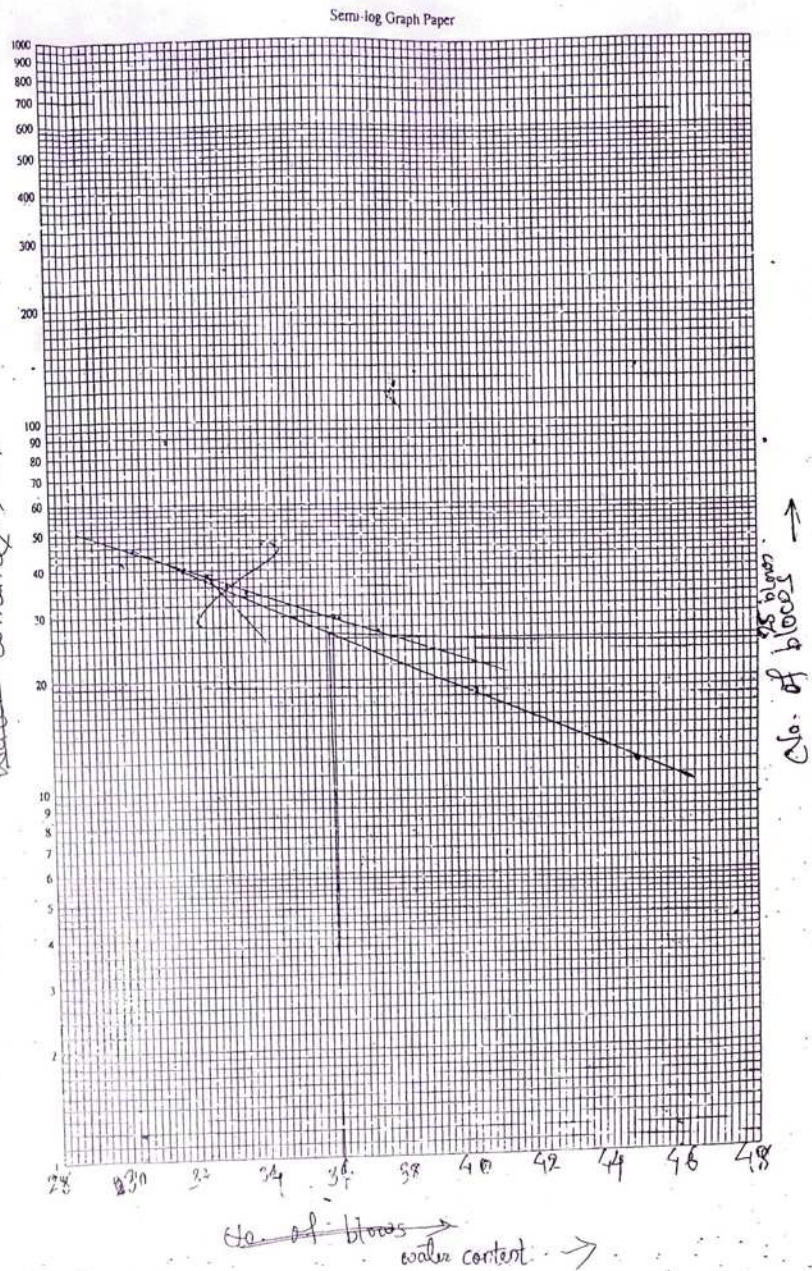
Additional Practise (2016 Fall) :- Solution same as A.N. 57.

Additional practise (2008 Fall, 2015 Fall)

In order to determine the liquid limit of silty clay, the following results were recorded.

Mass of container with wet soil, (gm)	19.62	21.26	19.5	21.28
" " " " dry soil, (gm)	16.74	18.39	17.24	19.08
Mass of container (gm)	10.46	11.21	10.81	11.62
No. of blows (N)	12	19	28	41

Plot the result in graph paper and determine the liquid limit for



the soil. If the plastic limit for the soil was 22% and natural water content 35%, find the plasticity index, liquidity index and consistency index.

→ Solution:-

Given that:-

Plastic limit (PL) = 22%

Natural water content (w) = 35%

Now,

water content (Y)	$\frac{(19.62 - 16.74)}{(16.74 - 10.46)} = 45.86$	$\frac{(21.26 - 18.34)}{(18.34 - 11.21)} = 39.97$	$\frac{(19.5 - 17.24)}{(17.24 - 10.87)} = 35.48$	$\frac{(21.28 - 19.08)}{(19.08 - 11.62)} = 29.49$
no. of blows	12	19	28	41

Now, plotting the graph between no. of blows Vs water content, we have,

From graph, water content at 25 blows = 37%

∴ Liquid limit = 37%

Now,

Plasticity index (PI) = L.L - PL = (37 - 22) = 15%

Liquidity Index (LI) = $\frac{w_n - PL}{PI} = \frac{35 - 22}{15} = 86.67\%$

and Consistency index (CI) = $\frac{LL - w_n}{PI} = \frac{37 - 35}{15} = 13.33\%$

Additional practice (2009 Spring)

The results of a liquid limit test are given below:

No. of blows (N)	48	38	29	20	14
water content (Y)	32.1	35.9	40.7	46.1	52.8

i) Determine the liquid limit for the soil

ii) If the plastic limit for the soil be 23%, find the plasticity index, flow index and toughness index. Hence comment the nature of the soil.

→ Solution -

a) From graph, we have,
water content corresponding to 25 blows = 43%

$$\therefore \text{Liquid limit (LL)} = 43\%$$

$$\text{Plastic limit (PL)} = 23\%$$

Now,

b) Plasticity index (PI) = LL - PL = (43 - 23)% = 20%

$$\text{Toughness index (I_t)} = \frac{PI}{I_f}$$

$$\text{flow index (I_f)} = \frac{w_p - w_L}{\log\left(\frac{100}{I_p}\right)} = \frac{w_p - w_L}{\log\left(\frac{100}{I_p}\right)}$$

from graph,

$$w_{10} = 54\% \quad w_{25} = 16\%$$

$$\therefore I_f = \frac{54 - 16}{1} = 38\%$$

$$\therefore I_t = \frac{20}{38} = 0.53$$

18. Two soils were tested for their consistency in the lab. following data were obtained.

	Soil A	Soil B	
No. of blows (N)			water content (%)
8	43	5	65
20	39	15	61
30	37	30	59
45	25	40	58
water content at which soil crumbled at 3mm ϕ	25	water content at which soil crumbled at 3mm diameter	30%

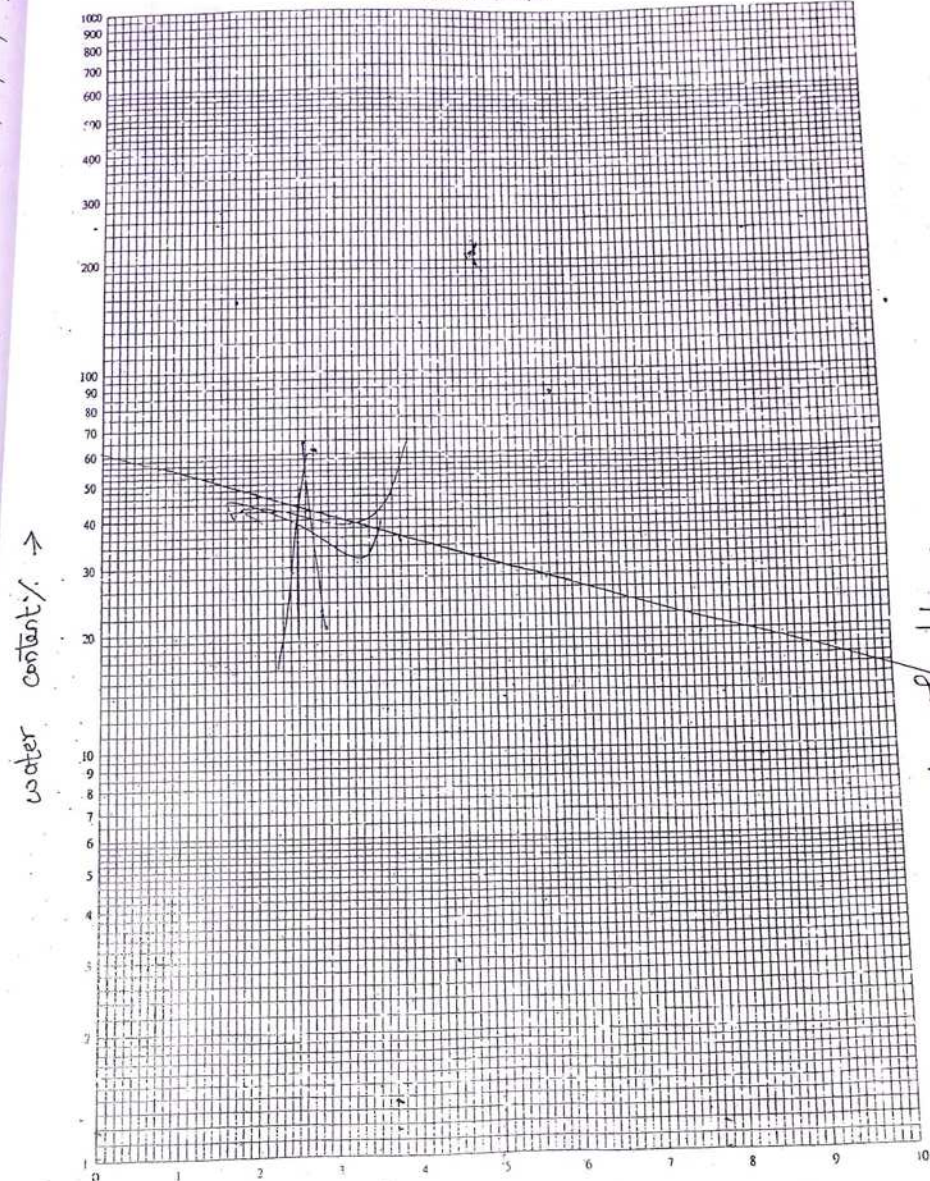
Natural water content % (40%)

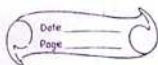
plasticity 50%

Determine: -

R.K. SHAH

Semi-log Graph Paper





3. For the sieve analysis of following grain size distribution has been obtained:

Sieve No.	4	10	20	40	100	200	Pan
Sieve opening (mm)	4.75	2	0.85	0.425	0.15	0.075	
Soil retained (gm)	100	180	120	140	160	200	100

- Determine the percent finer in each sieve.
- Without drawing the grain size distribution curve, determine the coefficient of uniformity and coefficient of curvature.
- According to the USCS should the soil be classified as sand or Gravel?

→ Solution:-

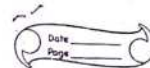
Sieve no.	Sieve Opening (mm)	Soil retained (gm)	Percent retained	Comm. % retained	% Passing
4	4.75	100	10	10	90
10	2	180	18	28	72
20	0.85	120	12	40	60
40	0.425	140	14	54	46
100	0.15	160	16	70	30
200	0.075	200	20	90	10

From the above table, we have $D_{10} = 0.075$, $D_{30} = 0.15$ mm and $D_{60} = 0.85$ mm.

$$b) \therefore C_u = \frac{D_{60}}{D_{10}} = \frac{0.85}{0.075} = 11.33$$

$$C_c = \frac{D_{30}^2}{D_{60} \times D_{10}} = \frac{(0.15)^2}{0.85 \times 0.075} = 0.353$$

- As more than 50% is retained on 0.075 mm sieve, the soil is coarse grained. Coarse fraction = 90%, Gravel fraction = 10% and sand fraction = 80%. As more than half of the coarse fraction is smaller than 4.75 mm sieve, the soil is sand.



20. The sieve analysis of a given sample of soil gave information that 57% of the particle pass through 75 micron sieve. The liquid and plastic limit of the soil were 62% and 28% respectively. Classify the soil as per the USCS.

→ Solution:-

Given that:-

% of particle passing through 0.075 mm sieve = 57%

Liquid limit (L.L.) = 62%

Plastic limit (P.L.) = 28%

$$P.I. = \frac{L.L. - P.L.}{100} = \frac{62 - 28}{100} = 0.34$$

$$P.I. = 0.73(L.L. - 20)$$

$$= 0.73(62 - 20) = 30.66$$

Since $P.I. > 25\%$ and $L.L. = 62\% > 50\%$.

So the soil is CH

Hence, the given soil is CH - Inorganic clay of high plasticity

21. 500 gm of dry soil was subjected to a sieve analysis. The weight of soil retained on each sieve is as follows:-

Sieve size	4.75 mm	2 mm	1 mm	425 μ	212 μ	150 μ	75 μ
Wt of soil retained on sieve (g)	10	165	100	85	40	30	50

Plot the grain size distribution curves and determine the following:

- Effective size, D_{10}
- Coefficient of uniformity, C_u
- Coefficient of curvature C_c
- The gradation of the soil.

→ Solution:-

The calculations have been calculated in the table below:-

Sieve Size (mm) retained	Soil retained	Percentage retained	Cum % retained	% Passing
4.75	10	2 ($\frac{10}{500} \times 100\%$)	2	98
2	165	33	35	65
1	100	20	55	45
0.425	85	17	72	28
0.212	40	8	80	20
0.150	30	6	86	14
0.075	50	10	96	4

From the particle size distribution curve, we have,

$$D_{10} = 0.13 \quad 0.13$$

$$D_{30} = 0.43 \quad 0.90$$

$$D_{60} = 0.50 \quad 1.80$$

a) $D_{10} = 0.13$

b) $C_u = \frac{D_{60}}{D_{10}} = \frac{0.50}{0.13} = 3.85 \approx 3.8$

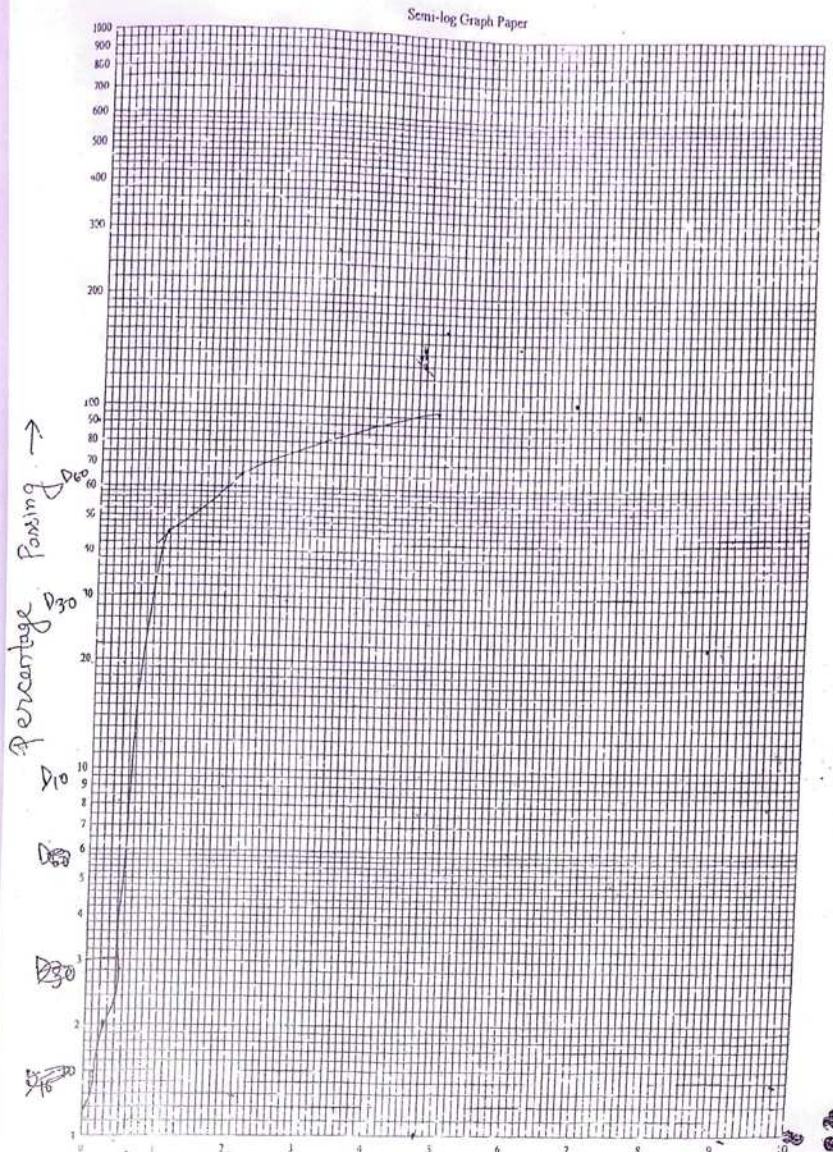
c) $C_c = \frac{D_{30}^2}{D_{60} \times D_{10}} = \frac{0.43^2}{0.50 \times 0.13} = 2.84 \approx 2.8$

And
The gradation of the soil is sand, soil is well graded.

Additional practise (2016 Spring).

> Solution same as above. expect for classification:-
According to IS: 1498-1959 above classification is

Clay Size	Silt size (mm)			Sand size (mm)			Gravel (mm)
	fine	Medium	Coarse	fine	Medium	Coarse	
	0.002	0.006	0.02	0.06	0.2	0.6	2



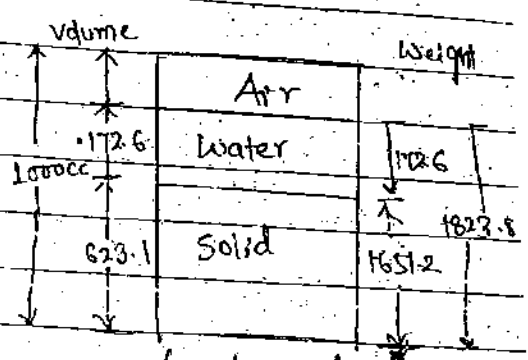
Grain size distribution (mm)

22. A 1000cc core cutter weighing 946.80 gm was used to find out the in-situ weight of an embankment. The weight of core cutter filled with soil was noted to be 2770.60 gm. Laboratory test on the sample indicates a water content of 10.45% and specific gravity of solids is 2.65. Determine the bulk unit weight, dry unit weight, void ratio and degree of saturation.

→ Solution.

Given that;

- Volume of cutter = 1000cc
- Empty weight of cutter = 946.80 gm
- Weight of cutter + soil = 2770.60 gm
- Sp. gravity (G) = 2.65



Weight of soil solid (W_s) = $W - W_{cutter} = 2770.6 - 946.8 = 1823.8$ gm

Weight of water (W_w) = $W - W_s = 2770.6 - 1823.8 = 946.8$ gm

$W_w = W - W_s = 1823.8 - 1651.2 = 172.6$ gm

$V_s = \frac{W_s}{G \times \gamma_w} = \frac{1651.2}{2.65 \times 1} = 623.1$ cc

$V_w = \frac{W_w}{\gamma_w} = \frac{172.6}{1} = 172.6$ cc

and, $V_v = V - V_s = 1000 - 623.1 = 376.9$ cc

Bulk unit weight (γ) = $\frac{W}{V} = \frac{1823.8}{1000} = 1.82$ g/cc

Dry unit weight (γ_d) = $\frac{W_s}{V} = \frac{1651.2}{1000} = 1.65$ g/cc

Void ratio (e) = $\frac{V_v}{V_s} = \frac{376.9}{623.1} = 0.61$

Degree of saturation (S_r) = $\frac{V_w}{V_v} = \frac{172.6}{376.9} = 0.458 = 45.8\%$

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23. In a proctor test, for one observation, mass of wet sample is missing. The oven dry mass of this sample was 1800 gm. The volume of the mould used is 900 cc. If the saturation of this sample was 80%, determine the moisture content and bulk density of the sample assuming specific gravity = 2.7.

→ Solution:-

Given that:-

$$\text{Oven dry mass } (M_d) = 1800 \text{ gm.}$$

$$\text{Volume of mould } (V) = 900 \text{ cc.}$$

$$\text{Saturation } (S) = 80\% = 0.8$$

$$\text{Moisture content } (w) = ?$$

$$\text{bulk density } (\gamma) = ?$$

$$\text{Sp. Gravity } (G) = 2.7$$

We have,

~~bulk density =~~

$$V_s = \frac{W_s}{G \times \gamma_w} = \frac{1800}{2.7 \times 1} = 666.67$$

And,

$$V_v = V - V_s = 900 - 666.67 = 233.33 \text{ cc.}$$

$$W_w = V_w \times \gamma_w = 233.33 \times 1 = 233.33 \text{ gm.}$$

$$\text{Water content } (w) = \frac{W_w}{W_s} \times 100\%$$

$$= \frac{233.33}{1800} \times 100\%$$

$$= 12.96\%$$

$$\text{And, } Se = wG \Rightarrow e = \frac{12.96 \times 2.7}{80} = 0.437$$

$$\text{And, } \gamma = \frac{(G + S_e) \gamma_w}{1 + e} = \frac{(2.7 + 0.8 \times 0.437) \times 1}{1 + 0.437} = 2.12 \text{ g/cc}$$

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24. A sample of clay with a mass of 680 gm was coated with paraffin wax. The combine mass of clay and wax was 692.5 gm. The volume of clay and wax was found by immersion in water to be 345 ml. The soil sample was then broken open. The moisture content and specific gravity of soil solid particle test gave 17% and 2.67 respectively. The specific gravity of paraffin wax was 0.88. determine: (i) bulk density (ii) degree of saturation (iii) void ratio (iv) dry density of the soil mass.

