Date / /
Page

Applied Mechanics I

Civil and Electrical and
Electronics

2nd Sem

Introduction (2hrs)

Chapter:→ 1

Date 07/5/18
Page 1

Applied Mechanics:

Applied Mechanics may be broadly defined as the branch of engineering science which deals with the study of different branch of mechanics and describe and predicts the condition of rest or motion of bodies under the action of force.

Scope:

In the present day, engineers are vitally engaged in planning, designing and construction of various types of structures and machines. In order to take up this job skillfully and effectively, an engineer must understand thoroughly the principles of applied mechanics along with their application to engineering problems.

Applied Mechanics
- Statics
- Dynamics
  - Kinetics
  - Kinematics

a) Statics:

Statics is the branch of rigid body which deals with the forces and their effect while acting upon the bodies at rest. In general, statics deals the equilibrium of stationary bodies.

b) Dynamics:

Dynamics is the branch of rigid body which deals with the forces and their effects while acting upon the bodies in motion. Dynamics is further divided into Kinetics and Kinematics.

- Kinetics is the branch of dynamic which deals with the bodies in motion due to the application of forces. In kinetics both the motion and its causes are considered.
- Kinematics is the branch of dynamics which deals with the bodies in motion, without any reference to the forces which are responsible for motion.

Concept of particle:

⇒ Particle is a material body which is so small that its dimension can be treated as negligible in comparision to other dimensions involved in the problem. It may also defined as an object that has infinite small volume (occupies negligible space) and has mass which can be considered to be concentrated at a point and a particle is sometimes called as mass point.

Rigid bodies:

Rigid bodies are the combination of particles which are so interconnected that they donot change their relative position in application of forces. If a body maintains its shape and size during motion of rigid body, various points can have different velocities and acceleration but due to invariance of relative position these quantities gets inter-related. For perfectly rigid body the change in shape and size of bodies due to the application of forces must be very much small

Deformable bodies:
Those bodies which undergoes change in shape and size of structure for application of force is called deformable bodies. eg. plastic, chewgum.

Fluid bodies:
Fluid bodies may be subdivided into incompressible fluid and compressible fluid. Incompressible fluid deals with problem involving liquid whereas compressible fluid deals with gases and vapour.

Fundamental concept and principles
The basic fundamental concept used in applied mechanics are space, time, mass and force. These concepts cannot be truely defined. They should be accepted on the basis of our intution and experience and used as a mental frame of reference for our study of mechanics.

- Space:- space is a region, which extends in all direction and contains everything in it. eg. sun, moon, star etc. In space, position of a body is located with respect

to a reference system.

- Time:- Time is a measure of succession events. To define a event, it is not sufficient to indicate its position in space. The time of the event should be also given.

- Mass:→ The concept of mass is used to characterize and compare bodies on the basis of certain fundamental mechanical experiments.

- Force:→ A force represents the action of one body on other. A force is characterized by its point of application, its magnitude and its direction.

System of Units:
Unit is a standard of measure for the quantitative measure of dimension. It is categorized as fundamental and derived unit.

i) Fundamental Unit / Primary Unit
The measurements of physical quantities is one of the most important operation

engineering. Every quantity is measured in terms of some arbitary, but internationally accepted units, called fundamental units. These units are independent of other units.
For eg. unit of mass is kilogram (kg), unit of length is meter (m), unit of time is second (sec).

b) Derived units / secondary unit
sometime, physical quantities are expressed in other units, which are derived from fundamental units and known as derived units. eg. unit of area m^2, unit of accleration m/s^2.

Types of system of units:
→ There are only three types of units which are commonly used and univesally recognised. They are C.G.S units, F.P.S units and M.K.S units.

a) C.G.S units :- In this system of units, the fundamental units of length, mass and time are centimeter, gram and second resp.

b) F.P.S units: In this system the fundamental units of length, mass and time are foot, pound and second resp.

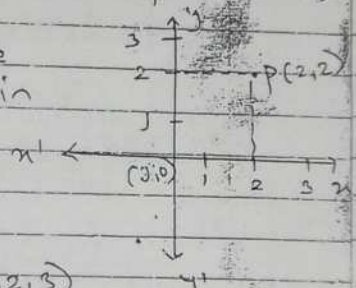
c) M.K.S units: The unit of length, mass and time in this system are meter, kilogram and second resp.

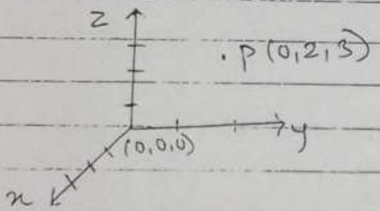
Date 1/5/19
Page 8

Review of co-ordinate system

(2007) a) Cartesian co-ordinate system:

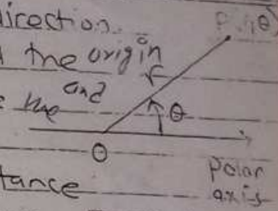
A cartesian co-ordinate system is a co-ordinate system that specifies each point uniquely in a plane by a pair of numerical co-ordinates which are the signed distance from the ~~point to~~ two fixed perp^n directed lines, measured in the same unit of length. Each reference line is called a co-ordinate axis and the point where they meet is its origin, ~~usually at ordered point~~ (0,0). ~~one~~ can use the same principle to specify the position of any point in three dimensional space by three cartesian co-ordinate.





(2006) (2008) b) Polar co-ordinate system:-

Polar co-ordinate system is a two-dimensional co-ordinate system in which each point on a plane is determined by a distance from a fixed point and an angle from a fixed direction. The fixed point is called the origin ~~pole and the ray from the pole with~~ and the fixed direction is the polar axis. The distance from the pole is called the ~~radial co-ordinate or~~ radius and the angle is ~~angular co-ordinate or~~ polar angle.

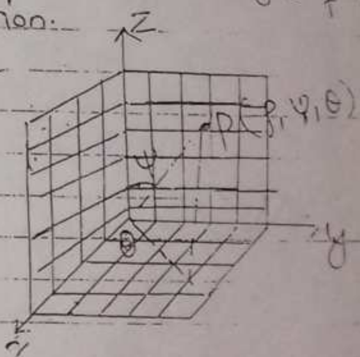


Date 1/5/19
Page 9

c) Spherical co-ordinate system:

Spherical co-ordinate system is a three dimensional co-ordinate system where the position of a point is ~~specified~~ determined by the three numbers; the radial distance of that point from a fixed origin; its polar angle measured from a fixed zenith direction and the azimuth angle of its orthogonal projection.

In the figure the position of a point is specified by three co-ordinates (ρ, φ, θ) where ρ = radial distance of point from fixed origin.

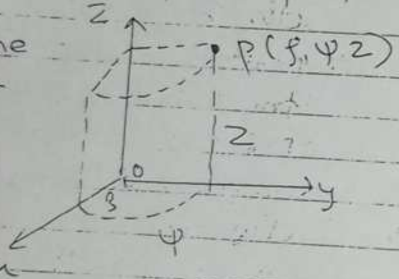


ψ = Zenith angle from positive z-axis
θ = Azimuth angle from positive x-axis.

d) Cylindrical co-ordinate system:

A cylindrical co-ordinate system is a three dimensional co-ordinate system that specifies point position by the distance from a chosen reference axis, the direction from the axis relative to a chosen reference direction, and the distance from a chosen reference plane perp^n to the axis.

In the figure, the position of a point is specified by three co-ordinate (ρ, φ, z), where



ρ = distance from the reference axis.
φ = direction from chosen reference axis
z = perp^n distance from plane.

Review of vector algebra:

$$\vec{F} = F_x i + F_y j + F_z k$$

$$|\vec{F}| = \sqrt{F_x^2 + F_y^2 + F_z^2}$$

Unit vector corresponding to $\vec{F} = \frac{\vec{F}}{|\vec{F}|}$

Vector addition / subtraction

$\vec{A} = 3i + 2j$
$\vec{B} = 2i - 3j$

Then,

$$\vec{P} = \vec{A} + \vec{B} = (3i + 2j) + (2i - 3j)$$
$$= 5i - j$$

$$|\vec{P}| = \sqrt{(5)^2 + (-1)^2} = 5.09 \text{ unit}$$

unit vector in the direction of $\vec{P}$

$$= \frac{5i - j}{5.09}$$

Dot (scalar product):

$$\vec{A} \cdot \vec{B} = (A_x i + A_y j + A_z k) \cdot (B_x i + B_y j + B_z k)$$

or, $\vec{A} \cdot \vec{B} = (A_x B_x + A_y B_y + A_z B_z)$

$i \cdot i = j \cdot j = k \cdot k = 1$
$i \cdot j = j \cdot k = k \cdot i = 0$

angle between $\vec{A}$ and $\vec{B}$ is $\cos\theta = \frac{\vec{A} \cdot \vec{B}}{|\vec{A}| |\vec{B}|}$

Cross product (vector product)

$$\vec{A} \times \vec{B} = \begin{vmatrix} i & j & k \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix} = i(A_y B_z - B_y A_z) - j(A_x B_z - B_x A_z) + k(A_x B_y - B_x A_y)$$

$$\sin\theta = \frac{\vec{A} \times \vec{B}}{|A| |B|}$$

Force acting on particles and rigid body

Date 15/20 Page 12

Unit 3

produce or tend to produce, destroy or tends motion.

Force:

Force is an important ~~factor~~ in the field of mechanics which may be defined as an agent which ~~produces~~ tends to change the speed or direction of a system. A force is applied whenever the system needs to be accelerated or deaccelerated. For eg. a horse applies force to pull a cart and to set it in motion.

Characteristics of force:

- Force is a vector quantity.
- It has magnitude and direction.
- It has point of application.
- Forces are transmissible vector. i.e. can be moved along its line of action.

Effects of a force:

A force may produce the following effect in a body, on which it acts,

- change its state of rest or motion.
- acclerate or retard its motion.
- Change its shape and size.
- turn or rotate it.
- Keep it in equilibrium.

Date 15/20 Page 13

4

Types of forces:-

a) External force: The external force represent the action of one body on the other. External force are entirely responsible for external behaviour of the rigid body. They will either cause it to move or ensure that it remains at rest.

b) Internal force: Internal force are the forces which hold together the particle forming rigid body. If rigid body is structurally composed of several parts, the forces holding the components together are defined as the internal forces.

Resolution and composition of force:

→ The process of splitting up the given force into a number of components without changing its effect on the body is called resolution of force. A force in a space can be represented by its projection on x, y and z axes.

i.e. $\vec{F} = F_x\hat{i} + F_y\hat{j} + F_z\hat{k}$ —— (1)

Where, $\hat{i}, \hat{j}$ and $\hat{k}$ are unit vectors along x, y and z axes.

The three components can be formed by the following scalar products:

$$F_x = \vec{F}\cdot\hat{i} = F\cos\theta_x$$
$$F_y = \vec{F}\cdot\hat{j} = F\cos\theta_y$$
$$F_z = \vec{F}\cdot\hat{k} = F\cos\theta_z$$

Where $\cos\theta_x$, $\cos\theta_y$ and $\cos\theta_z$ are the direction cosines of the force $\vec{F}$. Then

$$|\vec{F}| = \sqrt{F_x^2+F_y^2+F_z^2}$$

and the direction cosines are

$$\cos\theta_x = \frac{F_x}{|\vec{F}|} = \frac{F_x}{\sqrt{F_x^2+F_y^2+F_z^2}}$$

Similarly,

$$\cos\theta_y = \frac{F_y}{|\vec{F}|} \quad \text{and} \quad \cos\theta_z = \frac{F_z}{\sqrt{F_x^2+F_y^2+F_z^2}}$$

Imp

Principle of Transmissibility and Equivalent forces.

→ The principle of transmissibility states that "the condition of equilibrium or motion of a rigid body will remain unchanged if a force acting at given point of a rigid body is replaced by a force of the same magnitude and direction but acting at a different point provided that the two forces have same line of action".

Example:-

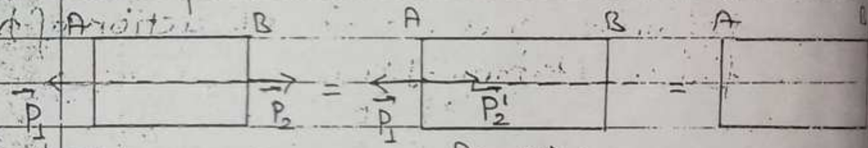


fig.(i)

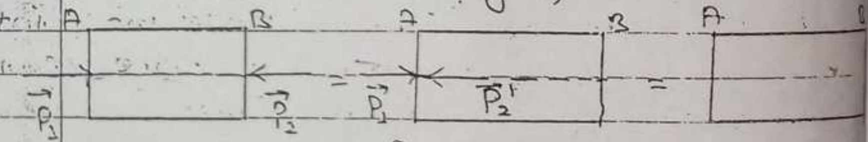


fig.(ii)

In figure (i), a rectangular bar AB is subjected to force $\vec{P_1}$ and $\vec{P_2}$ acting in opposite direction and having same magnitude. Then force $\vec{P_2}$ is replaced by $\vec{P_2'}$ having the same magnitude, line of action and direction, both acting at 'A' instead of B. Now, both forces are acting at a particle, can be added and their sum is equal to zero. Thus, the condition of body remains unchanged.

Moment:

Moment is the turning effect produced by a force, on the body on which it acts. The moment is equal to the product of force and the perpendicular distance of the point, about which the moment is required and the line of action of the force. Mathematically,

$$M = P \times d$$

Where, P = force acting on the body

d = perpr distance the point about which the moment is required.

Moment of force about a point

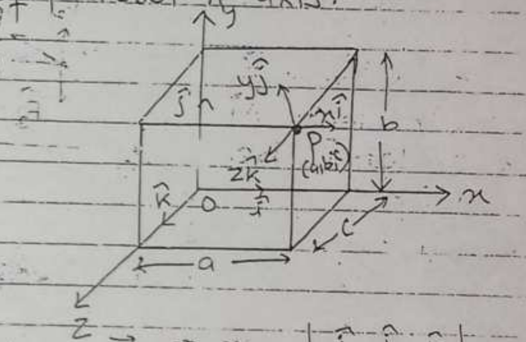
let $\vec{F} = x\hat{i} + y\hat{j} + z\hat{k}$ is a force acting at a point which has a position vector $\vec{r} = a\hat{i} + b\hat{j} + c\hat{k}$, then the moment of a force about a fixed point is given by

$$\vec{M} = \vec{r} \times \vec{F} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a & b & c \\ x & y & z \end{vmatrix}$$

or, $\vec{M} = \hat{i}(bz - cy) - \hat{j}(az - cx) + \hat{k}(ay - bx)$

Moment of a force about an axis:

Suppose $\vec{F} = x\hat{i} + y\hat{j} + z\hat{k}$ is acting at a point $P(a,b,c)$. Let us find out the moment of $\vec{F}$ about x-axis.



Now, $\vec{M} = \vec{r} \times \vec{F} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a & b & c \\ x & y & z \end{vmatrix}$

or, $\vec{M} = (bz - cy)\hat{i} - (az - cx)\hat{j} + (ay - bx)\hat{k}$

Moment of force $\vec{F}$ about x-axis is

$$\vec{M} \cdot \hat{i} = (bz - cy)\hat{i}\cdot\hat{i} - (az - cx)\hat{j}\cdot\hat{i} + (ay - bx)\hat{k}\cdot\hat{i}$$

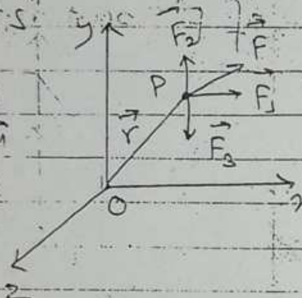
or, $\vec{M}\cdot\hat{i} = (bz - cy)$

similarly,

Moment of force $\vec{F}$ about y-axis is $\vec{M}\cdot\hat{j} = -(az - cx)$ and moment about z-axis $\vec{M}\cdot\hat{k} = (ay - bx)$

Varignon's Theorem:

→ Varignon's theorem states that "the moment of a force is equal to the sum of moments due to its components."



Consider a force $\vec{F}$ and its moment $\vec{M}$ about O. let $\vec{F}$ be resolved into number of forces $\vec{F_1}, \vec{F_2}, \vec{F_3} \ldots \vec{F_n}$ with respective position vectors $\vec{r_1}, \vec{r_2}, \vec{r_3}, \ldots \vec{r_n}$.

Then, the individual moments will be $\vec{r_1}\times\vec{F_1}$, $\vec{r_2}\times\vec{F_2}$, $\vec{r_3}\times\vec{F_3}$ ... $\vec{r_n}\times\vec{F_n}$

Now,

$$\vec{M} = \vec{r}\times\vec{F}$$

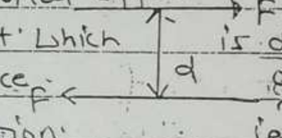
or, $\vec{M} = \vec{r}\times\left[\vec{F_1}+\vec{F_2}+\vec{F_3}+\ldots+\vec{F_n}\right]$

or, $\vec{M} = \vec{r}\times\vec{F_1} + \vec{r}\times\vec{F_2} + \vec{r}\times\vec{F_3} + \ldots + \vec{r}\times\vec{F_n}$

$\therefore \vec{M} = \vec{r_1}\times\vec{F_1} + \vec{r_2}\times\vec{F_2} + \vec{r_3}\times\vec{F_3} + \ldots + \vec{r_n}\times\vec{F_n}$

Couple:-

→ couple is defined as the combination of two equal and opposite force separated by a certain distance. The line of action of two forces is parallel. These forces will not translate the body because their sum is zero but they will tend to rotate a body. The rotational effect of a couple is measured by its moment which is defined as the product of either force and perpr distance betn the forces.



i.e. $M = F\times d$

Classification:

→ couple can be classified into two categories depending upon their direction, in which they tend to rotate the body, on which they act.

- → clockwise couple
- → Anticlockwise couple

i) Clockwise couple: A couple which tends to rotate the body in a clockwise direction on which it acts is known as a clockwise couple. Such couple is also called as positive couple.

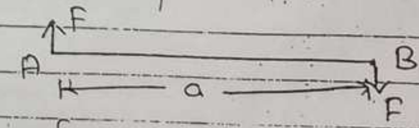


fig. clockwise couple

ii) Anti-clockwise couple: A couple which tends to rotate a body in an anticlockwise direction on which it acts is called anticlockwise couple. Such a couple is also called a negative couple.

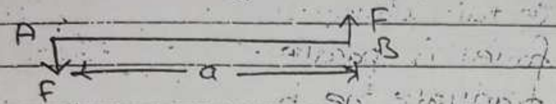


fig. Anti-clockwise couples

Characteristics:-

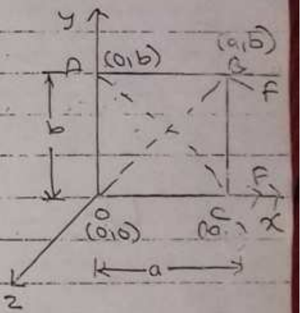
- a couple consists of a pair of equal and opposite forces separated by a definite distance.
- the algebraic sum of the forces constituting the couple is zero.
- the translatory effect of a couple on a body is zero.
- two couples will be equivalent if their moments are equal, both in magnitude and direction.

6

Prove couple is a free vector:-

Consider a rectangular plate OABC situated in xy-plane and two equal and opposite force of magnitude 'F' are applied on it. The distance of separation of parallel forces is 'b' which forms a couple having direction towards +ve z-axis.

As couple is a free vector so, moment about any point on the plate will be same.



i) Taking moment about O,

let, $\vec{OC} = \vec{r_1} = a\hat{i}$

$\vec{F_1} = F\hat{i}$

$\vec{OB} = \vec{r_2} = (a\hat{i} + b\hat{j})$

$\vec{F_2} = -F\hat{i}$

$\vec{M_o} = \vec{r_1} \times \vec{F_1} + \vec{r_2} \times \vec{F_2}$

or, $\vec{M_o} = a\hat{i} \times F\hat{i} + (a\hat{i} + b\hat{j}) \times (-F\hat{i})$

or, $\vec{M_o} = 0 + bF\hat{k} = bF\hat{k}$

ii) Taking moment about A,

$\vec{AC} = \vec{r_1} = (a\hat{i} - b\hat{j})$, $\vec{F_1} = F\hat{i}$

$\vec{AB} = \vec{r_2} = a\hat{i}$, $\vec{F_2} = -F\hat{i}$

Then,

$\vec{M_A} = \vec{r_1} \times \vec{F_1} + \vec{r_2} \times \vec{F_2}$

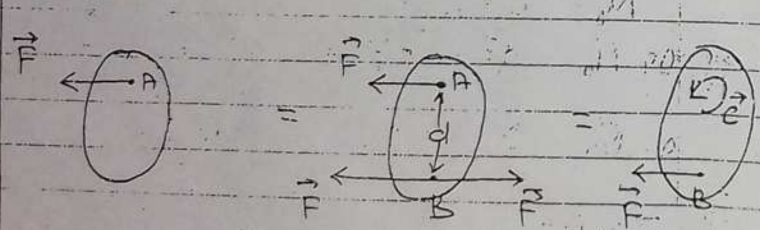
or, $\vec{M_A} = (a\hat{i} - b\hat{j}) \times (F\hat{i}) + (a\hat{i}) \times (-F\hat{i})$

or, $\vec{M_A} = bF\hat{k}$

Here, $\vec{M_A} = \vec{M_0}$ = moment about any other points = $bF\hat{k}$. Hence couple is a free vector.

4 ## Resolution of a force into forces and a couple.

→ Suppose $\vec{F}$ be the force acting at point A. If it is desired to transfer at B, we place equal and opposite force $\vec{F}$ at point B.



The system of forces now can be taken as a couple $\vec{C}$ of magnitude

Fd and a force $\vec{F}$ acting at B.

Resultant of force and moment for a system of force.

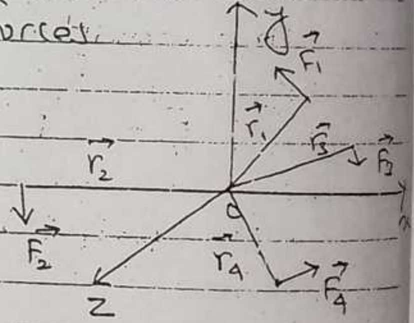
→ Resultant of force in space

→ Resultant of a co-planar force system

→ Resultant of parallel force system.

i) Resultant of force in space.

→ Consider a forces $\vec{F_1}$, $\vec{F_2}$, $\vec{F_3}$ and $\vec{F_4}$ acting in space with position vector $\vec{r_1}$, $\vec{r_2}$, $\vec{r_3}$ ... $\vec{r_n}$ resp. Then the resultant of given forces are given below.



$\vec{F_R} = (\Sigma F_x)\hat{i} + (\Sigma F_y)\hat{j} + (\Sigma F_z)\hat{k}$

and

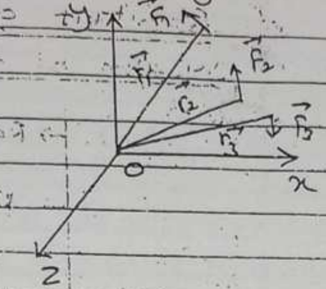
$\vec{M_R} = \vec{r_1} \times \vec{F_1} + \vec{r_2} \times \vec{F_2} + \dots + \vec{r_n} \times \vec{F_n}$

ii) Resultant of a coplanar force system.

→ Consider a forces $\vec{F_1}$, $\vec{F_2}$, $\vec{F_3}$ .... $\vec{F_n}$ are acting in x-y plane with position

Puspanjali
Date 15/1/24
Page 24

vector $\vec{r_1}, \vec{r_2}, \vec{r_3} \dots \vec{r_n}$ resp. Then the resultant of given forces are given below:



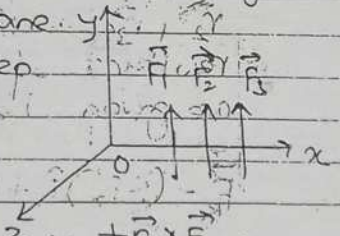
$$\vec{F_R} = (\Sigma F_x)\hat{i} + (\Sigma F_y)\hat{j}$$

and

$$\vec{M_R} = \vec{r_1} \times \vec{F_1} + \vec{r_2} \times \vec{F_2} + \vec{r_3} \times \vec{F_3} + \dots + \vec{r_n} \times \vec{F_n}$$

c) Resultant of parallel force system:

=> let us consider a force system which are parallel to y-axis and originating from x-z plane. The resultant is given by



$$\vec{F_R} = (\Sigma F_y)\hat{j}$$

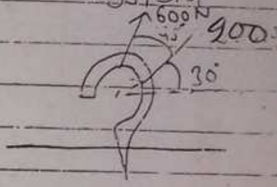
and

$$\vec{M_R} = \vec{r_1} \times \vec{F_1} + \vec{r_2} \times \vec{F_2} + \dots + \vec{r_n} \times \vec{F_n}$$

Date 15/1/24
Page 25

Numericals:

① Determine the magnitude and direction of the resultant of the two force system shown in figure.



=> Here,

Resolving the forces in x-and y-direction.

In x-direction:

$(\xrightarrow{+})$ $\Sigma F_x = 900 \cos 30° + 600 \cdot \cos 75° = 934.71\,N$

In y-direction:

$(+\uparrow)$ $\Sigma F_y = 900 \sin 30° + 600 \cdot \sin 75° = 1029.55\,N$

$\therefore$ Magnitude of resultant $= \sqrt{\Sigma F_x^2 + \Sigma F_y^2}$

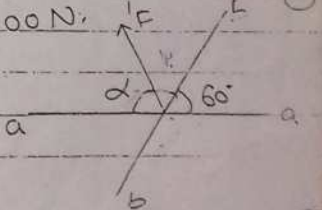
or, $F_R = 1350.56\,N$

and,

Direction $(\theta) = \tan^{-1}\left(\frac{\Sigma F_y}{\Sigma F_x}\right) = \tan^{-1}\left(\frac{1029.55}{934.71}\right)$

or, $\theta = 47.76°$

② The force 'F' of magnitude 1500 N is to be resolved into two components along line a-a and b-b. Determine the angle 'α', knowing that the components of F along line a-a to be 1200 N.



=> From fig.

$\theta = 180° - 60°$

or, $\theta = 120°$

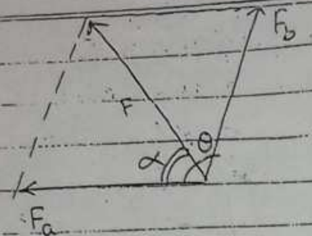
Given:

$F_a = 1200 N$

$F = 1500 N$

Then, $\alpha = ?$

We known that,



$F^2 = F_a^2 + F_b^2 + 2 F_a F_b \cos\theta$

or, $1500^2 = 1200^2 + F_b^2 + 2 \times 1200 \times F_b \times \cos 120°$

on solving,

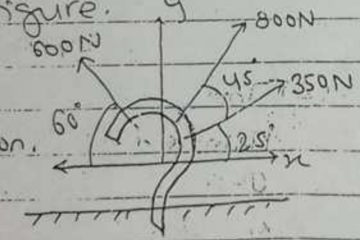
$F_b = 1681.67 N$

Now,

$\alpha = \tan^{-1}\left(\frac{F_b \sin\theta}{F_a + F_b \cos\theta}\right)$

or, $\alpha = \tan^{-1}\left(\frac{1681.67 \times \sin 120°}{1200 + 1681.67 \cos 120°}\right) = 76.15°$

(2009) 3) Determine the resultant of three forces as shown in the figure.



=> Here,

Resolving the forces in x and y direction.

$(\overset{+}{\rightarrow}) \Sigma F_x = 350 \cos 25° + 800 \cos 70° - 600 \cos 60°$

or, $\Sigma F_x = 290.824 N$

$(+\uparrow) \Sigma F_y = 350 \sin 25° + 800 \sin 70° + 600 \sin 60°$

$= 1419.286 N$

Resultant force $(R) = \sqrt{(\Sigma F_x)^2 + (\Sigma F_y)^2}$

or, $R = \sqrt{(290.824)^2 + (1419.286)^2}$

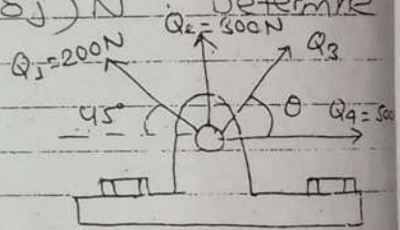
or, $R = 1448.776 N.$

Direction of resultant $(\theta) = \tan^{-1}\left(\frac{\Sigma F_y}{\Sigma F_x}\right)$

or, $\theta = \tan^{-1}\left(\frac{1419.286}{290.824}\right)$

or, $\theta = 78.42°$

4) The resultant of the four forces acting on the anchor shown is known to be $R = (559\hat{i} + 788\hat{j}) N$. Determine the force Q_3.



=> Here,

Resolving the forces horizontally,

$(\overset{+}{\rightarrow}) \Sigma F_x = 559 N$

or, $500 + Q_3 \cos\theta - 200 \cos 45° = 559$

or, $Q_3 \cos\theta = 200.421 \longrightarrow (i)$

Similarly,

Resolving the force vertically,

$Q_3 \sin\theta + 300 + 200 \sin 45° = 788$

or, $Q_3 \sin\theta = 346.579 \longrightarrow (ii)$

~~Dividing eqn (i) by (ii)~~

$\tan\theta = \frac{346.579}{200.421}$

जुन axis मा overlap हुन्छ त्यसको
अर्को axis को component
हुन्छ

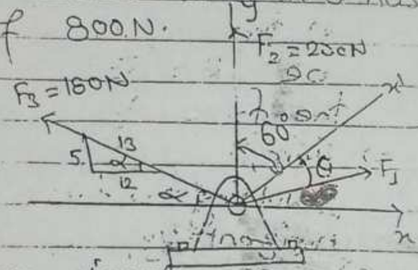
Date 15/25
Page

or, $\theta = 59.96$

$\therefore$ From eqn (i)

$$Q_3 = \frac{200.421}{\cos 59.96} = 400.358 \text{ N}$$

(5) Three forces act on the bracket. Determine the magnitude & direction θ of F_1 so that the resultant force is directed along the positive x axis and has a magnitude of 800 N.



$\Rightarrow$ From fig.

$\tan\alpha = 5/12$

$\therefore \alpha = 22.62°$

Resolving the forces in horizontal and vertical direction

$(\xrightarrow{+})\ \Sigma F_x = F_1 \sin(60+\theta) - 180 \cos 22.62$
$= -166.154 + F_1 \sin(60+\theta)$

$(+\uparrow)\ \Sigma F_y = F_1 \cos(60+\theta) + 200 + 180 \sin 22.62$

$= F_1 \cos(60+\theta) + 269.231$

But we have given that,

Date 15/25
Page

Resultant force $(R) = 800 \sin 60 \hat{i} + 800 \cos 60$

Now, equating the coeff. of $\hat{i}$ and $\hat{j}$

$-166.154 + F_1 \sin(60+\theta) = 800 \sin 60$

or, $F_1 \sin(60+\theta) = 858.974 \longrightarrow$ (i)

and

$F_1 \cos(60+\theta) + 269.231 = 800 \cos 60$

or, $F_1 \cos(60+\theta) = 130.769$ —— (ii)

Dividing eqn (i) by (ii)

$$\frac{F_1 \sin(60+\theta)}{F_1 \cos(60+\theta)} = \frac{858.974}{130.769}$$

or, $\tan(60+\theta) = 6.569$

or, $60+\theta = 81.344°$

or, $\theta = 21.344°$

from eqn (i)

$F_1 \sin(60+21.344) = 858.974$

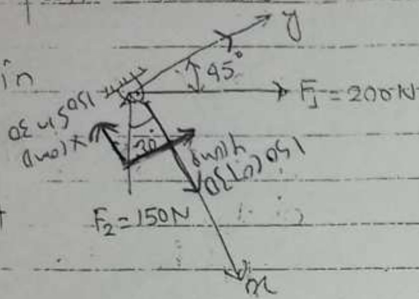
or, $F_1 = \frac{858.974}{\sin 81.344} = 868.87$ N

Date / /
Page

⑥ Determine the magnitude of the resultant force and its direction, measured counter-clockwise from the positive x-axis.

Here,

⇒ Resolving the force in x and y direction.



$\xrightarrow{+}(\Sigma F_x) = -150\cos 30° + 200\sin 45°$
$= 11.518N$

$+\uparrow(\Sigma F_y) = 150\sin 30° + 200\cos 45° = 216.421N$

Magnitude of resultant force $(R) = \sqrt{(\Sigma F_x)^2 + (\Sigma F_y)^2}$

or, $R = \sqrt{(11.518)^2 + (216.421)^2}$

or, $R = 216.727N$

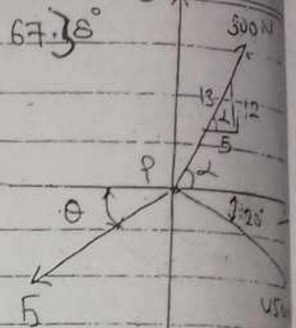
Direction $(\theta) = \tan^{-1}\left(\frac{\Sigma F_y}{\Sigma F_x}\right) = \left(\frac{216.727}{11.518}\right)$

or, $\theta = 86.95°$

⑦ Determine the magnitude and angle of F_1 so that particle 'p' is in equilibrium.

Date / /
Page 6

→ From fig, $\alpha = \tan^{-1}\left(\frac{12}{5}\right) = 67.38°$



Resolving the forces into its components.

$(\xrightarrow{+})\ \Sigma F_x = 0$

ie. $300\cos 67.38° + 450 \cos 20° - F_1\cos\theta = 0$

or, $F_1\cos\theta = 538.247 \longrightarrow ①$

$(+\uparrow)\ \Sigma F_y = 0$ ie. $300\sin 67.38° - F_1\sin\theta - 450\sin 20° = 0$

or, $F_1\sin\theta = 123.014 \longrightarrow ②$

Dividing eqⁿ ① by ⑪

$\frac{F_1\sin\theta}{F_1\cos\theta} = \frac{123.014}{538.247}$

or, $\tan\theta = 0.2285 \Rightarrow \theta = 12.873°$

from eqⁿ ①

$F_1 = \frac{538.247}{\cos 12.873°} = 552.124N$

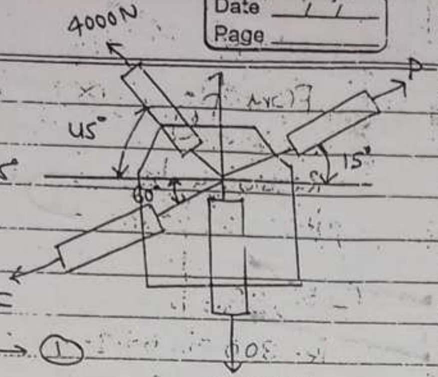
⑧ The force on the gusset plate of a joint in a bridge trusses act as shown. Determine the values of 'p' and 'F' to maintain the equilibrium of joint.

⇒ Resolving the forces in x and y directions.

(→+) $\Sigma F_x = 0$

or, $P\cos 15^\circ - 4000\cos 45^\circ - F\cos 60^\circ = 0$

or, $0.966P - 0.5F = 2828.427 \longrightarrow$ (i)



and

$+\uparrow(\Sigma F_y) = 0$

or, $P\sin 15^\circ + 4000\sin 45^\circ - F\sin 60^\circ - 3000 = 0$

or, $0.259P - 0.866F = \cancel{5828.427} \longrightarrow$ (ii)

Solving eqn (i) & (ii)

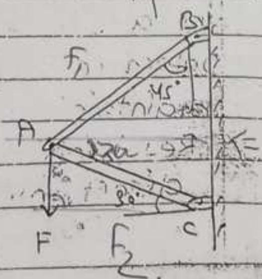
$P = -\cancel{657.368N}$ 3342.72N

$F = -\cancel{6926.38N}$ ~~800.70N~~

$F = 801.367\,N$

(9) The vertical force F = 500N acting downward at A on the two membered frame. Determine the magnitude of two components of F directed along the axes of AB & AC.

⇒ let us assume F_1 & F_2 be the magnitude of force on member AB and AC resp.

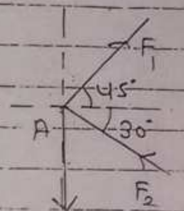


Resolving the forces in x and y direction

(→+) $\Sigma F_x = 0$

$F_1\cos 45^\circ - F_2\cos 30^\circ = 0$

or, $0.707F_1 - 0.866F_2 = 0 \rightarrow$ (i)



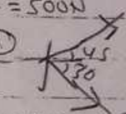
$+\uparrow \Sigma F_y = 0$

ie. $F_1\sin 45^\circ + F_2\sin 30^\circ - 500 = 0$

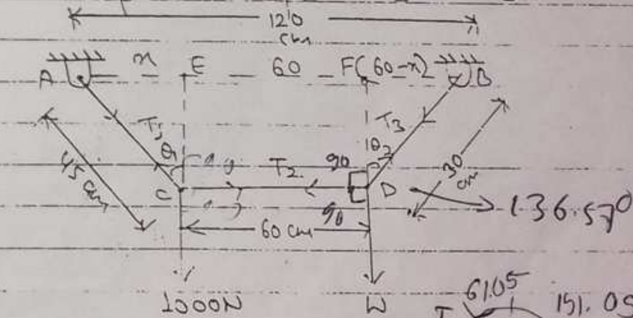
or, $0.707F_1 + 0.5F_2 = 500 \longrightarrow$ (ii)

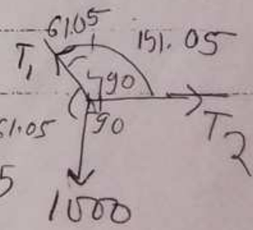
Solving eqn (i) and (ii)

$F_1 = 448.351\,N$ and $F_2 = 366.032\,N$



(10) A rope is connected between two points A and B 120cm apart at the same level. A load 1000 N is suspended from a point C on the rope 45 cm from A as shown. Find the load which should be suspended from the rope D 30cm from B, which will keep the rope CD level.





$360 - 180 - 61.05 = 118.95$

Date / /
Page

From fig; CE = DF, then squaring both side

$CE^2 = DF^2$

$(45)^2 - (x)^2 = 30^2 - (60-x)^2$

or, $2025 - x^2 = 900 - (3600 - 120x + x^2)$

or, $120x = 4725$

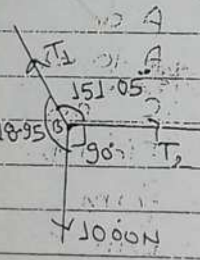
or, $x = 39.375$ cm.

$\theta_1 = \sin^{-1}\left(\frac{x}{45}\right) = \sin^{-1}\left(\frac{39.375}{45}\right) = 61.05°$

$\theta_2 = \sin^{-1}\left(\frac{60-x}{30}\right) = \sin^{-1}\left(\frac{20.625}{30}\right) = 43.43°$

At point C

Using Lami's theorem (01)



$\frac{T_1}{\sin 90°} = \frac{T_2}{\sin 118.95°} = \frac{1000}{\sin 151.05°}$

Taking 1st and 3rd fraction,

$\frac{T_1}{\sin 90°} = \frac{1000}{\sin 151.05°}$

$T_1 = 2065.92$ N

Taking 2nd and 3rd fraction,

$\frac{T_2}{\sin 118.95°} = \frac{1000}{\sin 151.05°}$

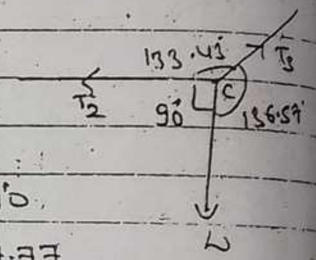
Date / /
Page

or, $T_2 = 1807.77$ N.

At point D

Using Lami's theorem.

$\frac{W}{\sin 133.43°} = \frac{T_3}{\sin 90°} = \frac{T_2}{\sin 136.57°}$

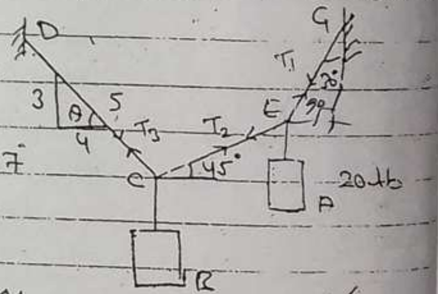


taking 1st and 3rd ratio.

$\frac{W}{\sin 133.43°} = \frac{1807.77}{\sin 136.57°}$

or, $W = 1909.66$ N.

(11) The cylinder at A has a weight of 20 lb determine the weight of B and the force in each cord needed to hold the system in equilibrium.



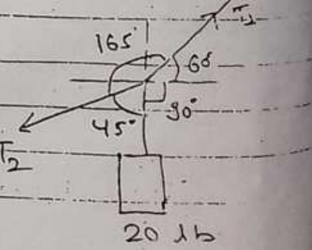
⇒ In figure,

$\theta = \tan^{-1}(3/4) = 36.87°$

At point E

Using Lami's theorem

$\frac{T_1}{\sin 45°} = \frac{T_2}{\sin 150°} = \frac{20}{\sin 165°}$



taking 1st & 3rd ratio

$\frac{T_1}{\sin 45°} = \frac{20}{\sin 165°}$

or, $T_1 = 54.64$ lb

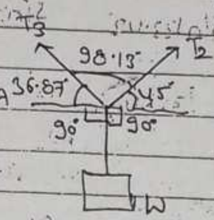
taking 2nd and 3rd ratio

$$\frac{T_2}{\sin 150^\circ} = \frac{20}{\sin 165^\circ} \Rightarrow T_2 = 38.64 \text{ lb}$$

At point C

Using lami's theorem

$$\frac{T_3}{\sin 135^\circ} = \frac{W}{\sin 98.13^\circ} = \frac{38.64}{\sin 126.87^\circ}$$



taking 1st and 3rd ratio

$$\frac{T_3}{\sin 135^\circ} = \frac{38.64}{\sin 126.87^\circ}$$

or, $T_3 = 34.15$ lb

and taking 2nd and 3rd ratio

$$\frac{W}{\sin 98.13^\circ} = \frac{38.64}{\sin 126.87^\circ}$$

or, $W = 48.81$ lb

(12) The directions of the 300N forces may vary, but the angle between the forces is always 45°. Determine the value of α for which the resultant of the forces acting at A is directed vertically upward.

→ Here,

The resultant is vertically upward, so the x-component of resultant force must be zero.

i.e. $\xrightarrow{+} \Sigma F_x = 0$

$1000 \sin 30^\circ - 300 \sin\alpha - 300 \sin(45^\circ + \alpha) = 0$

or, $1000 \sin 30^\circ - 300 \{\sin\alpha + \sin(45^\circ + \alpha)\} = 0$

or, $500 - 300 \times \left\{ 2 \times \sin\left(\frac{\alpha + 45 + \alpha}{2}\right) \cdot \cos\left(\frac{\alpha - 45 - \alpha}{2}\right) \right\}$

$\left[\because \sin A + \sin B = 2 \sin \frac{A+B}{2} \cdot \cos \frac{A-B}{2}\right]$

or, $500 - 600 \sin(\alpha + 22.5) \cdot \cos(-22.5) = 0$

or, $500 - 554.33 \sin(\alpha + 22.5) = 0$

or, $\sin(\alpha + 22.5) = \frac{500}{554.33}$

or, $\alpha + 22.5 = \sin^{-1}\left(\frac{500}{554.33}\right)$

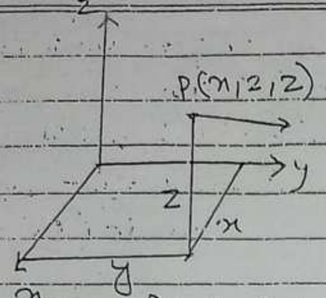
$\therefore \alpha = 64.42^\circ - 22.5^\circ = 41.92^\circ$

(13) A force $\vec{F} = (-5\hat{i} + 3\hat{j} - 4\hat{k})$ KN produces a moment $\vec{M_o} = (-17\hat{i} - 7\hat{j} + 16\hat{k})$ KNm about point O. If the forces acts at a point P, having a y-coordinate of $y = 2$. Determine x and z co-ordinate.

⇒ Given

$\vec{F} = (-5\hat{i} + 3\hat{j} - 4\hat{k})$ KN

$\vec{M_o} = (-17\hat{i} - 7\hat{j} + 16\hat{k})$ KNm suppose $P(x, 2, z)$ be the point where the force acts.



Then,

$\vec{M_o} = \vec{r} \times \vec{F}$

or, $(-17\hat{i} - 7\hat{j} + 16\hat{k}) = (x\hat{i} + 2\hat{j} + z\hat{k}) \times (-5\hat{i} + 3\hat{j} - 4\hat{k})$

or, $(-17\hat{i} - 7\hat{j} + 16\hat{k}) = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ x & 2 & z \\ -5 & 3 & -4 \end{vmatrix}$

or, $(-17\hat{i} - 7\hat{j} + 16\hat{k}) = (-8-3z)\hat{i} - (-4x+5z)\hat{j} + (3x+10)\hat{k}$

comparing the coeff. of like vector

$-17 = -8-3z \longrightarrow (i)$

$-7 = -(-4x+5z)$

or, $7 = -4x+5z \longrightarrow (ii)$

and $16 = 3x+10 \longrightarrow (iii)$

From eqn (iii)

$x = 6/3 = 2$

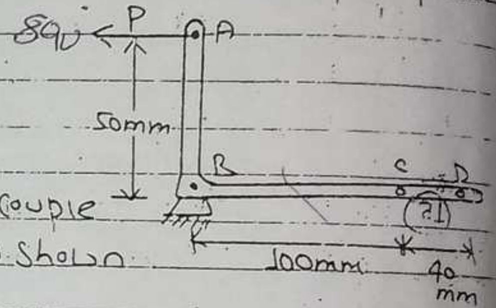
From eqn (ii) $7 = -4\times2+5z$

or, $z = 3$

Hence, req. value of x & z are 2 & 3 resp.

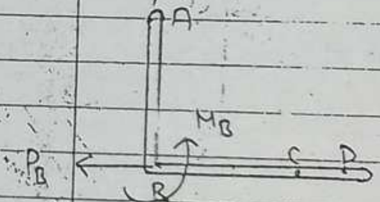
(2006) and the point is (2, 2, 3).

(14) The 80N horizontal force 'P' acts on a bell crank as shown. (a) Replace 'P' with an equivalent force-couple system at B. (b) Find the two vertical forces at C and D which are equivalent to the couple found in part (a).



⇒ Here,

The force 'P' is replaced by an equivalent force-couple system at B as shown in fig.



Then, $P_B = 80$ N.

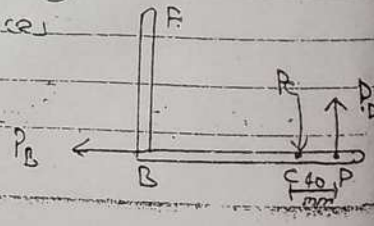
Also,

$M_B = P_B \times 50 = 80\times50$

or, $M_B = 4000$ Nmm

Hence, horizontal force $P = 80$kN can be replaced with an equivalent force-couple system of 4000Nmm at B.

Again, If two vertical forces are equivalent to M_B, they must be a couple. Then, the vertical forces P_C and P_D acting at point C and D.



are shown in the figure.

Taking moment at point D.

$M_D = P_c \times 40.$

or, $4000 = P_c \times 40$

or, $P_c = 100N.$

Also,

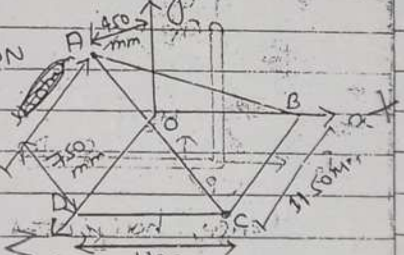
$+\uparrow \Sigma F_y = 0$

or, $-P_c + P_D = 0$

or, $P_D = P_c = 100N.$

(15) Knowing that the tension in cable AC is 2130N, Determine the component of the force exerted on plate at C.

=> Here,



Force on AC (T_{AC}) = 2130N

Co-ordinate of A $= (0, 750, 450)$

and,

Co-ordinate of C $= (1125, 0, 1600)$

Here,

$\vec{AC} = (1125-0, 0-750, 1600-450)$

$= (1125, -750, 1150)$

$= 1125\hat{i} - 750\hat{j} + 1150\hat{k}$

$|\vec{AC}| = \sqrt{(1125)^2 + (-750)^2 + (1150)^2}$

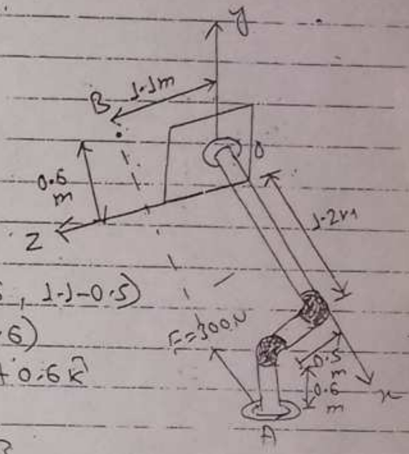
$= \cancel{1775N}$ $1522.53N$

Force on AC, $(\vec{T_{AC}}) = T_{AC} \cdot \hat{AC}$

$= 2130 \left[\frac{1125\hat{i} - 750\hat{j} + 1150\hat{k}}{\cancel{1775}\ 1522.53}\right]$

$= 1573.86\hat{i} - 1049.20\hat{j} + 979.20\hat{k}$ $= \cancel{1350\hat{i} - 900\hat{j} + 1380\hat{k}}$

(16) Find the moment of the force F shown about the origin.



=> Here,

Co-ordinate of

$O = (0,0,0)$

$A = (1.2, -0.6, 0.5)$

$B = (0, 0.6, 1.1)$

Now,

$\vec{AB} = (0-1.2, 0.6+0.6, 1.1-0.5)$

$= (-1.2, 1.2, 0.6)$

$= -1.2\hat{i} + 1.2\hat{j} + 0.6\hat{k}$

$|\vec{AB}| = 1.8N$

$\therefore$ Force $(\vec{F}_{AB}) = F \cdot \frac{\vec{AB}}{|\vec{AB}|}$

$= 300 \left[\frac{-1.2\hat{i} + 1.2\hat{j} + 0.6\hat{k}}{1.8}\right]$

$= -200\hat{i} + 200\hat{j} + 100\hat{k}$

Moment due to force about origin

$\vec{M_o} = \vec{r} \times \vec{F}$

Where, $\vec{OA} = \vec{r} = 1.2\hat{i} - 0.6\hat{j} + 0.5\hat{k}$

or, $\vec{M_o} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1.2 & -0.6 & 0.5 \\ -200 & 200 & 100 \end{vmatrix}$

determine component of force means $\vec{T_{AC}} = T_{AC} \cdot \hat{AC}$ (unit vector of AC)

Date / /
Page

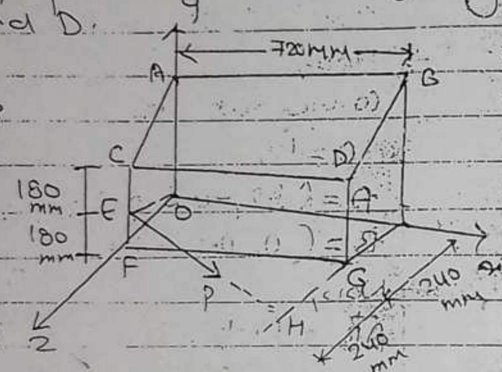
or, $\vec{M_o} = (-160\hat{i} - 220\hat{j} + 120\hat{k})$ Nm

(2008) Fall

(17) A force 'p' of magnitude 2600N acts on the frame shown at point E. Determine the moment of 'p' about a line joining points O and D.

⇒ Here,

Determining the co-ordinates



A (0, 360, 0)

B (720, 360, 0)

C (0, 360, 240)

D (720, 360, 240)

E (0, 180, 240)

H (720, 0, 480)

$\vec{EH} = 720\hat{i} - 180\hat{j} + 240\hat{k}$; $|\vec{EH}| = 780$

Unit vector along $\hat{EH} = \dfrac{720\hat{i} - 180\hat{j} + 240\hat{k}}{780}$

$= 0.923\hat{i} - 0.231\hat{j} + 0.308\hat{k}$

Now,

Force on EH $(\vec{F}) = P.\hat{EH}$

or, $\vec{F} = 2600\,(0.923\hat{i} - 0.231\hat{j} + 0.308\hat{k})$

or, $\vec{F} = 2400\hat{i} - 600\hat{j} + 800\hat{k}$

Now,

position vector $\vec{OE}$ $(\vec{r}) = +180\hat{j} + 240\hat{k}$

Page

Moment about O, $\vec{M_o} = \vec{r} \times \vec{F}$

or, $\vec{M_o} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & 180 & 240 \\ 2400 & -600 & 800 \end{vmatrix}$

or, $\vec{M_o} = \hat{i}\,(288000) + 576000\hat{j} - 432000$

Unit vector along OD $(\hat{OD}) = \dfrac{\vec{OD}}{|\vec{OD}|}$

or, $\hat{OD} = \dfrac{720\hat{i} + 360\hat{j} + 240\hat{k}}{840}$

∴ Moment of point 'p' about line OD

i.e. $\vec{M}_{OD} = \vec{M_o} \cdot$ (unit vector along OD)

$= \vec{M_o} \cdot \hat{OD}$

$= \{(288000)\hat{i} + 576000\hat{j} - 432000\} \cdot \left\{\dfrac{720\hat{i} + 360\hat{j} + 240\hat{k}}{840}\right\}$

or, $M_{OD} = 246857.14 + 246857.14 - 123428$

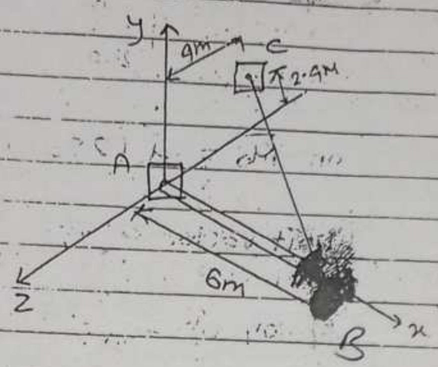
or, $M_{OD} = 370285.70$ Nmm

(2009) SP

(18) The 6m boom AB has a fixed end A. A steel cable is stretched from the free end B of the boom to a point C, located on the vertical wall. If the

tension in the cable is 1900N. Determine the moment about A of the force exerted by the cable at B.

→ Here,



AB = 6m

F_{BC} = 1900N

Co-ordinate of

A = (0,0,0)

B = (6,0,0)

C = (0, 2.4, -4)

Then,

Force on BC $(\vec{F}_{BC}) = F_{BC} \times \hat{BC}$

or, $\vec{F}_{BC} = 1900 \cdot \left[\frac{-6\hat{i}+2.4\hat{j}-4\hat{k}}{7.6}\right]$

or, $\vec{F}_{BC} = -1500\hat{i} + 600\hat{j} - 1000\hat{k}$

Now, $\vec{AB} = 6\hat{i} = \vec{r}$

Moment about A of the force exerted by the cable at B

$\vec{M}_A = \vec{r} \times \vec{F}_{BC}$

or, $\vec{M}_A = 6\hat{i} \times (-1500\hat{i} + 600\hat{j} - 1000\hat{k})$

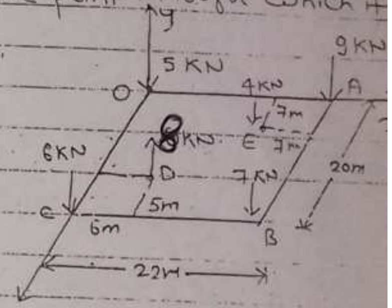
or, $\vec{M}_A = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 6 & 0 & 0 \\ -1500 & 600 & -1000 \end{vmatrix}$

First find BC (position of BC) and $F_{BC} = F_{BC} \times \hat{BC}$ and moment of the point & force

or, $\vec{M}_A = (6000\hat{j} + 36000\hat{k})$ Nm

(2066) SP

19. A concrete slab of supports six vertical loads as shown in figure. Determine the resultant of these forces and the point through which it acts.



=) Here,

Co-ordinate of six vertical loads are as follows:

O = (0,0,0)

A = (22,0,0)

B = (22,0,20)

C = (0,0,20)

D (6,0,15)

E = (15,0,7)

Resultant of force $(\vec{F}_R) = -5\hat{j} - 9\hat{j} - 4\hat{j} - 6\hat{j} + 8\hat{j} - 7\hat{j}$

or, $\vec{F}_R = -23\hat{j}$

Let the resultant passes through the point having position vector $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$

Taking moment about origin,

Then,

Moment due to resultant = Sum of moment due to individual force.

Varignon Theorem

Date / /
Page

$$\vec{r}\times\vec{F_R} = \vec{r_1}\times\vec{F_1} + \vec{r_2}\times\vec{F_2} + \vec{r_3}\times\vec{F_3} + \vec{r_4}\times\vec{F_4} + \vec{r_5}\times\vec{F_5} + \vec{r_6}\times\vec{F_6}$$

or, $(x\hat{i} + y\hat{j} + z\hat{k})\times(-23\hat{j}) = (0\hat{i}+0\hat{j}+0\hat{k})\times(-5\hat{j}) + (22\hat{i})\times(-9\hat{j}) + (22\hat{i}+20\hat{k})\times(-7\hat{j}) + (15\hat{i}+7\hat{k})\times(-4\hat{j}) + (6\hat{i}+15\hat{k})\times(8\hat{j}) + (20\hat{k})\times(-6\hat{j})$

or, $(-23x\hat{k} + 23z\hat{i}) = -198\hat{k} - 154\hat{k} + 140\hat{i} - 60\hat{k} + 28\hat{i} + 48\hat{k} - \cancel{120}\hat{i} + \cancel{120}\hat{i}$

or, $(-23x\hat{k} + 23z\hat{i}) = -364\hat{k} + 168\hat{i}$

comparing the coeff. of like vectors.

$-23x = -364 \Rightarrow x = 15.82m.$

and

$23z = 168 \Rightarrow z = 7.30m$

$\therefore$ co-ordinate of resultant $= (15.82, 0, 7.3)$

(2007) Fall

20. A 20 kg square plate shown in the figure is supported by three wires. Determine the tension in each wires.

$\Rightarrow$ Here, co-ordinate of

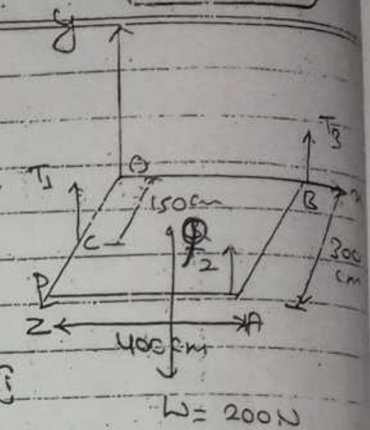
$O = (0,0,0)$

$A = (400, 0, 300)$

$B = (400, 0, 0)$

$C = (0, 0, 150)$

$Q = (200, 0, 150)$



$\vec{T_1} = T_1\hat{j}$, $\vec{T_3} = T_3\hat{j}$

$\vec{T_2} = T_2\hat{j}$, $\vec{W} = -200\hat{j}$

For equilibrium:-

$\uparrow\Sigma F_y = 0$ i.e. $\vec{T_1} + \vec{T_2} + \vec{T_3} + \vec{W} = 0$

or, $T_1\hat{j} + T_2\hat{j} + T_3\hat{j} - 200\hat{j} = 0$

or, $T_1 + T_2 + T_3 = 200 \longrightarrow (1)$

Also, $\Sigma \vec{M_O} = 0$

or, $\Sigma \vec{r}\times\vec{F} = 0$

or, $(150\hat{k}\times T_1\hat{j}) + (400\hat{i} + 300\hat{k})\times T_2\hat{j} + (400\hat{i})\times T_3\hat{j} + (200\hat{i} + 150\hat{k})\times(-200\hat{j}) =$

or, $-T_1 150\hat{i} + 400T_2\hat{k} - 300T_2\hat{i} + 400T_3\hat{k} - 40000\hat{k} + 30000\hat{i} = 0$

equating the coeff. of like vector

or, $-150T_1 - 300T_2 + 30000 = 0 \longrightarrow (ii)$

$i.j = k$ $\quad j.i = -k$

$j.k = i$ $\quad k.j = -i$

$k.i = j$ $\quad i.k = -j$

Date
Page

or, $T_1 + 2T_2 = 200$ ⟶ (iii)

and

$400 T_2 + 400 T_3 = 40000$

or, $T_2 + T_3 = 100$ ⟶ (iv)

from eqⁿ (i) and (ii)

$T_1 + 100 = 200 \Rightarrow T_1 = 100N$

from eqⁿ (iii)

$T_2 = 50N$ and

from eqⁿ (iv) $T_3 = 50N$

(21) Replace the two couple of the triangular block as shown in figure by a single resultant couple.

⇒ Here,

co-ordinate of

$O = (0,0,0)$

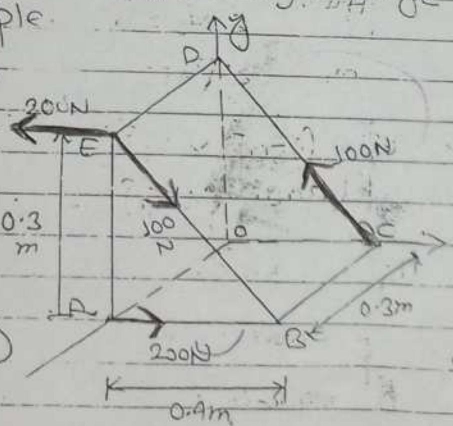
$D = (0, 0.3, 0)$

$E = (0, 0.3, 0.3)$

$C = (0.4, 0, 0)$

$B = (0.4, 0, 0.3)$

$A = (0, 0, 0.3)$



Date
Page

Force along CD $(\vec{F}_{CD}) = 100 \cdot \hat{CD}$

or, $\vec{F}_{CD} = 100 \cdot \left| \dfrac{-0.4\hat{i} + 0.3\hat{j}}{0.5} \right|$

or, $\vec{F}_{CD} = -80\hat{i} + 60\hat{j}$

$\vec{BC} = \vec{r}_{BC} = -0.3\hat{k}$

Again,

Force along AB $(\vec{F}_{AB}) = 200 \cdot \hat{AB}$

or, $\vec{F}_{AB} = 200 \left| \dfrac{0.4\hat{i}}{0.4} \right|$

or, $\vec{F}_{AB} = 200\hat{i}$

$\vec{EA} = \vec{r}_{EA} = -0.3\hat{j}$

Couple due to 100N force,

$\vec{C}_1 = \vec{r}_{BC} \times \vec{F}_{CD} = -0.3\hat{k} \times (-80\hat{i} + 60\hat{j})$

or, $\vec{C}_1 = 24\hat{j} + 18\hat{i}$

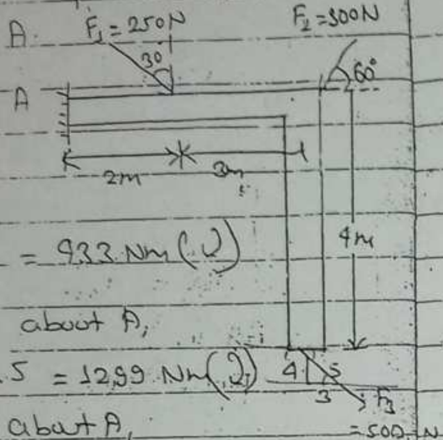
Couple due to 200N force,

$\vec{C}_2 = \vec{r}_{EA} \times \vec{F}_{AB}$

or, $\vec{C}_2 = -0.3\hat{j} \times (200\hat{i}) = 60\hat{k}$

Resultant couple $(\vec{C}) = \vec{C}_1 + \vec{C}_2 = 18\hat{i} + 24\hat{j} + 60\hat{k}$

(1) Determine the moment of each of the three forces about point A.



=> Here,

Moment of force F_1 about A,

$M_{A_1} = F_1 \cos 30° \times 2 = 433\ Nm$ (↺)

Moment of force F_2 about A,

$M_{A_2} = F_2 \sin 60° \times 5 = 1299\ Nm$ (↻)

Moment of force F_3 about A,

$M_{A_3} = -F_3 \sin\alpha \times 5 + F_3 \cos\alpha \times 4$

$= -500 \times 4/5 \times 5 + 500 \times 3/5 \times 4$

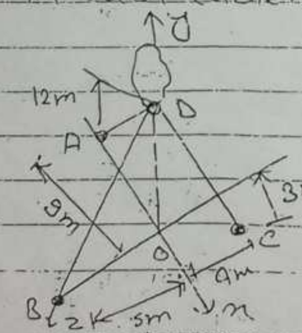
$= -800 Nm$

(2) Three cables DA, DB and DC are used to tie down a ballon at D as shown in figure. Knowing the ballon exerts as 640N forces at D, determine the tension in each cable.

=> Here,

Force exerted by ballon in upward direction (W) = 640N

let T_{DA}, T_{DB} and T_{DC}



be the tension in each cable DA, DB and DC resp.

co-ordinate : O (0,0,0)

A (−9, 0, 0)

B (0, 0, 5)

C (3, 0, −4)

D (0, 12, 0)

Then,

$\vec{T}_{DA} = T_{DA} \cdot \hat{DA} = T_{DA} \left| \frac{-9\hat{i} - 12\hat{j}}{15} \right|$

or, $\vec{T}_{DA} = (-0.6\hat{i} - 0.8\hat{j})\, T_{DA}$

Similarly, $\vec{T}_{DB} = T_{DB} \cdot \hat{DB}$

or, $\vec{T}_{DB} = T_{DB} \cdot \left| \frac{-12\hat{j} + 5\hat{k}}{13} \right| = (\bar{0.92}\hat{j}\ 0.38\hat{k})$

and

$\vec{T}_{DC} = T_{DC} \cdot \hat{DC}$

or, $\vec{T}_{DC} = T_{DC} \left| \frac{3\hat{i} - 12\hat{j} - 4\hat{k}}{13} \right|$

or, $\vec{T}_{DC} = (0.23\hat{i} - 0.92\hat{j} - 0.3\hat{k})\, T_{DC}$

For equilibrium

$+\uparrow \Sigma F_y = 0$ ie. $\vec{T}_{DA} + \vec{T}_{DB} + \vec{T}_{DC} + W = 0$

or, $(-0.6\hat{i} - 0.8\hat{j})\, T_{DA} + (\bar{0.92}\hat{j} + 0.38\hat{k}) \cdot T_{DB}$

$+ (0.23\hat{i} - 0.92\hat{j} - 0.3\hat{k}) \cdot T_{DC} + 640\hat{j} = 0$

Equating like vectors,

$-0.6\,T_{DA} + 0.23\,T_{DC} = 0 \longrightarrow$ (i)

$-0.8\,T_{DA} - 0.92\,T_{DC} + 640 - 0.92\,T_{DB} = 0$

or, $-0.8\,T_{DA} - 0.92\,T_{DC} = -640 \longrightarrow$ (ii)

and

$-0.3\,T_{DC} + 0.38\,T_{DB} = 0 \longrightarrow$ (iii)

solving eqn (i), (ii) & (iii)

$T_{DA} = 125.61\ N$

$T_{DB} = 258.71\ N$

$T_{DC} = 327.70\ N$

Basic concept of Static Equilibrium (2hr)

Date 6/2/

Equilibrium:

The word equilibrium employs a state of balance. The system is said to be in equilibrium if there is no unbalanced force acting on it. The principle of equilibrium states that "a stationary body which is subjected to coplanar forces will be in equilibrium if the algebraic sum of all external force is zero and also the algebraic sum of moments of all the external forces about any point in their plane is zero".

i.e. $\Sigma F = 0 \longrightarrow$ (i)

$\Sigma M = 0 \longrightarrow$ (ii)

The forces can be resolved into vertical and horizontal components so, eqn (i) can be written as

$\Sigma F_x = 0$, $\Sigma F_y = 0$.

(V.IMP) Hence, the three eqns of equilibrium are $\Sigma F_x = 0$, $\Sigma F_y = 0$, $\Sigma M = 0$

Free Body Diagram:

Free body diagram is a space diagram drawn in such a way that it shows all the applied forces, reactions and moment of the body.

i.e. FBD = [Diagram with body wt and load applied] − [Support & connection] +

[reaction from Support and connection]

In a free body diagram, all the supports (like walls, floors etc) are removed and replaced by the reactions which these supports exert on the body. A free body diagram can be drawn for a single body of a system, for any substance or for the entire system.

For Example:

consider a sphere resting on a frictionless surface as shown in the figure.

Here, the forces acting on the sphere are

- Force 'W' equal to weight of sphere
- Reaction 'RA' at the point of contact which acts upward normal to the surface.

Then the FBD is as shown in the fig below:

Points to be Remembered:

→ The following points should be kept in mind, while drawing FBD.

- object should be isolated from its supp and should be replaced by a reaction in an appropriate direction.
- The weight of body should act through its C.G in vertically downward direction.
- An appropriate co-ordinate system should be drawn.
- All the external forces should be place as per the given condition.
- All the reactions should be denoted by its magnitude or alphabets (for unknown rxn).

Load:-

→ To design a structure, a designer should have knowledge about the maxm expected load, their frequency, type material used and their properties. Also the nature and for the purpose of determining maxm stresses loads are of various types.

a) Dead load:- Dead load is the self load of the structure acting statically. Such load do not change their position and not vary in magnitude. i.e. self wt. of floor, beam.

b) Live load:- Live load is the load which is imposed in the structure when it fulfills its design purpose. The position and magnitude of live load may change. Such load aries due to the student in a class room, water in tank, books in a library etc.

c) Dynamic load:- Load imposed in a structure due to vibration, load imposed due to ground movement, during earthquake, load in a bridge due to moving vehicle are called dynamic load.

A/c to the intensity of load, it can be again divided into three groups.

a) Point/concentrated:- The load acting at a point on a beam or frame is known as a point load.

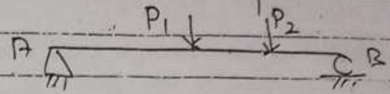


fig: point load

b) Uniformly distributed Load (UDL):- The load which is equally distributed over a part or the entire length of the beam or frame is known as UDL. for eg. self wt. of beam.