→ Solution:-

Given that,

$$\text{Weight of clay} = 680 \text{ gm.}$$

$$\text{Weight of clay + wax} = 692.5 \text{ gm.}$$

$$\therefore \text{Weight of wax} = 692.5 - 680 = 12.5 \text{ gm.}$$

$$\text{Displaced volume of water} = 345 \text{ ml.}$$

$$\therefore \text{Volume of clay and wax} = 345 \text{ ml.}$$

$$\text{Sp. gr. of paraffin wax } (G_{\text{wax}}) = 0.88$$

$$G_{\text{wax}} = \frac{W_{\text{wax}}}{V_{\text{wax}}} \Rightarrow V_{\text{wax}} = \frac{W_{\text{wax}}}{G_{\text{wax}}} = \frac{12.5}{0.88} = 14.2 \text{ cm}^3$$

$$\text{Volume of wax} = \frac{W_{\text{wax}}}{\gamma_{\text{wax}}} = \frac{12.5}{0.88} = 14.2 \text{ cm}^3$$

$$\text{Volume of clay} = 345 - 14.2 = 330.8 \text{ ml.}$$

$$\therefore \gamma_{\text{clay}} = \frac{W_{\text{clay}}}{V_{\text{clay}}} = \frac{680}{330.8} = 2.056 \text{ gm/cc.}$$

$$\gamma_d(\text{clay}) = \frac{\gamma_{\text{clay}}}{1 + w} = \frac{2.056}{1 + 0.17} = 1.75 \text{ gm/cc.}$$

$$\gamma_d = \frac{G \gamma_w}{1 + e} \Rightarrow 1.75 = \frac{2.67 \times 1}{1 + e} \Rightarrow e = 0.52$$

$$f \text{ } Se = wG$$

$$\therefore S = \frac{0.17 \times 2.67}{0.52} = 0.874$$

25. RAKESH KUMAR SAH
Classify the soil A and B with the following properties according to USCS.

Soil	L.L. %	P.I. %	% passing # 200 sieve (0.075mm)
A	62	30	60
B	38	17	82

→ Soil Solution:-

for soil A

$$P.I. = 0.73(L.L - 20)$$

$$= 0.73(62 - 20)$$

$$= 30.66\%$$

Since, P.I. of A = 30.66% > 25% and L.L > 50%,
so the soil is

for soil B

$$P.I. = 0.73(LL - 20)$$

$$= 0.73(38 - 20)$$

$$= 13.14\%$$

Additional practise (2014 Fall)
Solution same as above.

26. The following data refers to a light silty clay that was assumed to be saturated in the undisturbed condition. On the basis of these data, determine the liquidity index, sensitivity and void ratio of the saturated soil. Classify the soil according to the unified soil classification.

Index Properties	Unconfined Compressive strength (kN/m ²)	water content %	Liquid Limit, %	Plastic Limit, %	Shrinkage Limit, %	% passing no. 200 sieve
Undisturbed	244	22				
Remoulded	144	22	45	20	12	90

As per above data, soil is classified as A-7-6 according to AASHTO classification. Find the index of the soil.

→ Solution:-

We know that,

$$P.I. = L.L - P.L = 45\% - 20\% = 25\%$$

Now;

$$i) \text{ Liquid index (L.I.)} = \frac{w_n - P.L}{P.I} = \frac{22 - 20}{25} = 0.08$$

$$ii) \text{ Sensitivity (S}_t\text{)} = \frac{q_{u, \text{undisturbed}}}{q_{u, \text{remoulded}}} = \frac{244}{144} = 1.69$$

iii) Void ratio (e) = ?

$$s_e = G_w$$

$$\text{or, } 1 \times e = 2.68 \times 0.22$$

$$[\because S=1]$$

$$\therefore e = 0.5896$$

To classify according to USCS,

$$\text{from formula, } P.I. = 0.73(L.L - 20) = 18.25\%$$

Since, 25% > 18.25 and L.L = 45% < 50%, so the soil is CL.

To classify according to AASHTO,

$$G.I. = 0.2a + 0.005ac + 0.01bd \quad \dots (1)$$

$$a = 90 - 35 = 55 \Rightarrow 40$$

$$c = 45 - 40 = 5$$

$$b = 40\%$$

$$d = 25 - 10 = 15$$

Then eqn. (1) becomes,

$$G.I. = 0.2 \times 40 + 0.005 \times 40 \times 5 + 0.01 \times 40 \times 15$$

$$= 15$$

Hence, the soil is A-7-6(15) according to AASHTO.

— X — X — X —

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Rakesh K. S.

Tutorial-2

Ch-5 - Soil Compaction:-

1. The maximum dry density of a sample by light compaction test is 1.78 gm/cm^3 at optimum moisture content of 15%. Find the air voids and degree of saturation. What would be the corresponding value of dry density on zero air void line at OMC? Assume $G = 2.67$.

→ Solution:-

Given that:

Max. dry density (ρ_d) = 1.78 gm/cm^3 .

Moisture content (w) = 15%.

$G = 2.67$

Air voids (n_a) = ?

degree of saturation (S) = ?

We have,

$$\rho_d = \frac{G \rho_w (1 - n_a)}{1 + wG}$$

$$\text{or, } 1.78 = \frac{2.67 \times 1 (1 - n_a)}{1 + 0.15 \times 2.67}$$

$$\therefore n_a = 6.63\%$$

$$\text{and, } \rho_d = \frac{G \rho_w}{1 + wG} \Rightarrow 1.78 = \frac{2.67 \times 1}{1 + 0.15 \times 2.67}$$

$$\text{or, } 1.78 = \frac{G \rho_w}{1 + e}$$

$$\text{or, } \rho_d = \frac{G \rho_w (1 + w)}{1 + e}$$

$$\text{or, } 1.78 = \frac{2.67 \times 1 (1 + 0.15)}{1 + e} \Rightarrow e = 0.725$$

$$\therefore S = \frac{Gw}{e} =$$

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$$\text{And, } \rho_d = \frac{G \rho_w}{1 + wG} \Rightarrow 1.78 = \frac{2.67 \times 1}{1 + 0.15 \times 2.67}$$

$$\text{or, } 1.78 = \frac{2.67S}{S + 0.4005}$$

$$\therefore S = 80.5\%$$

And, For corresponding value of dry density on zero air void line at OMC, degree of saturation, $S = 1$.

$$\therefore \rho_d = \frac{G \rho_w}{1 + wG} = \frac{2.67 \times 1}{1 + 0.15 \times 2.67} = 1.906 \text{ gm/cm}^3$$

2. Earth is required to be excavated from pits for building an embankment. The unit weight of soil in wet condition is 18 kN/m^3 and its water content is 8%. In order to build a 4m high embankment with top width 2m and side slope 1:1, estimate the quantity of earth required to be excavated per meter length of embankment. The dry density required in the embankment is 15 kN/m^3 with the moisture content 10%. Take sp. gr. of solids as 2.67.

→ Solution:-

Given that:-

$$\gamma = 18 \text{ kN/m}^3 = \frac{W}{V}$$

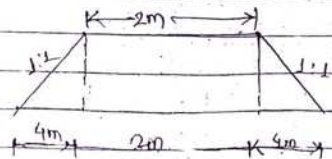
$$w = 8\%$$

$$\rho_d = \frac{15 \text{ kN/m}^3}{1 + 0.18} = 16.67 \text{ kN/m}^3$$

Required dry density of embankment (ρ_d)_{emb} = 15 kN/m^3 .

The volume of embankment soil per unit length = $\left(\frac{10+2}{2}\right) \times 4 \times 1$

$$= 24 \text{ m}^3$$



Dry wt. of 1m^3 excavated soil = 16.67KN

Dry wt. of 1m^3 embankment soil = 15KN

\therefore Volume of soil required for 1m^3 of embankment
 $= \frac{15}{16.67} = 0.899\text{m}^3$

\therefore Total quantity of soil required excavated per meter length of embankment
 $= 0.899 \times 24$
 $= 21.58\text{m}^3$

3. In a standard proctor test on a soil having specific gravity as 2.7, the following results were obtained

Water content (%)	5	8	10	12	15	20
Bulk unit wt, kg/m^3	1890	2130	2200	2210	2160	2080

Plot the dry density versus water content curve. Show zero air void line on the same curve. Determine the values of void ratio, porosity and degree of saturation for the soil at optimum moisture content condition.

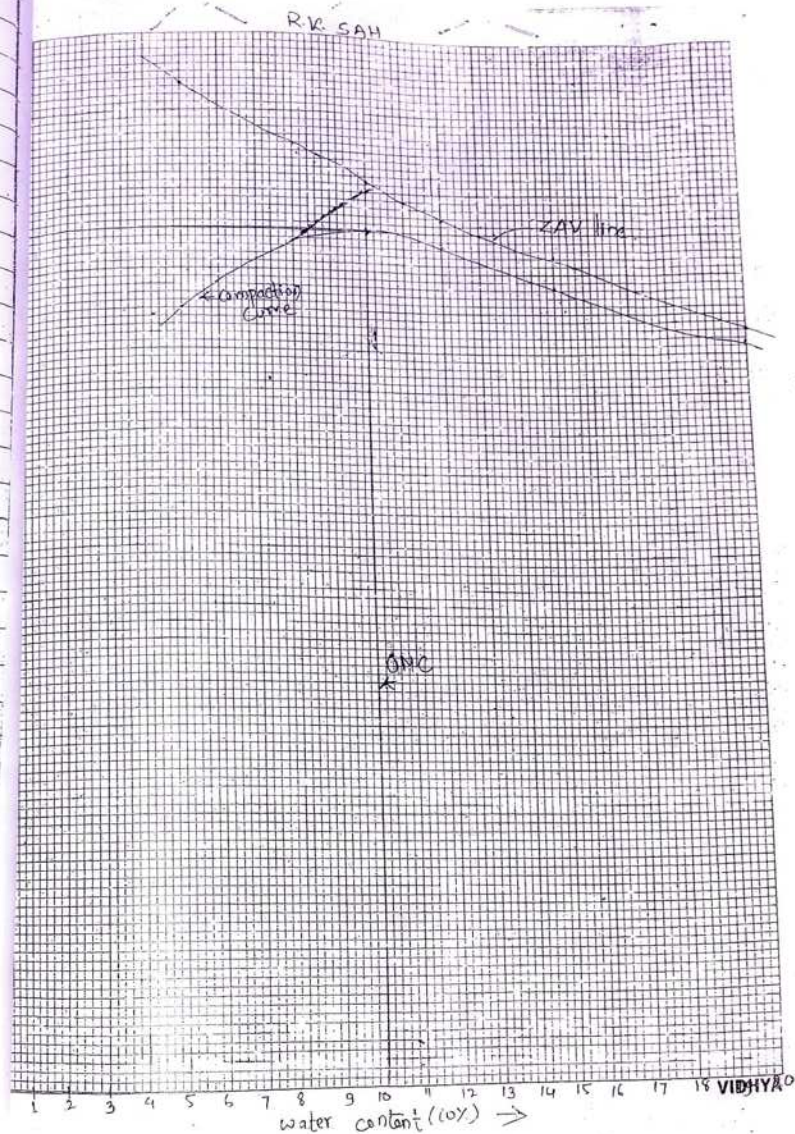
Solution -

Given that;

Sp. gr. of soil (G) = 2.7.

Now,

Water Content (%)	Bulk unit weight (γ) (kg/m^3)	Dry unit weight (kg/m^3) $\gamma_d = \frac{\gamma}{1+w}$	for zero air void line $\gamma_d = \frac{G \gamma_w}{1+wG}$
5	1890	1800	2378.86
8	2130	1972.22	2220.39



10	2200	2000	2125.98
12	2210	1973.21	2039.27
15	2160	1878.26	1921.71
20	2080	1733.33	1753.25

From the graph, we have,

OMC = 10%

and Max. dry unit weight $(\gamma_d)_{max} = 2000 \text{ kg/m}^3$.

Also,

$$\gamma_d = \frac{G \gamma_w}{1+e} \Rightarrow 2000 = \frac{2.7 \times 1000}{1+e} \Rightarrow e = 0.35$$

$$n = \frac{e}{1+e} = \frac{0.35}{1+0.35} = 25.92$$

And,

$$s_e = G_w$$

$$\Rightarrow s = \frac{0.1 \times 2.7}{0.35} = 77.14\%$$

Additional practice (2009 Spring):

The result of a laboratory Proctor test were shown below:

Nb. of test	1	2	3	4	5	6
Wt of mould & soil	3.526	3.711	3.797	3.906	3.924	3.882
Water content	7.33	9.40	11.23	15.20	16.92	19.39

The mould is 12.7cm high and has an internal diameter of 10cm.

The weight of the empty mould is 1.89 kg. Plot the moisture content & density curve and determine the optimum moisture content and maximum dry density. Given, $G = 2.68$.

→ Solution:-

We have,

$$\text{Volume of mould (V)} = \frac{\pi r^2 h}{4} = \frac{3989.82 \text{ cm}^3}{4}$$

Sl. No.	Water Content	Wt. of mould + soil	Wt. of soil only	$S = M$ $\frac{V}{\text{kg/cm}^3}$	$S_d = \frac{S}{1+W}$
1	7.33	3.526	1.636	4.1×10^{-4}	3.81×10^{-4}
2	9.40	3.711	1.821	4.56×10^{-4}	4.16×10^{-4}
3	11.23	3.797	1.907	4.77×10^{-4}	4.28×10^{-4}
4	15.20	3.906	2.016	5.05×10^{-4}	4.38×10^{-4}
5	16.92	3.924	2.034	5.097×10^{-4}	4.36×10^{-4}
6	19.39	3.882	1.992	4.99×10^{-4}	4.18×10^{-4}

Now,

from Graph we have,

OMC = 15.2% and,

Max. dry density = 4.38

4. An earthen embankment of $1 \times 10^6 \text{ m}^3$ volume is to be constructed with a soil having a void ratio of 0.8 after compaction. There are three borrow pit marked by A, B and C having soils with void ratios of 0.90, 1.50 and 1.80 respectively. The cost of excavation & transporting the soil is 1 Rs. 0.25, 0.23 and 0.18 per m^3 resp. Calculate the volume of soil to be excavated from each pit. Which borrow pit is most economical? Take sp. gr. of soil as 2.65.

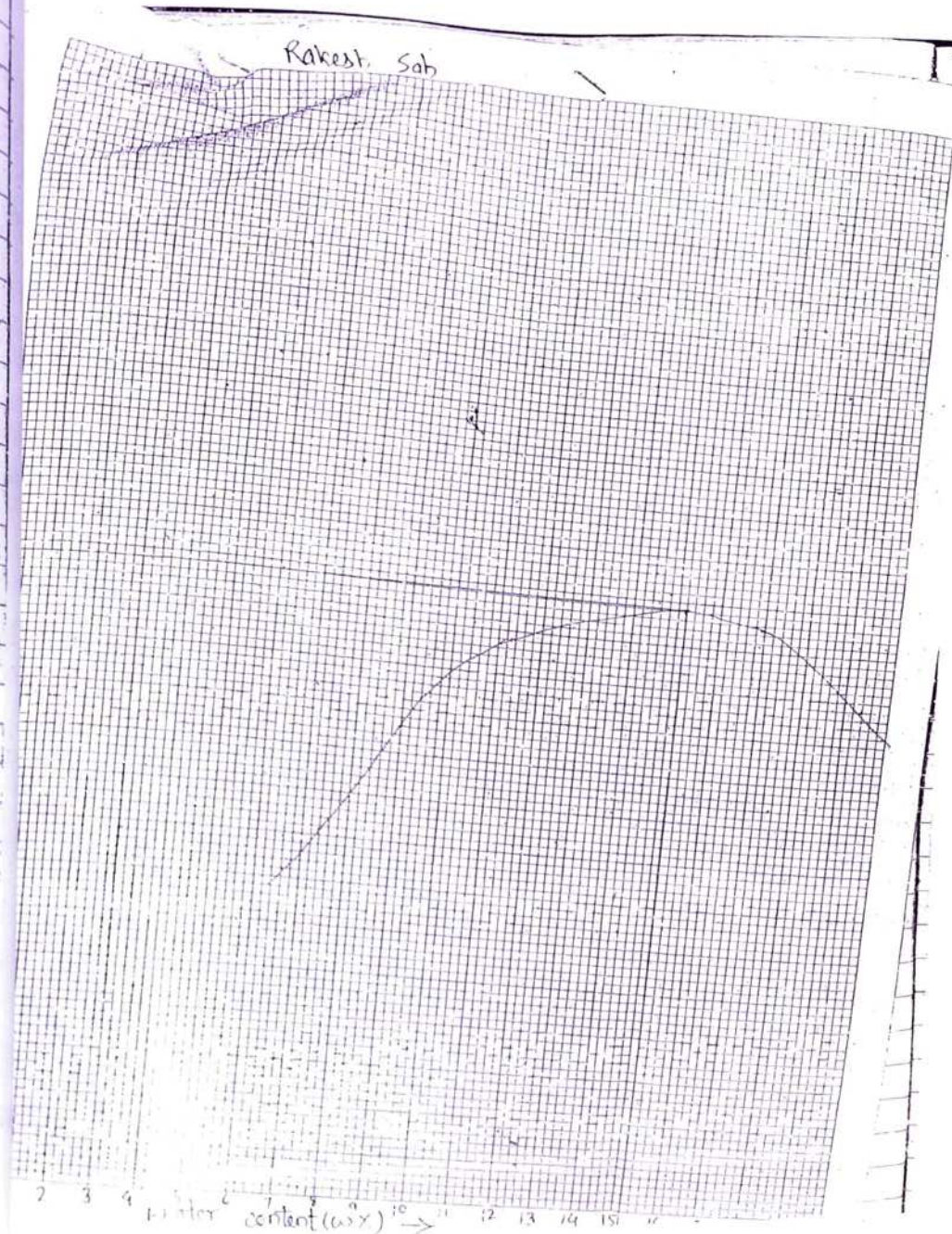
→ Solution:.

$$\text{We have, dry unit wt: } \gamma_d = \frac{G \gamma_w}{1+e} = \frac{2.65 \times 9.81}{1+0.8} = 14.14 \text{ kN/m}^3$$

$$\therefore (\gamma_d)_a = \frac{2.65 \times 9.81}{1+0.90} = 13.68 \text{ kN/m}^3$$

$$(\gamma_d)_b = \frac{2.65 \times 9.81}{1+1.5} = 10.39 \text{ kN/m}^3$$

$$(\gamma_d)_c = \frac{2.65 \times 9.81}{1+1.8} = 9.78 \text{ kN/m}^3$$



density and optimum moisture content. (iii) Find the degree of saturation at OMC (iv) If it is proposed to secure a relative compaction of 95% in the field, what is the range of water content that can be allowed?

→ Solution: -

Volume of mould (V) = 950 cc

Sp. gr. of soil (G) = 2.65

w%	Mass of wet sample (gm)	Density of dry sample $\rho_d = \frac{G}{1+w}$	100% saturation $\rho_d = \frac{G \rho_w}{1+wG}$	80% saturation $\rho_d = \frac{(G \rho_w)(1 + \frac{wG}{0.8})}{1+w}$	wet density (ρ)
8	1700	1.66	2.18	2.094	1.79
11.5	1900	1.79	2.03	1.92	2
14.5	2000	1.83	1.92	1.79	2.10
17.5	1980	1.78	1.81	1.68	2.089
19.5	1950	1.72	1.75	1.61	2.05
21.5	1920	1.66	1.688	1.55	2.02

→ Plotted in Graph paper

ii) From Graph,
Max. dry density = 1.83
OMC = 14.5%

$$\rho_{d \text{ field}} = \frac{G \rho_w}{1+wG}$$

$$\text{or, } 1.784 = \frac{2.65 \times 1}{1 + w \times 2.65}$$

$$\therefore w = 18.32\%$$

$$\text{for } \rho_{d \text{ lab } w = 16.61\%$$

iii) Degree of saturation at OMC =

$$(\rho_d)_{\text{max}} = \frac{G \rho_w}{1+e}$$

$$\therefore e = \frac{G \rho_w}{(\rho_d)_{\text{max}}} - 1 = \frac{2.65 \times 1}{1.84} - 1 = 0.44$$

$$\text{Now, } S = \frac{wG}{e} = \frac{0.145 \times 2.65}{0.44} = 0.8733 = 87.33\%$$

iv) R.C = 95% = $\frac{\rho_{d \text{ field}}}{\rho_{d \text{ lab}}}$ $\Rightarrow \rho_{d \text{ (field)}} = 1.84 \times 0.95 = 1.748 \text{ gm/cc}$

6. A sample of soil compacted according to standard proctor test has a unit weight of 20 kN/m^3 at 100% compaction and at optimum water content of 15%. What is the dry unit weight? What is the dry unit weight at zero air voids? If the voids becomes filled with water, what would be the saturated unit weight? Assume $G = 2.67$.