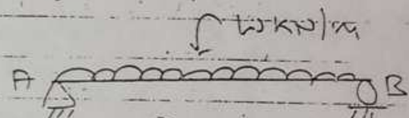


fig: UDL

© Uniformly varying load (UVL):- This load varies in intensity along the length of beam over which it is applied.

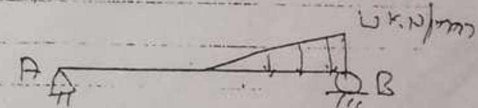


fig: UVL

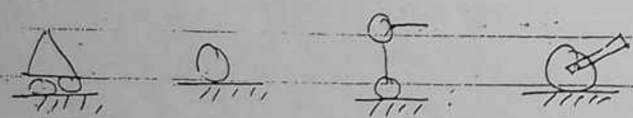
Types of supports:-

→ Different elements of a structure are connected by using support. The load applied on the structure is ultimately transmitted to the earth by means of structure support. There are three types of support.

→ Roller support
→ Hinge support
→ Fixed support.

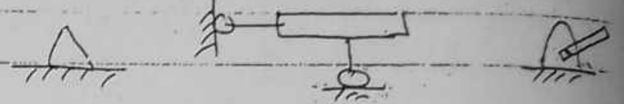
a) Roller support: The roller support is automatically known from its name i.e. the support used in the motion is roller. The type of support are free to move in x-direction and rotation about z-axis. It gives rise to one force reaction which is perpr to the plane supporting the roller. It has two degree of freedom.

Figure:



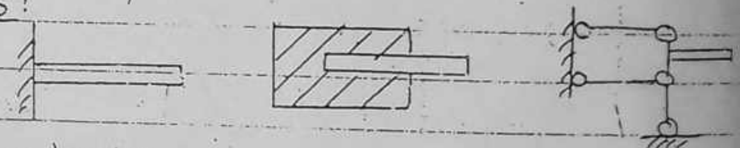
b) Hinge support: It gives rise to one force reaction whose direction is unknown but can be resolved into two direction. It results translatory motion both horizontal and vertical direction. It has one degree of freedom. [No. of independent ways by which a system can move without violating any restriction imposed on it]

Figure:



c) Fixed support: It give rise to one force reaction and one couple. It resists all forces i.e. rotatory and translatory as well as absorb moment. The degree of freedom is zero.

Figure:

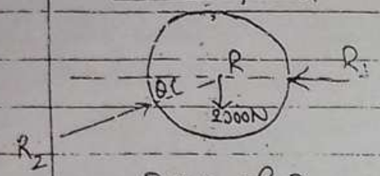


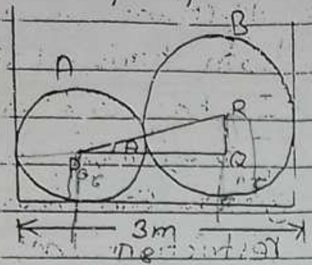
Numericals:-

1) The cylinders A and B weighing 1000N and 2000N resp. It is given that the radius of A is 0.6m and radius of B is 1.2m. Determine the force exerted at the point of contact.

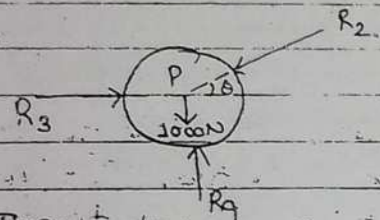
→ let θ be the angle made by the line PR with horizontal line PQ passing through center & p. of cylinder 'A'.

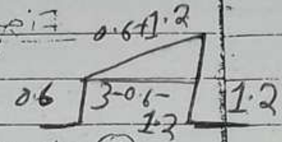
F.B.D of B





F.B.D of A





Then,

$$\cos\theta = \frac{PQ}{PR} = \frac{3-0.6-1.2}{0.6+1.2} \Rightarrow \theta = 48.19^\circ$$

For cylinder B

$+\uparrow \Sigma F_y = 0$ ie. $R_2 \sin 48.19' = 2000$

or, $R_2 = 2683.27N$ — (I)

$\rightarrow \Sigma F_x = 0$

ie. $R_2 \cos 48.19 - R_1 = 0$

or, $R_1 = 1788.43N$.

For cylinder A

$\xrightarrow{+} \Sigma F_x = 0$

ie. $R_3 - R_2 \cos 48.19 = 0$

or, $R_3 = 1788.83N$.

$+\uparrow \Sigma F_y = 0$

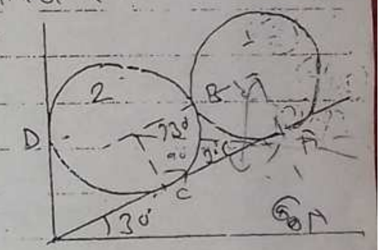
ie. $R_4 - 1000 - R_2 \sin 48.19' = 0$

or, $R_4 = 1000 + 2683.27 \times \sin 48.19$

or, $R_4 = 3000N$.

(2007) Fall

(2) Two identicals rollers, each of weight 500N are supported by an inclined plane and a vertical wall as shown in the figure. Find the reaction at the points of support A, B, and C.

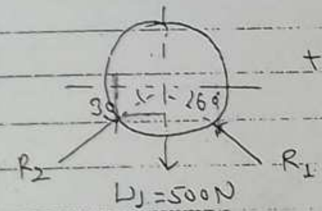


$\Rightarrow$ Here,

(i) Weight of two rollers

$W_1 = W_2 = 500N$

F.B.D of roller 1



$+\uparrow \Sigma F_y = 0$

ie. $R_2 \sin 30 + R_1 \sin 60 = 500 \longrightarrow$ (I)

and $\xrightarrow{+} \Sigma F_x = 0$

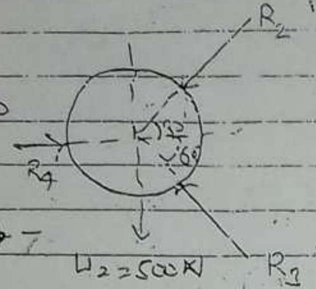
ie. $R_2 \cos 30 - R_1 \cos 60 = 0 \longrightarrow$ (II)

Solving eqn (1) and (2)

$R_2 = 250N$ and $R_1 = 433.01N$

Date / /
Page

F.B.D of roller 2



$\overset{+}{\rightarrow} \Sigma F_x = 0$

ie. $-R_2 \cos 30° - R_3 \cos 60° = 0$
$- R_4$ ⟶ (iii)

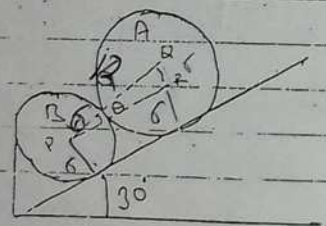
$+\uparrow \Sigma F_y = 0$

ie. $R_3 \sin 60° - R_2 \sin 30° - 500 = 0$ ⟶ (iv)

or, solving (iii) & (iv)
$R_3 = 721.7N$
$R_2 = 250N$
$R_4 = 577.37N$

(3) (2010) spring

Two spheres A and B are placed in container as shown in figure. Determine the reaction from the wall of the container.



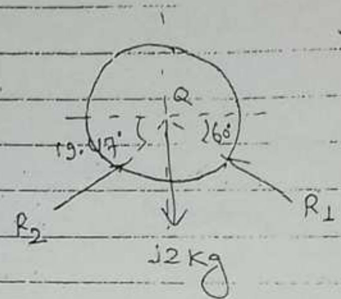
$r_A = 12cm$
$m_A = 12kg$

$r_B = 6cm$
$m_B = 10kg$

(4) let θ be the angle made by the line PQ with PR as shown in fig.
then, $\sin\theta = \frac{RQ}{PQ} = \frac{12-6}{12+6} \Rightarrow \theta = 19.47°$

Date / /
Page

F.B.D of A



$\overset{+}{\rightarrow} \Sigma F_x = 0$

ie. $R_2 \cos 19.47° - R_1 \cos$ ⟶ (i)

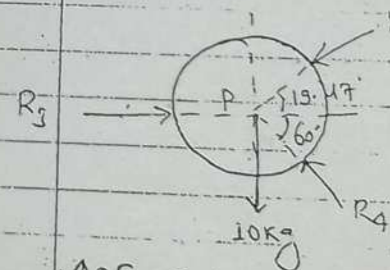
or,

$+\uparrow \Sigma F_y = 0$

ie. $R_2 \sin 19.47° + R_1 \sin 60° - 12 = 0$ ⟶ (ii)

solving eqn (i) and (ii)
$R_1 = 11.51\ kg$
$R_2 = 6.10\ kg$

F.B.D of B



$+\uparrow \Sigma F_y = 0$

ie. $R_4 \sin 60° - R_2 \sin 19.47° - 10 = 0$

or, $R_4 = \frac{6.10 \times \sin 19.47° + 10}{\sin 60°} = 13.89$

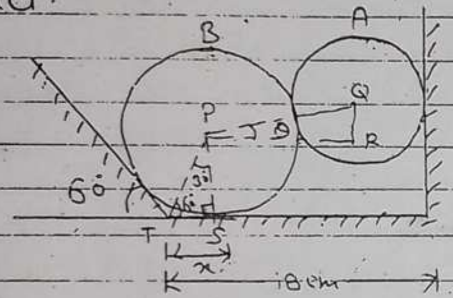
$\overset{+}{\rightarrow} \Sigma F_x = 0$

ie. $R_3 - R_2 \cos 19.47° - R_4 \cos 60° = 0$

or, $R_3 = 6.10 \times \cos 19.47° + 13.89 \cos 60° = 12.70$

(2012 Fall)

(4) Two cylinders A and B rest in a channel as shown in figure. 'A' has a diameter of 10cm and weight of 20kg and 'B' has diameter of 18cm and weight of 50kg. The channel is 18cm wide at the bottom with one side vertical and other at 120° as shown. Determine pressure at all four points of contact.



⇒ Here,

In rt. Δ PST

$\tan 30° = \frac{TS}{PS} = \frac{x}{9}$

or, $x = 5.20$ cm.

Then

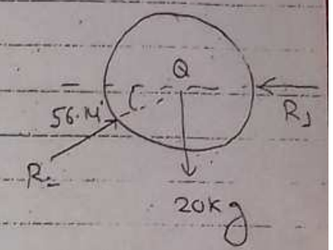
In rt. Δ PQR,

$\cos\theta = \frac{PR}{PQ}$

or, $\cos\theta = \frac{18-5.2-5}{9+5}$

or, $\theta = 56.14°$

F.B.D of cylinder A



$+\uparrow \Sigma F_y = 0$

i.e. $R_2 \sin 56.14° - 20 = 0$

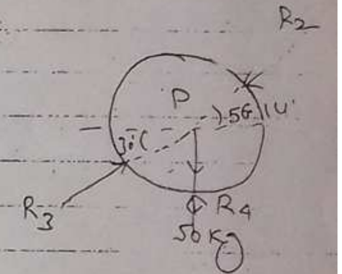
or, $R_2 = 24.08$ kg

$\xrightarrow{+} \Sigma F_x = 0$

i.e. $R_2 \cos 56.14° - R_1 = 0$

or, $R_1 = 13.42$ kg.

F.B.D of cylinder B



$\xrightarrow{+} \Sigma F_x = 0$

or, $R_3 \cos 30° - R_2 \cos 56.14° = 0$

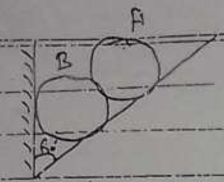
or, $R_3 = 15.45$ kg

$+\uparrow \Sigma F_y = 0$

i.e. $R_3 \sin 30° + R_4 - 50 - R_2 \sin 56.14°$

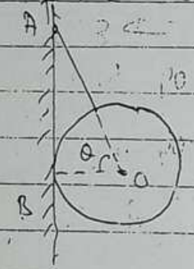
or, $R_4 = 62.25$ kg.

(5) Two identical rollers each of weight 100 N rest on an inclined plane which makes an angle 60° with the vertical wall. Find the reactions at all point of contact supports assuming all surface to be smooth.

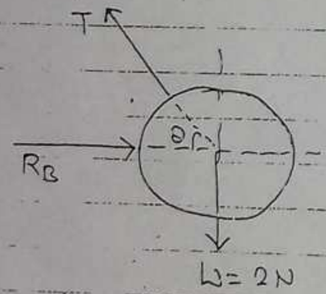


⑥ A smooth sphere of radius 15cm and weight 2N is supported in contact with a smooth vertical wall by a string whose length equals to the radius of the sphere. The string joins a point on the wall and a point on the surface of sphere. Workout inclination and the tension in the string and reaction of the wall.

⇒ Here,
Weight of sphere $(W) = 2N$
radius $(r) = 15cm$



The FBD is



In rt. Δ ABO, $\cos\theta = \frac{BO}{AO} = \frac{15}{30}$

or, $\theta = 60°$

$\rightarrow \Sigma F_x = 0$ i.e. $R_B - T\cos\theta = 0$

or, $R_B = T\cos\theta \longrightarrow ①$

$\uparrow \Sigma F_y = 0$

i.e. $T\sin\theta - W = 0$

or, $T\sin 60 - 2 = 0$

or, $T = 2.31N$

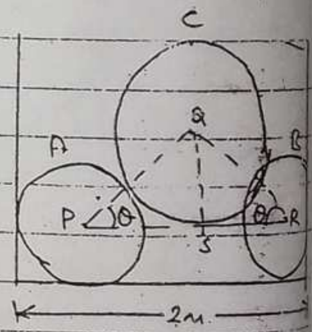
substituting the value of T in eqn ①

$R_B = 2.31 \times \cos 60°$

or, $R_B = 1.15N$

⑦ In the figure three spheres are arranged in a box whose width is 2m. The radius of sphere A and B is 0.3m while the radius of sphere C is 0.6m. Determine the force exerted at the contact points if ~~cylinder~~ sphere 'C' weights 2000N and sphere A and B weights 1000N.

⇒ Here,
let 'θ' be the angle made by the line PQ with PS.,
Then, In rt. Δ PQS

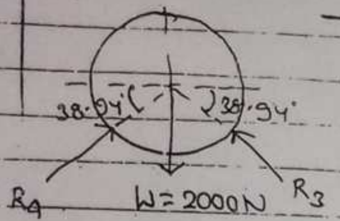


$\cos\theta = \frac{PS}{PQ} = \frac{PR/2}{PQ}$

or, $\theta = \cos^{-1}\left(\frac{(2-0.3-0.3)/2}{\cancel{2\times(0.3+0.6)}\ (0.3+0.6)}\right)$ or, $\theta = 38.94°$

Date / /
Page

F.B.D of ~~cylinder~~ sphere C



$\xrightarrow{+} \Sigma F_x = 0$

ie. $R_4 \cos 38.94' - R_3 \cos 38.94' = 0$

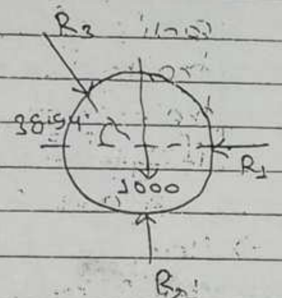
$+\uparrow \Sigma F_y = 0$

ie. $R_4 \sin 38.94 + R_3 \sin 38.94' - 2000 = 0$

Solving eq^n ① and ②

$R_4 = R_3 = 1591.07\,N$

F.B.D of sphere B



$\xrightarrow{+} \Sigma F_x = 0$

ie. $R_3 \cos 38.94 - R_1 = 0$

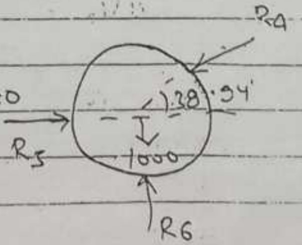
or, $R_1 = 1237.54\,N$

$+\uparrow \Sigma F_y = 0$

ie. $R_2 - 1000 - R_3 \sin 38.94' = 0$

or, $R_2 = 1999.99\,N$.

F.B.D of sphere A



$\xrightarrow{+} \Sigma F_x = 0$

ie. $R_5 \cancel{\cos 38.94'} - R_4 \cos 38.94' = 0$

or, $R_5 = 1237.54\,N$

18

Date / /
Page

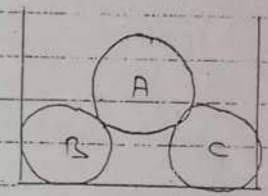
$+\uparrow \Sigma F_y = 0$

ie. $R_6 - R_4 \sin 38.94' - 1000 = 0$

or, $R_6 = 1999.99\,N$.

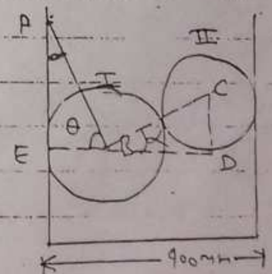
(2006) sp.

⑧ In the figure, three sphere each with 2K mass and each 350mm in diameter rest in a box 760mm wide. Find ⓐ the reaction of B on A, ⓑ the reaction of wall on C and ⓒ the reaction of floor on B.



(2013) Fall

⑨ Two spheres I and II of radii 150mm and 100mm resp are placed on a container as shown in figure. Sphere I of mass 80kg is suspended with a string AB of length 330mm and sphere II of mass 50kg is placed on top of it. Determine the tension in AB and the reaction from the container.



⇒ Here,

In rt. Δ ABE

$$\cos\theta = \frac{BE}{AB} = \frac{150}{330}$$

Date / /
Page

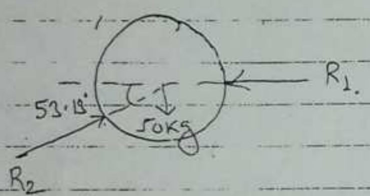
$\theta = 62.96^\circ$

And In rt. ΔBCD

$\cos\alpha = \frac{BD}{BC} = \frac{400-100-150}{100+150}$

or, $\alpha = 53.13^\circ$

F.B.D of sphere II



$+\uparrow \Sigma F_y = 0$ ie. $R_2 \sin 53.13 = 50$

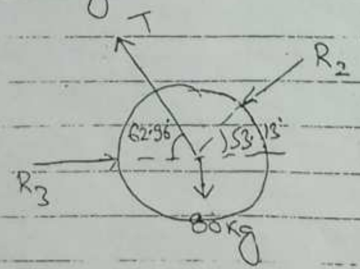
or, $R_2 = 62.5$ Kg

$\xrightarrow{+} \Sigma F_x = 0$ ie. $R_2 \cos 53.13' - R_1 = 0$

or, $R_1 = 62.5 \times \cos 53.13'$

or, $R_1 = 37.5$ Kg.

F.B.D of sphere I



$+\uparrow \Sigma F_y = 0$

ie. $T \sin 62.96' - 80 - R_2 \sin 53.13' = 0$

Date / /
Page

or, $T = \frac{80 + 62.5 \times \sin 53.13'}{\sin 62.96'} = 145.9$

$\xrightarrow{+} \Sigma F_x = 0$

ie. $R_3 - R_2 \cos 53.13' - T \cos 62.96' = 0$

or, $R_3 = 62.5 \times \cos 53.13' + 145.95 \times \cos$

or, $R_3 = 103.85$ Kg.

Friction:

→ When a body slides over another body, a force is exerted at the surface of contact by the stationary body on the moving body. This resisting force is called the force of friction or simply friction. It acts in a direction opposite to the direction of motion. The presence of friction would cause loss of power, wearing out of parts and huge economic loss. However, the working of many devices such as frictional brakes, belt and rope drives, holding and fastening devices depends on friction and there the presence of friction force is advantageous.

Causes of friction:-

→ If any two smooth surface comes in contact, there will be interlocking betw up and down projection of two surface. This interlocking creates friction. Highly smooth or highly polished surface have more friction in comparision to the smooth surface. When two highly polished surface comes in contact molecules of two surfaces comes so

close that intermolecular force of attraction develops there which increases the friction.

Types of Friction:

→ Friction can be classified into two types. They are
- → Static friction
- → Dynamic friction

a) Static friction:- It is the friction experienced by a body, when it is in rest. In other words it is the friction when the body tends to move. Max^m value of static friction is known as limiting friction.

b) Dynamic friction:- It is the friction experienced by a body when it is in motion. Dynamic friction comes into existence when body moves. It is also known as kinetic friction. It may be sliding friction, rolling friction.

Laws of friction:

→ The laws of friction are listed below:-

a) The force of friction always acts in a direction opposite to that in which body tends to move.

b) Till the limiting value is reached, the magnitude of friction is exactly equal to the force which tends to move the body.

c) The magnitude of limiting friction bears constant ratio to the normal reaction between the two surface of contact & this ratio is called coefficient of friction.

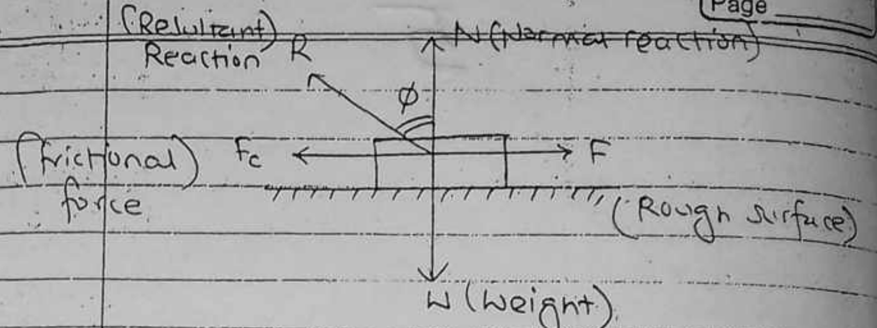
d) The friction force depends upon the roughness / smoothness of surface.

e) The force of friction is independent of area of contact between the two surface.

f) After the body starts moving the dynamic friction comes into play, the magnitude of which is less than that of limiting friction and it bears a constant ratio with the normal force. This ratio is called coeff. of dynamic friction.

Angle of friction:-

Angle of friction is defined as angle subtended by resultant of force of friction ~~and normal reaction~~ with normal reaction.



consider a smooth block placed on horizontal smooth surface as shown in fig. 'F_c' is the force of limiting ~~force~~ friction N is normal reaction, 'R' is resultant of F_c and N. Then the ratio ~~F_c/N~~ is called the angle of friction.

$\tan\phi = \frac{F_c}{N}$

coefficient of friction:-

coefficient of friction is defined as the ratio of force of friction to the normal reaction between the contact surface. It is denoted by μ.

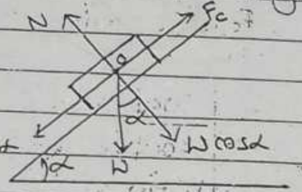
$$\text{ie. } \mu = \frac{F_c}{N} = \tan\phi$$

The coeff. of friction would be high if the contact surface are rough. Thus coeff. of friction measures the roughness between a pair of contact surfaces.

Angle of Repose:

→ Angle of repose is defined as the angle substended by a inclined plane with horizontal surface when a body kept at the inclined plane just starts to slide down.

Consider a block of mass 'm' placed on a inclined plane. Angle of inclination of inclined plane with horizontal is gradually increased until the block placed on it just starts to slide. let, this angle be 'α' which is angle of repose.



Weight 'W' of block acts vertically downward. It may be resolved into two rectangular components. component of W parallel to inclined plane is $W\sin\alpha$ while component perpⁿ to inclined plane is $W\cos\alpha$. 'N' is normal reaction perpʳ to inclined plane. 'F_c' is force of limiting friction.

In limiting friction,

$$F_c = W\sin\alpha \longrightarrow (i)$$

For equilibrium,

$$N = W\cos\alpha \longrightarrow (ii)$$

Dividing eqⁿ (i) by (ii)

$$\tan\alpha = \frac{F_c}{N}$$

But $\mu = \frac{F_c}{N}$, where, μ = coeff. of friction

$$\therefore \mu = \tan\alpha.$$

Thus, we can conclude that coeff of limiting friction is equal to the tangent of angle of repose.

But $\mu = \tan\phi$, where ϕ is angle of friction.

Thus, $\tan\phi = \tan\alpha$

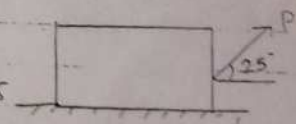
$$\Rightarrow \phi = \alpha$$

Hence, angle of friction and angle of repose are equal.

Numericals:

(1) A block having mass 300 N is resting on a smooth surface having coeff. of friction 0.35. Determine the force required to move the block inclined at 25°

Given:



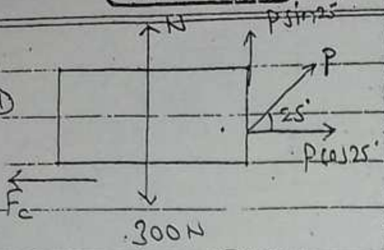
Mass of block (W) = 300 N

coeff. of friction (μ) = 0.35

Force required (P) = ?

Now,

Date / /
Page



$+\uparrow \Sigma F_y = 0$

i.e. $N - 300 + P\sin 25° = 0 \rightarrow (1)$

$\xrightarrow{+} \Sigma F_x = 0$

i.e. $P\cos 25° - F_c = 0$

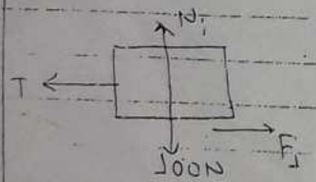
or, $P\cos 25° - \mu N = 0 \rightarrow (11) \left[\because \mu = \frac{F_c}{N} \right]$

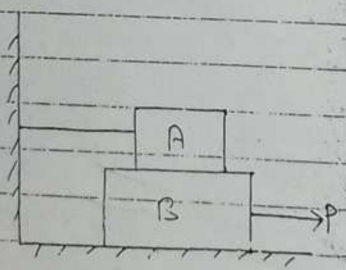
solving eqn (1) & (11)

$P = 99.6\ N$

(2006) ② Block 'A' weighing 100N rest over block B which weighs 200N as shown in fig. Block A is tied to wall by a horizontal string. If the coeff. of friction between block A and B is 0.25 and between B and floor is 1/3. What should be the value of 'p' to move block if 'p' is horizontal.

=> For block A





$+\uparrow \Sigma F_y = 0$ i.e. $N_1 - 100 = 0$

$\therefore N_1 = 100\ N$

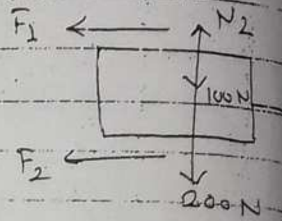
But $\mu_1 = \frac{F_1}{N_1}$ => $F_1 = 0.25 \times 100 = 25 N$

Date / /
Page

$\xrightarrow{+} \Sigma F_x = 0$

i.e. $F_1 - T = 0$ => $T = F_1 = 25 N$

For block B



$+\uparrow \Sigma F_y = 0$

i.e. $N_2 - 100 - 200 = 0$

or, $N_2 = 300N$.

But $F_2 = \mu_2 N_2$

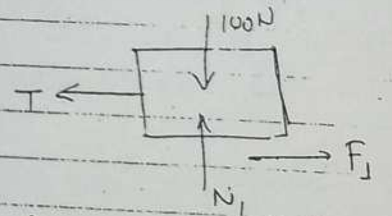
or, $F_2 = \frac{1}{3} \times 300 = 100N$

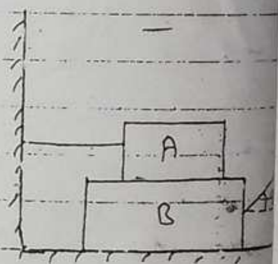
$\xrightarrow{+} \Sigma F_x = 0$ i.e. $P = F_1 + F_2$

or, $P = 25 + 100 = 125\ N$

③ If P is inclined at 30° upward with horizontal.

=> For Block A



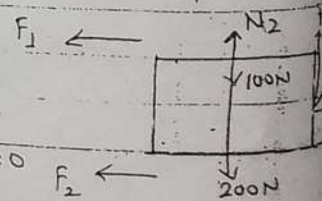


$+\uparrow \Sigma F_y = 0$ i.e. $N_1 - 100 = 0$ => $N_1 = 100N$

$F_1 = \mu_1 N_1 = 0.25 \times 100 = 25 N$

$\xrightarrow{+} \Sigma F_x = 0$ i.e. $F_1 - T = 0$ => $F_1 = T = 25N$

For block 3



$+\uparrow \Sigma F_y = 0$

i.e. $N_2 - 200 - 100 + P\sin 30° = 0$

or, $N_2 = 300 - 0.5P$

Also, $\mu_2 = F_2/N_2$

or, $F_2 = \frac{1}{3}(300 - 0.5P)$

or, $F_2 = \left(100 - \frac{0.5P}{3}\right)$ —— ①

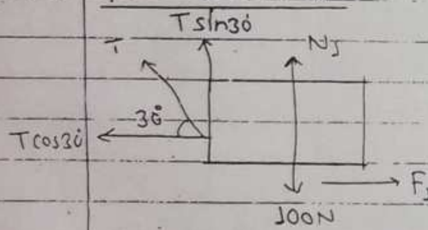
$\xrightarrow{+} \Sigma F_x = 0$ ie. $P\cos 30° - F_1 - F_2 = 0$

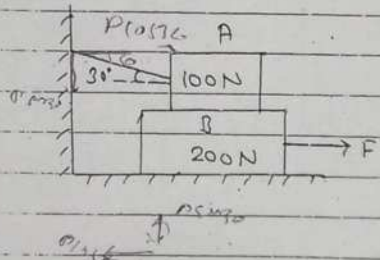
or, $P\cos 30° - 25 - 100 + \frac{0.5P}{3} = 0$

(2009) or, $P = 121.04$ N.

④ If the static coeff. of friction for all surfaces is 0.35, find the force F needed to start the 200N weight moving to right.

⇒ For block A





$\uparrow + \Sigma F_y = 0$ ie. $N_1 - 100 + T\sin 30° = 0$ → ①

Also, $\mu_1 = F_1/N_1$

or, $F_1 = 0.35 N_1$

$\xrightarrow{+} \Sigma F_x = 0$

ie. $F_1 - T\cos 30° = 0$

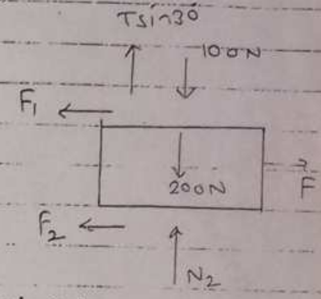
or, $0.35 N_1 - T\cos 30° = 0$ ⟶ ⑪

from eq^n ① & ⑪

$T = 33.62$ N

$F_1 = 29.12$ N

For block B



$\uparrow + \Sigma F_y = 0$

or, $T\sin 30° - 300 + N_2 = 0$

or, $N_2 = 300 - T\sin 30°$

or, $N_2 = 300 - 33.62 \times \sin 30°$

or, $N_2 = 283.19$ N.

Also, $\mu_2 = \frac{F_2}{N_2}$

or, $F_2 = 0.35 \times 283.19 = 99.12$ N.

$\xrightarrow{+} \Sigma F_x = 0$

ie. $F = F_1 + F_2$

or, $F = 29.12 + 99.12 = 128.24$ N.

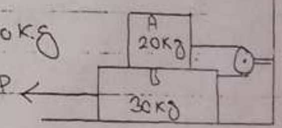
(2007)

⑤ The coeff. of friction are $\mu_s = 0.4$ and $\mu_k = 0.3$ between all surface of contact. Determine the force 'p' for which motion of block having mass 30kg is impending as shown in figure.

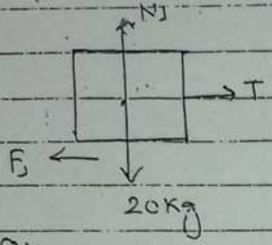
⇒ Here,

Mass of block A $(m_A) = 20$kg

mass of block B $(m_B) = 30$kg



Considering block A



$+\uparrow \Sigma F_y = 0$

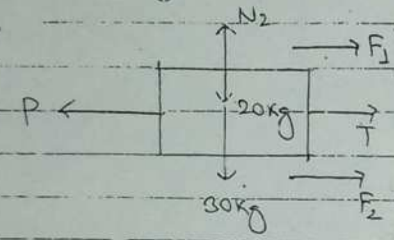
ie. $N_1 = 20kg$

But $\mu_s = \frac{F_1}{N_1}$

or, $F_1 = 0.4 \times 20 = 8kg$

$\xrightarrow{+} \Sigma F_x = 0$ ie. $F_1 = T = 8kg$

Considering block B



$+\uparrow \Sigma F_y = 0$

ie. $N_2 = 50kg$

But $\mu_s = \frac{F_2}{N_2}$

or, $F_2 = 0.4 \times 50 = 20kg$

$\xrightarrow{+} \Sigma F_x = 0$ ie. $P = T + F_1 + F_2$

or, $P = 8 + 8 + 20 = 36kg$.

6) Knowing that the coefficient of friction between the 25kg block and the incline is $\mu_s = 0.25$. Determine

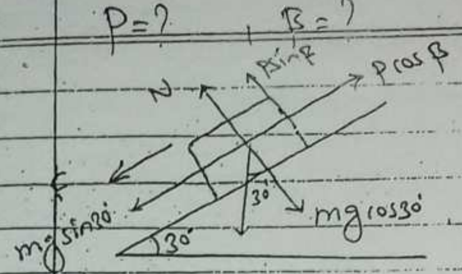
i) Smallest value of P required to start the block moving up the inclined.

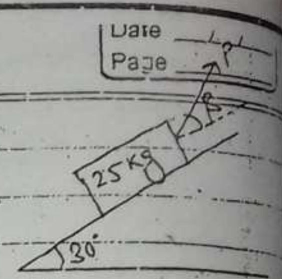
ii) corresponding value of β.

=> Here, Mass of block (M) = 25kg

$\mu_s = 0.25$

P = ?, β = ?





$\xrightarrow{+} \Sigma F_x = 0$

ie. $P\cos\beta = F + mg\sin 30°$

or, $P\cos\beta = 0.25N + 25 \times 9.81 \times \sin 30°$

or, $P\cos\beta = 0.25N + 122.625 \longrightarrow$ (i)

$+\uparrow \Sigma F_y = 0$

ie. $N + P\sin\beta = mg\cos 30°$

or, $N = 25 \times 9.81 \times \cos 30° - P\sin\beta$

or, $N = 212.39 - P\sin\beta \longrightarrow$ (ii)

Substituting value of N in eqn (i) we get,

$P\cos\beta = 0.25(212.39 - P\sin\beta) + 122.6$

or, $P(\cos\beta + 0.25\sin\beta) = 175.722$

or, $P = \frac{175.722}{\cos\beta + 0.25\sin\beta} \longrightarrow$ (iii)

A/c to question the value of 'P' should be smallest.

Hence, for smallest value of P $(\cos\beta + 0.25\sin\beta)$ should be maxm.

For maxima / minima

$\frac{d}{d\beta}(\cos\beta + 0.25\sin\beta) = 0$

Puspanjali
Date / /
Page

or, $-\sin\beta + 0.25\cos\beta = 0$

or, $\tan\beta = 0.25$

$\therefore \beta = 19°$

Putting the value of β in eqn (iii)

we get $P = 170.47$ N.