→ Solution:-

Given that:-

Unit weight $\gamma = 20 \text{ kN/m}^3$ at 100% compaction

$$w = 15\%$$

$$\gamma_d = ?$$

$$\gamma_d \text{ at zero air voids} = ?$$

$$\gamma_{\text{sat}} = ?$$

We have,

$$\gamma_d = \frac{\gamma}{1+w} = \frac{20}{1+0.15} = 17.39 \text{ kN/m}^3$$

At zero air voids (ZAV),

$$\gamma_d = \frac{G \gamma_w}{1+wG} = \frac{2.67 \times 9.81}{1+0.15 \times 2.67} = 18.7 \text{ kN/m}^3$$

When voids filled with water,

$$\gamma_d = \frac{G \gamma_w}{1+e} \Rightarrow 17.39 = \frac{2.67 \times 9.81}{1+e} \Rightarrow e = 0.506$$

Now,

$$\gamma_{\text{sat}} = \frac{(G+e) \gamma_w}{1+e} = \frac{(2.67+0.506) \times 9.81}{1+0.506}$$

$$= 20.69 \text{ kN/m}^3$$

Additional practice (2012 Fall)

Solution same as above.

7. A sample of soil is prepared by mixing a quantity of dry soil with 11% by weight of water. Find the weight of this wet mixture, which will be required to produce by static compaction, a cylindrical specimen 150mm diameter and 125mm deep with 5% air void. Take specific gravity of solids are 2.7. Also find the dry density and void ratio of soil.

→ Solution:-

Given that:-

$$\text{Water content } (w) = 11\% = 0.11$$

$$\text{Sp. gr. of solid } (G) = 2.7$$

$$\text{Degree of saturation } (S_r) = 100\% - 5\% = 95\% = 0.95$$

Now,

$$wG = S_r e$$

$$\therefore e = \frac{wG}{S_r} = \frac{0.11 \times 2.7}{0.95} = 0.313$$

$$\text{Again, } \gamma_d = \frac{G \gamma_w}{1+e} = \frac{2.7 \times 9.81}{1+0.313} = 20.06 \text{ gm/cm}^3$$

And,

$$\gamma_t = \frac{\gamma_d}{1+w}$$

$$\therefore \gamma_t = \gamma_d (1+w) = 20.06 (1+0.11) = 22.28 \text{ gm/cm}^3$$

$$\text{Volume of soil} = \frac{\pi d^2 b}{4} = \frac{\pi \times 15^2 \times 125}{4} = 2208.98 \text{ cm}^3$$

$$\text{Now, } \gamma = W \Rightarrow W = \frac{\gamma_t \times \text{Volume}}{\gamma_w} = \frac{22.28 \times 2208.98}{9.81} = 5036.37 \text{ gm}$$

$$\therefore \text{Weight of the wet mixture} = 5.036 \text{ kg}$$

8. The following data are available in connection with the construction of an embankment.

- i) Soil from borrow pit: natural density = 1.65 Mg/m^3 & $w = 10\%$.
 - ii) Soil after compaction: density = 2 Mg/m^3 & $w = 18\%$.
- Hence for every 50 m^3 of compacted soil of embankment estimate
- i) the quantity of soil to be excavated from the borrow pit.
 - ii) The amount of water to be added.

→ Solution:-

We have,

$$\rho_{d, \text{ nat}} = \frac{\rho}{1+w} = \frac{1.65}{1+0.10} = 1.473 \text{ Mg/cm}^3$$

$$\rho_{d, \text{ after}} = \frac{2}{1+0.18} = 1.695 \text{ Mg/cm}^3$$

$$\therefore \text{Volume of soil required for } 1 \text{ m}^3 \text{ compacted soil} = \frac{1.695}{1.473} = 1.15 \text{ m}^3$$

$$\therefore \text{for } 50 \text{ m}^3 \text{ compacted soil, volume of soil required} \\ = 50 \times 1.15 \\ = 57.53 \text{ m}^3$$

a) Now, in natural state, the moisture present in 1.695 Mg of dry soil would be

$$= 1.695 \times 0.12 = 0.2034 \text{ Mg/cm}^3$$

And, the moisture which the soil gains during compaction is $1.695 \times 0.18 = 0.3051 \text{ Mg/m}^3$

Hence,

$$\text{Mass of water to be added} = 0.3051 - 0.2034 \\ = 0.1017 \text{ Mg/m}^3$$

$$\therefore \text{for } 50 \text{ m}^3 \text{ soil, water to be added} = 50 \times 0.1017 \\ = 5.085 \text{ Mg} \\ = 50.85 \text{ kg}$$

Rakesh Kumar Sah

9. An earth embankment is to be compacted to a dry density of 1.84 gm/cc at moisture content of 15% . The bulk density and water content in the borrow pit are 1.77 and 8% respectively. How much excavation should be carried out in the borrow pit for each cubic meter of embankment?

→ Solution:-

Given that:

$$\text{Dry density } (\rho_d) = 1.84 \text{ gm/cc}$$

$$\text{Water content } (w) = 15\%$$

$$\text{Bulk density of borrow pit } (\rho_b) = 1.77$$

$$\text{Water content of " " } (w_b) = 8\%$$

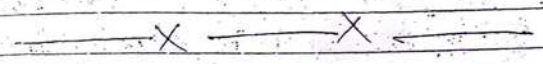
We have,

$$\rho_{d, b} = \frac{\rho_b}{1+w_b} = \frac{1.77}{1+0.08} = 1.64 \text{ gm/cc}$$

Here,

$$\text{for } 1 \text{ cm}^3 \text{ dry unit wt} = 1.64 \text{ gm}$$

$$\therefore \text{Volume of borrow pit} = \frac{1.84}{1.64} = 1.13 \text{ m}^3$$



Rakesh Kumar Sab
Tutorial - 3

Ch. 6 - Principle of effective stress, capilarity and Permeability on soil:-

1. A sample of coarse sand is 15cm high and 5.5cm diameter. It was tested in a constant head permeameter. Water permeated through the soil under a hydrostatic head of 50cm for 6 seconds. The water was collected and it was found to weigh 500 gms. Find the coefficient of permeability in cm/sec.

→ Solution:-

Given that:-

Height of sample (L) = 15cm.

Diameter of sample (d) = 5.5cm

Time (t) = 6sec.

Weight of water (W) = 500 gms.

Hydrostatic head (h) = 50cm.

Coefficient of permeability (K) = ?

We have,

$$K = \frac{qL}{Ah}$$

$$q = \frac{V}{t} = \frac{W}{\gamma \times t} = \frac{500}{1 \times 6} = 83.33 \text{ cm}^3/\text{s}$$

$$\therefore K = \frac{83.33 \times 15 \times 4}{\pi \times 5.5^2 \times 50} = 1.052 \text{ cm/sec. } \frac{2}{2}$$

Additional Practise (2016 Spring)

Calculate the coefficient of permeability of a soil sample 8cm height and c/s area 60cm². It is observed that in 12 minutes, 600ml of water passed down under constant head of 90cm.

The test specimen weigh 750 gm in dry condition. Take sp. Gr. of soil solid as 2.7. Calculate the seepage velocity.

Solution:-

Given that:-

Length of specimen, L = 80mm

Area of X-section, A = 6000 mm²

Quantity of flow, Q = 600ml = 600cc.

Time of flow t = 12 min. = 12 × 60 = 720 sec.

Constant head, H = 400mm.

$$Q = k_i A$$

$$\therefore \frac{Q}{t} = k \cdot H \cdot A$$

$$\therefore K = \frac{Q \cdot L}{A \cdot H \cdot t} = \frac{600 \times 80 \times 1000}{6000 \times 400 \times 720} = 0.028 \text{ mm/s}$$

$$\text{Discharge velocity, } \bar{v} = \frac{q}{A} = \frac{Q}{A \cdot t} = \frac{600 \times 1000}{6000 \times 720} = 0.139 \text{ mm/s}$$

Dry weight of specimen, W_d = 750 gm.

∴ Volume of specimen, V = 6000 × 80 = 4.8 × 10⁵ mm³.

$$\gamma_d = \frac{W_d}{V} = \frac{750 \times 1000}{4.8 \times 10^5 \times 1000} = 1.5625 \times 10^{-5} \text{ gm/mm}^3$$

$$e = \frac{G \gamma_w}{\gamma_d} - 1 = \frac{2.7 \times 9.8 \times 10^{-6}}{1.56 \times 10^{-5}} - 1 = 0.726$$

$$n = \frac{e}{1+e} = 0.42$$

$$\therefore \text{Seepage velocity, } V_s = \frac{\bar{v}}{n} = \frac{0.139}{0.42} = 0.33 \text{ mm/sec. } \frac{8}{8}$$

2. To find out the coefficient of permeability of clay the permeability test was carried out in a variable head permeameter having the diameter of 10cm. The initial head of water in the sand pipe was

found to be 45cm and it was observed to drop to 30cm in 3.5 minutes. If the height of the sample was 15cm and the diameter of the standpipe was 1.9cm, determine the coefficient of permeability.

→ Solution:-

Given that:-

Diameter of pipe (D) = 10cm

Length of sample (L) = 15cm

Diameter of standpipe (d) = 1.9cm

Initial height (h₁) = 45cm

Final height (h₂) = 30cm

Time (t) = 3.5 min = 210 sec.

Area of pipe (a) = $\frac{\pi d^2}{4} = 2.835 \text{ cm}^2$.

Coefficient of permeability (K) = ?

We know that,

$$K = \frac{aL}{At} \ln \left(\frac{h_1}{h_2} \right)$$

$$= \frac{2.835 \times 15 \times 4}{\pi \times 10^2 \times 210} \ln \left(\frac{45}{30} \right)$$

$$= 0.00105 \text{ cm/s}$$

$$\Rightarrow 0.907 \text{ m/day} \quad \&$$

Additional Practice (2013 Spring)

In falling head permeameter test, the initial head (t=0) is 40cm. The head drops by 5cm in 10 minutes. Calculate the time required to run the test for the final head to be at 20cm. If the sample is 6m in height and 50cm² in X-sectional area, calculate the coefficient of permeability, taking area of stand pipe as 0.5cm².

→ Solution:-

Given that:

Initial head (h₁) = 40cm

Final head (h₂) = 5cm

Time (t) = 10 min = 600 sec.

Area of stand pipe (a) = $\frac{\pi \times 0.5^2}{4} = 0.1963 \text{ cm}^2$ 0.5cm²

Length of sample (L) = 6m

Area of sample (A) = 50cm².

We have,

$$K = \frac{aL}{At} \log_2 \frac{h_1}{h_2}$$

$$= \frac{0.1963 \times 6}{50 \times 600} \log_2 \frac{40}{5}$$

$$= 8.16 \times 10^{-5} \text{ cm/sec.} \approx 2.079 \times 10^{-4} \text{ cm/sec.}$$

Now, For final head to be 20cm,

$$K = \frac{aL}{At} \log_2 \frac{h_1}{h_2}$$

$$\text{or, } 8.16 \times 10^{-5} = \frac{0.1963 \times 6}{50 \times t} \log_2 \frac{40}{20}$$

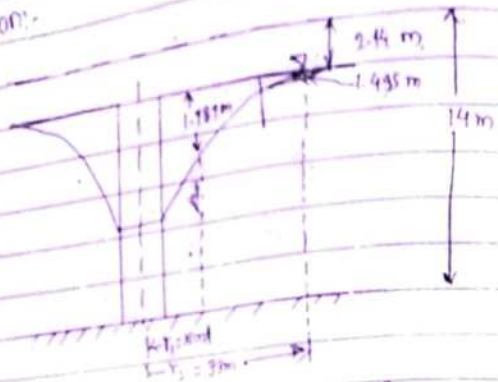
$$2.079 \times 10^{-4} = \frac{0.5 \times 6}{50 \times t} \ln \left(\frac{40}{20} \right)$$

$$\text{or, } t = 200 \text{ sec.} \quad \&$$

3. In order to determine the avg coefficient of permeability of a bed of sand 14meters thick, overlying on impermeable stratum, a well was sunk through the sand and a pumping test was carried out. After a certain interval, the discharge was 124 litres per second and draw down in the observation wells at 16meters and 33meters resp, from the pumping well were found to be 1.787 and 1.695m resp. If the GWT was originally at 2.14m below the ground surface

find (a) the permeability of sand layers and (b) an approximate value for the effective grain sizes.

→ Solution:-



$$\text{Discharge (q)} = 12.4 \text{ liter/sec} = 12.4 \times 10^{-3} \text{ m}^3/\text{sec.}$$

$$h_1 = (14 - 2.44 - 1.787) \text{ m} = 10.073 \text{ m}$$

$$h_2 = (14 - 2.44 - 1.495) \text{ m} = 10.365 \text{ m}$$

$$r_1 = 16 \text{ m}$$

$$r_2 = 33 \text{ m}$$

we have,

$$K = \frac{q}{\pi(h_2^2 - h_1^2)} \log \frac{r_2}{r_1}$$

$$= \frac{12.4 \times 10^{-3}}{\pi(10.365^2 - 10.073^2)} \times \log \left(\frac{33}{16} \right)$$

$$= 4.78 \times 10^{-4} \text{ m/sec.}$$

Again we have,

$$K = cD_0^2$$

$$\therefore D_0 = \sqrt{\frac{K}{c}} = \sqrt{\frac{4.78 \times 10^{-4}}{125}} \quad (\because c = 125)$$

$$= 0.02 \text{ mm}$$

Rakesh Kumar Sob

4. A horizontal stratified deposit consists of three layers with equal permeability values in horizontal as well as vertical direction. The permeability of the layers is $8 \times 10^{-4} \text{ cm/sec}$, $5 \times 10^{-4} \text{ cm/sec}$ and $15 \times 10^{-4} \text{ cm/sec}$ and their thickness are 6m, 3m and 18m respectively. Find the effective average permeability of the deposits in horizontal & vertical direction.

→ Solution:-

Equivalent permeability in horizontal direction is,

$$K_x = \frac{K_1 H_1 + K_2 H_2 + K_3 H_3}{H_1 + H_2 + H_3} = \frac{(8 \times 6 + 3 \times 5 + 15 \times 18) \times 10^{-4}}{6 + 3 + 18}$$

$$= 11.33 \times 10^{-4} \text{ cm/sec.}$$

Again, equivalent permeability in vertical direction is,

$$K_z = \frac{H_1 + H_2 + H_3}{\frac{H_1}{K_1} + \frac{H_2}{K_2} + \frac{H_3}{K_3}} = \frac{6 + 3 + 18}{\left(\frac{6}{8} + \frac{3}{5} + \frac{18}{15} \right) \times 10^{-4}} = 13.43 \times 10^{-4} \text{ cm/sec}$$

Additional practice (2008 Fall):

Same as above.

5. A layer of sand 6m thick lies beneath a clay stratum 5m thick and above a thick shale. In order to determine the permeability of sand a well was driven to the top of shale and water pumped out at the rate of $10 \times 10^3 \text{ m}^3/\text{sec}$. Two observation wells were driven through the clay at 15m and 30m from the pump well and the water was found to rise to levels of 3m and 2.5m below the ground surface respectively. Calculate the coefficient of permeability of the soil.

→ Solution:-

Given that:

Thickness of sand (H) = 6m

Discharge (q) = $10 \times 10^3 \text{ m}^3/\text{sec}$.

$$r_1 = 15m, r_2 = 30m$$

$$H_1 = (11-3)m = 8m$$

$$H_2 = (11-2.5)m = 8.5m$$

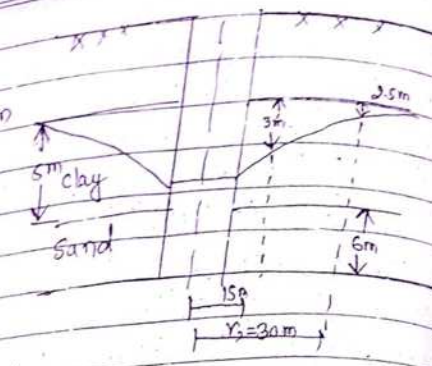
We have,

$$K = \frac{2.5 \times 9 \times \log \frac{r_2}{r_1}}{2\pi H (H_2 - H_1)}$$

$$= \frac{2.5 \times 10 \times 9 \times \log \frac{30}{15}}{2 \times \pi \times 6 \times (8.5 - 8)}$$

$$= 3.67 \times 10^{-4} \text{ m/sec}$$

$$\therefore K = 0.367 \text{ mm/sec}$$



6. During a pumping test, a well was sunk through a stratum of dense sand 10m deep overlying an impervious stratum. Observation wells were drilled at 6m and 12m from the well. Initially, the water level in the well was 2.5m below the ground surface. After pumping until steady conditions had been reached, the water level in observation wells had dropped 1.95m and 0.5m respectively. If the steady discharge was 5 litres/sec, determine the coefficient of permeability.

> Solution:-

Given that:-

$$q = 5 \text{ litres/sec} = 5 \times 10^{-3} \text{ m}^3/\text{s}$$

$$r_1 = 6m, r_2 = 12m$$

$$s_1 = 1.95m, s_2 = 0.5m$$

Initial position of water table is 2.5m below ground level

$$h_1 = 10 - 2.5 - 1.95 = 5.55m$$

$$h_2 = 10 - 2.5 - 0.5 = 7m$$

We have,

$$K = \frac{q \ln \left(\frac{r_2}{r_1} \right)}{\pi (h_2^2 - h_1^2)}$$

$$= \frac{5 \times 10^{-3} \ln \left(\frac{12}{6} \right)}{\pi (7^2 - 5.5^2)}$$

$$= 6.06 \times 10^{-5} \text{ m/s}$$

Additional practise (2011F)

> Solution: Same as above.

7. A sand layer 10m thick overlies an impervious stratum. The water table is in the sandy layer at a depth of 1.5m below the ground surface. Water is pumped out from a well at the rate of 100 litre per second and drawdown the water table at radial distance of 3.0m and 25m is 3.0m and 0.5m respectively. Determine the coefficient of permeability.

> Solution:-

$$q = 100 \text{ litres/sec} = 100 \times 10^{-3} \text{ m}^3/\text{s}$$

$$r_1 = 3m, r_2 = 25m$$

$$s_1 = 3.0m, s_2 = 0.5m$$

$$H_1 = 10 - 1.5 - 3 = 5.5$$

$$H_2 = 10 - 1.5 - 0.5 = 8$$

Now,

$$K = \frac{q \ln \left(\frac{r_2}{r_1} \right)}{\pi (h_2^2 - h_1^2)} = \frac{100 \times 10^{-3} \ln \left(\frac{25}{3} \right)}{\pi (8^2 - 5.5^2)}$$

$$= 1.99 \times 10^{-3} \text{ m/sec}$$

$$\therefore 1.99 \text{ mm/sec}$$

Rakesh Kumar Sah
8. What is the height of capillary rise and the capillary pressure in fine grained soil with effective size of 0.002 mm?

→ Solution:-

Given that:

$$\text{Effective size } (d_e) = 0.002 \text{ mm}$$

$$\text{Equivalent diameter of the tube } (d) = 0.2 * d_{10} \\ = 0.2 * 0.002 = 4 * 10^{-4} \text{ mm}$$

$$\text{Again, } h_c = \frac{0.3}{4 * 10^{-4}} = 7.5 \text{ m}$$

$$\text{And, Capillary pressure} = -h_c * \gamma_w = -7.5 * 9.81 \text{ } \gamma/\text{cm}^2$$

$$\therefore p_c = 735.75 \text{ kN/m}^2$$

9. Water table was found 1.6 m below the ground surface in a deep deposit of very fine sand. Above water table, the sand was found saturated by capillary water. The unit weight of saturated sand was 2.2 ton/m^3 . What is the effective vertical pressure on a horizontal plane at a depth of 4.5 m below the ground surface?