(2007)

⑦ Illustrate about conditions describing no friction, no motion, impeding motion.

→ Considering a body resting on a smooth surface and on a inclined plane as shown in fig ⓐ and fig ⓑ.

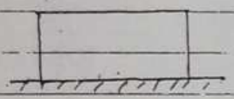


fig. ⓐ

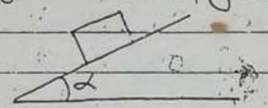


fig. ⓑ

Then,

① If the coefficient of friction is zero (ie. $\mu = 0$) then it can be said that there is no friction or surface is frictionless.

② If the applied force is less than the magnitude of frictional force acting in opposite direction or the angle of repose is less than it is required for a body to slide down then there will be no motion of the body.

Puspanjali
Date / /
Page 22

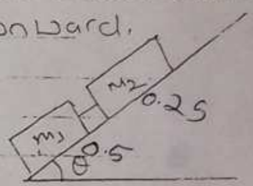
③ For impending motion, there should be sufficient angle of repose for a body placed in the inclined plane to slide down and applied force should be greater than frictional force.

(2011)SP (2013)Fall 2013(SP)

(2008) Fall ⑧ Two masses $m_1 = 22.5$ kg and $m_2 = 14$ kg are tied together by a rope parallel to the inclined plane. The coeff of friction between m_1 and plane is 0.25 while that of m_2 and plane is 0.5. Determine

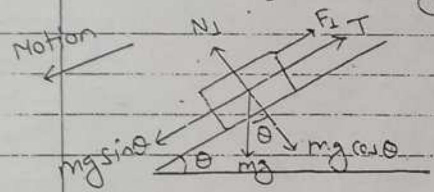
i) value of inclination of the plane surface θ for which the masses will just start sliding downward.

ii) tension in the rope.



Here, $\mu_1 = 0.25$, $\mu_2 = 0.5$

⇒ Considering block having mass $m_1 = 22.5$ kg



in question the motion is downward that is it is sliding downward so Friction is upward direction

→ $\Sigma F_x = 0$

ie. $T + F_1 = mg\sin\theta$

or, $T = -F_1 + mg\sin\theta$

Puspanjali
Date / /
Page

or, $T = -0.25N_1 + 22.5 \times 9.81 \sin\theta$ $[F_1 = \mu_1 N_1]$

or, $T = 220.725 \sin\theta - 0.25 N_1 \longrightarrow$ (I)

$+\uparrow \Sigma F_y = 0$

ie. $N_1 = mg\cos\theta$

or, $N_1 = 22.5 \times 9.81 \cos\theta$

or, $N_1 = 220.725 \cos\theta$

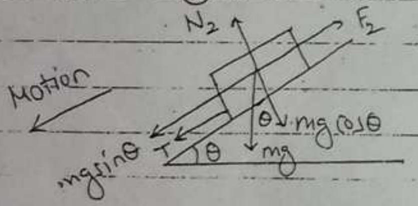
putting the value of N_1 in eqn (I)

We get,

$T = 220.725 \sin\theta - 0.25(220.725 \cos\theta)$

or, $T = 220.725 \sin\theta - 55.18 \cos\theta \longrightarrow$ (II)

considering block having mass $(m_2) = 14$ kg



$\xrightarrow{+} \Sigma F_x = 0$

ie. $F_2 = T + mg\sin\theta$

or, $T = 0.5N_2 - 14 \times 9.81 \sin\theta$ $[F_2 = \mu_2 N_2]$

or, $T = 0.5N_2 - 137.34 \sin\theta \longrightarrow$ (IV)

$+\uparrow \Sigma F_y = 0$ ie. $N_2 = mg\cos\theta$

Date / /
Page

or, $N_2 = 14 \times 9.81 \times \cos\theta = 137.34 \cos\theta$

putting the value of N_2 in eqn (IV)

$T = 0.5N_2 - 137.34 \sin\theta$

or, $T = 0.5(137.34 \cos\theta) - 137.34 \sin\theta$

or, $T = 68.67 \cos\theta - 137.34 \sin\theta \longrightarrow$

equating eqn (II) and (V)

$220.725 \sin\theta - 55.18 \cos\theta = 68.67 \cos\theta - 137.34 \sin\theta$

solving,

$\tan\theta = \dfrac{123.85}{358.065}$

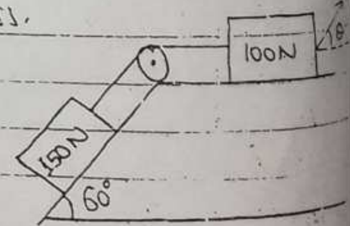
$\therefore \theta = 19.08°$

substituting the value of T in eqn (II)

$T = 20$ N.

(2010) (2013) fall

9. Referring to the figure shown, determine the least value of 'P' to cause the system to impend rightward. Assume coefficient of friction between the block and incline surface to be 0.2 and pully to be frictionless.



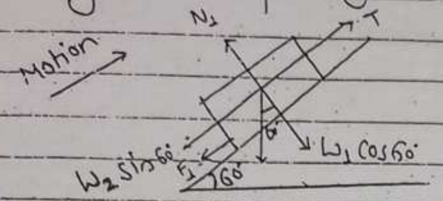
=> Here, $W_1 = 150$ N;

$W_2 = 100$ N

coefficient of friction $(\mu) = 0.2$

Date / /
Page

Considering block of weight 150 N



$\uparrow + \Sigma F_y = 0$

ie. $N_1 = W_1 \cos 60° = 150 \times \cos 60° = 75 N$

$\xrightarrow{+} \Sigma F_x = 0$

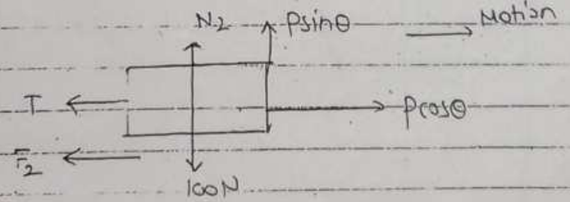
or, $T = W_1 \sin 60° + F_1$

or, $T = 150 \times \sin 60° + \mu N_1$ $\quad \left[\because \mu = \frac{F_1}{N_1}\right]$

or, $T = 150 \times \sin 60° + 0.2 \times 75$

or, $T = 144.90 N$

Considering block of weight 100 N



$\xrightarrow{+} \Sigma F_x = 0$

ie. $P\cos\theta = T + F_2$

or, $P\cos\theta = 144.90 + \mu N_2$ $\quad \left[\because \mu = \frac{F_2}{N_2}\right]$

Date / /
Page 2

or, $P\cos\theta = 144.90 + 0.2 N_2 \longrightarrow (I)$

$\uparrow \Sigma F_y = 0$

ie. $N_2 + P\sin\theta = 100$

or, $N_2 = 100 - P\sin\theta$

Substituting the value of N_2 in eqn (I)

$P\cos\theta = 144.90 + 0.2\,(100 - P\sin\theta)$

or, $P\cos\theta = 144.90 + 20 - 0.2 P\sin\theta$

or, $P(\cos\theta + 0.2\sin\theta) = 164.90$

or, $P = \dfrac{164.90}{\cos\theta + 0.2\sin\theta} \longrightarrow (II)$

A/c to the question for least value of 'p', the denominator $(\cos\theta + 0.2\sin\theta)$ should be maximum.

$\therefore$ For maxima | Minima

$\dfrac{d(\cos\theta + 0.2\sin\theta)}{d\theta} = 0$

or, $-\sin\theta + 0.2\cos\theta = 0$

$\therefore \tan\theta = 0.2 \implies \theta = 11.30°$

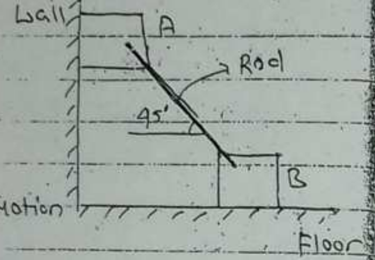
Substituting the value of 'θ' in eqn (II)

$P = 161.70 N$.

(2012) Fall

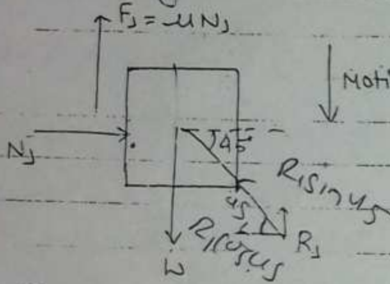
10) Two identical blocks A and B are connected by a rod and rest resp. against vertical wall and horizontal floor as shown in

Q. The sliding motion of blocks impends when the rod makes an angle of 45° with the horizontal. Estimate the coeff. of friction assuming it to be same both at the floor and wall.



=> Here,

considering block A



$\xrightarrow{+} \Sigma F_x = 0$ ie. $N_1 = R_1 \cos 45° \longrightarrow$ (i)

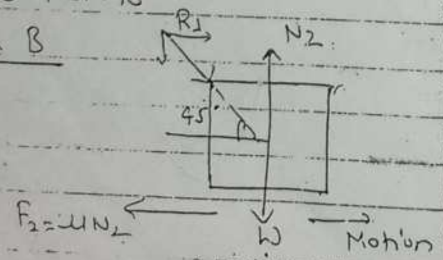
$+\uparrow \Sigma F_y = 0$ ie. $\mu N_1 + R_1 \sin 45° = W \longrightarrow$ (ii)

putting the value of N_1 in eqn (ii)

$\mu(R_1 \cos 45°) + R_1 \sin 45° = W$

or, $R_1 = \dfrac{W}{\mu \cos 45° + \sin 45°} \longrightarrow$ (iii)

considering block B



$\xrightarrow{+} \Sigma F_x = 0$

ie. $R_1 \cos 45° = F_2$

or, $\mu N_2 = R_1 \cos 45° \longrightarrow$ (iv)

$+\uparrow \Sigma F_y = 0$

ie. $N_2 = W + R_1 \sin 45°$

Putting the value of N_2 in eqn (iv)

$\mu(W + R_1 \sin 45°) = R_1 \cos 45°$

or, $\mu W + R_1 \mu \sin 45° = R_1 \cos 45°$

or, $R_1 = \dfrac{\mu W}{\cos 45° - \mu \sin 45°} \longrightarrow$ (v)

Equating eqn (iii) & eqn (v)

$$\frac{\cancel{W}}{\mu \cos 45° + \sin 45°} = \frac{\mu \cancel{W}}{\cos 45° - \mu \sin 45°}$$

or, $\dfrac{1}{\mu \times 0.707 + 0.707} = \dfrac{\mu}{0.707 - 0.707\mu}$

or, $\dfrac{1}{\cancel{0.707}(\mu+1)} = \dfrac{\mu}{\cancel{0.707}(1-\mu)}$

or, $\mu^2 + \mu = 1 - \mu$

or, $\mu^2 + 2\mu = 1$

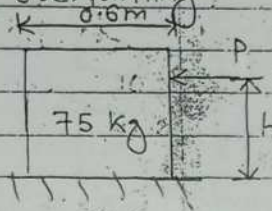
or, $\mu^2 + 2\mu - 1 = 0$ which is quadra[tic] eqn. solving,

$$\mu = \frac{-2 \pm \sqrt{4+4}}{2}$$

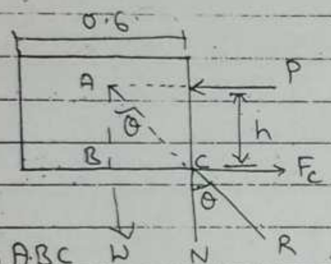
Neglecting negative sign.

$\therefore \mu = 0.414$

(11) A box of mass 75 kg rests on floor. The static coeff of friction for contact surface is 0.2. What is the highest position for a horizontal load 'p' that permits it to just move box without overturning?



⇒ Here,
Mass of box (m) = 75 kg



In rt. Δ ABC

$\tan\theta = \frac{BC}{AB} = \frac{0.6/2}{h}$

or, $\tan\theta = \frac{0.3}{h}$

or, $h = \frac{0.3}{\tan\theta}$ ⟶ ①

Also, $\mu = \tan\theta$

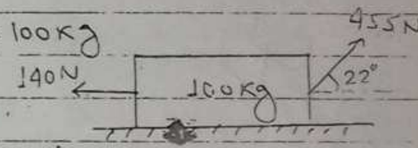
or, $\tan\theta = 0.2$

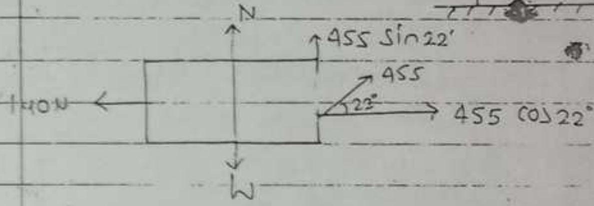
∴ eqⁿ ① become

$h = \frac{0.3}{0.2} = 1.5\,m.$

(12) Find the friction force for the block shown & state whether the block is in equilibrium or in motion. Also determine the additional force 'p' that must be added to 140 N force to just move the block to left. ($\mu_s = 0.2$)

→ Here,
Mass of block (m) = 100 kg
$\mu_s = 0.2$





$+\uparrow \Sigma F_y = 0$ $N = W - 455\sin 22$

or, $N = 100 \times 9.81 - 455\sin 22°$

or, $N = 810.55$ Newton

Frictional force (F_s) = μN

or, $F_s = 0.2 \times 810.55$

or, $F_s = 162.11$ Newton.

Since, Horizontal component of force 455 N is greater than sum of force 140 N and frictional force (F_s).

i.e. $455\cos 22° > (140 + 162.11)$

∴ Block is not in equilibrium. It moves right.

Date / / Page

let 'p' be the additional force

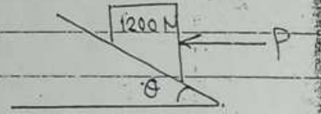
$\therefore 140 + P = F_s + 455\cos 22°$

or, $P = 162.11 - 140 + 455 \times \cos 22°$

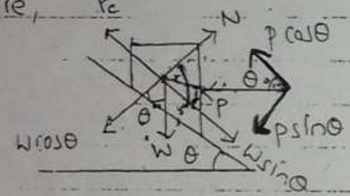
or, $P = 444.01$ N.

(2011) Fall (2014) Fall

(13) The coefficient of friction between the block and incline are $\mu_s = 0.35$ and $\mu_K = 0.25$. Determine whether the block is in equilibrium and find the magnitude and direction of frictional force when $\theta = 30°$ and $P = 150$ N



⇒ Here,



$+\uparrow \Sigma F_y = 0$

ie. $N - W\cos\theta - P\sin\theta = 0$

or, $N = 1200 \times \cos 30° + 150\sin 30°$

or, $N = 1114.23$ Newton

$\xrightarrow{+} \Sigma F_x = 0$

ie. $W\sin\theta - F_c - P\cos\theta = 0$

or, $F_c = 1200 \times \sin 30° - 150 \times \cos 30°$

or, $F_c = 470.096$ Newton

Date / / Page

But

$(F_c)_{max} = \mu_s N = 0.35 \times 1114.23$

or, $(F_c)_{max} = 389.98$ N.

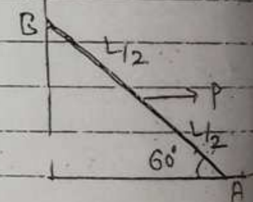
Here; $F_c > (F_c)_{max}$ which is impossible. So, the block moves downward.

$\therefore F = \mu_K N = 0.25 \times 1114.23$

or, $F = 278.55$ Newton $(\uparrow)$

(2011) SP

(14) A smaller ladder of uniform bar weigh 60kg is constructed to move in the smooth vertical guide. If $\mu_s = 0.80$, Determine horizontal force 'p' req. to initiate slipping at A.



⇒ Here,

Mass of bar = 60kg

$\mu_s = 0.80$

horizontal force (p) = ?

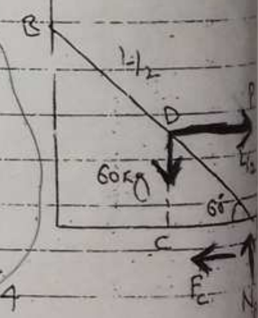
In rt. Δ ACD,

$\sin 60° = \frac{DC}{AD}$

or, $DC = \frac{L}{2} \times \sin 60° = \frac{\sqrt{3}L}{4}$

$\cos 60° = \frac{AC}{AD}$

or, $AC = L/2 \times \cos 60° = L/4$



since the rod is slipping at A,

$\xrightarrow{+} \Sigma F_x = 0$ ie. $P - F_c = 0 \longrightarrow$ ①

$+\uparrow \Sigma F_y = 0$ ie. $N_A - 60 = 0$

or, $N_A = 60\,kg$

$\circlearrowleft + \Sigma M_A = 0$

ie. $60 \times \frac{L}{4} - P \times \frac{\sqrt{3}L}{4} = 0$

or, $P = \frac{60}{\sqrt{3}} = 34.64\,kg$

From eqn ①, $P = F_c = 34.64\,kg$.

But,

$\therefore (F_c)_{max} = \mu_s \cdot N_A = 0.8 \times 60 = 48\,kg$

$\therefore F_c \not< (F_c)_{max}$. Hence, ladder slips at A.

⑮ A uniform ladder of length 5m and weighing 20N is placed against a smooth vertical wall with lower end 4m away from the wall. If the ladder is just to slip, determine ⓐ coeff. of friction ⓑ friction force acting on ladder at point of contact between ladder and floor.

Date / /
Page 2

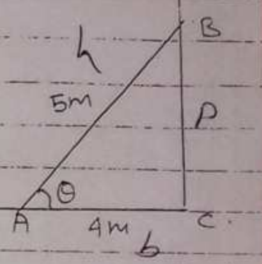
$\Rightarrow$ Here,

Wt. of ladder $(W) = 20N$

length of ladder $(AB) = 5m$

$AC = 4m$

$\therefore BC = \sqrt{5^2 - 4^2}$



or, $BC = 3m$

$\cos\theta = \frac{b}{h} = \frac{4}{5}$

In rt. ΔABC, $\sin\theta = \frac{BC}{AB}$

$= 36.87$

Now, or, $\sin\theta = \frac{3}{5}$

$+\uparrow \Sigma F_y = 0$

ie. $N_A + F_B - W = 0$ $\therefore \theta = 36.87°$

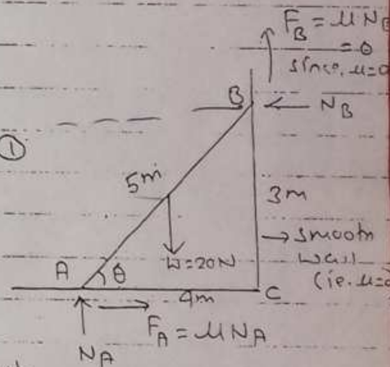
or, $N_A = 20N$

$\xrightarrow{+} \Sigma F_x = 0$

ie. $F_A - N_B = 0 \longrightarrow$ ①

or, $\mu N_A = N_B$

or, $\mu = \frac{N_B}{N_A} \longrightarrow$ ⑪



Taking Moment about point A.

$\circlearrowleft + \Sigma M_A = 0$

ie. $-W \times 2.5\cos\theta + N_B \times 3 = 0$

or, $N_B = \frac{20 \times 2.5 \times \cos 36.87}{3} = 13.33\,N$

Date __/__/__
Page ____

Putting the value of N_B in eqn (11)

$$\mu = \frac{13.33}{20} = 0.667$$

Frictional force $(F_A) = N_B$ [from eqn ①]

or, $F_A = 13.33\,N$.

Chapter 5

Date 2071/7/22
Page ____

Centre of Gravity, Centroid & Moment of Inertia

Centroid: The term centroid is used for the symmetrical figure like circle, triangle, etc which have only areas, but no mass. The point through which entire length, area & volume is assumed to be concentrated is called centroid. Centroid applies to the plane figure which have no mass.

Centre of Gravity: The point through which the whole weight of the body acts irrespective of the position of the body is known as centre of gravity. The position of centre of gravity depends upon the shape of the body and this may or may not necessarily be within the boundary of the body. A body has only one centre of gravity.

Axis of Reference: The centre of gravity (C.G.) is always calculated with reference to some assumed axis known as axis of reference. The axis of reference, of plane figures, is generally taken as the lowest line of figure for calculating $\bar{y}$ and left line of figure for calculating $\bar{x}$.

22

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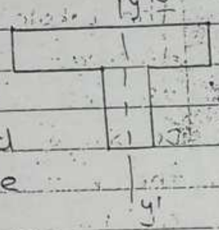
Date / /
Page

Symmetrical & Asymmetrical object:

An object that can be divided into two equal parts with the help of imaginary axis is known as symmetrical object, else it is known as asymmetrical object.

Axis of symmetry: The axis which divides a line or an area or a volume in two identically equal parts is known as axis of symmetry. In the homogeneous case the weight on one side of an axis of symmetry equal that on the other side and hence C.G lies on the axis of symmetry.

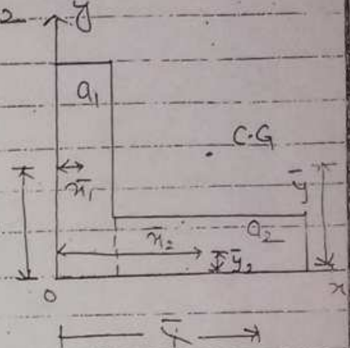


General formula for the Determination of centroid of composite figure

→ centroid of various types of built-up section eg L-section, T-section, channel section, I-section can be obtained by using simple formula.

Consider a L-section as shown in figure. Here, the total area

of L-section $a = a_1 + a_2$



Then,

first moment of whole area about y-axis
= summation of first moments of area about y-axis of two rectangles

ie. $a\bar{x} = a_1\bar{x}_1 + a_2\bar{x}_2$

or, $\bar{x} = \dfrac{a_1\bar{x}_1 + a_2\bar{x}_2}{a}$

or, $\bar{x} = \dfrac{a_1\bar{x}_1 + a_2\bar{x}_2}{a_1 + a_2}$

The generalize form will be,

$$\bar{x} = \frac{\sum_{i=1}^{n} a_i\bar{x}_i}{\sum_{i=1}^{n} a_i}$$

Similarly,

$$\bar{y} = \frac{a_1\bar{y}_1 + a_2\bar{y}_2}{a_1 + a_2}$$

The generalize form will be

$$\bar{y} = \frac{\sum_{i=1}^{n} a_i\bar{y}_i}{\sum_{i=1}^{n} a_i}$$

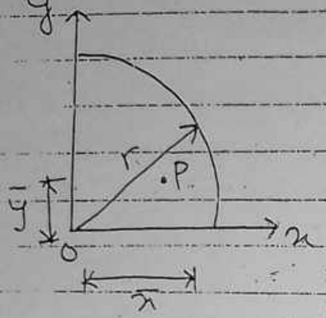
⊛ Centroid of common Shapes of lines:

a) Quarter circular arc:

$\bar{x} = \dfrac{2r}{\pi}$

$\bar{y} = \dfrac{2r}{\pi}$

length $= \dfrac{\pi r}{2}$

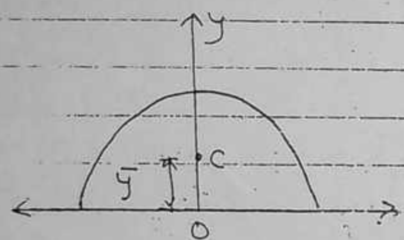


b) Semi Circular arc:

$\bar{x} = 0$

$\bar{y} = \dfrac{2r}{\pi}$

length $= \pi r$



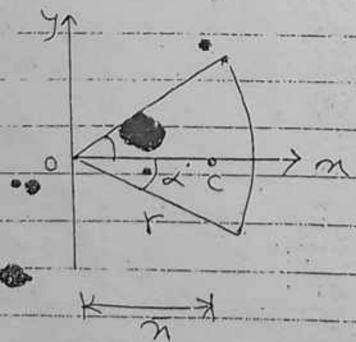
c) Arc of a circle:

$\bar{x} = \dfrac{r\sin\alpha}{\alpha}$

Where, 'α' in radian

$\bar{y} = 0$

length $= 2\alpha r$



⊛ centroid of common shape of area.

a) Quarter circular area:

$\bar{x} = \dfrac{4r}{3\pi}$

$\bar{y} = \dfrac{4r}{3\pi}$

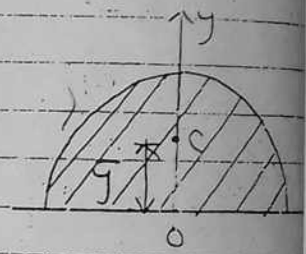
Area $= \dfrac{\pi r^2}{4}$

b) Semi circular area:

$\bar{x} = 0$

$\bar{y} = \dfrac{4r}{3\pi}$

Area $= \dfrac{\pi r^2}{2}$

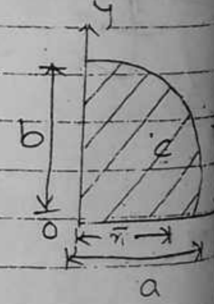


c) Quarter elliptical area:

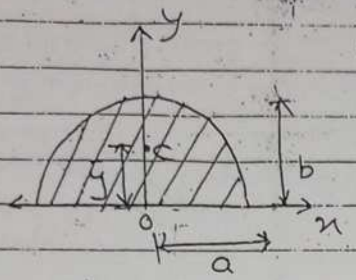
$\bar{x} = \dfrac{4a}{3\pi}$

$\bar{y} = \dfrac{4b}{3\pi}$

Area $= \dfrac{\pi ab}{4}$



d) Semi elliptical area:

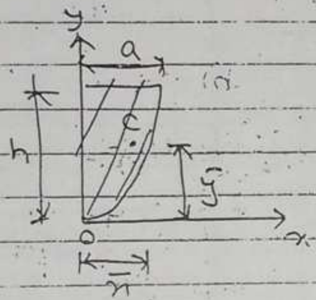


$\bar{x} = 0$

$\bar{y} = \frac{4b}{3\pi}$

Area $= \frac{\pi ab}{2}$

e) Semi parabolic area:

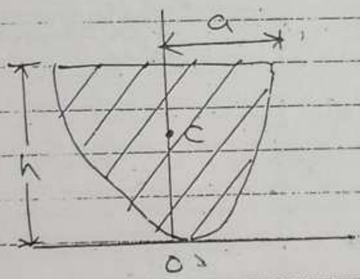


$\bar{x} = \frac{3a}{8}$

$\bar{y} = \frac{3h}{5}$

Area $= \frac{2ah}{3}$

f) Parabolic area:

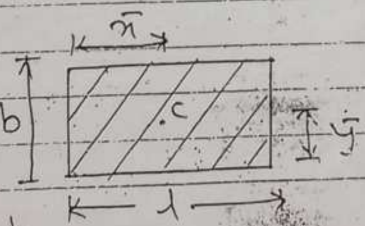


$\bar{x} = 0$

$\bar{y} = \frac{3h}{5}$

Area $= \frac{4ah}{3}$

g) Rectangle.

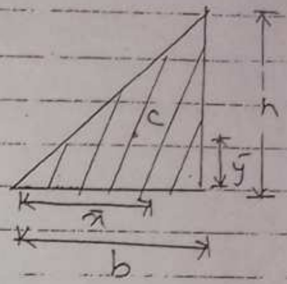


$\bar{x} = l/2$

$\bar{y} = b/2$

Area $= l \times b$

h) Triangle:

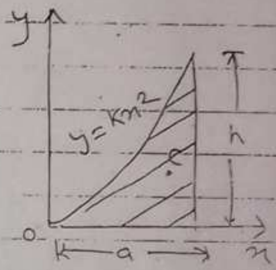


$\bar{x} = \frac{2b}{3}$

$\bar{y} = b/3$

Area $= \frac{1}{2} \times b \times h$

i) Parabolic spendrel:

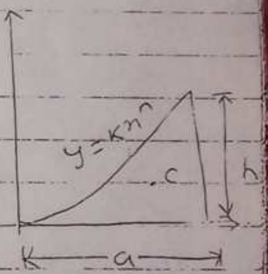


$\bar{x} = \frac{3a}{4}$

$\bar{y} = \frac{3h}{10}$

Area $= \frac{ah}{3}$

j) General spendrell

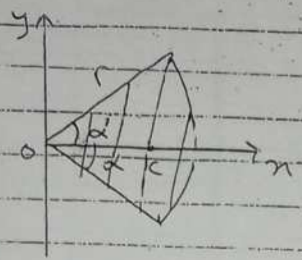


$\bar{x} = \frac{(n+1)a}{(n+2)}$

$\bar{y} = \frac{(n+1)h}{(4n+2)}$

Area $= \frac{ah}{(n+1)}$

k) Circular sector:

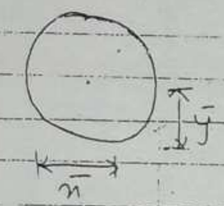


$$\bar{x} = \frac{2r\sin\alpha}{3\alpha}$$

$$\bar{y} = 0$$

$$\text{Area} = \alpha r^2$$

l) Circle:



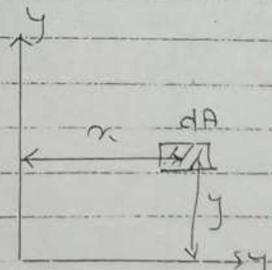
$$\bar{x} = r = d/2$$

$$\bar{y} = r = d/2$$

$$\text{Area} = \pi r^2 = \frac{\pi d^2}{4}$$

First Moment of Area:

The first moment of an element about any axis is the product of area of element and perp^n distance between the element and axis as shown.



ie. $dM_x = y \cdot dA$

$dM_y = x \cdot dA$

For finite area, Integrating above eq^n,

$$M_x = \int y \cdot dA$$

$$M_y = \int x \cdot dA$$

Second Moment of Area:

The product of area and its distance from an axis is known as first moment of area. If the quantity is again multiplied by the same distance, then the quantity obtained is called second moment of area.

First moment of area about y-axis $= \int x\,dA$

" " " " " x-axis $= \int y\,dA$

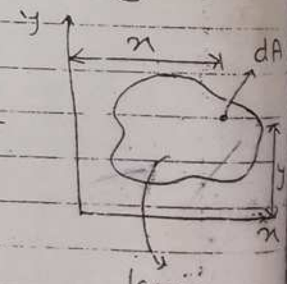
Second " " " " y-axis $= \int x^2 dA$

" " " " " x-axis $= \int y^2 dA$

Moment of Inertia:

The moment of Inertia (MoI) of any lamina is the second moment of all elemental areas dA comprising the lamina.

From figure,



I_{xx} = Moment of inertia about x-axis

or $I_{xx} = \Sigma (y\,dA) \cdot y$

or $I_{xx} = \Sigma y^2\,dA$

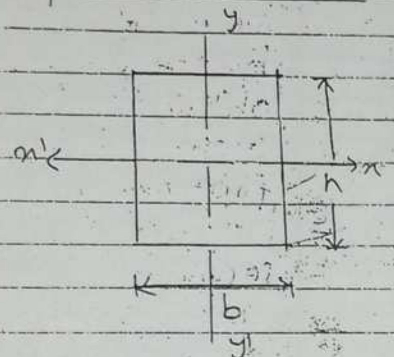
Similarly,

I_{yy} = Moment of inertia about y-axis

$= \Sigma x^2 dA$.

⊛ Moment of Inertia of common areas:

a) Rectangle:

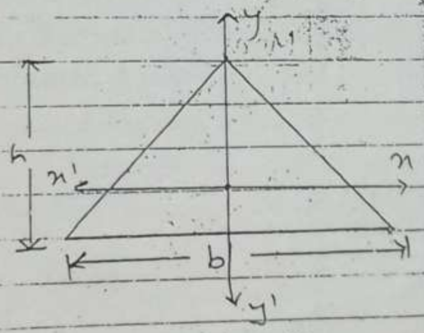


MOI about centroidal axis

$$I_{xx} = \frac{bh^3}{12}$$

$$I_{yy} = \frac{hb^3}{12}$$

b) Triangle

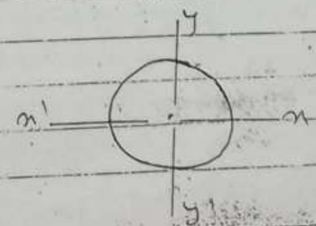


MOI about centroidal axis

$$I_{xx} = \frac{bh^3}{36}$$

$$I_{yy} = \frac{hb^3}{36}$$

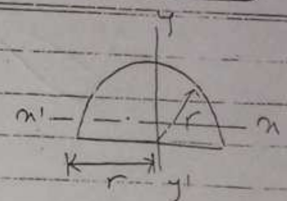
c) Circle:



MOI about centroidal axis

$$I_{xx} = I_{yy} = \frac{\pi d^4}{64}$$

d) Semi circle:

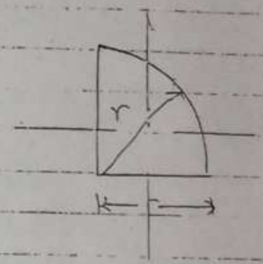


MOI about centroidal axis

$$I_{yy} = \frac{\pi r^4}{8}$$

$$I_{xx} = 0.11 r^4$$

e) Quarter circle:



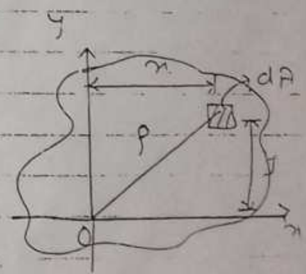
MOI about centroidal axis

$$I_{xx} = 0.055 r^4$$

$$I_{yy} = \frac{\pi r^4}{16}$$

Polar Moment of Inertia:

→ The moment of inertia taken from an axis perpr to the plane on which an object lies is called polar moment of inertia. Let 'ρ' be the distance from z-axis to an element, then



Polar Moment of Inertia $(I_{zz}) = \int \rho^2 dA$.

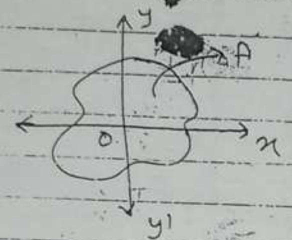
— or, $I_p = \int \rho^2 dA$ where, $\rho^2 = x^2 + y^2$

Date
Page

Radius of Gyration (K):

⇒ Radius of gyration is defined as the distance from the given axis at which all the elemental parts of the lamina would have to be place so that moment of inertia about the given axis is not changed.

consider a plane area as shown in fig. ⓐ and having moment of inertia I_x and I_y. let us replace this area by a strip and place to the axis at the distance K_x from the x-axis, such that the replaced area should have the same moment of inertia as that of plane lamina.



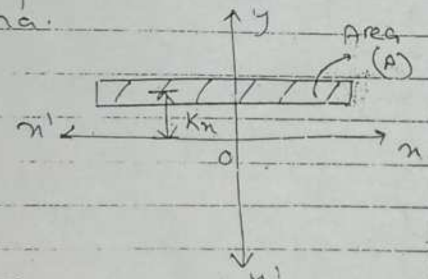


fig. (b)

$$I_x = K_x^2 A$$

or, $K_x = \sqrt{\frac{I_x}{A}}$

Similarly,

$$K_y = \sqrt{I_y/A}$$

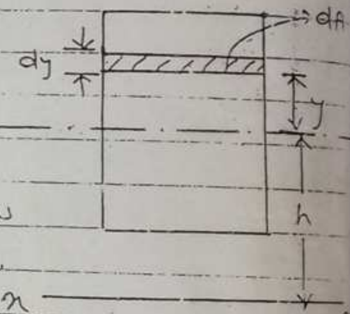
The distance K_x & K_y is known as radius of gyration of the area w.r.to x-axis and y-axis resp.

Date
Page

Parallel Axis Theorem:

→ It states that "the moment of inertia of a plane area about any axis parallel to the centroidal axis is equal to the sum of moment of inertia about a parallel centroidal axis and the product of the area and square of distance between the two axes."

let us consider a strip of elemental area dA at a distance 'y' from the centroidal axis as shown in the figure.



Then.

$$I_{xx} = \Sigma dA\, y^2$$

and $I_{xx'} = \Sigma dA\,(y+h)^2$

or $I_{xx'} = \Sigma dA\,(y^2 + 2hy + h^2)$

or, $I_{xx'} = \Sigma dA\, y^2 + \Sigma dA\, 2hy + \Sigma dA\, h^2$

or, $I_{xx'} = I_{xx} + 0 + Ah^2$

$= I_{xx} + Ah^2$ $\left[\because 2h \int dA \cdot y = 0 \right]$ First Moment of area about centroidal axis = 0

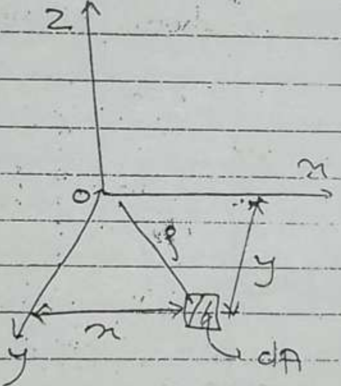
Hence proved

Perpendicular Axis Theorem:

It states that "the moment of inertia of a plane area about the axis perpn to the plane and passing through the intersection of other two axes x-x and y-y contained by the plane, is equal to the sum of the moment of inertia about x-x & y-y axes."

i.e. $I_{zz} = I_{xx} + I_{yy}$ ⟶ ①

Consider an elementary area dA at a distance of x and y from y-axis and x-axis resp. Then the distance of the elementary area from z-z axis is given by



$$\rho^2 = x^2 + y^2$$

Now,

$$I_{zz} = \int \rho^2 \, dA$$

or, $I_{zz} = \int (x^2 + y^2) \, dA$

or, $I_{zz} = \int x^2 dA + \int y^2 dA$

or, $I_{zz} = I_{yy} + I_{xx}$

$\therefore I_{zz} = I_{xx} + I_{yy}$

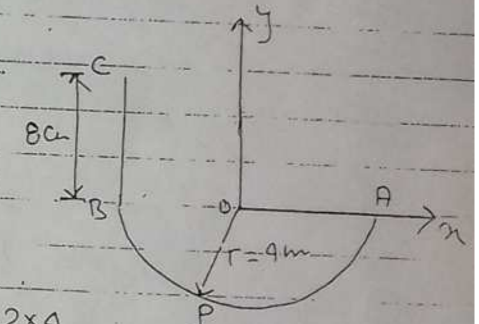
Where,

$\int x^2 dA = I_{yy}$

$\int y^2 dA = I_{xx}$

Numericals:

① Locate the centroid of the wire w.r.t. the given axes.



⟹ For portion $\overset{\frown}{APB}$

$l_1 = \pi r = \pi \times 4$

or, $l_1 = 12.566$ cm

$\bar{x}_1 = 0$

$\bar{y}_1 = -\frac{2r}{\pi} = \frac{-2 \times 4}{\pi} = -2.546$ cm.

For portion BC

$l_2 = 8$ cm, $\bar{x}_2 = -4$ cm, $\bar{y}_2 = \frac{8}{2} =$

Date / /
Page

$\therefore \bar{x} = \dfrac{l_1\bar{x}_1 + l_2\bar{x}_2}{l_1+l_2}$

or, $\bar{x} = \dfrac{12.566 \times 0 + 8\times(-4)}{12.566+8} = -1.556$ cm

And

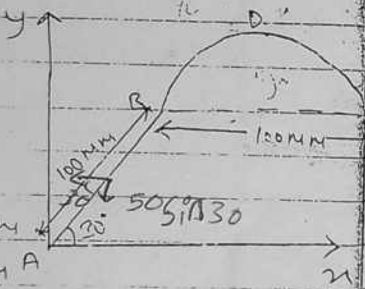
$\bar{y} = \dfrac{l_1\bar{y}_1 + l_2\bar{y}_2}{l_1+l_2}$

or, $\bar{y} = \dfrac{12.566\times(-2.546) + 8\times 4}{12.566+8} = 0$

Hence, coordinate of centroid $(\bar{x},\bar{y}) = (-1.556, 0)$

2) Locate the centroid of the bent wire as shown in figure.

=> Here,



considering portion AB.

$l_1 = 100$ mm

$\bar{x}_1 = 50\cos 30° = 43.30$ mm

$\bar{y}_1 = 50\sin 30° = 25$ mm

Considering portion $\overset{\frown}{BDC}$

length $(l_2) = \pi r = 50\pi = 157.07$ mm

$\bar{x}_2 = 100\cos 30° + 50 = 136.6$ mm

$\bar{y}_2 = 100\sin 30° + 31.83 = 81.83$ mm

Then,

Date / /
Page

centroid of the bent wire

$\bar{x} = \dfrac{l_1\bar{x}_1 + l_2\bar{x}_2}{l_1+l_2} = \dfrac{100\times 43.30 + 157.07\times\ldots}{100+157.07}$

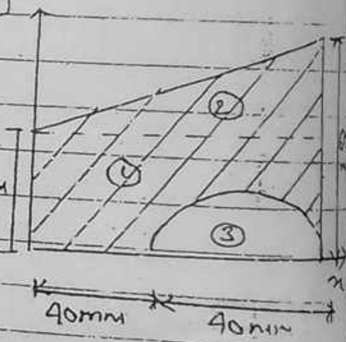
or, $\bar{x} = 100.10$ mm

$\bar{y} = \dfrac{l_1\bar{y}_1 + l_2\bar{y}_2}{l_1+l_2} = \dfrac{100\times 25 + 157\times 81.83}{100+157.07}$

or, $\bar{y} = 59.72$ mm

3) Locate the centroid of the following composite area as shown in figure.

=> This composite figure can be split into three parts as shown in figure.



For rectangle:

$a_1 = 30\times 80 = 2400$ mm^2

$\bar{x}_1 = 80/2 = 40$ mm

$\bar{y}_1 = 30/2 = 15$ mm

For triangle:

$a_1 = \frac{1}{2}\times 80\times 30 = 1200$ mm^2

$\bar{x}_2 = \frac{2}{3}\times 80 = 53.33mm$

$\bar{y}_2 = 30 + \frac{30}{3} = 40mm$

For semicircle (hole)

$a_3 = \frac{\pi r^2}{2} = \frac{\pi \times 20^2}{2} = 628.32\ mm^2$

$\Rightarrow x_3 = 40 + 40/2 = 60mm$

$\bar{y}_3 = \frac{4r}{3\pi} = \frac{4\times 20}{3\pi} = 8.49mm.$

Then,

$\bar{X} = \frac{a_1\bar{x}_1 + a_2\bar{x}_2 - a_3\bar{x}_3}{a_1 + a_2 - a_3}$

or, $\bar{x} = \frac{2400\times 40 + 1200\times 53.33 - 628.32\times 60}{2400 + 1200 - 628.32}$

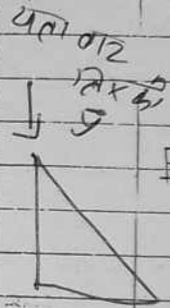
or, $\bar{x} = 91.15mm$

And,

$\bar{Y} = \frac{a_1\bar{y}_1 + a_2\bar{y}_2 - a_3\bar{y}_3}{a_1 + a_2 - a_3}$

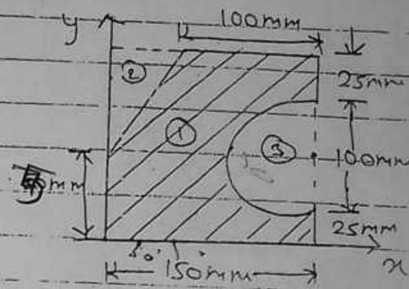
or, $\bar{y} = \frac{2400\times 15 + 1200\times 40 - 628.32\times 8.49}{2400 + 1200 - 628.32}$

or, $\bar{y} = 26.47\ mm$



4. locate the centroid of the area shown in figure.

$\Rightarrow$ Here, the given composite figure can be splitted into three parts.



For Rectangle:

$a_1 = 150\times 150 = 22500mm^2$

$\bar{x}_1 = \frac{150}{2} = 75mm$

$\bar{y}_1 = \frac{150}{2} = 75mm$

For triangle:

$a_2 = \frac{1}{2}\times 50\times 100 = 2500mm^2$

$\bar{x}_2 = 50/3 = 16.67mm$

$\bar{y}_2 = 50 + \frac{2\times 100}{3} = 116.67mm$

For semi sphere:

$a_3 = \frac{\pi r^2}{2} = \frac{\pi\times 50^2}{2} = 3926.99\ mm^2$

$\bar{x}_3 = 150 - \frac{4r}{3\pi} = 150 - \frac{4\times 50}{3\pi} = 128.77mm$

$\bar{y}_3 = 25 + 50 = 75mm$

Then,

$\bar{X} = \frac{a_1\bar{x}_1 - a_2\bar{x}_2 - a_3\bar{x}_3}{a_1 - a_2 - a_3}$

or, $\bar{x} = \frac{22500\times75 - 2500\times116.67 - 3926.99\times128.77}{22500 - 2500 - 3926.99}$

or, $\bar{x} = 70.99$ mm

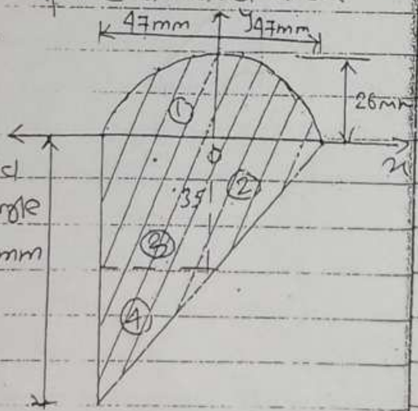
And,

$\bar{y} = \frac{a_1\bar{y}_1 - a_2\bar{y}_2 - a_3\bar{y}_3}{a_1 - a_2 - a_3}$

or, $\bar{y} = \frac{22500\times75 - 2500\times116.67 - 3926.99\times75}{22500 - 2500 - 3926.99}$

or, $\bar{y} = 68.52$ mm.

(2013) Fall (2012) Fall ⑤ Locate the centroid of the shaded area as shown in figure.



=> Here,

The given composite figure can be divided into semiellipse, rectangle and triangle

For semi ellipse: ①

$a_1 = \frac{\pi ab}{2}$ where, $a = 47$mm, $b = 26$ mm

or, $a_1 = \frac{\pi\times47\times26}{2} = 1919.51$ mm²

$\bar{x}_1 = 0$, $\bar{y}_1 = \frac{4b}{3\pi} = \frac{4\times26}{3\pi} = 11.$

For triangle: ②

$a_2 = \frac{1}{2}\times b\times h = \frac{1}{2}\times47\times35$

or, $a_2 = \cancel{3290\,mm^2}$ 822.5 mm²

$\bar{x}_2 = 47/3 = 15.67$ mm

$\bar{y}_2 = -35/3 = -11.67$

For rectangle: ③

$a_3 = 47\times35 = 1645$ mm²

$\bar{x}_3 = -47/2 = -23.5$ mm

$\bar{y}_3 = -\frac{35}{2} = -17.5$ mm

For triangle: ④

$a_4 = \frac{1}{2}\times47\times35 = 822.5$ mm²

$\bar{x}_4 = -\frac{2}{3}\times47 = -31.33$

यताबाट लिएको $\bar{y}_4 = -(35 + 35/3) = -46.67$.

Now,

$\bar{x} = \frac{a_1\bar{x}_1 + a_2\bar{x}_2 + a_3\bar{x}_3 + a_4\bar{x}_4}{a_1 + a_2 + a_3 + a_4}$

or, $\bar{x} = \frac{1919.51\times0 + 822.5\times15.67 + 1645\times(-23.5) + 822.5\times(-31.}{1919.51 + 822.5 + 1645 + 822.5}$

or, $\bar{x} = -9.89$ mm

and

$$\bar{y} = \frac{a_1\bar{y}_1 + a_2\bar{y}_2 + a_3\bar{y}_3 + a_4\bar{y}_4}{a_1 + a_2 + a_3 + a_4}$$

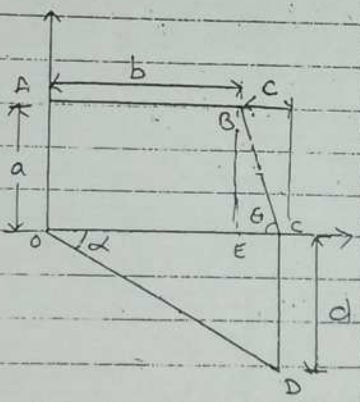
or, $$\bar{y} = \frac{1919.51\times 11.03 + 822.5\times(-11.67) + 1645\times(-17.5) + 822.5\times(-46.67)}{1919.51 + 822.5 + 1645 + 822.5}$$

or, $\bar{y} = -10.67$ mm.

(2010)P (2013)Fall

(6) Locate the centroid of the composite line shown where a = b = d = 60cm and c = 30cm.

⇒ Here,



For line OA

$l_1 = 60$cm

$\bar{x}_1 = 0$

$\bar{y}_1 = \frac{60}{2} = 30$cm.

For line AB

$l_2 = 60$cm

$\bar{x}_2 = \frac{60}{2} = 30$cm

$\bar{y}_2 = 60$cm.

For line BC

$l_3 = \sqrt{30^2 + 60^2} = 67.08$cm

$\bar{x}_3 = 90 - 15 = 75$ cm.

$\bar{y}_3 = 67.08\times 0.5\times\sin 63.43° = 30$ cm.

For line CD

[In rt. △BCE

$\sin\theta = \frac{AO}{BC}$

$\theta = \sin^{-1}\left[\frac{60}{67.08}\right]$

$\theta = 63.43°$]

$l_4 = 60$cm

$\bar{x}_4 = 60 + 30 = 90$cm

$\bar{y}_4 = -\frac{60}{2} = -30$cm.

For line OD

$l_5 = \sqrt{90^2 + 60^2} = 108.16$cm.

$\bar{x}_5 = \frac{108.16}{2}\times\cos 41.81°$

$\bar{x}_5 = 40.30$cm.

[In rt. △OCD

$\sin\alpha = \frac{CD}{OC}$

$\alpha = \sin^{-1}\left(\frac{60}{90}\right)$

$\alpha = 41.81°$]

$\bar{y}_5 = \frac{108.16}{2}\times\sin 41.81$

$\bar{y}_5 = 36.05$cm

Now,

$$\bar{X} = \frac{l_1\bar{x}_1 + l_2\bar{x}_2 + l_3\bar{x}_3 + l_4\bar{x}_4 + l_5\bar{x}_5}{l_1 + l_2 + l_3 + l_4 + l_5}$$

$$\bar{X} = \frac{60\times 0 + 60\times 30 + 67.08\times 75 + 60\times 90 + 108.16\times 40.30}{60 + 60 + 67.08 + 60 + 108.16}$$

$\bar{X} = 46.7$ cm

and

$$\bar{y} = \frac{A_1\bar{y}_1 + A_2\bar{y}_2 + A_3\bar{y}_3 + A_4\bar{y}_4 + A_5\bar{y}_5}{A_1 + A_2 + A_3 + A_4 + A_5}$$

$$\bar{y} = \frac{60\times30 + 60\times60 + 67.08\times30 + 60\times(-30) + 108.16\times36}{60 + 60 + 67.08 + 60 + 108.16}$$

or, $\bar{y} = 26.76$ cm

(2011) Fall (2011) SP (2014) Fall

7. Find the Centroid of the Shaded area by the method of integration.

⇒ Here,

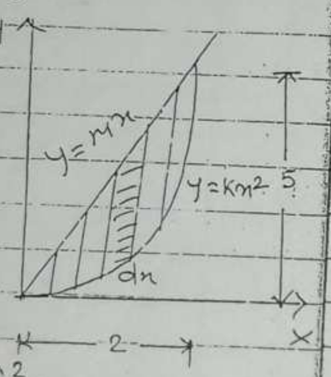
$y = mx$

or, $m = y/x = 5/2$

and, $y = kx^2$

or, $k = y/x^2$

or, $k = 5/2^2 = 5/4$



∴ Required Area $(A) = \int_0^2 dA$

or, $A = \int_0^2 (y_1 - y_2)\,dx$

or, $A = \int_0^2 (mx - kx^2)\,dx$

or, $A = \int_0^2 \frac{5}{2}x\,dx - \int_0^2 \frac{5}{4}x^2\,dx$

or, $A = \frac{5}{2}\left[\frac{x^2}{2}\right]_0^2 - \frac{5}{4}\left[\frac{x^3}{3}\right]_0^2$

or, $A = \frac{5}{2}\times 2 - \frac{5}{4}\times\frac{8}{3}$

or, $A = 5 - 10/3 = 5/3$ Sq. unit.

Now,

$$\bar{X} = \frac{1}{A}\int x\,dA = \frac{1}{A}\int x.y.dx$$

or, $\bar{X} = \frac{1}{A}\int_0^2 (y_1 - y_2)\,x\,dx$

or, $\bar{X} = \frac{1}{A}\int_0^2 (mx - kx^2)\,x\,dx$

or, $\bar{X} = \frac{1}{A}\left[\int_0^2 mx^2\,dx - \int_0^2 kx^3\,dx\right]$

or, $\bar{X} = \frac{3}{5}\left[\int_0^2 \frac{5}{2}x^2\,dx - \int_0^2 \frac{5}{4}x^3\,dx\right]$

or, $\bar{X} = \frac{3}{5}\left[\frac{5}{2}\left[\frac{x^3}{3}\right]_0^2 - \frac{5}{4}\left[\frac{x^4}{4}\right]_0^2\right]$

or, $\bar{X} = \frac{3}{5}\left[\frac{5}{2}\times\frac{8}{3} - \frac{5}{4}\times 4\right]$

or, $\bar{X} = \frac{3}{5}\left[\frac{20}{3} - 5\right]$

or, $\bar{X} = 1$ unit.

$\bar{y} = \frac{3}{5}\int_0^2 \frac{(mx+kx^2)(mx-kx^2)}{2}\,dx$

Again,

$\bar{y} = \frac{1}{A}\int y\,dA = \frac{1}{A}\int y.x\,dy$

$= 2$ unit

or, $\bar{y} = \frac{1}{A}\int_0^5 (x_2 - x_1)\,dy\,dy$

or, $\bar{y} = \frac{3}{5}\left[\int_0^5 y\cdot(y/k)^{1/2}\,dy - \int_0^5 \frac{y^2}{m}\,dy\right]$

or, $\bar{y} = \frac{3}{5}\left[\int_0^5 \frac{y^{3/2}}{k^{1/2}}\,dy - \int_0^5 \frac{y^2}{m}\,dy\right]$

or, $\bar{y} = \frac{3}{5}\left[\int_0^5 \frac{y^{3/2}}{(5/4)^{1/2}}\,dy - \int_0^5 \frac{y^2}{5/2}\,dy\right]$

or, $\bar{y} = \frac{3}{5}\left[\frac{1}{(5/4)^{1/2}}\left[\frac{y^{5/2}}{5/2}\right]_0^5 - \frac{1}{5/2}\left[\frac{y^3}{3}\right]_0^5\right]$

or, $\bar{y} = \frac{3}{5}\left[\frac{1}{(5/4)^{1/2}}\,\frac{5^{5/2}}{5/2} - \frac{1}{5/2}\times\frac{5^3}{3}\right]$

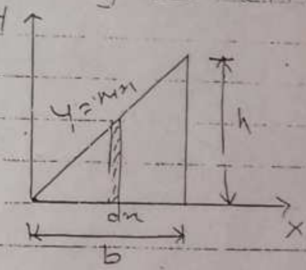
or, $\bar{y} = \frac{3}{5}\left[\frac{1}{\sqrt{5}/2}\times\frac{25\sqrt{5}}{5/2} - \frac{2}{5}\times\frac{125}{3}\right]$

or, $\bar{y} = \frac{3}{5}\left[2\times\frac{25\times 2}{5} - \frac{50}{3}\right]$

or, $\bar{y} = 2$ unit.

(2007) SP

(5) Locate the centroid of the area bounded by the line as shown in figure by the method of integration.



=> Here,

$y = mx$

or, $m = y/x = \frac{h}{b}$

$\therefore A = \int_0^b dA = \int_0^b y\cdot dx$

or, $A = \int_0^b mx\,dx = \int_0^b \frac{h}{b} x\,dx = \frac{h}{b}\left[\frac{x^2}{2}\right]_0^b$

or, $A = \frac{bh}{2}$

$\therefore$ Area $(A) = \frac{bh}{2}$

Again,

$$\bar{x} = \frac{1}{A}\int x\cdot dA = \frac{1}{A}\int_0^b x\cdot y\,dx$$

or, $\bar{x} = \frac{2}{bh}\int_0^b mx\cdot x\,dx = \frac{2}{bh}\int_0^b m x^2 dx$

or, $\bar{x} = \frac{2}{bh} \times \frac{h}{b}\left[\frac{x^3}{3}\right]_0^b$

or, $\bar{x} = \frac{2}{b^2} \times \frac{b^3}{3} = \frac{2}{3}\times b.$

Also,

$$\bar{y} = \frac{1}{A}\int y\cdot dA = \frac{1}{A}\int_0^b \cancel{y\cdot x\,dy}\ \frac{y}{2}\cdot dx$$

~~or, $\bar{y} = \frac{2}{bh}\int_0^b y\cdot\frac{y}{2}\,dy = \frac{2}{bh}\int_0^h \frac{y^2}{2}\,dy$~~

~~or, $\bar{y} = \frac{2}{bh} \times \frac{1}{2}\left[\frac{y^3}{3}\right]_0^b = \frac{2}{bh} \times \frac{b^3}{2\times 3}$~~

or, $\bar{y} = \frac{1}{A}\int_0^b \frac{y^2}{2}\,dx = \frac{1}{2A}\int_0^b m^2 x^2\,dx$

or, $\bar{y} = \frac{\cancel{2}}{bh\times\cancel{2}} \times \frac{h^2}{b^2}\left[\frac{x^3}{3}\right]_0^b$

or, $\bar{y} = \frac{h}{b^3} \times \frac{b^3}{3} = h/3$

(2) Locate the centroid of the shaded area by the method of integration.

$\Rightarrow$ Here,

$4y^2 = 9x$

~~or, $y = \frac{3}{2}\sqrt{x}$~~

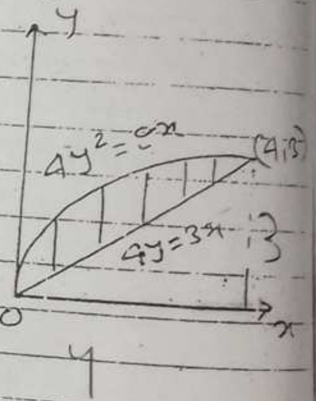
i.e. $y_1 = \frac{3}{2}\sqrt{x} \longrightarrow$ (i)

and,

$4y = 3x$

or, $y = \frac{3}{4}x$

or, $y_2 = 3/4\,x \longrightarrow$ (ii)



Total area $(A) = \int_0^4 dA = \int_0^4 y\cdot dx$

or, $A = \int_0^4 (y_1 - y_2)\,dx = \int_0^4 \left(\frac{3}{2}\sqrt{x} - 3/4\,x\right)dx$

or, $A = \frac{3}{2}\left[\frac{x^{3/2}}{3/2}\right]_0^4 - \frac{3}{4}\left[\frac{x^2}{2}\right]_0^4$

or, $A = 8 - 6 = 2$ sq. unit.

Now,

$$\bar{X} = \frac{1}{A}\int x\, dA = \frac{1}{A}\int_0^4 x.y.dA$$

or, $\bar{X} = \frac{1}{A}\int_0^4 \left(\frac{3}{2}\sqrt{x} - \frac{3}{4}x\right)x\, dx$

or, $\bar{X} = \frac{1}{2}\left[\frac{3}{2}\int_0^4 x\sqrt{x}\, dx - \frac{3}{4}\int_0^4 x^2 dx\right]$

or, $\bar{X} = \frac{1}{2}\left[\frac{3}{2}\times\left[\frac{x^{5/2}}{5/2}\right]_0^4 - \frac{3}{4}\left[\frac{x^3}{3}\right]_0^4\right]$

or, $\bar{x} = \frac{1}{2}\left[\frac{3}{2}\times\frac{4^{5/2}}{5/2} - \frac{3}{4}\times\frac{4^3}{3}\right]$

or, $\bar{X} = 8/5$

Again,

$$\bar{y} = \frac{1}{A}\int y.dA = \frac{1}{A}\int_0^4 y.\frac{(y_1+y_2)}{2}\, dx$$

$$\bar{y} = \frac{1}{A}\int_0^4 \left(\frac{3}{2}\sqrt{x} - \frac{3}{4}x\right)\times\frac{\left(\frac{3}{2}\sqrt{x} + \frac{3}{4}x\right)}{2}\, dx$$

$$\bar{y} = \frac{1}{2}\times\frac{1}{2}\int_0^4 \left(\frac{9x}{4} - \frac{9x^2}{16}\right) dx$$

$$\bar{y} = \frac{1}{4}\times\left[\frac{9}{4}\left[\frac{x^2}{2}\right]_0^4 - \frac{9}{16}\left[\frac{x^3}{3}\right]_0^4\right]$$

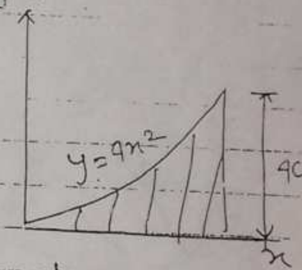
$$\bar{y} = 3/2$$

(2008) Fall

(10) Locate the Centroid of the area under the curves as shown by using Integration Method.

⇒ Here,

$y = 4x^2 \longrightarrow (1)$



Area (A) = $\int dA$

or, $A = \int_0^1 y\, dx$

or, $A = \int_0^1 4x^2 dx$

or, $A = 4\left[\frac{x^3}{3}\right]_0^1 = 4/3$ sq. unit.

Putting $y=4$ in $y = 4x^2$

$\therefore x = \pm 1$.

Date / /
Page

Now,

$\bar{x} = \frac{1}{A}\int \cancel{A}\cdot x\, dA = \frac{1}{A}\int_0^1 x\cdot y\cdot dx$

or, $\bar{x} = \frac{3}{4}\int_0^1 x\cdot 4x^2\, dx = \frac{3}{4}\int_0^1 4x^3\, dx$

or, $\bar{x} = \frac{3}{\cancel{4}}\times \cancel{4}\cdot\left[\frac{x^4}{4}\right]_0^1 = 3/4$ unit

Again,

$\bar{y} = \frac{1}{A}\int y\cdot dA = \frac{1}{A}\int_0^1 y\cdot y/2\, dx$

or, $\bar{y} = \frac{3}{4}\int_0^1 \frac{4x^2\cdot 4x^2}{2}\, dx$

or, $\bar{y} = 6\left[\frac{x^5}{5}\right]_0^1 = 6/5$ Unit.

(2007) Fall
(11) Locate the centroid of the area under the curve as shown in fig by using integration method.

=> Here,

$y^2 = x$

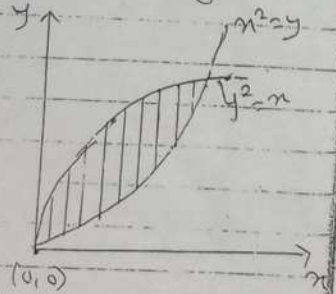
i.e. $y_1 = \sqrt{x}$ → (i)

and

$y = x^2$

or, $y = x^2$

i.e. $y_2 = x^2$ → (ii)



Area of shaded portion (A) = $\int dA$

or, $A = \int_0^1 y\, dx$

or, $A = \int_0^1 (y_1 - y_2)\, dx$

or, $A = \int_0^1 (\sqrt{x} - x^2)\, dx$. Where, $y = y_1 - y_2$

or, $A = \left[\frac{x^{3/2}}{3/2}\right]_0^1 - \left[\frac{x^3}{3}\right]_0^1$

or, $A = \frac{2}{3} - \frac{1}{3} = 1/3$ sq. unit

Solving $y^2 = x$ & $x^2 = y$ we get $x =$ $y =$

Now

$\bar{x} = \frac{1}{A}\int x\cdot dA = \frac{1}{A}\int_0^1 x\cdot y\cdot dx$

or, $\bar{x} = 3\int_0^1 (\sqrt{x} - x^2)\, x\, dx$

or, $\bar{x} = 3\times\left[\left(\frac{x^{5/2}}{5/2}\right)_0^1 - \left(\frac{x^4}{4}\right)_0^1\right]$

or, $\bar{x} = 3\times\left[\frac{2}{5} - \frac{1}{4}\right]$

or, $\bar{x} = 3\times\left[\frac{8-5}{20}\right] = \frac{9}{20}$ unit.

curve and line को लागि
X को लागि Y ''
X → x limit
Y → $\frac{y}{2}dx$
x-axis को Y limit
Y '' x को limit

or,
or,
or, $\bar{y}$
or, $\bar{y}$
(2008) Sp
(12) Determi... centroi... figure.
=> Here,
For ...
$a_1 = 4$...
$\bar{x}_1 =$
$\bar{y}_1 = 5$...

$$\bar{y} = \frac{1}{A}\int y \cdot dA = \frac{1}{A}\int_0^1 y\left(\frac{y_1+y_2}{2}\right) dx$$

$$\text{or, } \bar{y} = 3\int_0^1 (\sqrt{x}-x^2)\left(\frac{\sqrt{x}+x^2}{2}\right) dx$$

$$\text{or, } \bar{y} = \frac{3}{2}\int_0^1 (x-x^4)\, dx$$

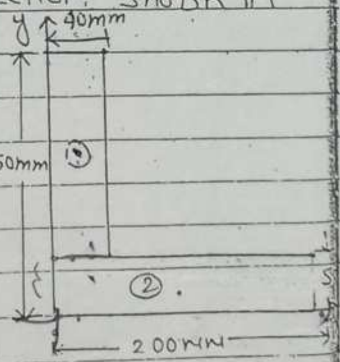
$$\text{or, } \bar{y} = \frac{3}{2}\left[\left[\frac{x^2}{2}\right]_0^1 - \left[\frac{x^5}{5}\right]_0^1\right]$$

$$\text{or, } \bar{y} = \frac{3}{2}\left[\frac{1}{2} - \frac{1}{5}\right]$$

$$\text{or, } \bar{y} = \frac{3}{2} \times \left(\frac{5-2}{10}\right) = \frac{3}{2}\times\frac{3}{10} = \frac{9}{20}$$

(2006) P

(2) Determine the moment of inertia about the centroidal axis of the section shown in figure.



=) Here,

For rectangle (1)

$a_1 = 40 \times 200 = 8000\, mm^2$

$\bar{x}_1 = \frac{40}{2} = 20\, mm$

$\bar{y}_1 = 50 + \frac{200}{2} = 150\, mm$

For rectangle (2)

$a_2 = 200 \times 50 = 10000\, mm^2$

$\bar{x}_2 = \frac{200}{2} = 100\, mm$

$\bar{y}_2 = 50/2 = 25\, mm$

Then, $\bar{x} = \frac{a_1\bar{x}_1 + a_2\bar{x}_2}{a_1+a_2}$

or, $\bar{X} = \frac{8000\times 20 + 10000\times 100}{8000+10000} = 64.44$

$\bar{y} = \frac{a_1\bar{y}_1 + a_2\bar{y}_2}{a_1+a_2} = \frac{8000\times 150 + 10000\times 25}{8000+10000}$

$\bar{y} = 80.55\, mm$

Then,

$$I_{xx} = \frac{b_1h_1^3}{12} + a_1(\bar{y}_1-\bar{y})^2 + \frac{b_2h_2^3}{12} + a_2(\bar{y}_2-\bar{y})^2$$

$$\text{or, } I_{xx} = \frac{40\times 200^3}{12} + 8000(150-80.55)^2 + \frac{200\times 50^3}{12} + 10000\times(25-80.55)^2$$

or, $I_{xx} = 98194445\, mm^4$

Similarly,

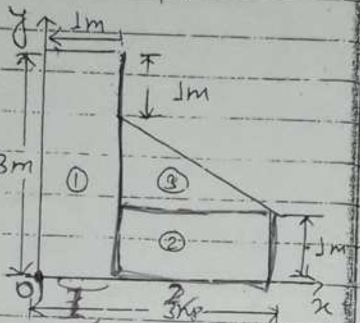
$$I_{yy} = \frac{h_1b_1^3}{12} + a_1(\bar{x}_1-\bar{x})^2 + \frac{h_2b_2^3}{12} + a_2(\bar{x}_2-\bar{x})^2$$

$$\text{or, } I_{yy} = \frac{200\times 40^3}{12} + 8000(20-64.44)^2 + \frac{50\times 200^3}{12} + 10000(100-64.44)^2$$

$\bar{y} = 62844444.8 \ mm^4$

(13) Determine the moment of inertia of the composite figure as shown in figure.