→ Solution:-

Given that:-

$$\gamma_{\text{sat}} = 2.2 \text{ tons/m}^3$$

$$\text{Effective vertical pressure } (\gamma') = ?$$

$$\text{Total pressure } (\sigma) = \gamma * 4 \\ = \sigma' = \sigma - u$$

At 4.5 m,

$$\sigma = 1.6 * 2.2 + (4.5 - 1.6) * 2.2 \\ = 9.9 \text{ tons/m}^2$$

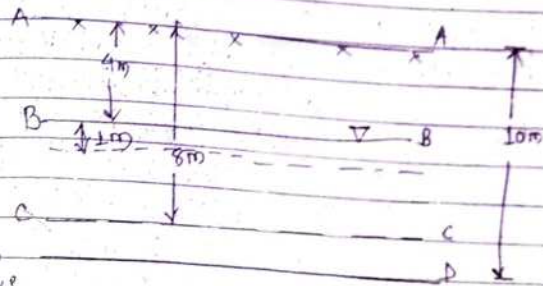
Also,

$$u = (4.5 - 1.6) * 9.81 \\ = 28.449 \text{ kN/m}^2 \\ = 2.9 \text{ tons/m}^2$$

$$\therefore \sigma' = 9.9 - 2.9 \\ = 7.0 \text{ tons/m}^2$$

10. A fine sand deposit is located between the ground surface to a depth of 10 m. The soil has an avg void ratio of 0.70 and specific gravity of 2.65. The water table is found at a depth of 4 m below the ground surface. Above the water table the degree of saturation of sand is 55%. Determine the total stress, pore water pressure and effective stress at a depth of 8 m below the ground surface. Calculate also the change in the effective stress if the soil gets saturated up to a height of 1 m above the water table due to capillary.

→ Solution:-



We have,

$$\gamma_t = \frac{(G + S_r e) \gamma_w}{1 + e} = \frac{(2.65 + 0.55 * 0.7) * 10}{1 + 0.7} = 17.85 \text{ kN/m}^3$$

$$\text{Similarly, } \gamma_{\text{sat}} = \frac{(G + e) \gamma_w}{1 + e} = \frac{(2.65 + 0.7) * 10}{1.7} = 19.71 \text{ kN/m}^3$$

(Now,

Case I:-

At C-C,

$$\sigma = (7.85 \times 4 + 19.71 \times 4) = 150.24 \text{ kN/m}^2$$

$$u = 10 \times 4 = 40 \text{ kN/m}^2$$

$$\therefore \bar{\sigma} = \sigma - u = (150.24 - 40) = 110.24 \text{ kN/m}^2$$

Case II

At C-C,

$$\sigma = (17.85 \times 3 + 19.71 \times 5) = 152.10 \text{ kN/m}^2$$

$$u = 10 \times 4 = 40 \text{ kN/m}^2$$

$$\therefore \bar{\sigma} = \sigma - u = (152.10 - 40) = 112.10 \text{ kN/m}^2$$

And, change in effective stress at 0.8 m = $(112.10 - 110.24) = 1.86 \text{ kN/m}^2$

Additional Practise (2012 Spring and 2011 Spring)

A 10m thick bed of sand is underlain by a layer of clay 6.5m thickness. The water table which was originally at the ground surface is lowered by drainage to a depth of 4.2m where the degree of saturation above the lowered water table reduces to 50%. Determine the increase in magnitude of effective vertical pressure at the mid of clay layer due to lowering of water table. The γ_{sat} of sand is 20.6, γ_{sat} for clay = 17.6 & γ of sand is 16.7 kN/m³.

→ Solution:-

11. A soil deposit consists of an upper layer of 3m thick having a unit weight of 18 kN/m³ and lower layer of 4m thick having a unit weight of 22 kN/m³. Determine the total stress, pore pressure and effective stress at the bottom of lower layer (i.e. at 7m below ground surface) if the water table is

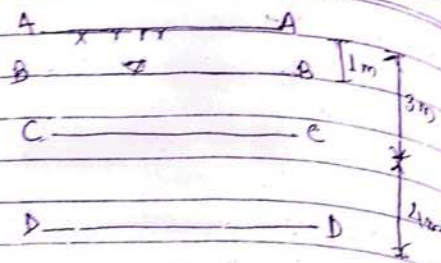
- ⊗ 1m below the ground surface.
- ⊗ Exactly at the ground surface
- ⊗ 1m above the ground surface.
- Solution:-

Case-I: at D-D

$$\sigma = (18 \times 3 + 22 \times 4) = 142 \text{ kN/m}^2$$

$$u = 10 \times 6 = 60 \text{ kN/m}^2$$

$$\begin{aligned} \therefore \bar{\sigma} &= \sigma - u \\ &= (142 - 60) \text{ kN/m}^2 \\ &= 82 \text{ kN/m}^2 \end{aligned}$$

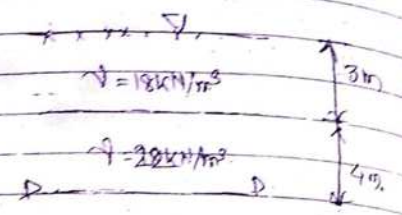


Case II: At D-D

$$\sigma = (18 \times 3 + 22 \times 4) = 142 \text{ kN/m}^2$$

$$u = 10 \times 7 = 70 \text{ kN/m}^2$$

$$\begin{aligned} \therefore \bar{\sigma} &= \sigma - u \\ &= 142 - 70 = 72 \text{ kN/m}^2 \end{aligned}$$



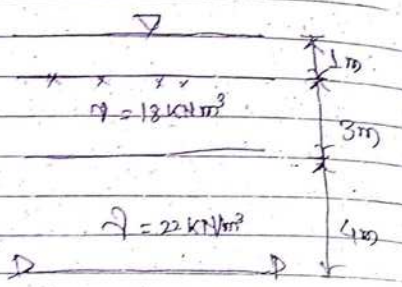
Case III: At D-D

$$\sigma = (10 \times 1 + 18 \times 3 + 22 \times 4)$$

$$= 152 \text{ kN/m}^2$$

$$u = 10 \times 8 = 80 \text{ kN/m}^2$$

$$\begin{aligned} \therefore \bar{\sigma} &= \sigma - u \\ &= 152 - 80 \\ &= 72 \text{ kN/m}^2 \end{aligned}$$

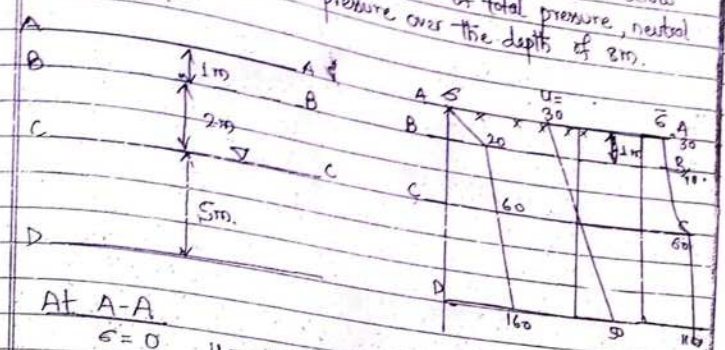


Additional Practice (2011 Fall)

The following informations were obtained from borehole test. At 0-3.5m depth - fine sand with saturated unit weight = 1620 kg/m^3 , 3.5 to 8.0m depth - clay with saturated unit weight = 2000 kg/m^3 and below 8.0m - medium sand. The water table was at the ground level, Calculate and draw diagrams showing the

12. The water table in a deposit of sand 8m thick is at a depth of 3m below the surface. The sand is saturated above the water table. Calculate the effective pressure at 1m, 3m and 8m below the surface. Also plot the variation of total pressure, neutral pressure and effective pressure over the depth of 8m.

→ Solution:-



At A-A

$$\begin{aligned} \sigma &= 0 \quad u = -10 \times 3 = -30 \text{ kN/m}^2 \\ \therefore \bar{\sigma} &= \sigma - u = 30 \text{ kN/m}^2 \end{aligned}$$

At B-B

$$\begin{aligned} \sigma &= 20 \times 1 = 20 \text{ kN/m}^2 \\ u &= -10 \times 2 = -20 \text{ kN/m}^2 \\ \therefore \bar{\sigma} &= \sigma - u = 40 \text{ kN/m}^2 \end{aligned}$$

At C-C:-

$$\begin{aligned} \sigma &= 20 \times 3 = 60 \text{ kN/m}^2 \neq u = 0 \\ \therefore \bar{\sigma} &= 60 \text{ kN/m}^2 \end{aligned}$$

At D-D

$$\begin{aligned} \sigma &= 20 \times 3 + 20 \times 5 = 160 \text{ kN/m}^2 \\ u &= 10 \times 5 = 50 \text{ kN/m}^2 \\ \sigma - u &= \bar{\sigma} = 110 \text{ kN/m}^2 \end{aligned}$$

14 Calculate the critical hydraulic gradient in a saturated soil with a specific gravity of 2.68 and water content of 50%.

→ Solution:-

Given that:

Specific gravity (G) = 2.68

Water content (w) = 50% = 0.5

We have,

$$wG = Sr e$$

$$\therefore e = \frac{wG}{Sr} = \frac{0.5 \times 2.68}{1} = 1.34$$

Now, Critical hydraulic gradient is

$$i_c = \frac{\gamma'}{\gamma_w} = \frac{G-1}{1+e} = \frac{2.68-1}{1+1.34} = 0.72$$

15. A trench is excavated in fine sand up to a depth of 4m. The excavation is carried out by providing necessary side supports and pumping water. The position of water table is 1m below the ground surface at the sides of excavation and there is no water inside the excavation. Examine the possibility of quick sand condition if G = 2.64 and e = 0.7.

→ Solution:-

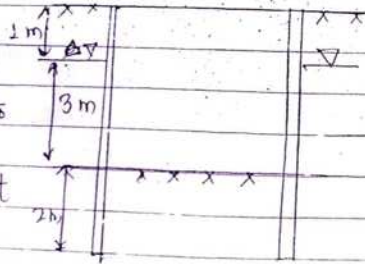
Given that:-

Sheeting is taken at 2m depth below the bottom of the trench to increase the seepage path.

The equation for critical gradient is

$$i_c = \frac{G-1}{1+e}$$

If the trench is to be stable, the hydraulic gradient i is



$$i = \frac{h}{L}$$

There will be no quick condition if, $\frac{h}{L} < \frac{G-1}{1+e}$

For the given data,

$$i_c = \frac{G-1}{1+e} = \frac{2.64-1}{1+0.7} = 0.96$$

$$\therefore \frac{h}{L} = \frac{3}{2} = 1.5$$

Here, $h > i_c$, hence quick sand condition occurs.

If, L is more than 4m, then $i = \frac{h}{L} = \frac{3}{4} = 0.75 < 0.96$, So, No quick sand condition occurs.

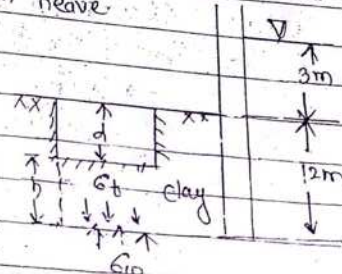
16. A 12m thick layer of relatively impervious stratum of clay lies over a bed of gravel aquifer. Piezometer tubes 3m above the top surface of the clay strata. The properties of the clay are $e = 1.2$ and $G = 2.7$. Determine
 a) The effective stress at the top level of gravel strata.
 b) The depth of excavation that can be made in the clay.

→ Solution:-

$$\gamma_{sat} = \frac{(G + Sr e) \gamma_w}{1+e}$$

$$= \frac{(2.7 + 1.2) \times 10}{1+1.2}$$

$$= 17.73 \text{ kN/m}^3$$



At the top level of gravel strata,

Gravel

$$\sigma = 12 \times 17.73 = 212.76 \text{ kN/m}^2$$

$$u = 15 \times 10 = 150 \text{ kN/m}^2$$

$$\therefore \bar{\sigma} = \sigma - u = (212.76 - 150) = 62.76 \text{ kN/m}^2$$

ii) Depth of excavation must be such that, $\bar{\sigma} = 0$ i.e. $\sigma = u$
 Let h be the height of the bottom of excavation from gravel layer
 $\therefore \sigma = \gamma_{\text{sat}} \times h = 17.73h \text{ kN/m}^2$

Again, $u_{\text{at}} = 150 \text{ kN/m}^2$

$$\therefore 17.73h = 150$$

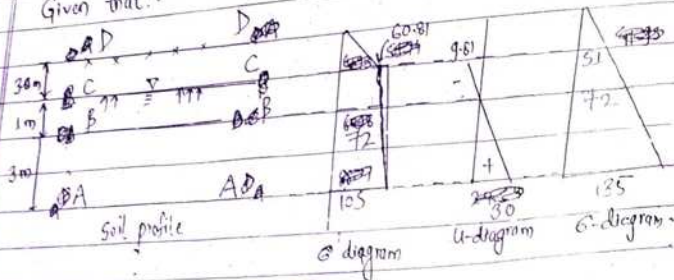
$$\therefore h = 8.5 \text{ m}$$

$$\therefore \text{Depth of excavation} = (12 - 8.5) \text{ m} = 3.5 \text{ m}$$

17. A sand deposit consists of two layers. The top layer is 3.0 m thick ($\gamma = 17 \text{ kN/m}^3$) and the bottom layer is 4 m thick ($\gamma_{\text{sat}} = 21 \text{ kN/m}^3$). The water table is at a depth of 4.0 m from the surface and the zone of capillary saturation is 1 m above the water table. Draw the diagrams, showing the variation of total stress, neutral stress and effective stress.

> Solution:-

Given that:-



$$\gamma = 17.0967 \times 9.81 \times 10^{-3} = 16.77 \text{ kN/m}^3$$

$$\gamma_{\text{sat}} = 2064.52 \times 9.81 \times 10^{-3} = 2025 \text{ kN/m}^3$$

At AA

$$\sigma = 0, u = 0 \therefore \bar{\sigma} = \sigma - u = 0$$

At BB (immediately above)

Solution:-

At AA:-

$$\sigma = 3.0 \times 17 + 4 \times 21 = 135 \text{ kN/m}^2$$

$$u = 3 \times 10 = 30 \text{ kN/m}^2$$

$$\therefore \bar{\sigma}' = (\sigma - u) = 135 - 30 = 105 \text{ kN/m}^2$$

At BB:-

$$\sigma = 3.0 \times 17 + 1 \times 21 = 72 \text{ kN/m}^2$$

$$u = 0$$

$$\therefore \bar{\sigma}' = \sigma - u = 72 \text{ kN/m}^2$$

At CC (immediately above)

$$\sigma = 3 \times 17 = 51 \text{ kN/m}^2$$

$$u = 0$$

$$\therefore \bar{\sigma}' = 51 \text{ kN/m}^2$$

At CC (immediately below)

$$\sigma = 3.0 \times 17 = 51$$

$$u = -1 \times 9.81 = -9.81 \text{ kN/m}^2$$

$$\therefore \bar{\sigma}' = (\sigma - u) = 51 + 9.81 = 60.81 \text{ kN/m}^2$$

At D-D

$$\sigma = 0, u = 0 \therefore \bar{\sigma}' = 0$$

18. A sand deposit is 12m thick and overlies a bed of soft clay. The ground water table is 4m below the ground surface. If the sand above the water table has a degree of saturation of 40%, plot the diagram showing

The variation of total stress, pore pressure and effective stress. The void ratio of sand is 0.8 and $G = 2.7$.

> Solution:-

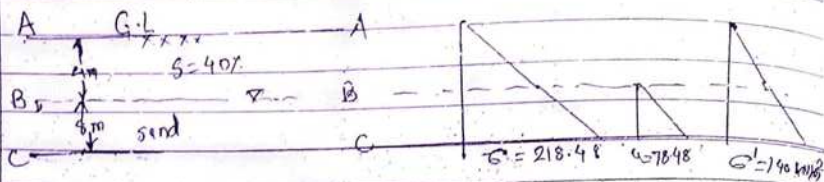
Given that:-

$$G = 2.7, e = 0.8 \quad S = 40\%$$

Now,

$$\gamma = \frac{(G+se)\gamma_w}{1+e} = \frac{(2.7+0.4 \times 0.8) \times 9.81}{1+0.8} = 16.46 \text{ kN/m}^3$$

$$\gamma_{sat} = \frac{(G+se)\gamma_w}{1+e} = \frac{(2.7+0.8) \times 9.81}{1+0.8} = 19.08 \text{ kN/m}^3$$



Total stress (σ) =

At point A-A,

$$\sigma = u = \sigma' = 0$$

At point B-B (4m below G.L.)

$$\sigma = 16.46 \times 4 = 65.84 \text{ kN/m}^2$$

$$u = 0$$

$$\sigma' = \sigma - u = 65.84 \text{ kN/m}^2$$

At C-C (12m below G.L.)

$$\sigma = 16.46 \times 4 + 19.08 \times 8 = 218.48 \text{ kN/m}^2$$

$$u = 9.81 \times 8 = 78.48 \text{ kN/m}^2$$

$$\therefore \bar{\sigma} = \sigma - u$$

$$= 218.48 - 78.48$$

$$= 140 \text{ kN/m}^2$$

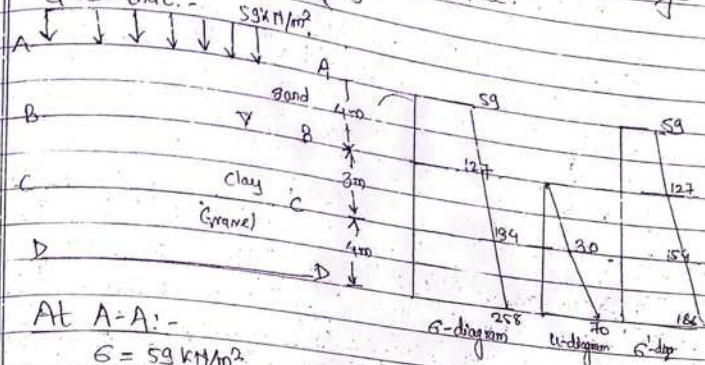
Additional Practice (2015 Spring) (Same as above)

13.

A soil profile consists of surface layer of sand 4.0m thick ($\gamma = 17 \text{ kN/m}^3$) and intermediate layer of gravel 3.0m thick ($\gamma = 19 \text{ kN/m}^3$), and the bottom layer of clay 4.0m thick ($\gamma = 18.5 \text{ kN/m}^3$). The water level is at the upper surface of the clay layer. Determine the effective pressure at various levels immediately after the placement of a surcharge load of 59 kN/m^2 to the ground surface.

> Solution:-

Given that:-



At A-A:-

$$\sigma = 59 \text{ kN/m}^2$$

$$u = 0$$

$$\therefore \bar{\sigma} = \sigma - u = 59 \text{ kN/m}^2$$

At B-B

$$\sigma = (59 + 17 \times 4) = 127 \text{ kN/m}^2$$

$$u = 0$$

$$\therefore \bar{\sigma} = \sigma - u = 127 \text{ kN/m}^2$$

At C-C:

$$\sigma = 59 + 17 \times 4 + 3 \times 19 = 184 \text{ kN/m}^2$$

$$u = 3 \times 10 = 30 \text{ kN/m}^2$$

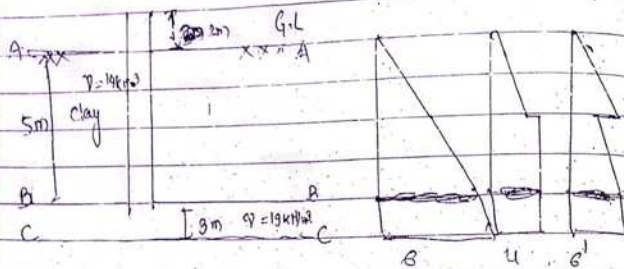
$$\therefore \bar{\sigma} = \sigma - u = 154 \text{ kN/m}^2$$

At D-D

$$\sigma = (59 + 17 \times 4 + 19 \times 3 + 18.5 \times 4) = 258 \text{ kN/m}^2 \quad u = 10 \times 7 = 70 \text{ kN/m}^2$$

20. A soil profile consists of a surface layer of clay 5m thick with unit weight 14 kN/m^3 and a sand layer 3m thick with unit weight 19 kN/m^3 overlying an impermeable rock. The water table is at the ground surface. If the water table in a standpipe driven into the sand layer rises 2m above the ground surface then, draw the plot showing the variation of total stress, pore water pressure and effective stress.

→ Solution:-



At A-A $G = \bar{G} = u = 0$.