=> Here



For rectangle ①

$a_1 = 3 \times 1 = 3m^2$

$\bar{x}_1 = 1/2 = 0.5m$

$\bar{y}_1 = 3/2 = 1.5m$

For rectangle ②

$a_2 = 2 \times 1 = 2m^2$

$\bar{x}_2 = 1 + 2/2 = 2m$

$\bar{y}_2 = 1/2 = 0.5m$

$\boxed{\text{~~Taking from~~ centre}}$

For triangle ③

$a_3 = \frac{1}{2} \times 2 \times 1 = 1m^2$

$\bar{x}_3 = 1 + 2/3 = 5/3 \ m$

$\bar{y}_3 = 1 + 1/3 = 4/3 \ m$

Now

$$\bar{x} = \frac{a_1\bar{x}_1 + a_2\bar{x}_2 + a_3\bar{x}_3}{a_1 + a_2 + a_3}$$

or, $\bar{x} = \frac{3 \times 0.5 + 2 \times 2 + 1 \times 5/3}{3+2+1} = 1.19m$

$$\bar{y} = \frac{a_1\bar{y}_1 + a_2\bar{y}_2 + a_3\bar{y}_3}{a_1+a_2+a_3} = \frac{3 \times 1.5 + 2 \times 0.5 + ...}{3+2+1}$$

or, $\bar{y} = 1.13m$

Again,

$$I_{xx} = \frac{b_1h_1^3}{12} + a_1(\bar{y}_1 - \bar{y})^2 + \frac{b_2h_2^3}{12} + a_2(\bar{y}_2 ...$$
$$+ \frac{b_3h_3^3}{36} + a_3(\bar{y}_3 - \bar{y})^2$$

or, $I_{xx} = \frac{1 \times 3^3}{12} + 3(1.5 - 1.13)^2 + \frac{2 \times 1^3}{12} + 2 \times (0...$
$+ \frac{2 \times 1^3}{36} + 1 \times (4/3 - 1.13)^2$

or, $I_{xx} = 3.71 \ m^4$

And

$$I_{yy} = \frac{h_1b_1^3}{12} + a_1(\bar{x}_1 - \bar{x})^2 + \frac{h_2b_2^3}{12} + a_2(...$$
$$+ \frac{h_3b_3^3}{36} + a_3(\bar{x}_3 - \bar{x})^2$$

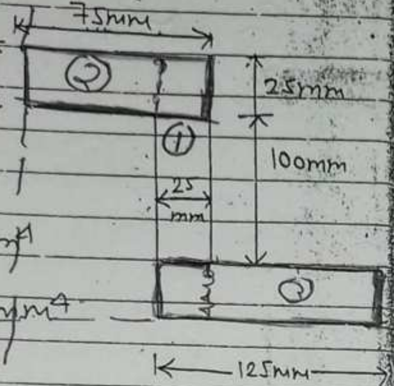
or, $I_{yy} = \frac{3 \times 1^3}{12} + 3 \times (0.5 - 1.19)^2 + \frac{1 \times 2^3}{12}$
$2 \times (2 - 1.19)^2 + \frac{1 \times 2^3}{36} + 1 \times (5/3 ...$

or, $I_{yy} = 4.10 m^4$

Date
Page

2013

(14) Determine the moment of inertia of the given figure.



⇒ Ans:

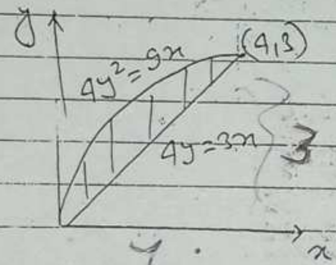
$\bar{x} = 77.08mm$

$\bar{y} = 64.60mm$

$I_{xx} = 21.06 \times 10^6 \ mm^4$

$I_{yy} = 12.456 \times 10^6 \ mm^4$

(15) Locate the centroid and MoI of the area bounded by curve.



⇒ Here,

$4y^2 = 9x$

or, $y = 3/2 \sqrt{x}$

$\therefore y_1 = \frac{3}{2}\sqrt{x} \longrightarrow (1)$

and

$4y = 3x$

or, $y = 3/4 x$

$\therefore y_2 = 3/4 x \longrightarrow (11)$

Then,

$I_{xx} = \int y^2 dA$

Date
Page 35

$$I_{xx} = \int_0^3 y^2 \cdot x \cdot dy = \int_0^3 y^2 \cdot (x_2 - x_1) dy$$

$$\text{or, } I_{xx} = \int_0^3 y^2\left(\frac{4}{3}y - \frac{4}{9}y^2\right)dy$$

$$\text{or, } I_{xx} = \int_0^3 \left(\frac{4}{3}y^3 - \frac{4}{9}y^4\right)dy$$

$$\text{or, } I_{xx} = \frac{4}{3} \times \left[\frac{y^4}{4}\right]_0^3 - \frac{4}{9}\left[\frac{y^5}{5}\right]_0^3$$

$$\text{or, } I_{xx} = \frac{4}{3} \times \frac{3^4}{4} - \frac{4}{9} \times \frac{3^5}{5} = \frac{27}{5}$$

And,

$$I_{yy} = \int x^2 dA = \int_0^4 x^2 \cdot y \, dx$$

$$\text{or, } I_{yy} = \int_0^4 x^2 \cdot (y_1 - y_2) dx$$

$$\text{or, } I_{yy} = \int_0^4 x^2 \cdot \left(\frac{3}{2}\sqrt{x} - 3/4 x\right) dx$$

$$\text{or, } I_{yy} = \int_0^4 \left(\frac{3}{2}x^{5/2} - \frac{3}{4}x^3\right) dx$$

Date 2071/8/1
Page

or $I_{yy} = \frac{3}{2} \times \left[\frac{x^{7/2}}{7/2}\right]_0^4 - \frac{3}{4}\left[\frac{x^4}{4}\right]_0^4$

or, $I_{yy} = \frac{3}{2} \times \frac{4^{7/2}}{7/2} - \frac{3}{4} \times \frac{4^4}{4}$

or, $I_{yy} = \frac{48}{7}$

(5hrs)

Chapter: → 7

Analysis of Beam

1 Beam:

A beam is a structural element that is capable of withstanding load primarily by resisting bending. The beam is supported along its length and is acted upon by a ~~straight~~ system of loads at right angle to its axis. Due to external loads and couples, shear force and bending moment develop at any section of the beam. Therefore, for a design of a beam information about the shear force and bending moment is desired.

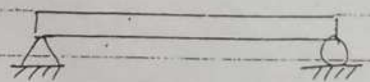


fig. simply supported beam.

Date
Page

2 → Types of Beam:

According to the support condition be are classified as below.

a) simply supported Beam: A beam which are supported on the ends and rests freely on support is known as simply supported Beam.

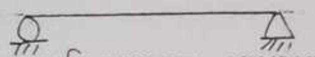


fig. simply supported Beam

b) Overhanging Beam: A beam having or both ends extended over the support is known as overhanging Beam.

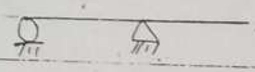


fig. overhanging Beam

c) Cantilever Beam: A beam having one end fixed and another end free to deflect is known as Cantilever beam. There is not rotation or deflection at fixed end.

fig. cantilever beam

d) <u>Fixed Beam:</u> A beam having its both of the ends fixed is known as fixed beam.

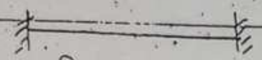


fig: Fixed beam

e) <u>continous Beam:</u> A beam provided with more than two supports is known as continuous beam. Such beam may or may not be overhanging.

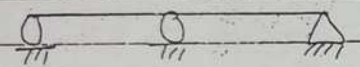


fig: Continuous Beam

f) <u>Fixed simply supported Beam:</u> A beam having its one end fixed and other end rests freely on support is called fixed simply supported beam.

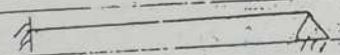


fig: fixed simply supported Beam

d) Fixed Beam: A beam having its both of the ends fixed is known as fixed beam.

fig: Fixed beam

e) Continuous Beam: A beam provided with more than two supports is known as continuous beam. Such beam may or may not be overhanging.

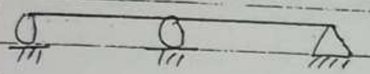


fig: Continuous Beam

f) Fixed simply supported Beam: A beam having its one end fixed and other end rests freely on support is called fixed simply supported beam.

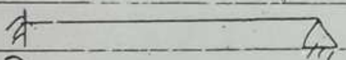


fig. fixed simply supported Beam

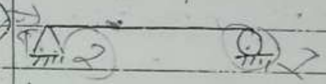
Static determinancy and Stability of beams:

If 'm' be the number of members, 'r' be the number of unknown reaction, 'j' be the no. of joints in a beam then the following three conditions may arise.

- $3m+r=3j$ → Beam is stable & determinate
- $3m+r>3j$ → Beam is indeterminate.
- $3m+r<3j$ → Beam is unstable.

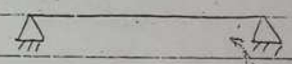
For examples:

a)



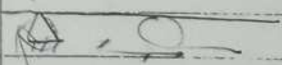
Here, $m=1$
$r=3$
$j=2$
$\therefore 3m+r = 3\times1+3=6$
$3j=3\times2=6$
$\therefore 3m+r=3j$
Hence given beam is stable and determinate

b)

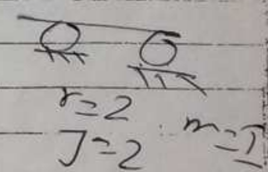


Here, $m=1$
$r=4$
$j=2$
$3m+r=3\times1+4=7$
$3j=3\times2=6$
$\therefore 3m+r>3j$
Hence, given beam is indeterminate

c)



Here, $m=1$, $r=2$, $j=2$
$3m+r = 3\times1+2=5$
$3j = 3\times2=6$
$\therefore 3m+r<3j$ Hence, given beam is unstable.



Axial force, Shear force and Bending moment diagram.

a) **Axial force:-** It is the algebric sum of all forces acting parallel to the axis on either side of the section. A tensile force is assumed to be positive while compressive force is negative.

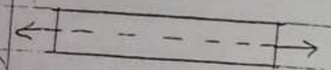


fig. Tensile axial force (+ve)

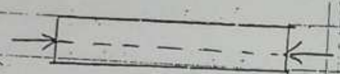


fig. compressive axial force (-ve)

b) **Shear force:-** It is the algebric sum of all the forces acting transverse to the member on either side of the section. Shear force is (+ve) when left ward portion of the section tends to move upward and right ward portion tends to move downward and vice-versa.

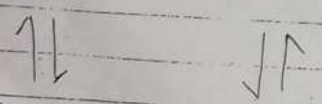


fig. (+ve) Shear (-ve) Shear

c) **Bending Moment:** The bending moment at the cross section of a beam may be defined as the algebric sum of moments of all forces on either side of the section. Sagging moment is taken as +ve & hogging moment is taken as negative. In general B.M is said to be negative when it is acting in a clockwise direction to the left and, positive when it is acting in anticlockwise direction.

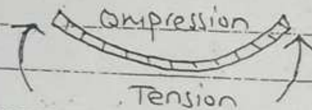


fig. Sagging Moment (+ve)

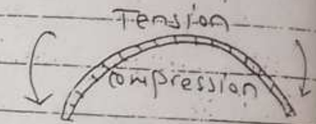
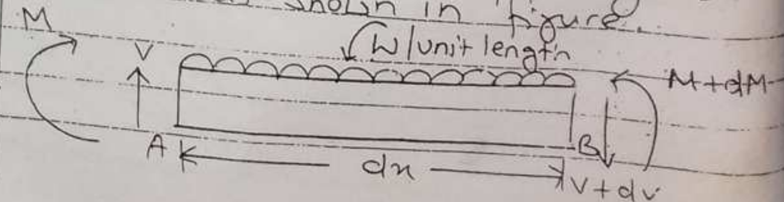


fig. Hogging Moment (-ve)

IMP

Relation between load, Shear force and Bending Moment.

Consider an element 'dx' along the length of the beam with load intensity 'W' per unit length. V and (V+dv) are the shear forces at two ends and M and (M+dM) are corresponding bending moment as shown in figure.



Date / /
Page

Now,

Considering vertical equilibrium.

$\uparrow^{+} \Sigma F_y = 0$

ie. $V - (V + dv) - w \times dx = 0$

or, $dv = -w dx$

or, $\frac{dv}{dx} = -w \longrightarrow ①$

eqⁿ ① indicates that the slope of the shear curve is negative and numerically equal to the load per unit length at any point.

Again,

considering moment equilibrium about right end of the beam section.

$\circlearrowleft^{+} \Sigma M_B = 0$

ie. $M + V \times dx - w \times \frac{dx^2}{2} - (M + dM) = 0$

~~Neglecting~~ 2nd order of dx

or, $\cancel{M} + V dx - \cancel{M} - dM = 0$

or, $\frac{dM}{dx} = V \longrightarrow ②$

eqⁿ ② indicates the slope of the bending moment diagram is equal to the value of the shear at that section.

Date / /
Page 34

Integrating both side of eqⁿ ① and eqⁿ ② from point A to B.

$$\int_A^B dv = -w \int_A^B dx$$

or, $V_B - V_A = -\int_A^B w \, dx$

or, $V_B - V_A = -$ (Area under the load curve between A and B.)

And,

$$\int_A^B dM = V \int_A^B dx$$

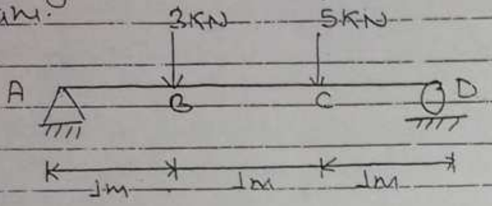
or, $M_B - M_A = \int_A^B V \, dx$

or, $M_B - M_A =$ (Area under the shear curve between A & B.)

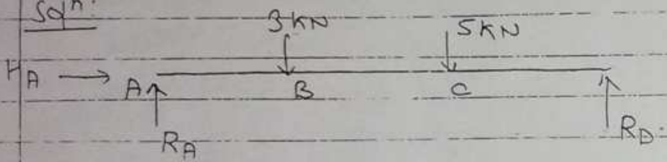
Hence, Change in shear between any two points is equal to the area under the load curve between the same points and the change in moment between any two points is equal to the area under the shear curve.

Numericals

① Draw the Axial force, Shear force and Bending Moment diagram for a given beam.



⇒ Soln:



⊛ Calculation of support reaction.

$\xrightarrow{+} \Sigma F_x = 0$

ie. $H_A = 0$

$+\uparrow \Sigma F_y = 0$ ie. $R_A + R_D = 8 \longrightarrow ①$

पालमा लिने जोड्दा

$\circlearrowright^{+} \Sigma M_A = 0$

ie. $3\times1 + 5\times2 - R_D \times 3$

or, $R_D = 4.33\ kN$

and $R_A = 3.67\ kN$

⊛ Calculation of Axial force

For section AD, A.F = 0

Left हुँदा त्यो point बाहेकको reaction लिने

Right हुँदा त्यो point को [illegible] reaction लिने

⊛ Calculation of Shear force:

(+↑) At point A
Just left S.F = 0
Just right S.F = 3.67 kN (+ve)

At point B
Just left S.F = 3.67 kN (+ve)
Just right S.F = 3.67 − 3
= 0.67 kN (+ve)

At point C
Just left S.F = 0.67 kN (+ve)
Just right S.F = 0.67 − 5 = 4.33 kN (−ve)

At point D
Just left S.F = 4.33 kN (−ve)
Just right S.F = −4.33 + 4.33 = 0

⊛ Calculation of bending Moment:

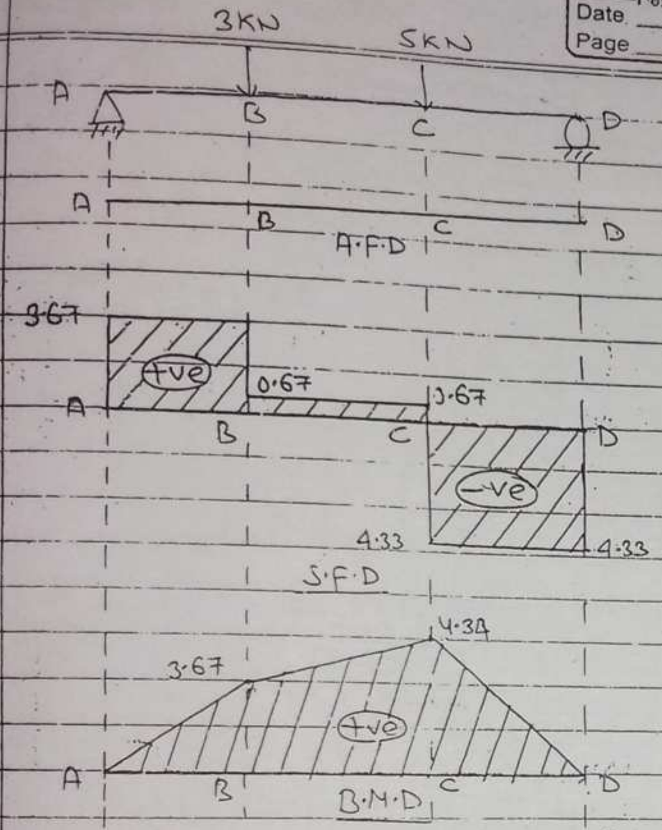
(+↻) At point A, B.M = 0

At point B,
B.M = 3.67 × 1 = 3.67 kNm (+ve)

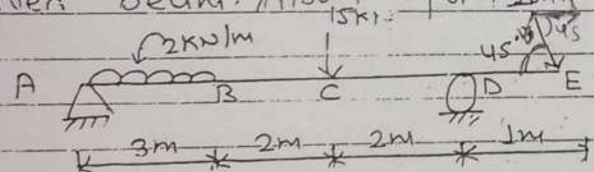
At point C,
B.M = 3.67×2 − 3×1
or, B.M = 4.34 kNm (+ve)

At point D
B.M = 3.67×3 − 3×2 − 5×1 = 0

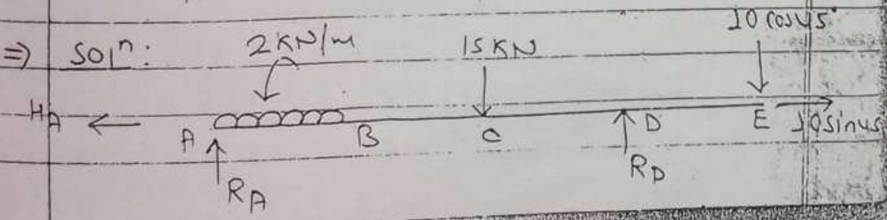
इसीले पुल्नदाइने → यहाँ हुँदैन आउको हो ∴ BM at A = 0



② Draw the A.F, S.F and B.M diagram of given beam. Also find point of contraflexure.



⇒ Soln:



* Calculation of support reaction:

$\xrightarrow{+} \Sigma F_x = 0$ ie. $-H_A + 10\sin 45 = 0$

or, $H_A = 7.07$ KN

$+\uparrow \Sigma F_y = 0$ ie. $R_A - 2\times3 - 15 + R_D - 10\cos45 = 0$

or, $R_A + R_D = 28.07 \longrightarrow$ ①

$+\circlearrowright \Sigma M_A = 0$

ie. $3\times2\times\frac{3}{2} + 15\times5 - R_D\times7 + 7.07\times8 = 0$

or, $R_D = 20.08$ KN

From eqn ①

$R_A = 7.99$ KN

* Calculation of Axial force,

For Section AE, A.F = 7.07 KN (+ve)

[∵ 7.07 ← A —— E → 7.07]

Tensile axial force, so, it is taken as positive.

* Calculation of shear force:

(+↑)

At point A, Just left S.F = 0

Just right S.F = 7.99 KN (+ve)

At point B, Just left, S.F = 7.99 − 3×2 = 1.99 KN (+ve)

~~Just right~~ Just right S.F = 1.99 KN (+ve)

At point C, Just left, S.F = 1.99 KN (+ve)

Just right S.F = 1.99 − 15 = 13.01 KN (−ve)

At point D

Just left S.F = 13.01 KN (-ve)

Just right S.F = -13.01 + 20.08 = 7.07 KN (+ve)

At point E

Just left S.F = 7.07 KN (+ve)

Just right S.F = 7.07 - 7.07 = 0

* Calculation of bending Moment:

(+) At point A, B.M = 0

At point B, $B.M = \left(7.99 \times 3 - 2 \times 3 \times 3/2\right)$

$= 14.97$ KNm (+ve)

At point C, $B.M = 7.99 \times 5 - 3 \times 2 \times \left(\frac{3}{2} + 2\right)$

or, B.M = 19.95 KNm (+ve)

At point D,

$B.M = 7.99 \times 7 - 3 \times 2 \times \left(\frac{3}{2} + 4\right) - 2 \times 15$

or, B.M = 7.07 kNm (-ve)

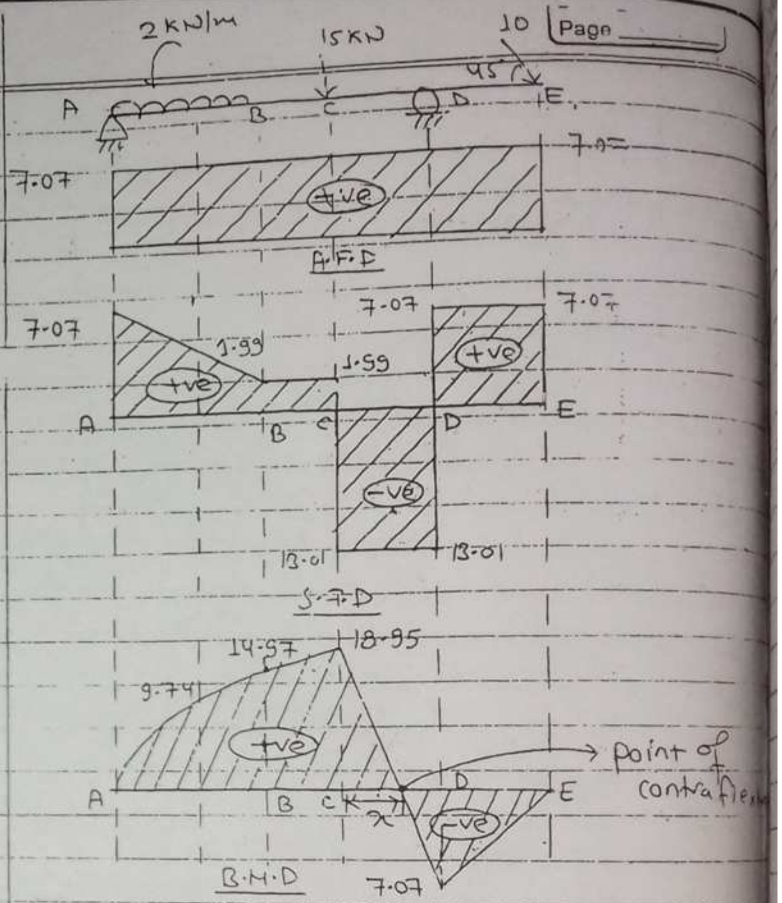
At point E

$B.M = 7.99 \times 8 - 3 \times 2\left(\frac{3}{2} + 5\right) - 15 \times 3 + 20.08 \times 1$

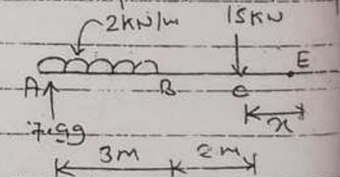
B.M = 0

~~B.M at mid of~~ AB (i.e. at 1.5m from pt. A)

$B.M = 7.99 \times 1.5 - 2 \times 1.5 \times \frac{1.5}{2} = 9.74$ kNm (+ve)



let x be the point of contraflexure from point C.



Then,

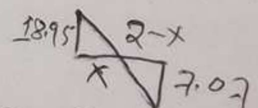
(+) $\Sigma M_E = 0$

or, $7.99 \times (5 + x) - 3 \times 2\left(\frac{3}{2} + 2 + x\right) - 15 \times x = 0$

~~or, x = 1.45m~~

∴ Point of contraflexure lies at 1.45m from

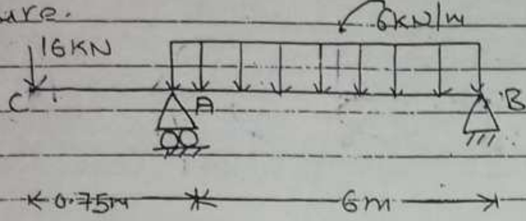
$\frac{x}{18.95} = \frac{2 - x}{7.07}$



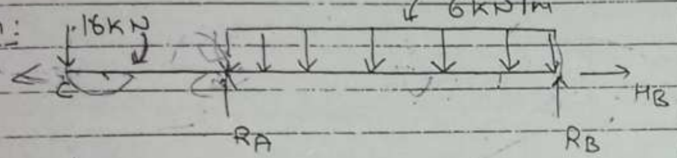
⇒ x = 1.45 m from C or 0.54 from D

(2006) Fall Support आउँदा ठाउँमा BM लिने

③ Draw the shear force and bending moment diagram for the beam as shown in figure.



⇒ Soln:



⊛ calculation of support reaction:

$\rightarrow \Sigma F_x = 0$ ie. $H_B = 0$

$+\uparrow \Sigma F_y = 0$ ie. $R_A + R_B = 52 \longrightarrow ①$

$\Sigma M_A = 0$

ie. $-16 \times 0.75 + 6 \times 6 \times \frac{6}{2} - R_B \times 6 = 0$

or, $R_B = 16KN$

From eqn ① $R_A = 52 - 16 = 36KN$

⊛ calculation of shear force:

(+) At point C, Just left , S.F = 0

Just right S.F = 16KN (-ve)

At point A

Just left , S.F = 16KN (-ve)

Just right S.F = $-16 + 36 = 20KN$ (+ve)

At point B

Just left S.F = $20 - 6 \times 6 = 16KN$ (-ve)

Just right S.F = $-16 + 16 = 0$

⊛ calculation of Bending Moment:

(+) At point C B.M = 0

At point A,

B.M = $-16 \times 0.75 = 12KNm$ (-ve)

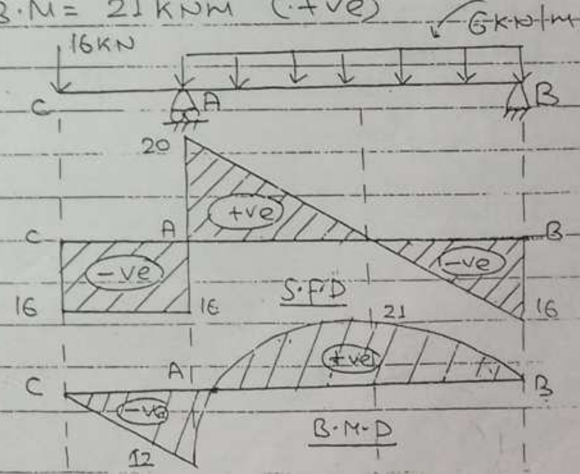
At point B, B.M = $-16 \times 6.75 + 36 \times 6 - 6 \times 6 \times 6/2$

B.M = 0

B.M at mid of AB (ie. at 3.75m from point C)

B.M = $-16 \times 3.75 + 36 \times 3 - 6 \times 3 \times 3/2$

B.M = 21 KNm (+ve)



$21.x = -36 + 12$

$\frac{x}{+12} = \frac{3-x}{21}$

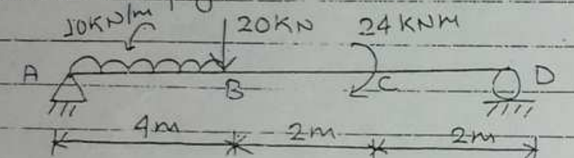
+12 x 3-x 21

Note:

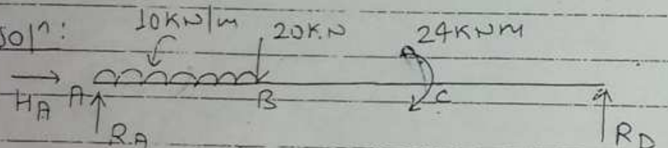
Type of Load	S.F Diagram	B.M diagram
a) point load	st. line	oblique line
b) UDL	ob'lique line	square parabola
c) UVL	square parabola	cubic parabola

(2006)SP

④ Draw the shearforce and Bending moment diagram for the beam as shown in figure.



⇒ Sol^n:



⊛ Calculation of support reaction:

$\xrightarrow{+} \Sigma F_x = 0$ ie. $H_A = 0$

$+\uparrow \Sigma F_y = 0$ $\quad R_A + R_D = 60 \longrightarrow ①$

$\circlearrowleft \Sigma M_A = 0$ ie. $10 \times 4 \times 4/_2 + 20 \times 4 + 24 - R_D \times 8 = 0$

or, $R_D = 23\,KN$

From eq^n ① $\quad R_A = 37\,KN$

⊛ Calculation of shearforce:

(+↑) At point A, Just left S.F = 0

Just right S.F = 37 KN (+ve)

At point B, Just left S.F = 37 − 10×4

S.F = 3 KN (−ve)

Just right S.F = −3 − 20 = 23 KN (−ve)

At point C, Just left S.F = −23 KN = 23 KN (−ve)

Just right S.F = 23 KN (−ve)

At point D, Just left S.F = 23 KN (−ve)

Just right S.F = −23 + 23 = 0

⊛ Calculation of Bending Moment.

(+) At point A, B.M = 0

At point B, $B.M = 37 \times 4 - 10 \times 4 \times 4/_2$

B.M = 68 KNm (+ve)

At point C,

Just left $B.M = 37 \times 6 - 10 \times 4 \times \left(\frac{4}{2} + 2\right) - 20 \times 2$

B.M = 22 KNm (+ve)

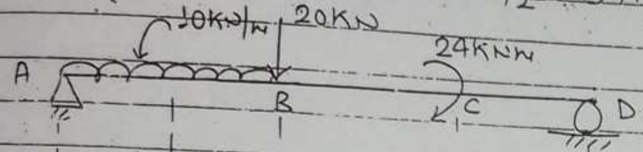
Just right B.M = 22 + 24 = 46 KNm (+ve)

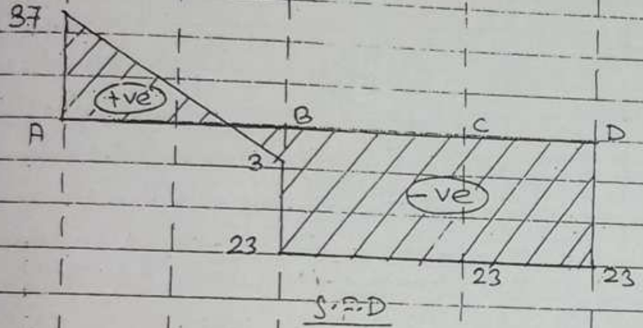
At point D

$B.M = 37\times8 - 10\times4\times(\frac{4}{2}+4) - 20\times4 + 24 = 0$

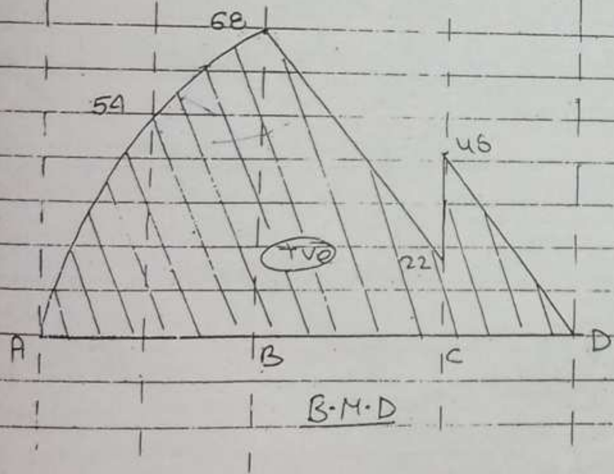
B.M at Mid of AB (ie. at 2m from A)

$B.M = 37\times2 - 10\times2\times2/_2 = 54\,KNm$ (+ve)





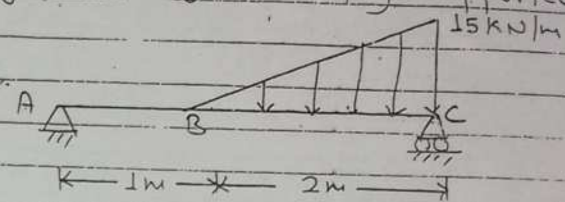
S.F.D



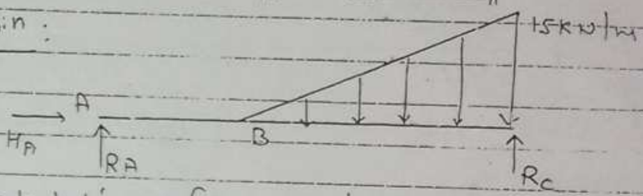
B.M.D

(2010) Fall

⑤ Draw the shear force and bending moment diagram of given simply supported beam.



⇒ Soln:



(*) Calculation of support reaction:

$\xrightarrow{+} \Sigma F_x = 0$

ie. $H_A = 0$

$+\uparrow \Sigma F_y = 0$ ie. $R_A + R_C = \frac{1}{2}\times2\times15$

or, $R_A + R_C = 15 \longrightarrow$ ①

$\Sigma M_A = 0$ ie. $\frac{1}{2}\times2\times15\times(1+\frac{2}{3}\times2) - R_C\times3 = 0$

or, $R_C = 11.67\,KN$

∴ From eqn ① $R_A = 3.33\,KN$

(*) Calculation of shear force:

(A) At point A, — (यो point सम्म)

Just left $S.F = 0$

Just right $S.F = 3.33\,KN$ (+ve)

(यो point को मानि [+↑])

At point B

Just left $S.F = 3.33\,KN$ (+ve)

left moment लिंदा गर्दा clockwise लाई +ve

right मा moment लिंदा anticlock wise लाई -ve

BM at support is equal to zero

Date / /
Page

Just right S.F = 3.33 KN (+ve)

At point C

Just left S.F = $3.33 - \frac{1}{2} \times 2 \times 15$

or, S.F = 11.67 KN (-ve)

Just right S.F = $-11.67 + 11.67 = 0$

S.F at Mid of BC (ie. at 2m from point A)

S.F = $3.33 - \frac{1}{2} \times 1 \times 7.5 = 0.42$ KN (+ve)

* Calculation of Bending Moment.