At B-B: 5m below G.L

$$G = 5 \times 14 = 70 \text{ kN/m}^2$$

$$u = 9.81 \times 5 = 49.05 \text{ kN/m}^2$$

$$G' = 70 - 49.05 = 20.95 \text{ kN/m}^2$$

At B-B (5m below G.L) for sand layer.

$$G = 14 \times 5 = 70 \text{ kN/m}^2$$

$$u = 9.81 \times (5+2) = 68.48 \text{ kN/m}^2$$

$$\therefore \bar{G} = 70 - 68.48 = 1.52 \text{ kN/m}^2$$

At C-C (8m below G.L)

$$G = 14 \times 5 + 19 \times 3 = 127 \text{ kN/m}^2$$

$$u = 9.81 \times (7+2) = 98.1 \text{ kN/m}^2$$

$$\bar{G} = G - u = 28.9 \text{ kN/m}^2$$

21. A masonry dam has pervious sand as foundation. Determine the maximum permissible upward gradient if a factor of safety of 5 against quick sand condition, take porosity, $n=45\%$ and $G_s = 2.65$.

→ Solution:-

Critical hydraulic gradient, $i_c = \frac{G-1}{1+e}$

$$\therefore e = \frac{D}{1-D} = \frac{0.45}{1-0.45} = 0.82$$

$$\therefore i_c = \frac{2.65-1}{1+0.82} = 0.9$$

Hence, permissible hydraulic gradient = $\frac{i_c}{FS} = \frac{0.9}{5} = 0.18$

22. In a site reclamation project, 2.5m of graded fill ($\gamma = 22 \text{ kN/m}^3$) were laid in the compacted layers over an existing layer of silty clay ($\gamma = 18 \text{ kN/m}^3$) which has 3m thick. This was underlain by a 2m thick layer of gravel ($\gamma = 20 \text{ kN/m}^3$). Assuming the water table remains at the surface of silty clay, draw the effective stress profiles for case after the fill has been placed.

→ Solution:-

RAKESH KUMAR SAH

Tutorial- 4
Ch-7: Seepage Analysis through soils.

1. A concrete dam is constructed across a river over permeable strata of the soil of limited thickness. The water head on U/s side is 16m and on the d/s side is 2m. The flow net constructed under the dam gives $N_f = 7$, $N_d = 21$. Calculate the seepage loss through the subsoil if the average value of coefficient of permeability is 6×10^{-3} cm/sec horizontally and 3×10^{-4} cm/sec vertically. Calculate the exit gradient if the average length of the last field is 0.9m.

→ Solution:-

Given that:-

No. of flow channels (N_f) = 7

No. of equipotential drops (N_d) = 21

Water level on U/s side (h_1) = 16m

Water level on d/s side (h_2) = 2m

∴ Total hydraulic head causing flow (h) = $h_1 - h_2 = 16 - 2 = 14$ m

Permeability in horz. direction (k_H) = 6×10^{-3} cm/s.

Permeability in vertical direction (k_V) = 3×10^{-4} cm/s

$$k_e = \sqrt{k_H \cdot k_V} = \sqrt{6 \times 10^{-3} \times 3 \times 10^{-4}} = 1.34 \times 10^{-3} \text{ cm/sec}$$

Again,

$$q = k_e h \frac{N_f}{N_d} = 1.34 \times 10^{-3} \times 14 \times \frac{7}{21} = 6.26 \times 10^{-3} \text{ cm/s}$$

$$\text{Head loss per potential drop} = \Delta h = \frac{h}{N_d} = \frac{14}{21} = 0.67$$

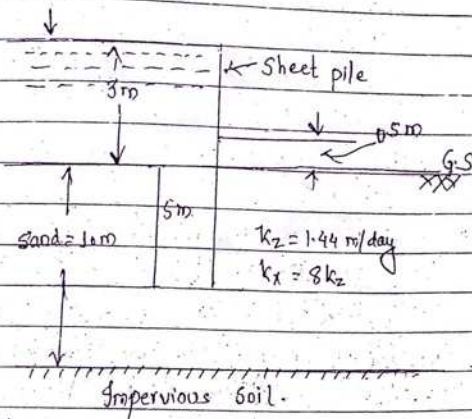
$$\therefore \text{The exit gradient } (i) = \frac{\Delta h}{z_e} = \frac{0.67}{0.9} = 0.74$$

→ A sand deposit 10m thick overlies an impervious soil. A vertical

sheet pile penetrates halfway into the sand deposit. The water level on one side of the wall is 30m and on the other side is 0.5m above the ground level. The sand stratum has a vertical permeability (k_z) 1.44m per day, and the horizontal permeability equals 8 times the permeability in the vertical direction. A flow net construction reveals that there are 12 flow channels and 26 potential drops. Determine the seepage of flow per day.

Solution:-

Given that:



$$k_x = 8k_z$$

$$k_z = 1.44 \text{ m/day}$$

∴ The equivalent coefficient of permeability,

$$k_e = \sqrt{k_x \cdot k_z} = \sqrt{8k_x \cdot k_z} = \sqrt{8} k_z = \sqrt{8} \times 1.44$$

$$\therefore k_e = 4.07 \text{ m/day}$$

The seepage is given by

$$q = k_e h \frac{N_f}{N_d}$$

$$= 4.07 (3 - 0.5) \times \frac{12}{26} = 4.7 \text{ m}^3/\text{s/m}$$

3. The section of homogeneous earth dam is shown in fig. 1. The coefficient of permeability in the horizontal and vertical directions are $8 \times 10^{-7} \text{ cm/sec}$ and $3.6 \times 10^{-7} \text{ cm/sec}$ respectively. Estimate the seepage through the dam section.

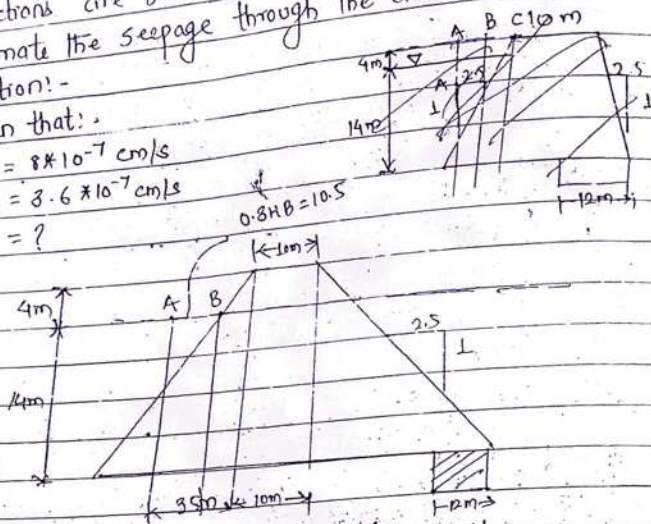
> Solution:-

Given that:-

$$K_x = 8 \times 10^{-7} \text{ cm/s}$$

$$K_z = 3.6 \times 10^{-7} \text{ cm/s}$$

$$q = ?$$



$$K = \sqrt{K_x \cdot K_z} = \sqrt{8 \times 10^{-7} \times 3.6 \times 10^{-7}} = 5.36 \times 10^{-7} \text{ cm/s} \approx 5.36 \times 10^{-9} \text{ cm/s}$$

$$b = (45 - 12) + 10 + 10 + 10.5 = 63.5 \text{ m}$$

$$S = \sqrt{b^2 + h^2} - b = \sqrt{63.5^2 + 14^2} - 63.5 = 1.52$$

Now,

$$q = K S$$

$$= 5.36 \times 10^{-9} \times 1.52$$

$$= 8.14 \times 10^{-9} \text{ m}^3/\text{s/m}$$

Additional Practice (2003 Spring, 2011 Spring)
Solution same as above question.

4. A homogeneous earth dam 30m high has free board of 1.5m. A flow net was constructed and the following results were noted: nos of potential drop-12, nos of flow channels-3. The dam at its downstream end has a 18m long horizontal filter. Calculate the seepage loss across the dam per day, if the width of dam is 200m and the coefficient of permeability of the soil is $3.35 \times 10^{-4} \text{ cm/sec}$.

> Solution:-

Given that, e:

$$Cd = 12$$

$$Cf = 3$$

$$K = 3.35 \times 10^{-4} \text{ cm/sec}$$

∴ Head loss,

$$H = 30 - 1.5$$

$$= 28.5 \text{ m}$$

Flow, $q =$

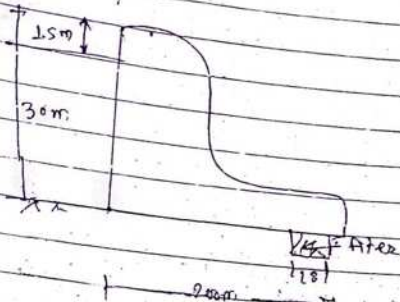
We know that,

$$\text{Discharge (Q)} = \frac{K N_f C_d}{C_d} H$$

$$= 3.35 \times 10^{-4} \times 28.5 \times \frac{3}{12}$$

$$= 2.38 \times 10^{-3} \text{ m}^3/\text{sec}$$

$$\approx 2.06 \text{ m}^3/\text{day}$$



5. A 30m high dam has a top width of 10m and a free board of 5m. The dam is made of coarse sand having coefficient of permeability 4.5×10^{-4} cm/sec. A ϕ 60m long horizontal drainage blanket is placed near the downstream end of the dam. Determine the quantity of seepage loss if the bottom width of the dam be 185m.

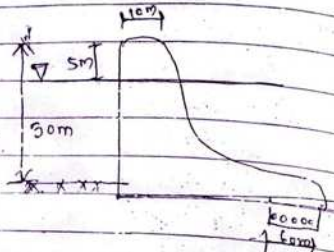
→ Solution:-

Given that,

$$K = 4.5 \times 10^{-4} \text{ cm/sec}$$

$$H = 30 + 5 = 35 \text{ m}$$

$$\text{Length of dam (L)} = 185 \text{ m}$$



Here,

$$\alpha = \frac{87.5}{30}$$

$$\therefore \alpha : 1 = 2H : \sqrt{L} = 291 : 1 = 98 : 1$$

$$\therefore HB = \frac{25 \times 35}{12} = 72.917 \text{ m}$$

$$AB = 0.3HB = 0.3 \times 72.917 = 21.875 \text{ m}$$

$$b = 185 - 60 - 0.7 \times 72.917 = 73.958$$

$\&$

$$S = \sqrt{b^2 + h^2} - b$$

$$= \sqrt{73.958^2 + 25^2} - 73.958$$

$$= 4.11 \text{ m}$$

Now, seepage discharge $q = KS$

$$= 4.5 \times 10^{-6} \times 4.11$$

$$= 1.85 \times 10^{-5} \text{ m}^3/\text{sec/m}$$

$$\approx 18.5 \text{ cm}^3/\text{sec/m}$$

6. A flow net for flow around a single row of sheet pile in a permeable soil is shown in fig below. If $K_1 = K_2 = 5 \times 10^{-2} \text{ m/s}$ determine:
- How high (above ground surface) will the water rise, if measured by piezometers placed at A, B, C and D?
 - What will be the rate of seepage through the flow channel - I per unit length of the sheet pile?
 - What is total seepage per unit length through permeable layer?
- Solution:-

Tutorial - 5
Ch-8 - Vertical Stresses below applied load.

1. A water tower has a circular foundation of 10m diameter. If the total weight of the tower, including the foundation is 2×10^5 kN. Calculate the vertical stress at a depth of 3m below the centre of foundation.

→ Solution:-

Given that;

Radius (R) = $10/2 = 5$ m.

Total weight = 2×10^5 kN

depth (z) = 3 m

Vertical stress (G_z) = ?

We have,

$$I_c = 1 - \frac{1}{\left[1 + \left(\frac{R}{z}\right)^2\right]^{3/2}} = 1 - \frac{1}{\left[1 + \left(\frac{5}{3}\right)^2\right]^{3/2}} = 0.8638$$

$$q = \frac{\text{load}}{\text{area}} = \frac{2 \times 10^5}{\frac{1}{4} \pi 10^2} = 2546.48 \text{ kN/m}^2$$

$$\therefore G_z = I_c \cdot q = 0.8638 \times 2546.48 = 2199.67 \text{ kN/m}^2$$

Additional Practice (2012 Fall)

A Raft foundation of 8m dia is placed on the ground surface and carries a load of 10,000 kN including self-weight. Calculate the vertical stress increment at the centre of clay layer using Boussinesq's theory. The clay layer is 2m thick and it exist 5m below ground surface.

→ Solution:-

Given that,

Diameter of foundation (d) = 8 m
Load (Q) = 10,000 kN.

Now,

$$q = \frac{Q}{A} = \frac{10,000}{\frac{1}{4} \pi 8^2} = 198.94 \text{ kN/m}^2$$

Now, vertical stress increment is given by

$$G_v = q \left[1 - \frac{1}{\left[1 + \left(\frac{R}{z}\right)^2\right]^{3/2}} \right]$$

$$= 198.94 \left[1 - \frac{1}{\left[1 + \left(\frac{4}{5}\right)^2\right]^{3/2}} \right] \quad \because z = 5 + 1 = 6$$

$$= 84.34 \text{ kN/m}^2$$

2. A water tower weighing 15,000 kN is to be considered as concentrated load acting on the ground surface. Compute the vertical stress at a depth of 8 meters below the surface. Also the center of the water tower. 7m away from

→ Solution:-

Given that:-

load of water tower (Q) = 15,000 kN.

z = 8 m r = 0

$G_z = ?$

We have,

Vertical stress at a depth 8m,

$$G_z = I_B \cdot \frac{Q}{z^2} \quad \dots \dots (1)$$

$$I_B = \frac{3}{2\pi} \cdot \frac{1}{\left[1 + \left(\frac{r}{z}\right)^2\right]^{5/2}} = \frac{3}{2\pi} \cdot \frac{1}{\left[1 + 0\right]^{5/2}} = 0.477$$

from eqn (1),

$$G_z = \frac{0.444 \times 15000}{8^2} = 111.90 \text{ kN/m}^2$$

Case-II:-
 $r = 4 \text{ m}$ $z = 8 \text{ m}$ $Q = 15,000 \text{ kN}$

$$I_B = \frac{3}{2\pi} \left[\frac{1}{\left[1 + \left(\frac{z}{r}\right)^2\right]^2} \right]^{3/2} = 0.115$$

\therefore Vertical stress at a distance 7m from centre of the water tower.

$$G_z = \frac{0.115 \times 15000}{8^2} = 27.015 \text{ kN/m}^2$$

Additional Practice [2015 Spring] :-
 Solution same as above.

3. A strip footing is given in plan as shown in fig. 1. The load per unit area is 300 kN/m^2 . Determine the intensity of vertical stress at a point 5m directly below point A.

\rightarrow Solution:-

Given that,

$$q = 300 \text{ kN/m}^2$$

$$B = 20 \text{ m}$$

$$z = 5 \text{ m}$$

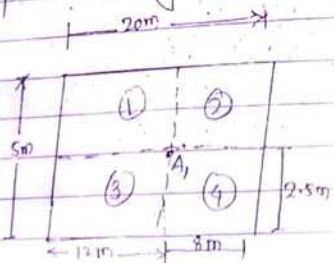
Now,

$$Q = q \times A$$

$$= 300 \times 20 \times 5 = 30,000 \text{ kN}$$

For 1, $m = \frac{r}{z} = \frac{10}{5} = 2.4 \Rightarrow I_1 = 0.13$

$n = \frac{R}{z} = \frac{2.5}{5} = 0.5 \Rightarrow I_2 = 0.125$



for 2,

$$m = \frac{r}{z} = 1.6$$

$$n = \frac{2.5}{5} = 0.5$$

$$\Rightarrow I_3 = 0.125$$

for 3,

$$m = \frac{r}{z} = 2.4$$

$$n = \frac{2.5}{5} = 0.5$$

$$\Rightarrow I_4 = 0.13$$

for 4,

$$m = \frac{r}{z} = 1.6$$

$$n = \frac{2.5}{5} = 0.5$$

$$\Rightarrow I_4 = 0.125$$

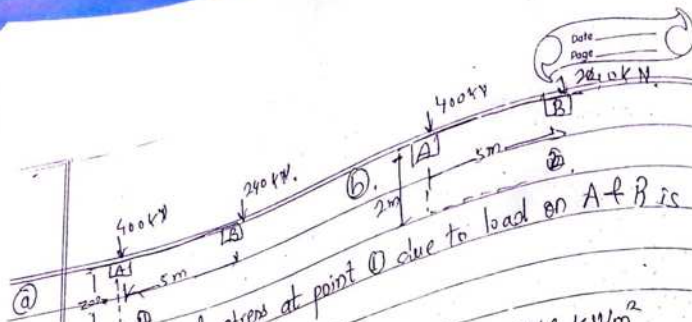
$$\therefore I = I_1 + I_2 + I_3 + I_4 = 0.51$$

\therefore Vertical stress (G_z) = $q \cdot I$

$$= 300 \times 0.51$$

$$= 153 \text{ kN/m}^2$$

Two columns A and B are placed at meter center to center. Through point A, a load of 400kN is acting and from point B a load of 240kN. Calculate the vertical stress due to these loads on a horizontal plane 2m below the ground surfaces at points: (a) vertically below the points A and B (b) 5m horizontally away from point A.



Now, vertical stress at point O due to load on A & B is

$$\text{Due to A, } I_B = \frac{3}{2\pi}$$

$$(G_z)_A = I_B \cdot Q = \frac{3}{2\pi} \cdot \frac{400}{2^2} = 47.746 \text{ kN/m}^2$$

$$= 4.96 \text{ t/m}^2$$

$$\text{Due to B, } I_B = \frac{3}{2\pi} \left[1 + \left(\frac{R}{z}\right)^2\right]^{3/2}$$

$$= \frac{3}{2\pi} \left[1 + \left(\frac{5}{2}\right)^2\right]^{3/2} = 3.373 \times 10^{-3}$$

$$(G_z)_B = I_B \cdot A = 3.373 \times 10^{-3} \times 240 = 0.202 \text{ kN/m}^2$$

\therefore Total stress vertically below point A is

$$(G_z)_A = (G_z)_A + (G_z)_B = 47.948 \text{ kN/m}^2 = 4.887 \text{ t/m}^2$$

Again, vertical stress at point O due to load A & B,

$$\text{Due to load B, } I_B = \frac{3}{2\pi} \cdot \frac{240}{2^2} = 28.647 \text{ kN/m}^2$$

$$(G_z)_B = I_B \cdot A = \frac{3}{2\pi} \cdot \frac{240}{2^2} = 28.647 \text{ kN/m}^2$$

$$\text{Due to load A, } I_B = \frac{3}{2\pi} \left[1 + \left(\frac{R}{z}\right)^2\right]^{3/2} = \frac{3}{2\pi} \left[1 + \left(\frac{5}{2}\right)^2\right]^{3/2} = 3.373 \times 10^{-3}$$

$$(G_z)_A = I_B \cdot A = 3.373 \times 10^{-3} \times 400 = 0.337 \text{ kN/m}^2$$

\therefore Total stress vertically below point B is

$$(G_z)_B = (G_z)_A + (G_z)_B$$

$$= 28.984 \text{ kN/m}^2$$

$$\approx 2.955 \text{ t/m}^2$$

Rakesh Kumar Sah

A water tank in three pillars is in the form of an equilateral triangle in plan and the sides of the triangle are 10 meters. The total weight of water tower is 1000 kN. Find the vertical stress 10m below the ground surface under any one of the legs.

Solution:-

Given that:-

Total weight of tower = 1000 kN

Since, load is coming equally on three legs of the tower,

\therefore Load on each leg = $\frac{1000}{3}$

$$= 333.33 \text{ kN}$$

Now, vertical stress at 10m below one of the leg (say below A) is caused by three point loads, coming through the three A, B, C legs.

Vertical stress caused by load on leg A, below which the stress is computed, $Q_A = 333.33 \text{ kN}$, $z = 10$ & $r = 0$

$$\therefore G_{z1} = \frac{3Q_A}{2\pi z^2} \left[\frac{1}{1 + \left(\frac{r}{z}\right)^2} \right]^{3/2}$$

$$= \frac{3 \times 333.33}{2\pi \times 10^2} \left[\frac{1}{1 + 0^2} \right]^{3/2} = 1.59 \text{ kN/m}^2$$

Vertical stress caused by load on either of the other two legs B and C, $\therefore Q_B = 333.33$, $z = 10$ & $r = 10$

$$\therefore G_{z2} = 0.2813 \text{ kN/m}^2 = G_{z3}$$

$$\therefore G_z = G_{z1} + G_{z2} + G_{z3} = (1.59 + 0.28 + 0.28) \text{ kN/m}^2$$

$$= 2.15 \text{ kN/m}^2$$

6. A point load of 132 kN is applied at the ground surface. Construct a pressure bulb when the stress imposed becomes 20% of the applied load.

Solution.

Given that:-

Load (Q) = 132 kN.

We have,

$$G_z = I_B \frac{Q}{z^2}$$

or, $0.2Q = I_B \frac{Q}{z^2}$

$$\therefore I_B = 0.2 z^2 = \frac{3}{27 \left[1 + \left(\frac{r}{z} \right)^2 \right]^{3/2}}$$

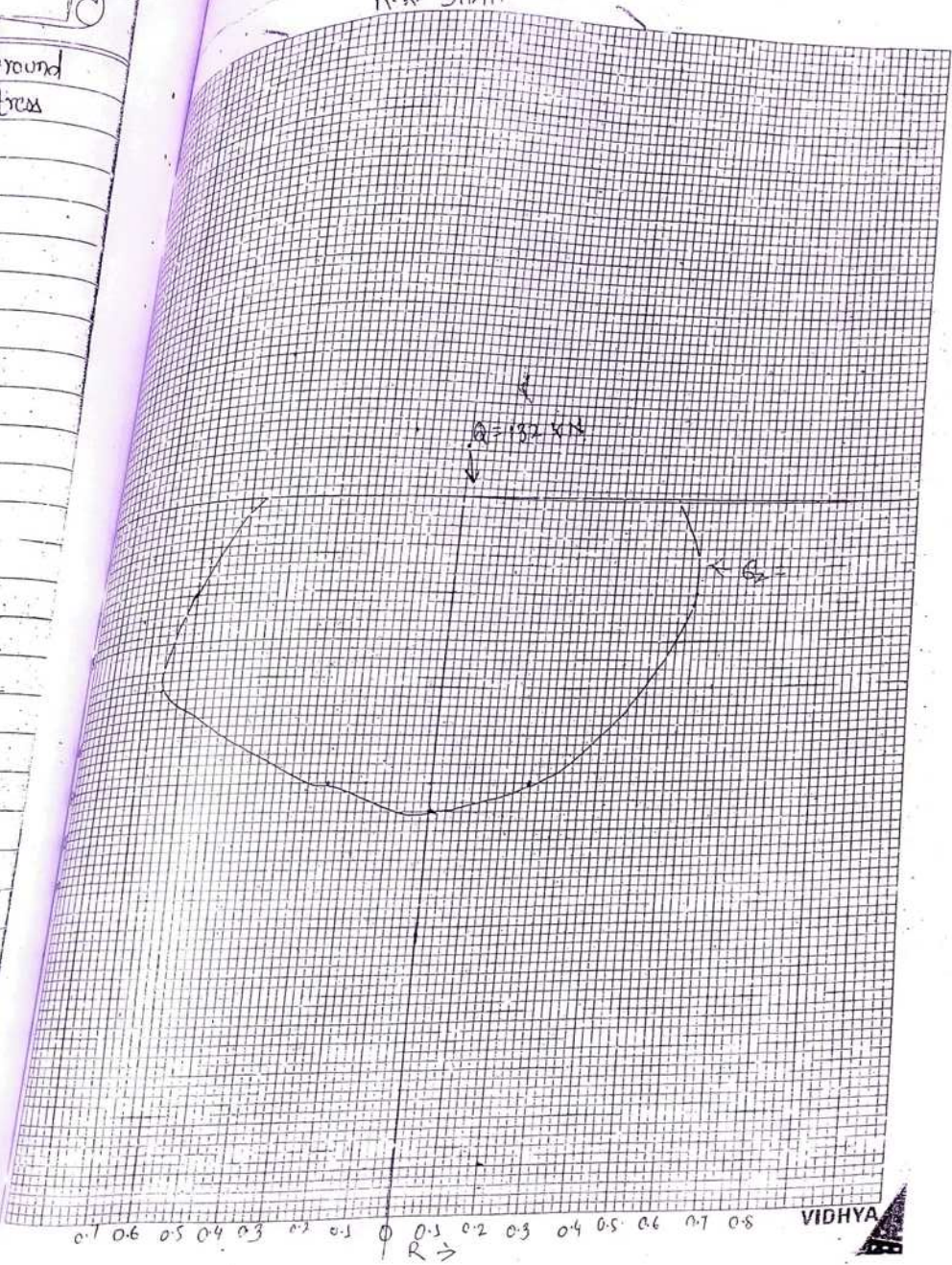
Putting $r=0$ in above eqn, $z=1.545$. We can get the values of r for different values of z as tabulated below.

z	0.5	1	1.5	1.545
I_B	0.05	0.2	0.45	0.477
r/z	1.21	0.645	0.154	≈ 0.00
r	0.60	0.645	0.232	0.00

Now,

The above data is plotted in graph.

- Additional practise (2013 Spring)
 - Additional practise (2016 Spring)
- Solution same as above.



7. A water tower has circular foundation of diameter 9m. The total weight of the tower including the foundation is 17,500 kN. A very weak foundation capacity of 95 kN/m² lies 3m below the foundation level. Calculate the stress due to foundation at the top of the weak stratum. Give your comment with regard to the feasibility of the foundation construction at the top surface of the weak layer.

→ Solution:-

Given that,

Diameter of foundation (d) = 9m

$$\text{Foundation Area (A)} = \frac{\pi d^2}{4} = \frac{\pi \times 9^2}{4} = 63.62 \text{ m}^2$$

$$\text{Load per unit area (q)} = \frac{17500}{63.62} = 275.07 \text{ kN/m}^2$$

$$\text{Again, } R_z = \frac{4.5}{3} = 1.5$$

$$\therefore G_z = q \left[\frac{1 - \left(\frac{1}{1 + (R_z/2)^2} \right)^{3/2}}{1 + (R_z/2)^2} \right] = 275.07 * \left[\frac{1 - \left(\frac{1}{1 + 1.5^2} \right)^{3/2}}{1 + 1.5^2} \right]$$

$$= 228.12 \text{ kN/m}^2$$

∴ Pressure increment at the top of the weak stratum due to construction of the water tower will be 228.12 kN/m².

While the bearing capacity of the stratum is 95 kN/m² only. Hence, it is not safe to construct the water tower with the proposed foundation size.

8. A concentrated point load of 250 kN acts at the ground surface. Find the intensity of vertical pressure at depth of 5m below the ground surface at the point on the axis of loading using Boussinesq analysis. What will be the difference in the

Rakesh Kumar Sah
vertical pressure at the same point if the load shifted to a distance of 2m from its original position horizontally.

Solution:

Case I:-

At point P, $z = 5m$,

$Q = 250kN$, $r = 0$

$$\therefore G_z = 3Q$$

$$= \frac{2\pi \times 250}{2\pi \times 5^2} [1 + \frac{0}{5}]^2 \times \frac{5}{2}$$

$$= 3 \times 250$$

$$= \frac{2\pi \times 250}{2\pi \times 5^2} [1 + 0]^2 \times \frac{5}{2}$$

Case II:- $z = 5m$, $r = 2m$.

$$\therefore G_z = 3Q$$

$$= \frac{2\pi \times 250}{2\pi \times 5^2} [1 + \frac{4}{25}]^2 \times \frac{5}{2}$$

$$= 3 \times 250 = 3.29 kN/m^2$$

$$\therefore \text{Difference in vertical pressure} = 4.77 - 3.29 = 1.48 kN/m^2$$

9) Using Fadum's chart, determine the increase in vertical stress at 2m depth below A and B due to uniformly distributed load of $40 kN/m^2$ on the ground surface shown by the area in the fig. 2.

Solution:

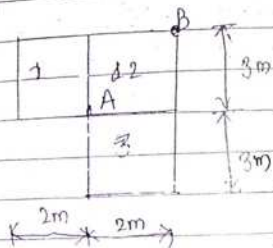
Given that:-

for point A,

$z = 3m$, $q = 40 kN/m$

for loaded area I,

$$m = \frac{B}{z} = \frac{2}{3} = 1$$



$$n = \frac{L}{z} = \frac{2}{3} = 1$$

For $m = 1$ & $n = 1$, from Fadum chart, $I_N = 0.18$

$$\therefore G_z = 3 \times q \times I_N = 3 \times 40 \times 0.18 = 21.6 kN/m^2$$

for the point B,

$$A_1 = m = \frac{B}{z} = \frac{4}{3} = 2 \quad \& \quad n = \frac{L}{z} = \frac{2}{3} = 1, \quad I_{N1} = 0.24$$

$$(Area)_2 = m = \frac{B}{z} = \frac{2}{3} = 1 \quad \& \quad n = \frac{L}{z} = \frac{4}{3} = 2, \quad I_{N2} = 0.21$$

$$(Area)_3 = m = \frac{B}{z} = \frac{2}{3} = 1 \quad \& \quad n = \frac{L}{z} = \frac{2}{3} = 1, \quad I_{N3} = 0.21$$

$$\begin{aligned} \therefore G_z &= q (I_{N1})_1 + (I_{N2})_2 + (I_{N3})_3 \\ &= 40 (0.21 + 0.24 + 0.21) \\ &= 9.6 kN/m^2 \end{aligned}$$

10) A uniformly distributed load ABCDEF having a plan area as shown in fig. 3 is placed on the ground surface. The intensity of the load is $20 kN/m^2$. Determine the increase in the vertical stress due to the load at 6m depth below the corner points C and D.

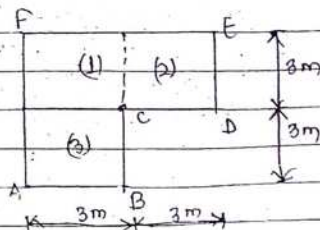
Solution:

Below corner point C,

for loaded area 1,

$$m = \frac{B}{z} = \frac{3}{6} = 0.5$$

$$n = \frac{L}{z} = \frac{3}{6} = 0.5$$



for $m = 0.5$ & $n = 0.5$, from Fadum chart, $I_{N1} = 0.0818$.

$$\therefore G_z = 3 \times q \times I_{N1} = 3 \times 20 \times 0.0818 = 4.91 kN/m^2$$

12) A rectangular foundation $2m \times 1.5m$ carries a uniform load of 40 kN/m^2 . Determine the vertical stress at P which is $3m$ below the ground surface as shown in fig. 5. Use equivalent point load method.

Solution:-

Given that:

Rectangular area = $2 \times 1.5 \text{ m}^2$

Load intensity (q) = 40 kN/m^2

Depth (z) = $3m$.

The area is divided into 6 small area units as shown in the following figure.

∴ load on each area = $40 \times (1 \times 0.5) = 20 \text{ kN/m}^2$

The stress at point P are determined due to 6 points.

Using Boussinesq's Solⁿ for loads (1), (2), (3) & (4)

$$r = \sqrt{0.5^2 + 0.25^2} = 0.559 \text{ m}$$

$$z = 3 \text{ m}$$

for loads (5) & (6)

$$r = \sqrt{0.5^2 + 0.75^2} = 0.901 \text{ m}$$

$$\therefore G_z = \sum_{i=1}^6 \frac{3Q_i}{2\pi r_i^2} \left[\frac{1}{1 + \left(\frac{z}{r_i}\right)^2} \right]^{5/2}$$

$$= \frac{3 \times 20}{2\pi \times 9^2} \left[\frac{4}{1 + \left(\frac{0.559}{3}\right)^2} \right]^{5/2} + \frac{2}{1 + \left(\frac{0.901}{3}\right)^2} \right]^{5/2}$$

$$= 5.609 \text{ kN/m}^2$$

13) The plan of a three legged tower forms an equilateral triangle of side $5m$. If the total weight of the tower is 600 kN and equally carried by all the legs, compute the vertical stress increase in the soil by the tower at a depth of $4m$ directly below the center of equilateral triangle.

Solution:-

Given that:-

Total weight of tower (Q) = 600 kN

height ($\frac{\sqrt{3}}{2}$) = 4 m

$$\therefore G_z = I_B \cdot \frac{Q}{z^2} \quad \dots (1)$$

$$I_B = \frac{3}{2\pi \left[\frac{1}{2} \right]^2} \right]^{5/2}$$

$$= \frac{3}{2\pi \left[1 + \left(\frac{2.88}{4} \right)^2 \right]^{5/2}} = 0.168$$

from (1),

$$G_z = 0.168 \times 600 = 6.3 \text{ kN/m}^2$$

$$\therefore \text{Total stress} = 3 G_z = 3 \times 6.3 = 18.9 \text{ kN/m}^2$$

Now,

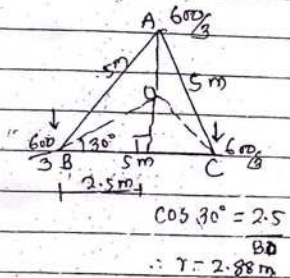
Vertical stress (G_z) due to load at A below $4m$ is

$$(G_z)_A =$$

$$\text{Total stress} = 3 \times (G_z)_A$$

Additional practise (2012 Spring)

Solution same as above



14. A circular ring foundation for an overhead tank transmits a constant pressure of 300 kN/m^2 . Its internal diameter is 6 m and external diameter is 10 m . Compute the vertical stress on the central line of the footing at a depth of 6.5 m below the ground level.

→ Solution:-

Given that:-

$$\text{Pressure } (q) = 300 \text{ kN/m}^2$$

$$\text{Internal diameter} = 6 \text{ m}$$

$$\text{External diameter} = 10 \text{ m}$$

$$\text{Vertical stress} = ? \text{ for } z = 6.5 \text{ m}$$

We have,

$$G_z = I_c \cdot q$$

$$I_c = 1 - \frac{1}{\left\{1 + \left(\frac{R}{z}\right)^2\right\}^{3/2}}$$

At $R = 5 \text{ m}$

$$I_{c1} = 1 - \frac{1}{\left\{1 + \left(\frac{5}{6.5}\right)^2\right\}^{3/2}} = 0.502$$

$$\therefore G_{z1} = I_{c1} \cdot q = 0.502 \times 300 = 150.6$$

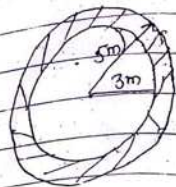
At $R = 3 \text{ m}$,

$$I_{c2} = 1 - \frac{1}{\left\{1 + \left(\frac{3}{6.5}\right)^2\right\}^{3/2}} = 0.251$$

$$\therefore G_{z2} = I_{c2} \cdot q = 0.251 \times 300 = 75.45$$

Hence,

$$\text{Required stress } (G_z) = G_{z1} - G_{z2} \\ = 150.6 - 75.45 = 75.16 \text{ kN/m}^2$$



R.K. SHAH

Find vertical stress at 5 m below point P due to uniformly loaded hatched area of 300 kN/m^2 intensity as shown in fig. 6.

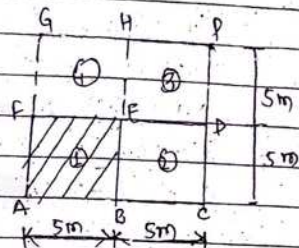
→ Solution:-

Given that,

$$q = 300 \text{ kN/m}^2$$

$$z = 5 \text{ m}$$

$$G_z = ?$$



$$G_z = q [(I_H)_1 - (I_H)_2 - (I_H)_3 + (I_H)_4]$$

$$(I_H)_1 [\text{rect. ACPG}]: m = 10/5 = 2 \quad \left. \begin{array}{l} \text{from Fadum's chart} \\ n = 10/5 = 2 \end{array} \right\} (I_H)_1 = 0.235$$

$$(I_H)_2 [\text{rect. GFDP}]: m = 10/5 = 2 \Rightarrow (I_H)_2 = 0.2 \\ n = 5/5 = 1$$

$$(I_H)_3 [\text{rect. BHP}]: m = 10/5 = 2 \quad (I_H)_3 = 0.2 \\ n = 5/5 = 1$$

$$(I_H)_4 [\text{rect. HPDE}]: m = 5/5 = 1 \quad (I_H)_4 = 0.18 \\ n = 5/5 = 1$$

$$\therefore G_z = q [(I_H)_1 - (I_H)_2 - (I_H)_3 + (I_H)_4] \\ = 300 [0.235 - 0.2 - 0.2 + 0.18] \\ = 4.5 \text{ kN/m}^2$$

15. A ring footing of external diameter 8 m and internal diameter 4 m rests at a depth 2 m below the ground surface. It carries a load intensity of 150 kN/m^2 . Find the vertical stress at depth of 2 m & 4 m

along the axis of footing base. Neglect the effect of excavation on the stress.

→ Solution:-

$$q = 150 \text{ kN/m}^2$$

$$r_1 = r_2 = 4 \text{ m} \quad r_2 = 4/2 = 2 \text{ m}$$

$$z_1 = 2 \text{ m} \quad \& \quad z_2 = 4 \text{ m}$$

For circular area, $G_z = q \cdot I_c$

$$G_z \text{ due to } r_1 = 4 \text{ m, } z_1 = 2 \text{ m,}$$

$$I_{c1} = 1 - \frac{1}{\left\{1 + \left(\frac{z_1}{r_1}\right)^2\right\}^{3/2}} = 0.911$$

$$\therefore G_{z1} = 150 \times 0.911 = 136.65 \text{ kN/m}^2$$

$$G_z \text{ due to } r_2 = 2 \text{ m } \& \quad z_2 = 2 \text{ m,}$$

$$I_{c2} = 1 - \frac{1}{\left\{1 + \left(\frac{z_2}{r_2}\right)^2\right\}^{3/2}} = 0.646$$

$$\therefore G_{z2} = 150 \times 0.646 = 96.9 \text{ kN/m}^2$$

∴ Resultant vertical stress at $z = 2 \text{ m}$ is,

$$G_z = G_{z1} - G_{z2} = 136.65 - 96.9 = 39.75 \text{ kN/m}^2$$

Another Case:-

At $z = 4 \text{ m}$,

$$G_z \text{ due to } r_1 = 4 \text{ m, } I_{c1} = 1 - \frac{1}{\left\{1 + \left(\frac{z_1}{r_1}\right)^2\right\}^{3/2}} = 0.646$$

$$\therefore G_{z1} = 150 \times 0.646 = 96.9 \text{ kN/m}^2$$

$$G_z \text{ due to } r_2 = 2 \text{ m, } I_{c2} = 1 - \frac{1}{\left\{1 + \left(\frac{z_2}{r_2}\right)^2\right\}^{3/2}} = 0.284$$

$$\therefore G_{z2} = 150 \times 0.284 = 42.6 \text{ kN/m}^2$$

∴ Resultant vertical stress at $z = 4 \text{ m}$ is

$$G_z = G_{z1} - G_{z2}$$

$$= 96.9 - 42.6$$

$$= 54.3 \text{ kN/m}^2$$

17. A water tank is supported by a ring foundation of external diameter 10 m and internal diameter of 7.5 m. The ring foundation transmits uniform load intensity of 160 kN/m². Compute the vertical stress induced at a depth of 4 m below the centre of ring foundation, using

1) Boussinesq analysis

2) Westergaard's analysis.

→ Solution:-

Given that:

1) Using Boussinesq analysis,

$$G_v = q \left[1 - \frac{1}{\left\{1 + \left(\frac{R_0}{z}\right)^2\right\}^{3/2}} \right] - q \left[1 - \frac{1}{\left\{1 + \left(\frac{R_1}{z}\right)^2\right\}^{3/2}} \right]$$

$$= \frac{160}{2} \left[1 - \frac{1}{\left\{1 + \left(\frac{10}{4}\right)^2\right\}^{3/2}} \right] - 2160 \left[1 - \frac{1}{\left\{1 + \left(\frac{3.75}{4}\right)^2\right\}^{3/2}} \right]$$

$$= 23.11 \text{ kN/m}^2$$

1) Using Westergaard's analysis

RAKESH KUNAR SHAH

Tutorial - 6
Ch-9 Compressibility and Consolidation of Soil

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1. Two clay samples A and B have initial void ratio of 0.55 and 0.632 respectively under a pressure 100 kN/m^2 . The pressure increased to 150 kN/m^2 . With the application of the pressure, the void ratio decreases to 0.495 and 0.616 respectively. The time taken by the specimen A to reach 50% consolidation is one-third of that required by the specimen B for the same consolidation. Find the ratio of the coefficient of the permeability of the two specimens A and B if their thickness was 40mm and 30mm resp.