⇒ At point A, B.M = 0

At point B,

B.M = $3.33 \times 1 = 3.33$ KNm (+ve)

At point C,

B.M = $3.33 \times 3 - \frac{1}{2} \times 2 \times 15 \times \left(\frac{2}{3}\right)$

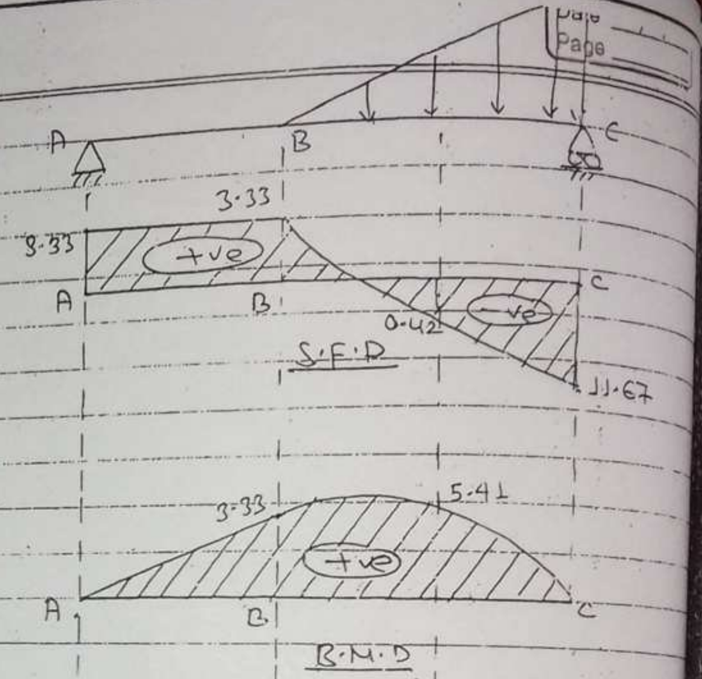
or, B.M = $-0.01 \approx 0$

B.M at Mid of BC (ie. at 2m from pt. A)

B.M = $3.33 \times 2 - \frac{1}{2} \times 1 \times 7.5 \times 1/3$

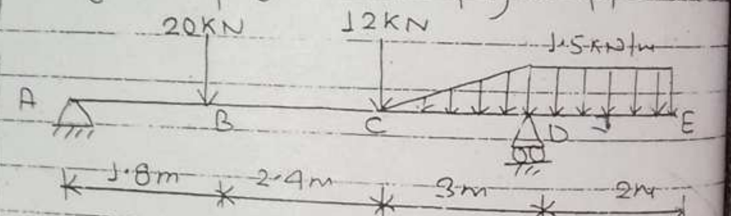
or, B.M = 5.41 KNm (+ve)

Date
Page

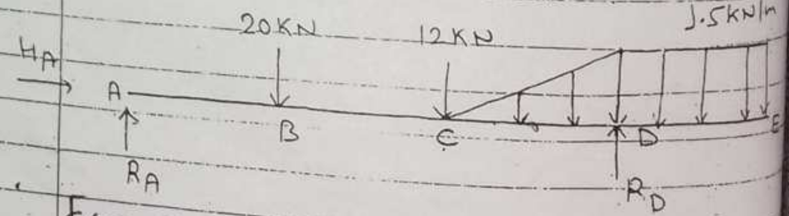


(2013) SP

⑥ Draw the shearforce and Bending moment diagram of given simply supported beam



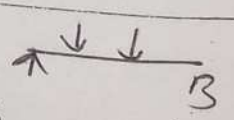
⇒ Soln:



For BM

upto point B

(मो अपवाद (लको) गारिनु



2 (को) point को zero

⊛ Calculation of support reaction:

$\xrightarrow{+} \Sigma F_x = 0$ ie. $H_A = 0$

$+\uparrow \Sigma F_y = 0$ ie. $R_A + R_D = 20 + 12 + \frac{1}{2} \times 1.5 \times 3 + 1.5 \times 2$

or, $R_A + R_D = 37.25 \longrightarrow (1)$

$\Sigma M_A = 0$

ie. $20 \times 1.8 + 12 \times 4.2 + \frac{1}{2} \times 3 \times 1.5 \times \left(4.2 + \frac{2}{3} \times 3\right) - R_D \times 7.2 + 1.5 \times 2 \times \left(7.2 + \frac{2}{2}\right) = 0$

or, $R_D = 17.35$ KN

From eqn (1) $R_A = 37.25 - 17.35$

or, $R_A = 19.9$ KN

⊛ Calculation of shear force:

(+↑) At point A,

Just left $S.F = 0$

Just right $S.F = 19.9$ KN (+ve)

At point B

Just left $S.F = 19.9$ KN (+ve)

Just right $S.F = 19.9 - 20 = 0.1$ KN (-ve)

At point C

Just left $S.F = 0.1$ KN (-ve)

Just right $S.F = -0.1 - 12 = 12.1$ KN (-ve)

At point D

Just left $S.F = -12.1 - \frac{1}{2} \times 3 \times 1.5$

$= 14.35$ KN (-ve)

Just right $S.F = -14.35 + 17.35$

$= 3$ KN (+ve)

At point E

Just left $S.F = 3 - 1.5 \times 2 = 0$

Just right $S.F = 0$

S.F at Mid of CD (ie. at 5.7m from pt. A)

$S.F = 19.9 - 20 - 12 - \frac{1}{2} \times 1.5 \times 0.75$

or, $S.F = 12.66$ KN (-ve)

⊛ Calculation of Bending Moment

At point A, $B.M = 0$

At point B, $BM = 19.9 \times 1.8 = 35.82$ KNm (+ve)

At point C, $B.M = 19.9 \times 4.2 - 20 \times 2.4 = 35.58$ KNm (+ve)

At point D

$B.M = 19.9 \times 7.2 - 20 \times 5.4 - 12 \times 3 - \frac{1}{2} \times 3 \times 1.5 \times 3/3$

$B.M = 2.97$ KNm (-ve)

At point E

$B.M = 19.9 \times 9.2 - 20 \times 7.4 - 12 \times 5 - \frac{1}{2} \times 3 \times 1.5 \left(\frac{3}{3} + 2\right) + 17.35 \times 2 - 1.5 \times 2 \times 2/2$

$B.M = 0.03 \approx 0$

यो त्रिकोणको SF मा निकाल्नु पर्छ middle को

Or middle को SF निकाल्नु पर्छ

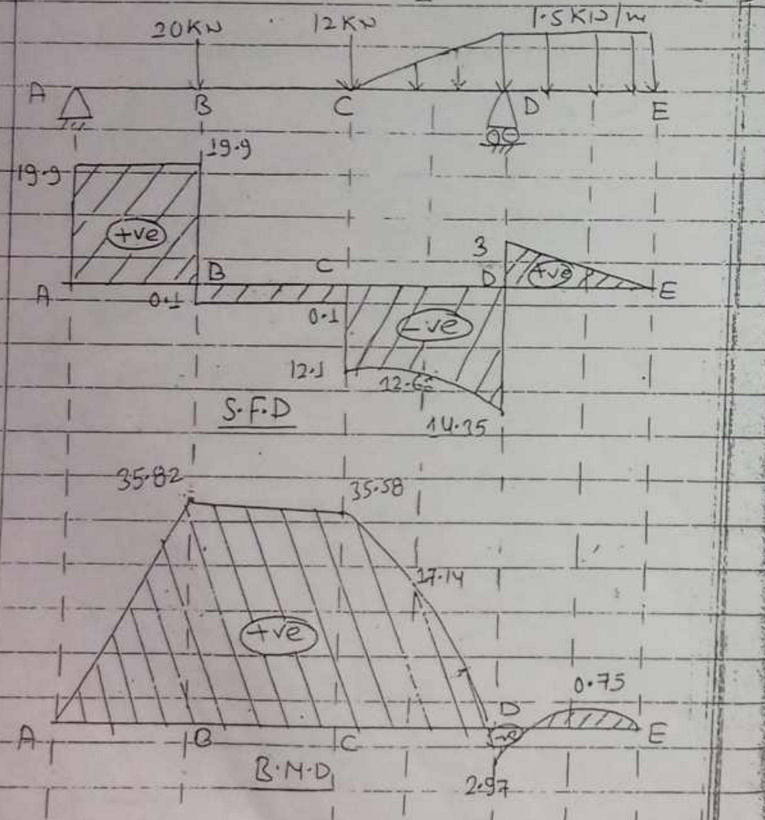
B.M at Mid of CD (ie at 5.7m from pt. A)

$$B.M = 19.9\times5.7 - 20\times3.9 - 12\times1.5 - \frac{1}{2}\times1.5\times0.75\times\left(\frac{1.5}{3}\right)$$

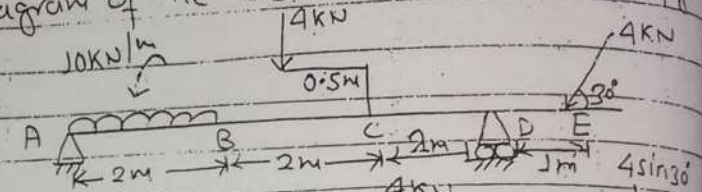
$B.M = 17.14$ KNm (+ve)

Also, B.M at Mid of DE (ie. At ~~0.5~~ 1m from E)

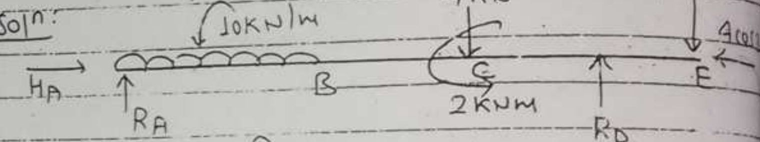
$$B.M = 1.5\times1\times\frac{1}{2} = 0.75 \text{ KNm (+ve)}$$



(7) Draw the shear force and bending moment diagram of the beam as shown in fig.



⇒ Soln:



(*) Calculation of support reaction.

$\xrightarrow{+} \Sigma F_x = 0$ ie. $H_A = 4\cos30° = 3.46$ KN

$+\uparrow \Sigma F_y = 0$ ie. $R_A + R_D = 10\times2 + 4 + 4\sin30°$

or, $R_A + R_D = 26 \longrightarrow (1)$

$+\circlearrowright \Sigma M_A = 0$

ie. $10\times2\times\frac{2}{2} + 4\times4 - 2 - R_D\times6 + 4\sin30°\times\cancel{8}7 = 0$

or, $R_D = 8$ KN

$\therefore$ From eqn (1) $R_A = 26 - 8 = 18$ KN

(*) Calculation of shear force:

(+↑) At point A

Just left S.F = 0

Just right S.F = 18KN

At point B

Just left S.F = 18 − 10×2

= 2 KN (+ve)

Just right

At point C

Just

Just

At point D

Just

Ju

At point E

calculatio

At point

At point

At point C

Just

Just

At Poin

B.M =

nding mome
shown in
= 3.46 KN
2 + 4 + 4 sin30
⟶ ①
−2 − R_D × 6 +
30 × 7 = 0
26 − 8 = 18
ce:
F = 0
S.F = 18 KN
= 18 − 10×2
= 2 KN (−ve)

Just right S.F = 2KN (−ve)

At point C

Just left S.F = 2KN (−ve)

Just right S.F = −2 − 4 = 6KN (−ve)

At point D

Just left S.F = 6KN (−ve)

Just right S.F = −6 + 8 = 2KN (+ve)

At point E

Just left S.F = 2KN (+ve)

Just right S.F = 2 − 4 sin30° = 0

Calculation of Bending Moment.

At point A, B.M = 0

At point B B.M = $18\times 2 - 10\times 2\times \frac{2}{2}$

B.M = 16 KNm (+ve)

At point C

Just left B.M = $18\times 4 - 10\times 2\times\left(\frac{2}{2}+2\right)$

= 12 KNm (+ve)

Just right B.M = 12 − 2 = 10 KNm (+ve)

At point D

B.M = $18\times 6 - 10\times 2\times\left(\frac{2}{2}+4\right) - 4\times 2 - 2$

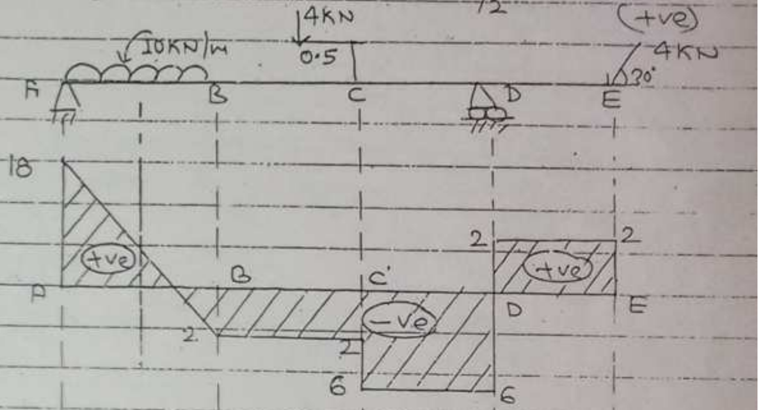
= 2 KNm (−ve)

At point E

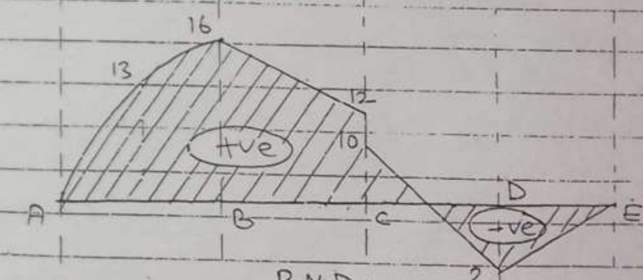
B.M = $18\times 7 - 10\times 2\left(\frac{2}{2}+5\right) - 4\times 3 - 2 + 8\times 1 = 0$

B.M at mid of AB (ie. at 1m from point A)

B.M = $18\times 1 - 10\times 1\times \frac{1}{2}$ = 13 KNm (+ve)



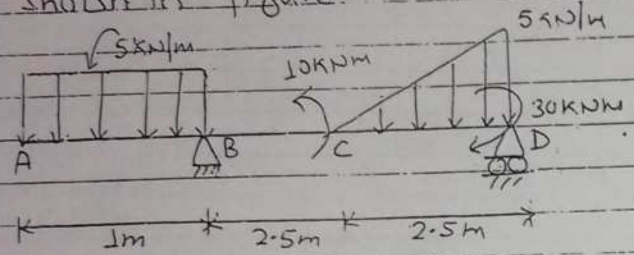
S.F.D



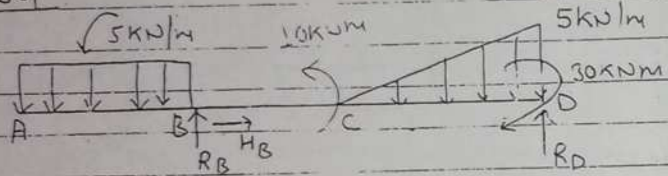
B.M.D

Date / /
Page

(8) Draw the shearforce and bending moment diagram for the beam as shown in figure.



⇒ Solⁿ



(*) Calculation of Support reaction:

$\xrightarrow{+} \Sigma F_x = 0$ ie. $H_B = 0$

$+\uparrow \Sigma F_y = 0$ ie. $-5\times1 + R_B + R_D - \frac{1}{2}\times2.5\times5 = 0$

or, $R_B + R_D = 11.25 \longrightarrow (1)$

$\overset{+}{\curvearrowleft} \Sigma M_B = 0$

ie. $-5\times1\times\frac{1}{2} - 10 + \frac{1}{2}\times2.5\times5\times\left(2.5+\frac{2}{3}\times2.5\right)$

$+30 - R_D\times5 = 0$

or, $R_D = 8.7$ KN

From eqⁿ (1)

$R_B = 2.55$ KN

Date / /
Page

(*) Calculation of Shear force

At point A   ~~Just left S.F = 0~~
~~Just right S.F = 0~~

At point B   Just left S.F $= -5\times1 = 5$KN(-ve)
Just right S.F $= -5\times1 + 2.55$
$= 2.45$KN(-ve)

At point C
Just left S.F = 2.45KN (-ve)
Just right S.F = 2.45KN (-ve)

At point D   Just left S.F $= -2.46 - \frac{1}{2}\times2.5\times5$
S.F = 8.7 KN (-ve)

Just right S.F $= -8.7 + 8.7 = 0$

S.F At Mid of CD (ie. at 4.75m from A)

$S.F = -5\times1 + 2.55 - \frac{1}{2}\times1.25\times2.5$

or, S.F = 4.01 KN (-ve)

(*) Calculation of Bending Moment.

At point A   B.M = 0
At point B   B.M $= -5\times1\times\frac{1}{2} = 2.5$KNm (-ve)

At point C
Just left B.M $= -5\times1\times\left(\frac{1}{2}+2.5\right) + 2.55\times2.5$
$= 8.62$ KNm (-ve)

Date / /
Page

Just right B.M $= -8.62 - 10 = 18.62$ KNm (−ve)

At point D

Just left B.M $= -5\times1\times\left(\frac{1}{2}+5\right)+2.55\times5-10-\frac{1}{2}\times2.5\times5\times\frac{2.5}{3} = 30$ KN (−ve)

Just right B.M $= -30+30 = 0$

B.M at Mid of AB (ie. at 0.5m from A)

B.M $= -5\times0.5\times\frac{0.5}{2} = 0.625$ KNm (−ve)

B.M at mid of CD (ie. at 4.75m from A)

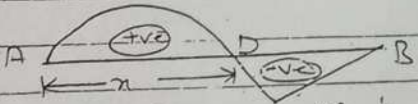
B.M $= -5\times1\times\left(\frac{1}{2}+2.5+1.25\right)+2.55\times3.75-10-\frac{1}{2}\times1.25\times2.5\times\frac{1.25}{3} = 22.33$ KNm (−ve)

Note:-

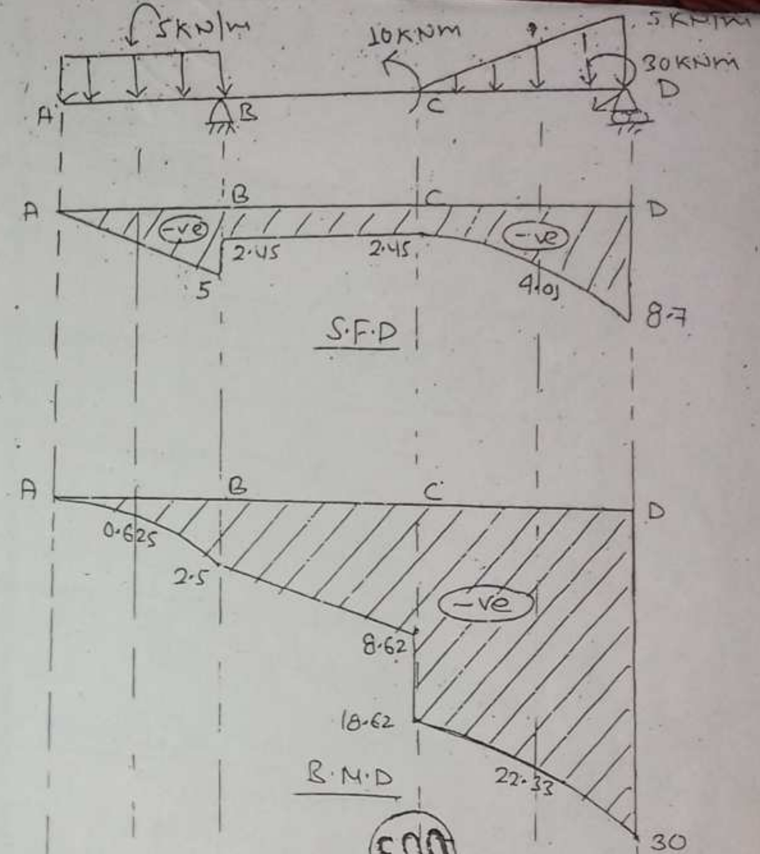
* Point of contraflexture:

The point having zero bending moment (ie. BM changes its sign from a point) is known as point of contraflexture.

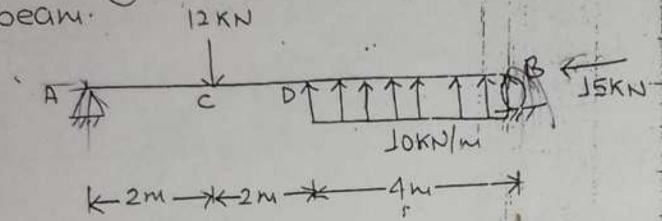
Fig:



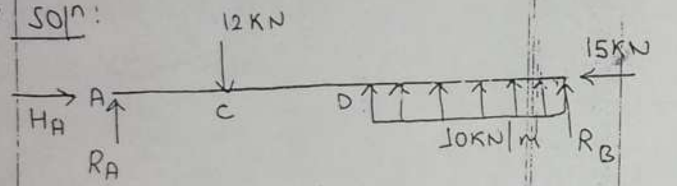
In above B.M diagram 'D' is the point at a distance of 'x' from point 'A' when the B.M is zero and that point is known as point of contraflexture.



⑨ Draw the axial force, shear force and bending moment diagram of given beam.



⇒ Soln:



⊛ Calculation of support reaction:

$\xrightarrow{+} \Sigma F_x = 0$

ie. $H_A - 15 = 0$

$\therefore H_A = 15 KN$

$+\uparrow \Sigma F_y = 0$

ie. $R_A - 12 + 10 \times 4 + R_B = 0$

or, $R_A + R_B = -28 \longrightarrow ①$

$+\circlearrowright \Sigma M_A = 0$

ie. $12 \times 2 - 10 \times 4 \times (4 + \frac{2}{2}) + R_B \times 8 = 0$

or, $R_B = -27 KN$

$\therefore R_B = 27 KN (\downarrow)$

From eqn ①

$R_A = -1 KN$

or, $R_A = 1 KN (\downarrow)$

⊛ Calculation of Axial force:

For section $\overline{AB}$ axial force = 15KN(-

⊛ Calculation of Shear force.

At point A,

Just left S.F = 0

Just right S.F = −1 KN

= 1 KN (−ve)

At point C

Just left S.F = −1 KN = 1 KN (−ve)

Just right S.F = −1 − 12 = 13 KN (−

At point D

Just left S.F = 13 KN (−ve)

Just right S.F = 13 KN (−ve)

At point B

Just left S.F = −13 + 40 = 27 KN (+ve)

Just right S.F = 27 − 27 = 0

= 15 kN(
-1 kN
= 1 kN(-ve
= 1 kN(-ve
= 13 kN(
e)
(-ve)
o = 27 kN (+ve
= 0

(*) Calculation of Bending Moment.

(+↺) At point A, B·M = 0

At point C, $B \cdot M = -1 \times 2 = 2$ kNm (-ve)

At point D, $B \cdot M = -1 \times 4 - 12 \times 2$
$= 28$ kNm (-ve)

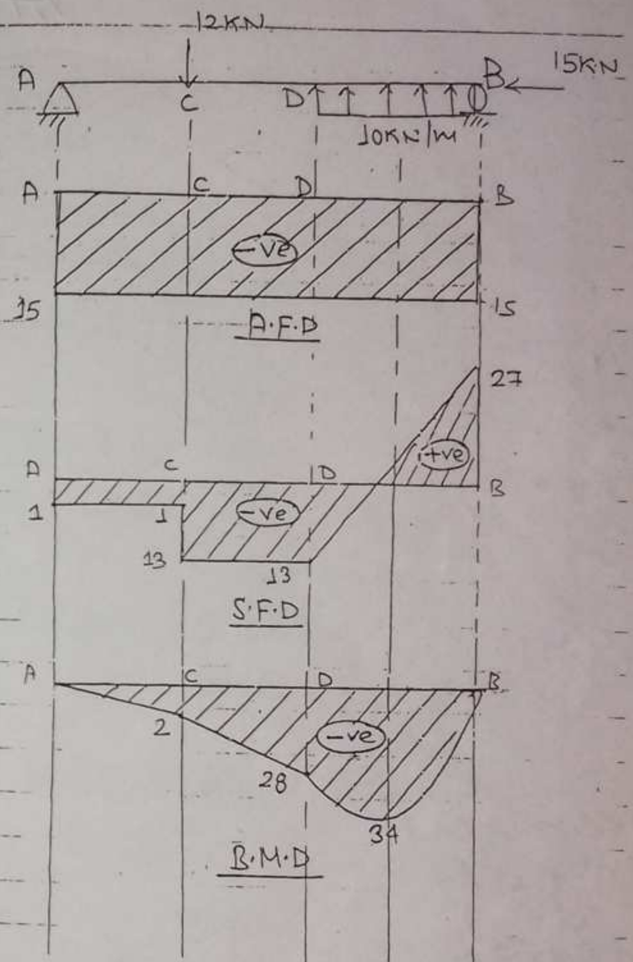
At point B, $B \cdot M = -1 \times 8 - 12 \times 6 + 4 \times 10 \times \frac{4}{2}$

or, $B \cdot M = 0$

B·M at Mid of DB (ie. at 6m from pt. A)

$B \cdot M = -1 \times 6 - 12 \times 4 + 10 \times 2 \times 2/2$

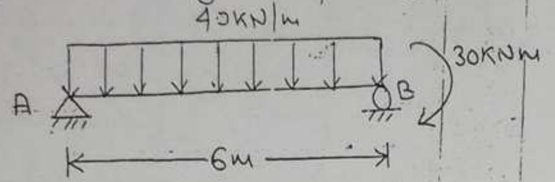
or, $B \cdot M = 34$ kNm (-ve)



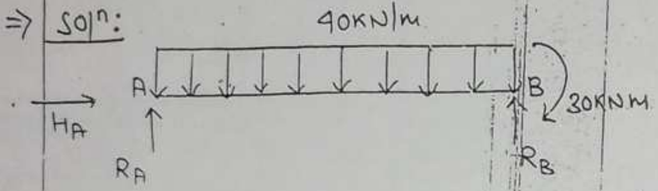
gives two reaction & & gives one reaction.

(2007) fall

(10) Draw the shear force and Bending moment diagram of the given beam.



⇒ Sol^n:



⊛ calculation of support reaction.

$\xrightarrow{+} \Sigma F_x = 0$

ie. $H_A = 0$

$+\uparrow \Sigma F_y = 0$ ie. $R_A + R_B = 240 \longrightarrow ①$

$+\circlearrowright \Sigma M_A = 0$

ie. $40 \times 6 \times \frac{6}{2} - R_B \times 6 + 30 = 0$

or, $R_B = 125 KN$

From eq^n ①

$R_A = 115 KN$

take moment only at supports

take shear force $+\uparrow$ and $\rightarrow \leftarrow$ does not matter

⊛ calculation of shear force

(+↑) At point A

Just left $S.F = 0$

Just right $S.F = 115 KN (+ve)$

At point B

Just left, $S.F = 115 - 40 \times 6$

$= 125 KN$ (-

Just right, $S.F = -125 + 125 = 0$

⊛ calculation of Bending Moment.

At point A, $B.M = 0$

At point B, Just left $B.M = 115 \times 6 - 40 \times 6 \times \frac{6}{2}$

or, $B.M = 30 KNm (-ve)$

Just right $B.M = -30 + 30 = 0$

B.M at mid of AB. (ie. at 3m from

$B.M = 115 \times 3 - 40 \times 3 \times 3/2$

or, $B.M = 165 KNm (+ve)$

s one
n.

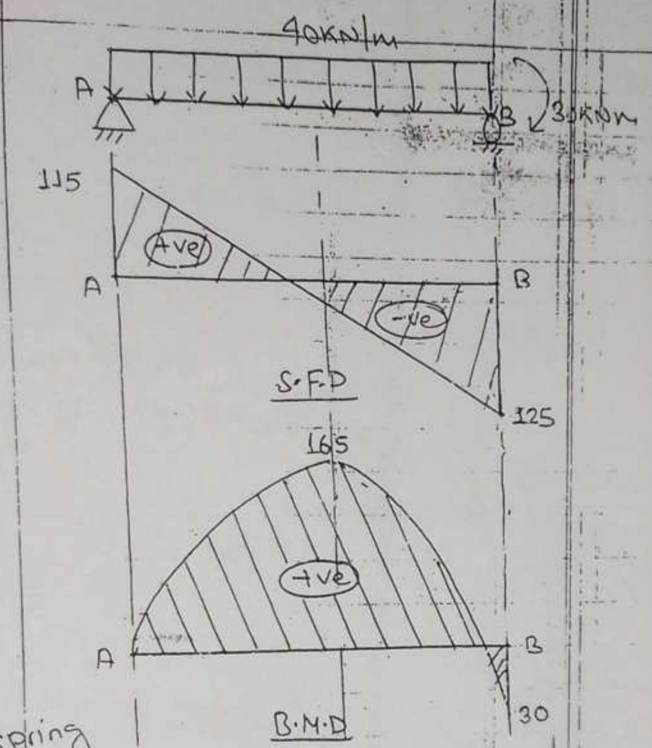
15 KN (+ve)

15 - 40×6
12.5 KN (-ve)
+125 = 0

oment.

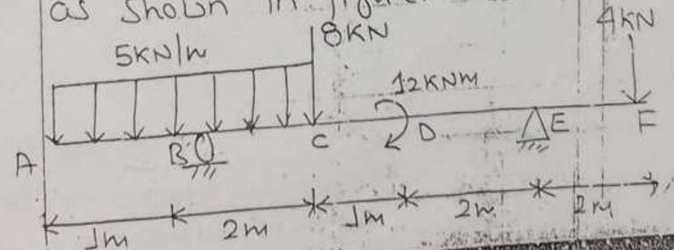
0 × 6 × 6/2
m (-ve)
= 0

at 3m from

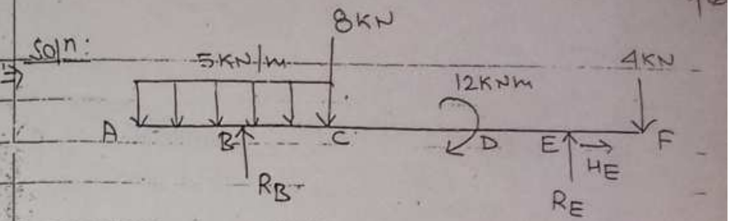


(2011) spring

(11) Draw the shear force and Bending moment diagram of beam loaded as shown in figure.



Soln:



* calculation of support reaction.

$\xrightarrow{+} \Sigma F_x = 0$ ie. $H_E = 0$

$+\uparrow \Sigma F_y = 0$ ie. $R_B + R_E = 5\times3 + 8 + 4$

or, $R_B + R_E = 27 \longrightarrow (1)$

$\Sigma M_B = 0$

ie. $-5\times1\times\frac{1}{2} + 5\times2\times\frac{2}{2} + 8\times2 + 12 + 4\times7 - R_E\times5 = 0$

or, $R_E = 12.7$ KN

∴ From eqn (1) $R_B = 14.3$ KN

* calculation of shear force.

At point A,
Just left S.F = 0
Just right S.F = 0

At point B
Just left S.F = $-5\times1 = 5$KN (-ve)
Just right S.F = $-5 + 14.3 = 9.3$ KN (+ve)

For ΣM_B we have to look at +) mean this is towards B & do -ve ative respect

For BM.
only look from one side and take moment from any one side from where we have started.
if we take $+\circlearrowright$ then $\uparrow$ $\leftarrow$ +ve, $\downarrow$ $\rightarrow$ −ve.

जहाँ moment हो उसी left & right ...

At point C
Just left S.F = 9.3 − 5×2 = 0.7 KN (+ve)

Just right S.F = −0.7 − 8 = 8.7 KN (−ve)

At point D
Just left S.F = 8.7 KN (−ve)
Just right S.F = 8.7 KN (−ve)

At point E
Just left S.F = 8.7 KN (−ve)
Just right S.F = −8.7 + 12.7
= 4 KN. (+ve)

At point F
Just left S.F = 4 KN (+ve)
Just right S.F = 4 − 4 = 0

* calculation of bending moment.

$+\circlearrowright$

At point A,
B.M = 0

At point B
B.M = −5×1×1/2 = 2.5 KNm (−ve)

for $\Sigma M = 0$

At point C
B.M = $-5\times3\times\frac{3}{2} + 14.3\times2$ = 6.1 KNm (+ve)

At point D
Just left B.M = $-5\times3\times\left(1+\frac{3}{2}\right) +$
$14.3\times3 - 8\times1$

or, B.M = −2.6 KNm = 2.6 KNm (−ve)
Just right, B.M = −2.6 + 12
or, B.M = 9.4 KNm (+ve)

At point E
B.M = $-5\times3\times\left(\frac{3}{2}+3\right) + 14.3\times5 - 8\times3$
$+12$ = 8 KNm (+ve)

At point F
B.M = $-5\times3\times\left(\frac{3}{2}+5\right) + 14.3\times7 - 8\times5 + 12$
$+ 12.7\times2$ = 0

B.M at Mid of AB (ie. at 0.5m from A)

B.M = $-5\times0.5\times\frac{0.5}{2}$ = 0.625 KNm (−ve)

$+\circlearrowright$ at the time of taking look where is the destination if same then +ve

if opposite then −ve

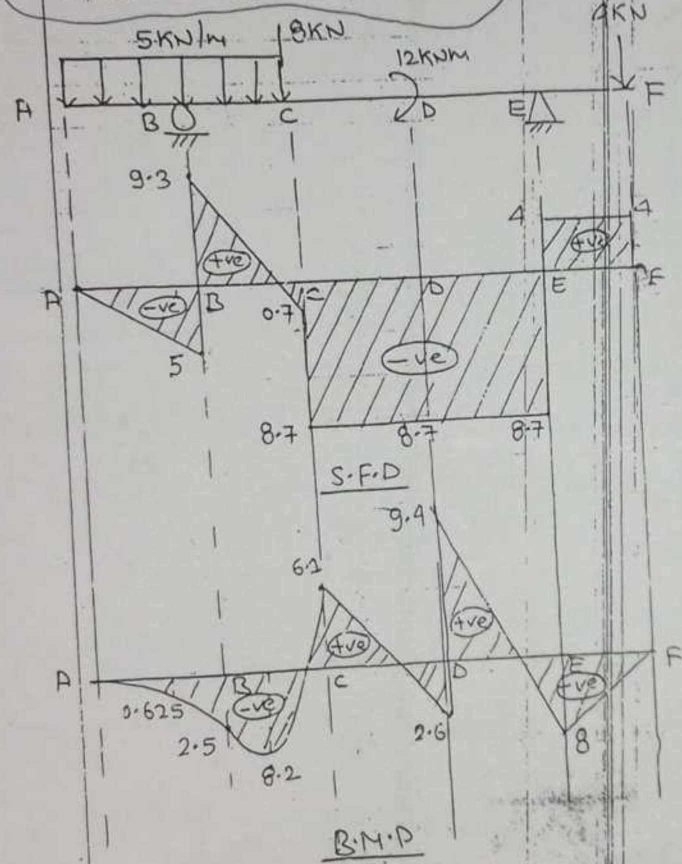
$+5\times1\times1.5+14.3\times1-5\times1\times0.5$

$=4.3\ kNm\ (+ve)$

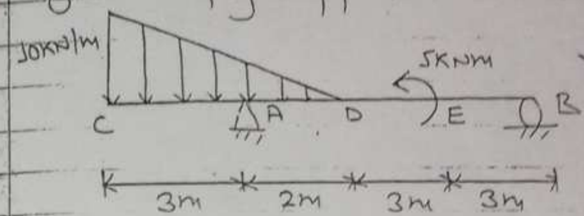
B.M at mid of BC (ie. at 2m from A)

$B.M = -5\times3\times\frac{3}{2}+14.3\times1$

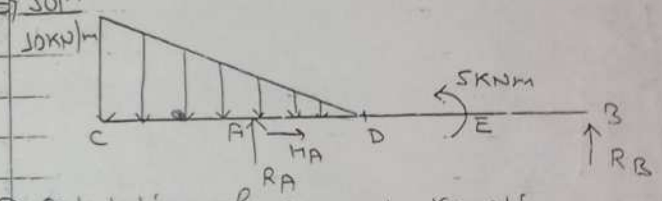
or, $B.M = 8.2$ KNm (-ve)



(12) Draw the S.F and B.M diagram of the given simply supported beam.



⇒ Soln



(*) Calculation of support reaction.

$\xrightarrow{+}\Sigma F_x = 0$ ie. $H_A = 0$

$+\uparrow \Sigma F_y = 0$ ie. $R_A + R_B = \frac{1}{2}\times5\times10$

or, $R_A + R_B = 25 \longrightarrow (1)$

$\Sigma M_B = 0$

ie. $-\frac{1}{2}\times5\times10\times\left(6+\frac{2}{3}\times5\right)+R_A\times8-5=0$

or, $R_A = 29.79$ KN

∴ From eqn (1)

$R_B = 4.79$ KN (-ve)

⊛ calculation of Shear force.