→ Solution:-

Given that:-

Sample A
 $e_{0A} = 0.55$
 initial, $P_{0A} = 100 \text{ kN/m}^2$
 $P_f = 150 \text{ kN/m}^2$
 final void ratio (e_A) = 0.495
 degree of consolidation (U) = 50%
 time t (t_A) = $t/3$
 thickness/ height of sample (H_A) = 40mm
 $\frac{k_A}{k_B} = ?$

Sample B
 $e_{0B} = 0.632$
 initial pressure (P_{0B}) = 100 kN/m^2
 final " $P_f = 150 \text{ kN/m}^2$
 final void ratio (e_B) = 0.616
 $U = 50\%$
 time (t_B) = t
 $H_B = 30 \text{ mm}$

Since the degree of consolidation is same,
 $T_{vA} = T_{vB}$ [∵ $T_v = \text{time factor}$]

or, $\frac{C_v t_A}{d^2} = \frac{C_v t_B}{d^2}$... (1)

$C_v = \frac{k}{m_v \gamma_w}$ $m_v = \frac{a_v}{1 + e_0}$ $a_v = \frac{\Delta e}{\Delta P}$

(Now,

▷ $C_{vA} = \frac{-\Delta e_A}{\Delta P_A} = -\frac{(0.495 - 0.55)}{150 - 100} = 0.0011$

$C_{vB} = \frac{-\Delta e_B}{\Delta P_B} = -\frac{(0.616 - 0.632)}{150 - 100} = 0.00032$

▷ $m_{vA} = \frac{0.0011}{1 + 0.55} = 7.09 \times 10^{-4}$

$m_{vB} = \frac{0.00032}{1 + 0.632} = 1.96 \times 10^{-4}$

▷ $C_{vA} = \frac{k_A}{m_{vA} \gamma_w} = \frac{k_A}{7.09 \times 10^{-4} \times 9.81} = \frac{k_A}{7.09 \times 10^{-4} \gamma_w}$

$C_{vB} = \frac{k_B}{m_{vB} \gamma_w} = \frac{k_B}{1.96 \times 10^{-4} \gamma_w}$

From eqⁿ (1), we have,
 $\frac{k_A}{7.09 \times 10^{-4} \gamma_w} \times \frac{1}{40^2} = \frac{k_B}{1.96 \times 10^{-4} \gamma_w} \times \frac{1}{30^2}$

or, $\frac{k_A}{k_B} = \frac{7.09 \times 40^2 \times 3}{1.96 \times 30^2} = 19.3$

∴ Req. ratio = $\frac{k_A}{k_B} = 19.3$

2. The following information was obtained from the soil investigation work. Two layers of clay of thickness 2m each and sandwiched between two layers of dense sand was encountered at 5m and 14m depths respectively. The compression index, water content and specific gravity of both clays were 0.38, 30% and 2.7 respectively. The sand weighed 2.5 gm/cc and was completely submerged. Compute the total settlement under a uniformly distributed load of 3.5 kg/cm^2 on a large area over ground surface.

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→ Solution:-

Given that:-

Case I:-

We will first calculate the settlement in the upper clay layer.
Effective overburden pressure at section A-A i.e. centre of upper clay layer is,

$$G'_v = \gamma_{\text{sub.sand}} \times 5 + \gamma_{\text{sub.clay}} \times 1 \quad [\because e = wq = 0.3 \times 2.7 = 0.81]$$

$$\gamma_{\text{sub.clay}} = \frac{G_s - 1}{1 + e} \times \gamma_w = \frac{2.7 - 1}{1 + 0.81} \times 10 = 9.39 \text{ kN/m}^3$$

$$\gamma_{\text{sub.sand}} = (\gamma_{\text{sat}} - \gamma_w) = (21 - 10) = 11 \text{ kN/m}^3$$

Now,

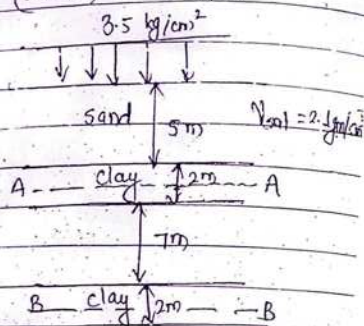
$$G'_0 = 11 \times 5 + 9.39 \times 1$$

$$= 64.39 \text{ kN/m}^2$$

$$\therefore \Delta H_1 = \frac{C_c \cdot H_0}{1 + e_0} \log_{10} \left(\frac{G'_0 + \Delta G'_v}{G'_0} \right)$$

$$= \frac{0.38 \times 200}{1 + 0.81} \log_{10} \left(\frac{64.39 + 350}{64.39} \right)$$

$$= 33.95 \text{ cm.}$$



Case II:- settlement in the lower clay layer.

Effective overburden pressure at section B-B i.e. centre of lower clay layer is

$$G'_v = \gamma_{\text{sub.sand}} \times (5+7) + \gamma_{\text{sub.clay}} \times (2+1)$$

$$= 11 \times 12 + 9.39 \times 3$$

$$= 160.17 \text{ kN/m}^2$$

$$\therefore \Delta H_2 = \frac{C_c \cdot H_0}{1 + e_0} \log_{10} \left(\frac{G'_0 + \Delta G'_v}{G'_0} \right)$$

$$= \frac{0.38 \times 200}{1.81} \log_{10} \left(\frac{160.17 + 350}{160.17} \right) = 21.13 \text{ cm.}$$

$$\therefore \text{Total settlement} = \Delta H_1 + \Delta H_2$$

$$= 33.95 + 21.13$$

$$= 55.1 \text{ cm}$$

3. A saturated soil stratum 5m thick lies above an impervious stratum and below a pervious stratum. It has a compression index of 0.25 and a coefficient of permeability of $3.2 \times 10^{-4} \text{ cm/sec}$. It has a void ratio of 1.9 at a stress of 1.5 kg/cm^2 .

(a) the change void ratio due to increase of stress of 2 kg/cm^2 .

(b) settlement of soil stratum due to above increase of stress.

(c) time required for 50% consolidation, the time factor for 50% consolidation may be assumed to be 0.2.

→ Solution:-

Given that.

Compression index $(C_c) = 0.25$

Coeff. of permeability $(K) = 3.2 \times 10^{-4} \text{ cm/s}$

$e_0 = 1.9$

Stress $(G_0) = 1.5 \text{ kg/cm}^2$.

a) $\Delta e = ?$ for $\Delta G = 2 \text{ kg/cm}^2$

b) settlement = ?

c) $t_{50} = ?$ time factor $(T_v) = 0.2$

We have

$$C_c = - \Delta e$$

$$G_0 = 1.5 \text{ kg/cm}^2$$

$$\log_{10} \left(\frac{G}{G_0} \right)$$

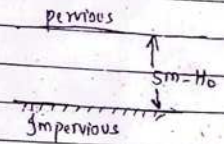
$$G = G_0 + \Delta G = 3.5 \text{ kg/cm}^2$$

$$\text{or, } 0.25 = - \Delta e$$

$$\log_{10} \left(\frac{3.5}{1.5} \right)$$

$$\Delta e = 0.211$$

Also,



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$$\begin{aligned} b) \Delta H &= \frac{C_c H_0}{1+e_0} \log \left(\frac{\sigma_0 + \Delta \sigma}{\sigma_0} \right) \\ &= \frac{0.25 \times 5}{1+1.9} \log \left(\frac{1.5+2}{1.5} \right) \\ &= 0.365 \text{ m} \end{aligned}$$

$$\begin{aligned} c) T_v &= \frac{C_v t}{d^2} \quad \therefore d = 5 \text{ m} = 500 \text{ cm} \\ a_v &= \frac{-\Delta e}{\Delta \sigma} = \frac{-(-0.211)}{2} = 0.1055 \text{ cm}^2/\text{kg} \\ m_v &= \frac{a_v}{1+e_0} = \frac{0.1055}{1+1.9} = 0.0363 \text{ cm}^2/\text{kg} \end{aligned}$$

$$C_v = \frac{k}{m_v \gamma_w} = \frac{3.2 \times 10^{-4}}{0.0363 \times 10^{-3}} = 8.815 \text{ cm}^2/\text{s}$$

Flow, $\left. \begin{aligned} \gamma_w &= 9.81 \text{ kN/m}^3 \\ &= 1 \times 10^{-3} \text{ kg/cm}^3 \end{aligned} \right\}$

$$T_v = \frac{C_v t_{90}}{d^2}$$

$$\text{or, } 0.2 = \frac{8.815 \times t_{90}}{500^2} \Rightarrow t_{90} = 5671.875 \text{ sec} \approx 94.53 \text{ min.}$$

Additional practise (2015 fall)

Solution same as above process.

4. A clay layer 2.5m thick is overlain by a layer of sand 4m in thickness. The clay has a compression index of 0.22 and saturated unit weight of 18 kN/m³. The initial void ratio of clay is 1.30. Calculate the final settlement of the clay layer due to an increase in pressure of 30 kN/m² at center of clay layer. Also, calculate the settlement when the table water rises to the ground surface. Unit weight of sand is 20 kN/m³.

Solution:-

Given that:-

Initial pressure at the center of clay layer,

$$\begin{aligned} \sigma_0 &= 4 \times 20 + 1.25 \times 18 \\ &= 102.5 \text{ kN/m}^2 \end{aligned}$$

$$\therefore \Delta H = \frac{C_c H_0}{1+e_0} \log \left(\frac{\sigma_0 + \Delta \sigma}{\sigma_0} \right)$$

$$\begin{aligned} &= \frac{0.22 \times 2.5}{2.3} \log_{10} \left(\frac{102.5 + 30}{102.5} \right) \\ &= 0.0266 \text{ m} \approx 2.67 \text{ cm} \end{aligned}$$

When water table rises to the ground surface,

$$\sigma_0' = 4 \times 20 + 1.25 \times 18(1.5 - 1.0) = 50 \text{ kN/m}^2$$

$$\therefore \Delta H = \frac{0.22 \times 2.5}{2.3} \log_{10} \left(\frac{50 + 30}{50} \right) = 0.0488 \text{ m} \approx 4.88 \text{ cm}$$

Additional Practice (2015 Spring)

Additional practice (2014 Spring)

Additional practice (2011 Fall)

} Solⁿ same as above process.

5. A strata of normally consolidated clay of thickness 3m is drained on both sides. If it has a coefficient of permeability $k = 5 \times 10^{-8}$ cm/sec and coefficient of volume compressibility $= 125 \times 10^{-2}$ cm²/kN. Determine the total compression of the strata for a change in stress of 25 t/m². Also determine the time required for 80% consolidation.

⇒ Solution:-

Given that:-

$$\text{Coefficient of permeability (k)} = 5 \times 10^{-8} \text{ cm/s}$$

$$\text{Coeff. of volume compressibility (m}_v) = 125 \times 10^{-2} \text{ cm}^2/\text{kN}$$

$$\text{Total compression } (\Delta H) = ?$$

$$\Delta \sigma = 25 \text{ t/m}^2 = 25 \times 1000 \text{ kg/m}^2 = 25000 \times 9.81 \times 10^{-3} \text{ kN/cm}^2 \times 10^4$$

We have;

$$\Delta H = m_v H_0 \Delta \bar{\sigma}$$
$$= 1.75 \times 10^{-2} \times 300 \times 0.0345$$
$$= 9.2 \text{ cm}$$

$$T_v = \frac{C_v t_{90}}{d^2} \quad \dots (1)$$

for $U = 80\% = 0.8$

$$T_v = -0.933 \log_{10} (1-U) - 0.085$$
$$= -0.933 \log_{10} (1-0.8) - 0.085$$
$$= 0.567$$

$$C_v = \frac{k_v}{\gamma_w H_0} = \frac{5 \times 10^{-8}}{125 \times 10^{-2} \times 9.81 \times 10^{-6}} = 4.07 \times 10^{-3} \text{ cm}^2/\text{s}$$

Now, from (1),

$$0.567 = \frac{4.07 \times 10^{-3} \times t_{90}}{150^2}$$

$$\therefore d = \frac{H_0}{2} = \frac{300}{2} = 150 \text{ cm}$$

$$\therefore t_{90} = 3134520.885 \text{ sec}$$
$$\approx 36.28 \text{ days}$$

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$$\gamma_w = 9.81 \text{ KN/m}^3$$
$$= \frac{9.81}{1 \times 10^6} \text{ KN/cm}^3$$
$$= 9.81 \times 10^{-6} \text{ KN/cm}^3$$

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$$\bar{\sigma} = 18 \times 5 \times 1.5 + 6 \times 18 + 17.5 \times 0.5 - 9.81 \times 4.5$$
$$= 100.36 \text{ KN/m}^2$$

and initial $\bar{\sigma}_0 = 6 \times 18 +$

$$17.5 \times 0.5 - 9.81 \times 4.5$$
$$= 72.61 \text{ KN/m}^2$$

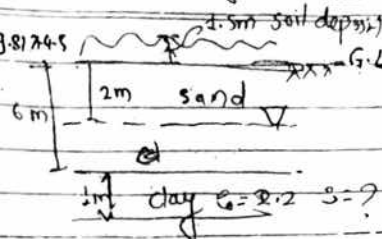
$$\therefore \Delta \bar{\sigma} = 100.36 - 72.61 = 27.75 \text{ KN/m}^2$$

Now,

$$S = \frac{C_v H_0 \log_{10} \left(\frac{\bar{\sigma}_0 + \Delta \bar{\sigma}}{\bar{\sigma}_0} \right)}{1 + C_v}$$

$$= \frac{0.4 \times 1 \times \log_{10} \left(\frac{100.36}{72.61} \right)}{1 + 2.2}$$

$$= 0.01757 \text{ m} \approx 17.57 \text{ mm}$$



Here,

We have,

$$s = \frac{C_c}{1 + e_0} H_0 \log_{10} \left(\frac{\bar{\sigma}_0 + \Delta \bar{\sigma}}{\bar{\sigma}_0} \right)$$

Now,

$$\bar{\sigma}_0 = 5 \times 17.5 + 1.0 \times 17.0 - 6 \times 9.81$$

5m thick s = 31.98 m

$$= 45.64 \text{ kN/m}^2$$

f

$$\Delta \bar{\sigma} = (2 \times 19 + 45.64) - 45.64$$

$$= 38 \text{ kN/m}^2$$

$$\therefore s = \frac{0.5}{1 + 0.6} \times 1 \times \log_{10} \left(\frac{45.64 + 38}{45.64} \right)$$

$$= 0.03654 \text{ m}$$

$$= 36.54 \text{ mm}$$

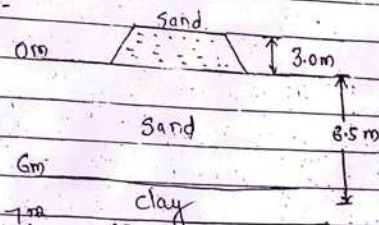
Rakesh Kumar Shah.

8. A clay layer of 1m thick is found between 6m and 7m below the ground surface. The clay layer is sandwiched between sand layers. The coefficient of volume change of the clay is $0.40 \times 10^{-3} \text{ m}^3/\text{kn}$. A 3m high sand embankment of circular plan area diameter 20m is placed on the ground surface. The unit weight of the sand is 20 kN/m^3 . Determine the settlement of the clay layer under the center of the circular shape of radius r which can be calculated according to the given formula below.

$$\Delta \sigma' = \left[1 - \frac{1}{(1 + (z/r)^2)^{1.5}} \right] \times q$$

→ Solution:-

Given that:-



Unit weight of sand (γ_{sand}) = 20 kN/m^3 .

Height of embankment (H) = 3m.

\therefore Stress (q) = $20 \times 3 = 60 \text{ kN/m}^2$

Now,

$$\Delta \sigma'_v = q \left[1 - \frac{1}{(1 + (z/r)^2)^{1.5}} \right] = 60 \left[1 - \frac{1}{(1 + (z/r)^2)^{1.5}} \right]$$

$$= 7.6 \text{ kN/m}^2$$

We have,

$$\Delta H = m_v \times \Delta \sigma'_v \times H_0$$

$$= 0.4 \times 10^{-3} \times 7.6 \times 1$$

$$= 3.05 \text{ mm}$$

9. A consolidation test was performed on a 25mm thick undisturbed clay sample, 50% consolidation occurred in 5 minutes. In the field, a clay layer is 2m thick and is underlain by rock stratum. There is a sand layer above the clay layer. Determine the coefficient of consolidation. Also compute the number of days required for the field stratum to reach 50% consolidation.

→ Solution:-

Given that:-

In lab

$$\text{Sample ht (h)} = 25 \text{ mm}$$

$$\text{time} = 5 \text{ m}$$

$$C_v = ?$$

$$C_v = \frac{T_v d^2}{t}$$

$$= \frac{0.196 \times 1.25^2}{5}$$

$$= 0.06125 \text{ cm}^2/\text{min}$$

$$= 61.25 \times 10^{-3} \text{ cm}^2/\text{min}$$

for U = 50%

$$(T_v)_{\text{lab}} = (T_v)_{\text{field}}$$

$$\frac{C_v t_{\text{lab}}}{d_{\text{lab}}^2} = \frac{C_v t_{\text{field}}}{(d_{\text{field}})^2}$$

$$\text{or, } \frac{5}{t_{\text{field}}} = \frac{1.25^2}{200^2}$$

$$\therefore t_{\text{field}} = 128000 \text{ min} \\ \approx 88.89 \text{ days} \quad \&$$

10. Find the time required for 50% and 80% consolidation in a soil stratum, 10m thick with a pervious strata on top and bottom.

Also find the coefficient of consolidation if, $K = 1 \times 10^{-9} \text{ m/s}$, $e_0 = 1.5$ and $a_v = 0.003 \text{ m}^2/\text{kN}$.

→ Solution:-

Given that:-

$$t_{50} = ?$$

$$t_{80} = ?$$

$$C_v = ?$$

$$K = 1 \times 10^{-9} \text{ m/s}$$

We have,

$$m_v = \frac{a_v}{1+e_0}$$

$$= \frac{0.003}{1+1.5}$$

$$= 0.0012 \text{ m}^2/\text{kN}$$

$$\therefore C_v = \frac{K}{m_v \gamma_w} = \frac{1 \times 10^{-9}}{0.0012 \times 9.81} = 8.494 \times 10^{-8} \text{ m}^2/\text{s}$$

$$\text{For } U = 50\%, T_v = \frac{\pi U^2}{4} = \frac{\pi \times 0.5^2}{4} = 0.196$$

$$\therefore C_v = \frac{T_v d^2}{t_{50}}$$

$$\text{or, } 8.494 \times 10^{-8} = \frac{0.196 \times 5^2}{t_{50}}$$

$$\therefore t_{50} = \frac{57687779.61}{60 \times 60 \times 24} = 667.68 \text{ days}$$

$$\text{For } U = 80\%, T_v = -0.933 \log(1-U) - 0.085$$

$$= -0.933 \log(1-0.8) - 0.085$$

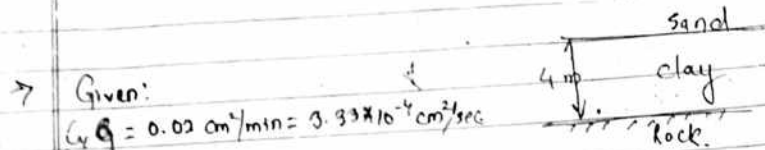
$$= 0.567$$

$$\therefore C_v = \frac{T_v d^2}{t_{80}}$$

$$\therefore \text{or, } 8.494 \times 10^{-8} = \frac{0.567 \times 5^2}{t_{80}}$$

$$\therefore t_{80} = \frac{166882805.3}{60 \times 60 \times 24} = 1931.51 \text{ days} \quad \&$$

11.



→ Given:
 $C_v = 0.02 \text{ cm}^2/\text{min} = 3.33 \times 10^{-4} \text{ cm}^2/\text{sec}$

$S_f = 15 \text{ cm}$

For settlement of 5m (S_s), time req. = ?

For 80% settlement, time req. = ?

Here,

$$S_s = 5 \text{ m}, \frac{S}{15} \times 100 = 33.33\%$$

For $U = 33.33\%$,

$$T_v = \frac{\pi}{4} H^2 U^2 = \frac{\pi}{4} \times (4 \times 333)^2 = 0.0872$$

$H_0 = d = 4 \text{ m}$ (one way down)

Now, we have,

$$t_{33.33} = \frac{T_v d^2}{C_v} = \frac{0.0872 \times (4 \times 100)^2}{3.33 \times 10^{-4}} = 4189789.9 \text{ sec.}$$

$$t \approx 484.93 \text{ days.}$$

For $U = 80\%$,

$$T_v = -0.933 \log_{10}(1-0.8) - 0.085 = 0.567$$

$$\therefore t_{80} = \frac{0.567 \times 400^2}{3.33 \times 10^{-4}}$$

$$= 272432432.4 \text{ sec}$$

$$\approx 3153.15 \text{ days}$$

→ Solution.

Given that:-

$H_0 = d = 10/2 = 5 \text{ m}$ (for two way down)

$C_v = 0.025 \text{ cm}^2/\text{min} = 4.167 \times 10^{-4} \text{ cm}^2/\text{sec}$

$S_f = 8 \text{ cm}$

For 50% settlement, $t_{50} = ?$

Settlement in 6 months = ?

Here,

$$T_v = \frac{\pi}{4} U^2 \text{ for } U = 50\%$$

$$= \frac{\pi}{4} \times 0.5^2 = 0.196$$

$$\therefore t_{50} = \frac{T_v d^2}{C_v} = \frac{0.196 \times 500^2}{4.167 \times 10^{-4}} = 117590592.8 \text{ sec.}$$

$$\therefore t_{50} = 3.73 \text{ yrs} \approx 3.73 \times 12 \text{ months}$$

Similarly,

For 50% settlement = 0.196

For time (t_s) = 6 months, (T_v) is calculated as

We have,

$$\frac{T_v}{(T_v)_s} = \frac{t}{t_s}$$

$$\Rightarrow Tv_1 = \frac{0.196 \times 6}{3.70712} = 0.0263$$

For $(Tv)_1 = 0.0263$,

we have,

$$U^2 = (Tv)_1 \times \frac{4}{\pi}$$

$$= \frac{(0.0263) \times 4}{\pi}$$

$$= 0.033486$$

$$\therefore U = 0.18299$$

$$\approx 18.30\%$$

In 6 months, there is 18.30% settlement.

Then total settlement in 6 months,

$$S = U \times S_f$$

$$= 0.183 \times 8$$

$$= 1.464 \text{ cm}$$

Hence, total settlement = 1.464 cm

→ Solution:

Given that,

$$m_v = 0.25 \times 10^{-3} \text{ m}^2/\text{kN}$$

$$\gamma_{\text{sand}} = 20 \text{ kN/m}^3$$

$$\gamma_{\text{clay}} = 18 \text{ kN/m}^3$$

Initial pressure at centre of clay when WT is at G.L.

$$\bar{\sigma}_0 = 5 \times 20 + 1 \times 18 - 6 \times 9.81$$

$$= 59.14 \text{ kN/m}^2$$

When WT is lowered by 2m from G.L permanently,

final pressure,

$$\bar{\sigma} = 5 \times 20 + 1 \times 18 - 4 \times 9.81$$

$$= 78.76 \text{ kN/m}^2$$

∴ Change in pressure

$$\Delta \bar{\sigma}_0 = 78.76 - 59.14$$

$$= 19.62 \text{ kN/m}^2$$

$$\therefore H_0 = 2 \text{ m}$$

∴ Now,

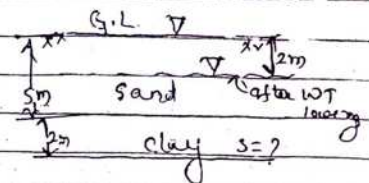
Settlement at centre of clay is given by

$$S = m_v H_0 \Delta \bar{\sigma}_0$$

$$= 0.25 \times 10^{-3} \times 2 \times 19.62$$

$$= 9.81 \times 10^{-3} \text{ m}$$

$$= 9.81 \text{ mm}$$



Rakesh Kumar Sah

14. A stratum of clay is 2m thick and has an initial overburden pressure of 50 kN/m^2 at its middle. Determine the final settlement due to an increase of 40 kN/m^2 at the middle of the clay layer. The clay is over consolidated, with a preconsolidation pressure of 75 kN/m^2 . The values of the coefficients of recompression and compression index are 0.05 and 0.25 respectively. Take initial void ratio as 1.40.

→ Solution:-

Given that:

Thickness of clay layer (H_0) = 2m

Initial overburden pressure (σ'_0) = 50 kN/m^2

Increase in pressure ($\Delta \sigma'$) = 40 kN/m^2

Pre-compression pressure (σ'_c) = 75 kN/m^2

Coefficient of recompression (C_r) = 0.05

Compression index (C_c) = 0.25

Initial void ratio (e_0) = 1.4

Since, clay layer is over-consolidated and $\sigma'_0 < \sigma'_c < (\sigma'_0 + \Delta \sigma')$

The final settlement is given by:

$$H = \frac{C_r}{1+e_0} \cdot H_0 \log_{10} \left(\frac{\sigma'_c}{\sigma'_0} \right) + \left(\frac{C_c}{1+e_0} \right) H_0 \log_{10} \left(\frac{\sigma'_0 + \Delta \sigma'}{\sigma'_c} \right)$$

$$= \frac{0.05}{1+1.4} \times 2 \times \log_{10} \left(\frac{75}{50} \right) + \frac{0.25}{1+1.4} \times 2 \times \log_{10} \left(\frac{50+40}{75} \right)$$

$$= 0.02384$$

$$\therefore H = \approx 23.84 \text{ mm}$$

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→ Solution:-

Given that: time (t_1) = 6 months = 180 days

$$U_1 = \frac{\text{settlement in 6 months}}{\text{Total settlement}} \times 100\% = \frac{1.6}{6.5} \times 100\% = 24.6\%$$

Now,

$$\text{For } U_1 = 24.6\%, (Tv)_1 = \frac{1}{4} \times (0.246)^2 = 0.0475$$

$$(i) U_2 = 50\% \quad (Tv)_2 = \frac{1}{4} \times 0.5^2 = 0.1963$$

$$\therefore \frac{t_2}{t_1} = \frac{(Tv)_2}{(Tv)_1} = \frac{0.1963}{0.0475} = 4.1326$$

$$\text{i.e. } t_2 = t_1 \times 4.1326 = 180 \times 4.1326 = 743.87 \text{ days} \\ \approx 2.038 \text{ yrs}$$

(ii) Also, when time = 6 months i.e. $t = 6 + 6 = 12$ months

$$Tv = \left(\frac{t}{t_1} \right) \times (Tv)_1 = \frac{12}{6} \times 0.0475$$

$$\therefore Tv = 0.095$$

$$\text{For } Tv = 0.095, U_1 = \sqrt{\frac{1}{4} \times \frac{4}{\pi}} = \sqrt{\frac{0.095 \times 4}{\pi}} = 34.78\%$$

\therefore Settlement in 12 months,

$$S = U_1 \times S_f$$

$$= 0.3478 \times 6.5$$

$$= 2.26 \text{ cm}$$

\therefore Settlement in 12 months (next 6 months add)

$$= 2.26 \text{ cm}$$

\therefore Total settlement in next 6 months

$$= 2.26 - 1.6$$

$$= 0.66 \text{ cm}$$

16) A saturated clay stratum 4m thick lies below above an impervious stratum and below a pervious stratum. It has a void ratio of 1.2 at an initial pressure of 200 kN/m². Compute the change in void ratio due to an increase in stress of 50 kN/m² assuming that soil is normally consolidated with compression index 0.2. What would be the time required for 50% consolidation. Take coefficient of permeability = 3×10^{-4} cm/s.

→ Solution:-

Given that:-

$$H_0 = 4 \text{ m (one way drain)} \quad \sigma_0 = 200 \text{ kN/m}^2$$

$$C_c = 0.2 \quad \sigma_1 = 50 \text{ kN/m}^2$$

$$k = 3 \times 10^{-4} \text{ cm/s.}$$

$$e_0 = 1.2$$

$$C_c = \frac{\Delta e}{\Delta \log \sigma} = \frac{e_0 - e_1}{\log \sigma_1 - \log \sigma_0}$$

$$\therefore e_0 - e_1 = C_c \log \left(\frac{\sigma_0 + \Delta \sigma}{\sigma_0} \right) = 0.2 \log \left(\frac{250}{200} \right) = 0.0194$$

$$b) \Delta H = \left(\frac{C_c}{1 + e_0} \right) H_0 \log_{10} \left(\frac{\sigma_1}{\sigma_0} \right)$$

$$a) C_v = \frac{\Delta e}{\Delta \sigma} = \frac{0.0194}{50} = 3.88 \times 10^{-4} \text{ m}^2/\text{kN}$$

$$c) \text{ For } U = 50\%, T_v = \frac{1}{4} \times U^2 = \frac{1}{4} \times 0.5^2 = 0.196$$

$$m_v = \left(\frac{\Delta e}{1 + e_0} \right) \frac{1}{\Delta \sigma} = \frac{a_v}{1 + e_0} = \frac{3.88 \times 10^{-4}}{1 + 1.2} = 1.7636 \times 10^{-4} \text{ m}^2/\text{kN}$$

$$C_v = \frac{k}{m_v \gamma_w} = \frac{3 \times 10^{-4}}{9.81 \times 1.7636 \times 10^{-3}} = 1.794 \times 10^{-5} \text{ m}^2/\text{s}$$

Also,

$$t_{50} = \frac{T_v \times d^2}{C_v} = \frac{0.196 \times 4^2}{1.794 \times 10^{-5}} = 1908.54 \text{ sec} \approx 30.14 \text{ minutes}$$

17.

→ Solution:-

Given that:-

$$\sigma_{\text{clay, sat}} = 18.5 \text{ kN/m}^2$$

$$C_c = 0.22$$

$$e_0 = 1.4$$

$$\Delta \bar{\sigma} = 40 \text{ kN/m}^2$$

Case I:-

$$S = \frac{C_c}{1 + e_0} H_0 \log_{10} \left(\frac{\bar{\sigma}_0 + \Delta \bar{\sigma}}{\bar{\sigma}_0} \right) \quad H_0 = 3.5 \text{ m}$$

$$\text{where, } \bar{\sigma}_0 = 1.75 \times 18.5 = 32.375 \text{ kN/m}^2$$

$$S = \frac{0.22}{1 + 1.4} \times 3.5 \log_{10} \left(\frac{32.375 + 40}{32.375} \right)$$

$$= 112.09 \text{ mm}$$

Case II:-

when σ_{clay} rises to G.L.,
Settlement = ?

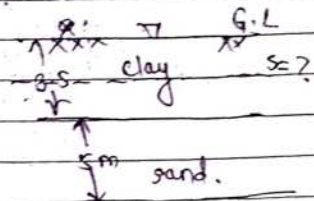
Here,

$$\bar{\sigma}_0 = 1.75 \times 18.5 = 1.75 \times 19.81$$

$$= 15.208 \text{ kN/m}^2$$

Now,

$$S = \frac{C_c}{1 + e_0} H_0 \log_{10} \left(\frac{\bar{\sigma}_0 + \Delta \bar{\sigma}}{\bar{\sigma}_0} \right)$$



$$= \frac{0.22 \times 3.5 \times \log_{10} \left(\frac{15.208 + 90}{15.208} \right)}{1 + 1.9}$$

$$= 0.17969 \text{ m}$$

$$\approx 179.64 \text{ mm}$$

Additional practise (2016 Spring) same as above solution.

Tutorial-8

Ch-11 Stability of Slopes:-

1. Calculate the safe height of an embankment rising 70° to the horizontal and to be made with the clayey soil having unit weight of 2.1 gm/cc , $\phi = 15^\circ$ and a cohesion of 0.2 kg/cm^2 . The factor of safety as 2.5 and value of stability number corresponding to slope angle 70° and $\phi = 15^\circ$ is 0.14.

→ Solution:-

Given that:-

Since the factor of safety given in the problem is with respect to shear strength, it is applicable to both C and ϕ .

$$\gamma = 2.1 \text{ gm/cc} = 21 \text{ kN/m}^3$$

$$\phi = 15^\circ$$

$$c = 0.2 \text{ kg/cm}^2 = 20 \text{ kN/m}^2$$

$$F = 2.5$$

$$S_n = 0.14 \text{ for } i = 70^\circ$$

We have, $F = \frac{\tan \phi}{\tan \phi_m}$

$$\therefore \phi_m = \tan^{-1} F \tan \phi$$

$$S_n = \frac{C_m}{\gamma H_c} = \frac{C}{F \gamma H_c}$$

$$\text{or, } H_c = \frac{C}{F \gamma S_n} = \frac{20}{2.5 \times 21 \times 0.14} = 2.721 \text{ m}$$

Hence, the safe height of an embankment is 2.721 m.

2. Calculate the factor of safety w/c cohesion of a clay slope laid at $1 \text{ in } 2$ to a height of 10 m , if the angle of internal friction $\phi = 10^\circ$, $C = 25 \text{ kN/m}^2$ and $\gamma = 19 \text{ kN/m}^3$. What will be the critical height of the slope in this soil?

→ Solution:-

Given that:-

$\phi = 10^\circ$
 $C = 25 \text{ kN/m}^2$ $\gamma = 19 \text{ kN/m}^3$ $H = 10 \text{ m}$ and $\tan i = \frac{1}{3}$
 $\Rightarrow i = 26.5^\circ$

For $\phi = 10^\circ$ & $i = 26.5^\circ$ from table, $S_n = 0.064$.

Also, we have,

$$S_n = \frac{C}{F_c \gamma H}$$

$$\therefore F_c = \frac{C}{S_n \gamma H} = \frac{25}{0.064 \times 19 \times 10} = 2.06$$

Again,

$$H_c = \frac{C}{\gamma S_n} = \frac{25}{19 \times 0.064} = 20.6 \text{ m}$$

3. A slope is to be constructed at an inclination of 30° with the horizontal. Determine the safe height of the slope at factor of safety of 1.5. The soil has following properties.
 $\phi = 22.5^\circ$, $C = 15 \text{ kN/m}^2$ & $\gamma = 19 \text{ kN/m}^3$.

→ Solution:-

We have, for $\phi = 22.5^\circ$ & $i = 30^\circ$, $S_n = 0.18$

$$S_n = \frac{C}{F_c \gamma H} = \frac{15}{1.5 \times 19 \times H}$$

$$\text{or, } 0.18 = \frac{15}{1.5 \times 19 \times H}$$

$$\text{or, } H =$$

4. The soil at a site has $C = 15 \text{ kN/m}^2$ and $\gamma = 18 \text{ kN/m}^3$. A hard stratum exists at a depth of 9m below the ground surface. If a cutting 6m deep is to be made in the soil, find factor of safety if a 30° slope is safe. What will be the slope angle if factor of safety of 1.5 is required?

→ Solution:-

Given that:-

Depth factor (D_f) = $\frac{9}{6} = 1.5$

From stability chart,
 for $i = 30^\circ$ & $D_f = 1.5$, $S_n = 0.18$

$$S_n = \frac{C}{F_c \gamma H}$$

$$\therefore F_c = \frac{C}{S_n \gamma H} = \frac{15}{0.18 \times 18 \times 6} = 0.77$$

Therefore, the proposed slope is unsafe.

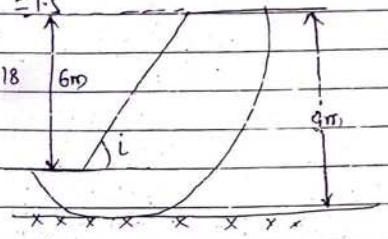
For $F_c = 1.5$, the required stability number is

$$S_n = \frac{C}{F_c \gamma H} = \frac{15}{1.5 \times 18 \times 6} = 0.092$$

Now,

for $S_n = 0.092$ and $D_f = 1.5$

slope angle (i) = 9°



5.

6). A granular soil has $\gamma_{sat} = 19 \text{ kN/m}^3$, $\phi = 30^\circ$. A slope has to be made of this soil. If a factor of safety of 1.2 needed against slope failure, determine the safe angle of the slope, when the slope is dry or submerged without seepage.

i) If seepage occurs at and parallel to the slope.

ii) If seepage occurs parallel to the slope with water table at a depth of 1.5m, what is the factor of safety available on a slope parallel to the ground surface at a depth of 4m? Assume a slope angle of 28° .

→ Solution:-

Given that,

$$C = 0 \text{ kN/m}^2$$

$$\gamma_{sat} = 19 \text{ kN/m}^3$$

$$\phi = 30^\circ$$

$$F = 1.2$$

→ When the slope is dry.

$$F = \frac{\tan \phi}{\tan i}$$

$$\text{or, } 1.2 = \frac{\tan 30^\circ}{\tan i}$$

$$\text{or, } i = 25.69^\circ$$

ii) When water level is at surface and seepage occurs parallel to surface,

$$F = \frac{\gamma'}{\gamma_{sat}} \times \frac{\tan \phi}{\tan i}$$

$$\text{or, } \tan i = \frac{19 - 9.81}{19} \times \frac{\tan 30^\circ}{1.2} =$$

$$\therefore i = 13.1^\circ$$

And,

11. A new canal is excavated to a depth of 5m below ground level, through a soil having the following characteristics: $C = 14 \text{ kN/m}^2$, $\phi = 15^\circ$, $e = 0.8$ and $G = 2.70$. The slope of the banks is 1 in 1. Calculate the factor of safety wrt cohesion when the canal runs full. If it is suddenly and completely emptied, what will be factor of safety?

→ Solution:-

$$\gamma_{sat} = \left(\frac{G + e}{1 + e} \right) \gamma_w = \frac{2.70 + 0.8}{1 + 0.8} \times 9.81 = 19.075 \text{ kN/m}^3$$

$$\gamma' = (\gamma_{sat} - \gamma_w) = 19.075 - 9.81 = 9.265 \text{ kN/m}^3$$

$$i = 45^\circ$$

$$\phi = 15^\circ$$

→ Submerged condition,

$$\text{for } i = 45^\circ \text{ \& } \phi = 15^\circ, S_n =$$

Now,

$$S_n = \frac{C}{\gamma' H}$$

$$\therefore C = S_n \gamma' H$$

\therefore Factor of safety wrt cohesion,

$$F_c = \frac{C}{C_m} = 14$$

ii) Rapid emptied.

$$\phi_w = \frac{\gamma'}{\gamma_{sat}} \times \phi = \frac{9.265}{19.075} \times 15^\circ = 7.285^\circ$$

$$\text{for } i = 45^\circ \text{ \& } \phi = 7.28^\circ, S_n =$$

$$\therefore S_n = \frac{C}{\gamma_{sat} H}$$

$$\text{or, } C =$$

\therefore Factor of safety wrt cohesion,
 $F_c = \frac{C}{C_m} = 1.4$

12.

Tutorial-7

Ch-10 - Shear strength of soil

1. Two triaxial tests were done on a soil sample. In the first test, all round pressure was 2.4 kg/cm^2 and the failure occurred at axial stress of 7.5 kg/cm^2 . In another test, all round pressure was 4 kg/cm^2 and the failure occurred at total axial strength stress of 16 kg/cm^2 . Determine the value of cohesion and angle of internal friction at failure.

\rightarrow Solution:-

Given that:-

First test

$$\sigma_3 = 7.5 \text{ kg/cm}^2$$

$$\sigma_3 = 2.4 \text{ kg/cm}^2$$

$$\sigma_1 = 7.5 + 2.4 = 9.9$$

2nd test

$$\sigma_3 = 4 \text{ kg/cm}^2$$

$$\sigma_1 = 16 \text{ kg/cm}^2$$

$$\sigma_1 = 20$$

We know,

$$\sigma_1 = \sigma_3 \tan^2(45^\circ + \frac{\phi}{2}) + 2c \cot(45^\circ + \frac{\phi}{2})$$

$$\text{or, } 9.9 = 2.4 \tan^2(45^\circ + \frac{\phi}{2}) + 2c \cot(45^\circ + \frac{\phi}{2}) \quad \text{--- (1)}$$

$$20 = 4 \tan^2(45^\circ + \frac{\phi}{2}) + 2c \cot(45^\circ + \frac{\phi}{2}) \quad \text{--- (2)}$$

Subtracting (1) from (2), we get,

$$10.1 = 1.6 \cot^2(45^\circ + \frac{\phi}{2})$$

$$\therefore \cot^2(45^\circ + \frac{\phi}{2}) = 6.3125$$

$$\text{or, } \tan^2(45^\circ + \frac{\phi}{2}) = 0.157$$

$$\text{or, } 45^\circ + \frac{\phi}{2} = 68.31$$

$$\therefore \phi = 46.62$$

2. An embankment is planned to be constructed from a soil with $c' = 4.0 \text{ kg/cm}^2$ and $\phi' = 26^\circ$. Evaluate the shear strength of the material on a horizontal plane at a point 10m below the surface of the embankment. The bulk density of the soil is 2.3 gm/cc and the pore pressure at that point is 1.8 kg/cm^2 .

→ Solution! -

Given that,

Normal pressure caused by the weight of the fill at a depth of 10m,

$$\therefore \sigma_z = \gamma \times h = \frac{2.3 \times 1000}{1000} = 2.3 \text{ kg/cm}^2$$

$$\text{Pore water pressure (u)} = 1.8 \text{ kg/cm}^2.$$

$$\therefore \text{Effective normal pressure } (\bar{\sigma}) = \sigma_z - u = 2.3 - 1.8 = 0.5 \text{ kg/cm}^2.$$

Now, shear strength,

$$\tau_f = c + \bar{\sigma} \tan \phi$$

$$= 4 + 0.5 \tan 26^\circ$$

$$= 4.244 \text{ kg/cm}^2 \quad ?$$

3. Keeping the minor principal stress constant as 2 kg/cm^2 , the major principal stress on a cylindrical soil sample was increased till failure occurred. If the cohesion of the soil was 2.5 kg/cm^2 and the angle of internal friction as 20° . Calculate (a) max. axial load at failure (b) shear and normal stress along the failure plane (c) Angle made by the failure plane with horizontal.

→ Solution! -

Given that! -