At point C

Just left S.F = 0

Just right S.F = 0

At point A

Just left $S.F = -\frac{1}{2}\times 6\times 3 - 4\times 3$

$= 21\ KN\ (-ve)$

Just right $S.F = -21 + 29.79$

$= 8.79\ KN\ (+ve)$

At point D

Just left $S.F = -\frac{1}{2}\times 5\times 10 + 29.79$

$= 4.79\ KN\ (+ve)$

Just right $S.F = 4.79\ KN\ (+ve)$

At point E

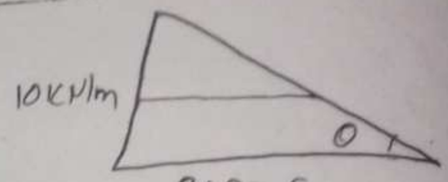
Just left $S.F = 4.79\ KN\ (+ve)$

Just right $S.F = 4.79\ KN\ (+ve)$

At point B

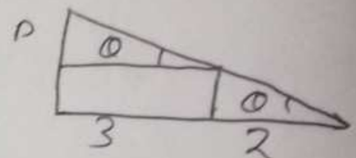
Just left $S.F = 4.79\ KN\ (+ve)$

Just right $S.F = 4.79 - 4.79 = 0$



$\therefore \theta = \tan^{-1}\left(\frac{10}{5}\right) = 63.4$

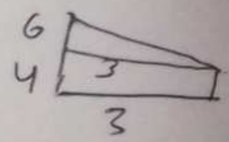
now



$\frac{p}{b} = \tan\theta$

$\therefore p = \tan(63.494)\times 3$

$= 6$



Calculation of Shear force.

At point C

Just left $S.F = 0$

Just right $S.F = 0$

At point A

Just left $S.F = -\frac{1}{2}\times 6\times 3 - 4\times 3$

$= 21\,KN(-ve)$

Just right $S.F = -21 + 29.79$

$= 8.79\,KN\,(+ve)$

At point D

Just left $S.F = -\frac{1}{2}\times 5\times 10 + 29.79$

$= 4.79\,KN\,(+ve)$

Just right $S.F = 4.79\,KN\,(+ve)$

At point E

Just left $S.F = 4.79\,KN\,(+ve)$

Just right $S.F = 4.79\,KN\,(+ve)$

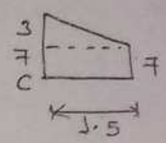
At point B

Just left $S.F = 4.79\,KN\,(+ve)$

Just right $S.F = 4.79 - 4.79 = 0$

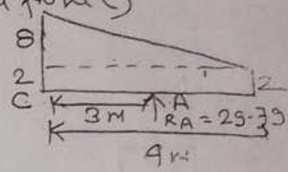
S.F at Mid of CA (ie. at 1.5m from pt. C)

$S.F = -\frac{1}{2}\times 1.5\times 3 - 7\times 1.5$



or, $S.F = 12.75\,KN(-ve)$

S.F at Mid of AD (ie. at 4m from C)

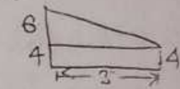


$S.F = -\frac{1}{2}\times 4\times 8 - 2\times 4 + 29.79$

or, $S.F = 5.79\,KN\,(+ve)$

Calculation of Bending Moment.

At point C, $B.M = 0$



At Point A $B.M = -\frac{1}{2}\times 3\times 6\times \frac{2}{3}\times 3 - 4\times 3\times \frac{3}{2}$

or, $B.M = 36\,KNm\,(-ve)$

At point D

$B.M = -\frac{1}{2}\times 5\times 10\times \frac{2}{3}\times 5 + 29.79\times 2$

or, $B.M = 23.75\,KNm\,(-ve)$

At Point E

Just left, $B.M = -\frac{1}{2}\times 5\times 10\times \left(\frac{2}{3}\times 5 + 3\right)$

$+ 29.79\times 5$

or, $B.M = 9.38\,KNm\,(-ve)$

Nard

Just right

$B.M = -9.38 - 5$

or, $B.M = 14.38$ KNm (-ve)

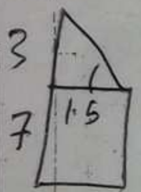
At point B

$B.M = -\frac{1}{2}\times5\times10\times\left(\frac{2}{3}\times5+6\right) + 29.79\times8 - 5$

or, $B.M = -0.01 \simeq 0$

Again,

B.M at Mid of CA (ie. at 1.5m from C)



$B.M = -\frac{1}{2}\times3\times1.5\times\frac{2}{3}\times1.5 - 7\times1.5\times\frac{1.5}{2}$

$\tan(63.494)\times1.5 = 3$

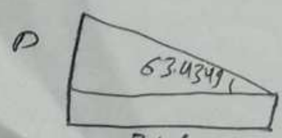
or, $B.M = 10.12$ KNm (-ve)

Also,

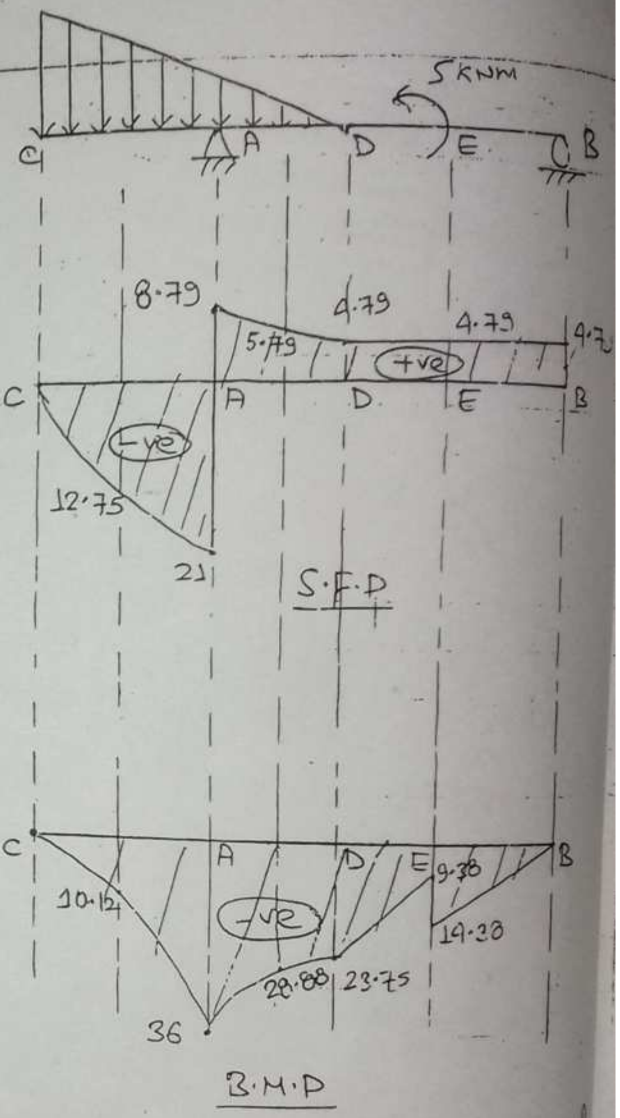
B.M at Mid of AD (ie. at 4m from C)

$B.M = -\frac{1}{2}\times4\times8\times\frac{2}{3}\times4 - 2\times4\times\frac{4}{2} + 29.79\times1$

or, $B.M = 28.88$ KNm (-ve)



$P = \tan\theta\times4 = 8$



Chapter → 8 (4hrs)

Analysis of Truss

Truss:

The truss is one of the major type of engineering structures. They are made of slender members pin connected at ends and designed to support load at joints. Members of a truss are acted upon by two equal and opposite force directed along the member ie. axial force. Hence truss members has no shear force and bending moment but only has axial force. Such structural system is used extensively in bridges, towers, large span building roofs etc.

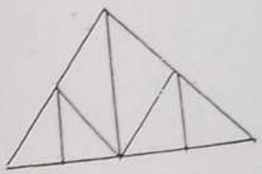


fig. Roof Truss

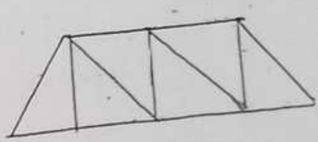


fig. Bridge Truss

Assumption of Ideal truss:

→ The following assumption are made in the analysis of pin jointed truss.

- The members are connected by smooth frictionless pins.
- The members are capable of transmitting only axial tension or axial compression.
- The load acts only at the joint.
- Self weight of members are neglected.
- Generally, all the members have uniform cross-sectional area.

Static determinancy and stability of plane Truss.

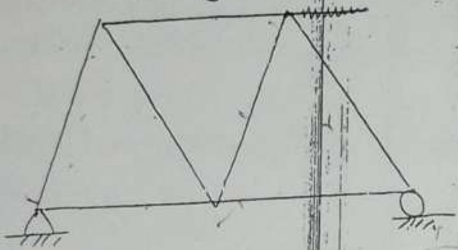
→ Suppose 'm' be the number of members of a truss, 'r' be the no. of reaction & j be the number of joints in a truss then the following condition may arise.

a) $m + r = 2j$ → Truss is Statically determinate & stable

b) $m + r > 2j$ → Truss is Statically indeterminate

c) $m + r < 2j$ → Truss is unstable.

a) $m+r=2j$ Such type of truss is also known as perfect truss. This type of truss has got sufficient number of member to resist the load without undergoing appreciable deformation.



Here,

$m=7$

$r=3$. and

$j=5$

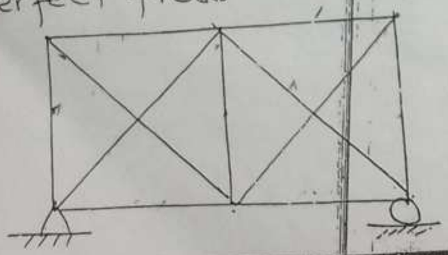
Then,

$m+r=2j$

or, $7+3=2\times5 \Rightarrow 10=10$

So, truss is stable and determinate.

b) $m+r > 2j$ These type of truss are also known as redundant truss. The number of member in it are more than the required no for a perfect truss



Here,

$r=3$

$m=11$

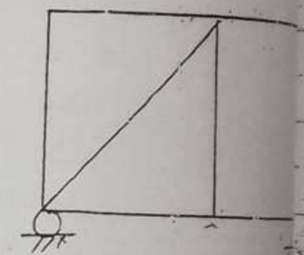
$j=6$

Then, $m+r = 3+11 = 14$

$2j = 2\times6 = 12$

$\therefore m+r > 2j$ so, the truss is indeterminate. Such type of truss cannot be analysed by using eq of equilibrium.

c) $m+r < 2j$ These type of truss are also known as deficient truss. A truss is said to be deficient if the number of members in it are less than required for a perfect truss. These truss cannot retain their shape when loaded.



Here, $m=8$, $r=3$, $j=6$

$m+r = 8+3 = 11$

$2j = 2\times6 = 12$

$\therefore m+r < 2j$ Hence truss is unstable.

so, the
n type of
using eqn

truss are
s. A truss

truss is

Method of analysis of Truss

→ There are two methods to find out the forces in the members of a truss. They are

- → Method of Joint
- → Method of section (Method of Moments)

a) Method of Joint

In this method each and every joint is treated separately as shown in figure below.

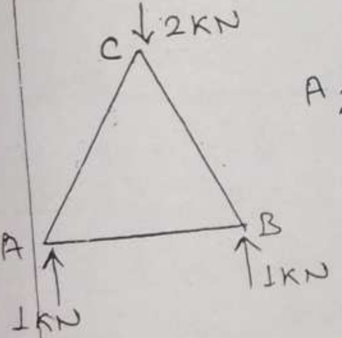


fig. simple Truss

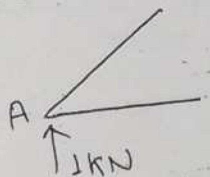


fig. Joint A

fig. Joint B

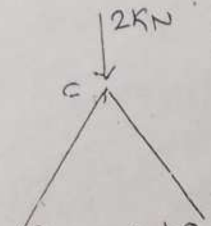


fig. Joint C

The value of unknown forces are then determined by equilibrium equations.

ie. $\Sigma F_x = 0$, $\Sigma F_y = 0$

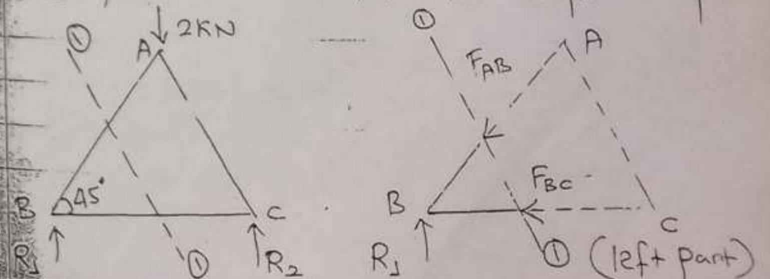
Before isolating the joint, support reactions must be calculated taking equilibrium of whole structure.

ie. $\Sigma F_x = 0$, $\Sigma F_y = 0$, $\Sigma M = 0$

The joint having maximum two unknown force should be selected first.

b) Method of section

This method is convenient, when the force in a few members of a truss are required to be found out. In this method after determining support reaction, a section line is drawn passing through not more than three members in which force are not known such that the frame is cut into two separate parts.



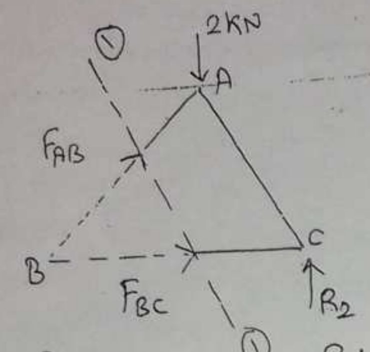


fig. Right part of truss

Each part should be in equilibrium under the action of loads, reaction and member forces. A part of the structure, on any side of the section line, is then treated as a free body in equilibrium under the action of external forces. The unknown forces are then found out by the application of equilibrium or the principle of statics.

ie. $\Sigma F_x=0$, $\Sigma F_y=0$ & $\Sigma M=0$

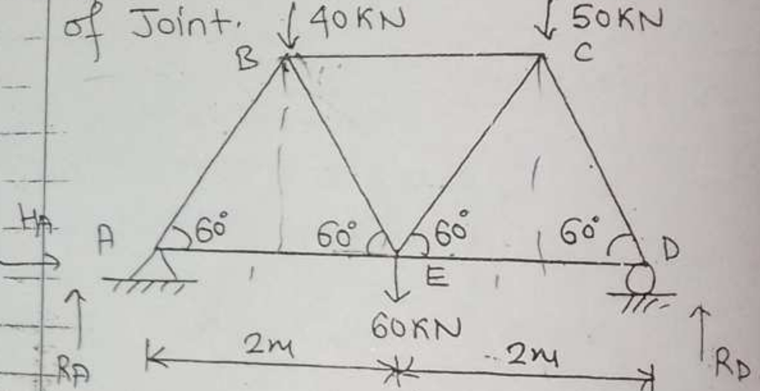
Note:

Under the following two situations, the method of section is preferred over the method of joint

- In the analysis of large truss in which force in few members are re...
- If the method of joint fails to sta... or proceed with analysis for not g... a joint with only two unknown for...

Numericals:

① Determine the forces in all the mem... of a truss as shown in fig by meth... of Joint.



⇒ Calculation of support reaction

$\xrightarrow{+} \Sigma F_x=0$ ie. $H_A=0$

$+\uparrow \Sigma F_y=0$ ie. $R_A+R_D=150$ ⟶ ①

$\Sigma M_A=0$ ie. $40\times1+50\times3+60\times2-4\times R_D=0$

or, $R_D=77.5\,KN$

ers are required
ails to start
for not getting
known force.

the members
by method

R_D

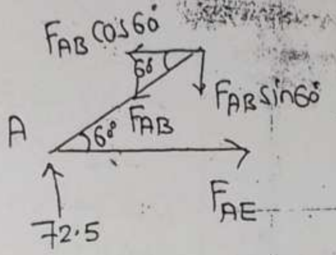
$\longrightarrow$ ①

$+60\times2-$
$R_D = 0$

From eqn ①

$R_A = 72.5\,KN$

At joint A



$+\uparrow \Sigma F_y = 0$

ie. $72.5 - F_{AB}\sin60^\circ = 0$

or, $F_{AB} = \dfrac{72.5}{\sin60^\circ}$

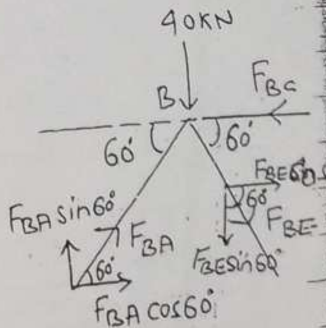
or, $F_{AB} = 83.72\,KN$ (C)

$\xrightarrow{+} \Sigma F_x = 0$

$F_{AE} - F_{AB}\cos60^\circ = 0$

or, $F_{AE} = 83.72\times\cos60^\circ = 41.86\,KN$ (T)

At Joint B



$+\uparrow \Sigma F_y = 0$

ie. $-40 + F_{BA}\sin60^\circ$
$- F_{BE}\sin60^\circ = 0$

or, $-40 + 83.72\sin60^\circ$
$- F_{BE}\sin60^\circ = 0$

or, $F_{BE} = 37.53\,KN$ (T)

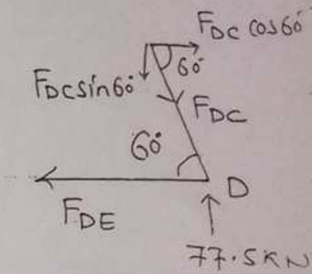
$\xrightarrow{+} \Sigma F_x = 0$

ie. $F_{BA}\cos60^\circ + F_{BE}\cos60^\circ = F_{BC}$

or, $F_{BC} = 83.72\times\cos60^\circ + 37.53\cos60^\circ$

or, $F_{BC} = 60.63\,KN$ (C)

At Joint D



$+\uparrow \Sigma F_y = 0$

ie. $77.5 - F_{DC}\sin60^\circ = 0$

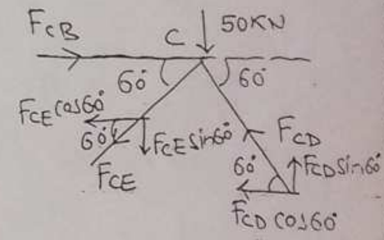
or, $F_{DC} = 89.49\,KN$ (C)

$\xrightarrow{+} \Sigma F_x = 0$

ie. $F_{DC}\cos60^\circ - F_{DE} = 0$

or, $F_{DE} = 89.49\times\cos60^\circ = 44.75\,KN$ (T)

At Joint C



$+\uparrow \Sigma F_y = 0$

ie. $-50 + F_{CD}\sin60^\circ -$
$F_{CE}\sin60^\circ = 0$

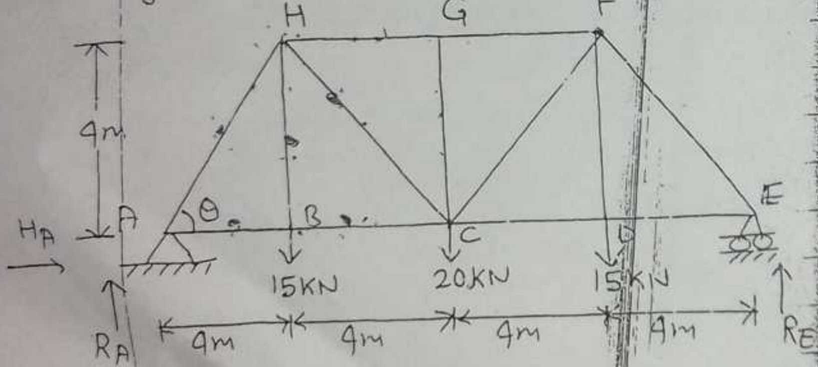
or, $F_{CE} = \dfrac{-50 + 89.49\times\sin60^\circ}{\sin60^\circ}$

or, $F_{CE} = 31.75\,KN$ (T).

The force in all the member of a truss are tabulated below.

SN	Member	Magnitude of force (KN)	Nature
1	AB	83.72	C
2	AE	41.86	T
3	DE	44.74	T
4	DC	89.49	C
5	BE	37.53	T
6	BC	60.63	C
7	CE	31.75	T

(2) Solve the following truss using the joint Method.



⇒ Calculation of support rxn

$\xrightarrow{+} \Sigma F_x = 0$ ie. $H_A = 0$

$+\uparrow \Sigma F_y = 0$ ie. $R_A + R_E = 50 \longrightarrow ①$

$\Sigma M_A = 0$ ie. $15\times4 + 20\times8 + 15\times12 = R_E \times 16$

or, $R_E = 25\,KN$

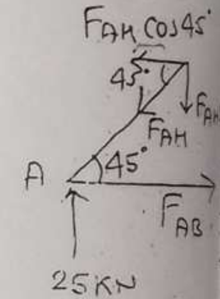
from eqn ① $R_A = 50 - 25 = 25\,KN$

In rt. Δ ABH

$\tan\theta = 4/4 \Rightarrow \theta = 45°$

The given truss is symmetrical in structure and loading hence let us calculate only member force about left part of GC.

At Joint A



$+\uparrow \Sigma F_y = 0$

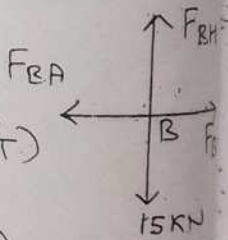
ie. $25 = F_{AH}\sin45°$

or, $F_{AH} = 35.36\,KN$ (C)

$\xrightarrow{+} \Sigma F_x = 0$ ie. $F_{AB} = F_{AH}\cos45°$

or, $F_{AB} = 35.36 \times \cos45° = 25\,KN$ (T)

At Joint B



$\xrightarrow{+} \Sigma F_x = 0$

ie. $F_{BC} = F_{BA} = 25\,KN$ (T)

$+\uparrow \Sigma F_y = 0$

ie. $F_{BH} = 15\,KN$ (T)

+15x12=R_E

5 = 25 kN

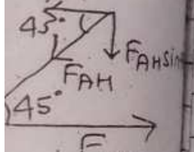
45°

al in

ce let us

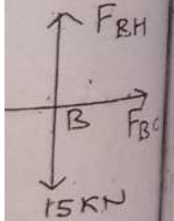
bout left

$F_{AH}\cos 45°$

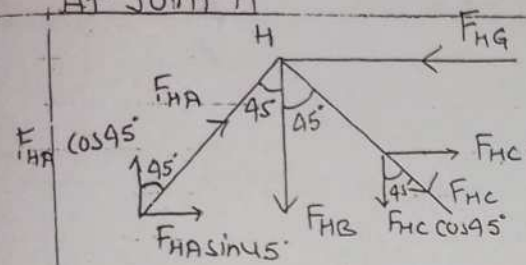


kN

25 kN (T)



At Joint H



$+\uparrow \Sigma F_y = 0$

ie. $F_{HA}\cos 45° - F_{HB} - F_{HC}\cos 45° = 0$

or, $35.36\cos 45° - 15 = F_{HC}\cos 45°$

or, $F_{HC} = 14.15$ kN (T)

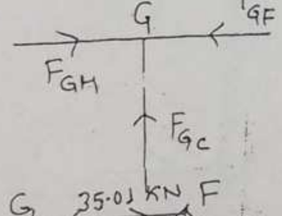
$\xrightarrow{+} \Sigma F_x = 0$

ie. $F_{HA}\sin 45° + F_{HC}\sin 45° = F_{HG}$

or, $F_{HG} = 35.36\cos 45° + 14.15\sin 45°$

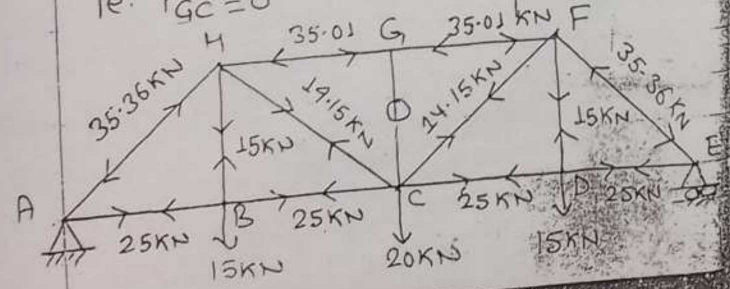
or, $F_{HG} = 35.01$ kN (C)

At Joint G



$+\uparrow \Sigma F_y = 0$

ie. $F_{GC} = 0$



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(3) Analyse the force in all the member of given truss using method of Joint.

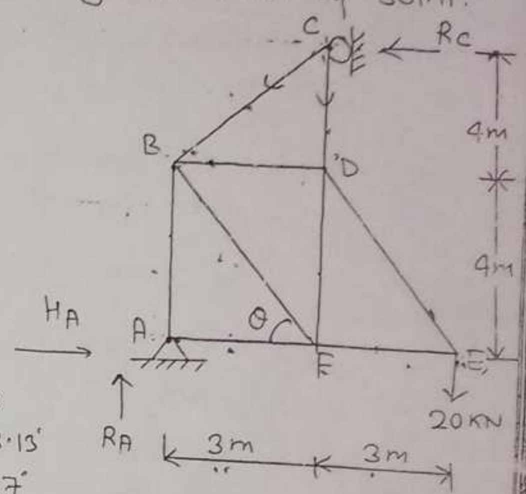
⇒ Here,

In rt. △ AFB

$\tan\theta = \frac{AB}{AF}$

or, $\tan\theta = \frac{4}{3}$

∴ $\theta = 53.13°$



Then,

∠ABF = $90° - \theta$

$= 90° - 53.13°$

$= 36.87°$

Calculation of support reaction

$\xrightarrow{+} \Sigma F_x = 0$

ie. $H_A - R_C \Rightarrow H_A = R_C \longrightarrow (1)$

$+\uparrow \Sigma F_y = 0$ ie. $R_A = 20$ kN

$\Sigma M_A = 0$ ie. $20 \times 6 - R_C \times 8 = 0$

or, $R_C = 15$ kN

From eqn (1)

$R_C = H_A = 15$ kN

direction लाई

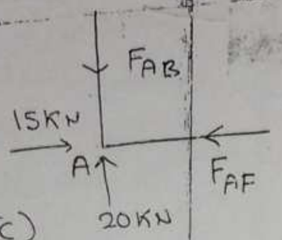
Tension भए outward जाने

compression भए inward जाने

At Joint A

$\xrightarrow{+} \Sigma F_x = 0$

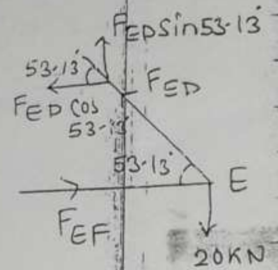
ie. $F_{AF} = 15KN$ (C)



$+\uparrow \Sigma F_y = 0$

ie. $F_{AB} = 20KN$ (C)

At Joint E



$+\uparrow \Sigma F_y = 0$

ie. $F_{ED} \sin 53 \cdot 13 = 20$

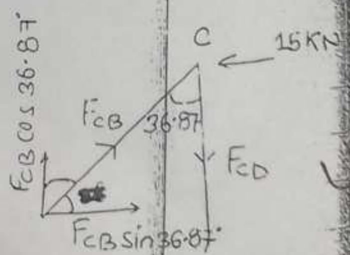
or, $F_{ED} = 25KN$ (T)

$\xrightarrow{+} \Sigma F_x = 0$

ie. $F_{EF} = F_{ED} \cos 53 \cdot 13'$

or, $F_{EF} = 25 \times \cos 53 \cdot 13' = 15KN$ (C)

At Joint C



$\xrightarrow{+} \Sigma F_x = 0$

ie. $F_{CB} \sin 36 \cdot 87 = 15$

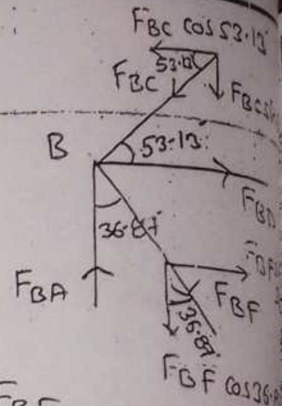
or, $F_{CB} = 25KN$ (C)

$+\uparrow \Sigma F_y = 0$

ie. $F_{CB} \cos 36 \cdot 87' = F_{CD}$

or, $F_{CD} = 25 \cos 36 \cdot 87' = 20KN$ (T)

At Joint B



$+\uparrow \Sigma F_y = 0$

ie. $F_{BA} - F_{BC} \sin 53 \cdot 13' - F_{BF} \cos 36 \cdot 87' = 0$

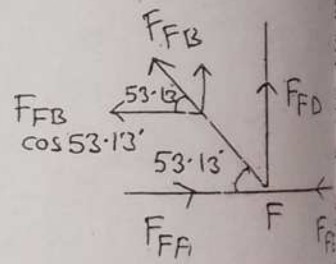
or, $\dfrac{20 - 25 \times \sin 53 \cdot 13'}{\cos 36 \cdot 87'} = F_{BF}$

or, $F_{BF} = 0$

$\xrightarrow{+} \Sigma F_x = 0$ ie. $F_{BF} \sin 36 \cdot 87' + F_{BD} - F_{BC}$

or, $0 + F_{BD} = 25 \times \cos 53 \cdot 13' = 15KN$

At Joint F



$+\uparrow \Sigma F_y = 0$

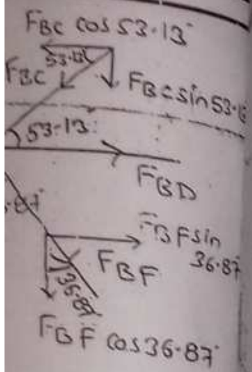
ie. $F_{FB} \sin 53 \cdot 13' + F_{FD} = 0$

or, $0 + F_{FD} = 0$

$\therefore F_{FD} = 0$

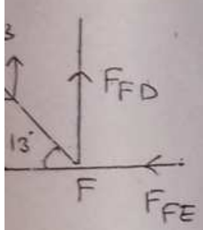
(2013) Fall

4. Using the method of Joints, Determine the force in each member of the truss as shown in figure.

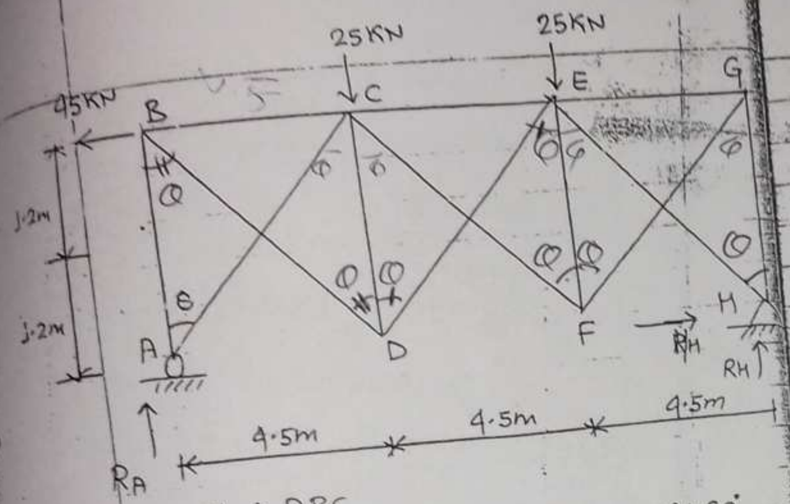


$+ F_{BD} - F_{BC}\cos 53.13° ...$

$= 15 KN (T)$



Determine of the



$\Rightarrow$ In rt. Δ ABC

$\tan\theta = \frac{BC}{AB} = \frac{4.5}{2.4} \Rightarrow \theta = 61.92°$

Calculation of support reaction

$\xrightarrow{+} \Sigma F_x = 0$

ie. $R_H = 45 KN$

$+\uparrow \Sigma F_y = 0$

ie. $R_A + R_H = 50 \longrightarrow (1)$

$+\circlearrowright \Sigma M_A = 0$

ie. $-45\times 2.4 + 25\times 4.5 + 25\times 9 - R_H \times 13.5 ...$

or, $R_H = 17 KN$

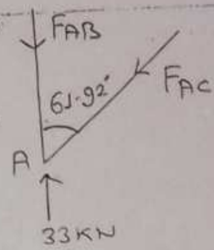
$\therefore$ From eq^n (1)

$R_A = 50 - 17 = 33 KN$

At Joint A $\xrightarrow{+} \Sigma F_x = 0$

$\therefore F_{AC}\cos 61.92° = 0$

or, $F_{AC} = 0$



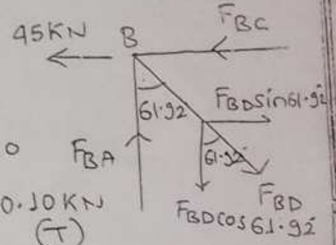
$+\uparrow \Sigma F_y = 0$

ie. $33 - F_{AB} - F_{AC}\sin 61.92° = 0$

or, $33 - F_{AB} - 0 = 0$

or, $F_{AB} = 33 KN$ (C)

At Joint B



$+\uparrow \Sigma F_y = 0$

ie. $F_{BA} - F_{BD}\cos 61.92° = 0$

or, $F_{BD} = \frac{33}{\cos 61.92°} = 70.10 KN$ (T)

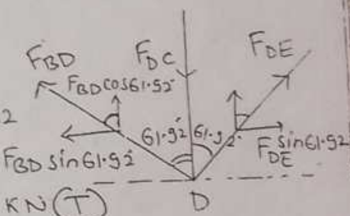
$\xrightarrow{+} \Sigma F_x = 0$

ie. $-45 - F_{BC} + F_{BD}\sin 61.92° = 0$

or, $-45 + 70.10 \times \sin 61.92° = F_{BC}$

or, $F_{BC} = 16.84 KN$ (C)

At Joint D



$\xrightarrow{+} \Sigma F_x = 0$

ie. $F_{DE}\cancel{\sin 61.92°} = F_{BD}\cancel{\sin 61.92}$

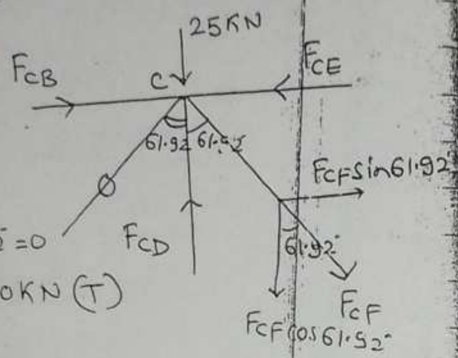
or, $F_{DE} = F_{BD} = 70.10 KN$ (T)

$+\uparrow \Sigma F_y = 0$ ie. $F_{DE}\cos 61.92° + F_{BD}\cos 61.92° - F_{DC} = 0$

or, $2 \times 70.10 \times \cos 61.92 = F_{DC}$

or, $F_{DC} = 66\,KN$ (C)

At Joint C



$+\uparrow \Sigma F_y = 0$

ie. $F_{CD} - 25 - F_{CF} \cos 61.92° = 0$

or, $F_{CF} = 87.10\,KN$ (T)

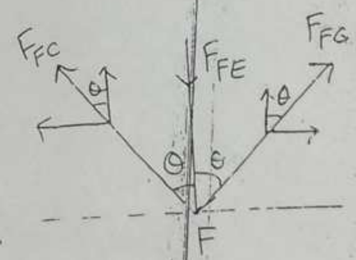
$\xrightarrow{+} \Sigma F_x = 0$

ie. $F_{CB} - F_{CE} + F_{CF} \sin 61.92° = 0$

or, $16.84 - F_{CE} + 87.10 \times \sin 61.92° = 0$

or, $F_{CE} = 93.69\,KN$ (C)

At joint F



$\xrightarrow{+} \Sigma F_x = 0$

ie. $F_{FG} \sin 61.92° = F_{FC} \sin 61.92°$

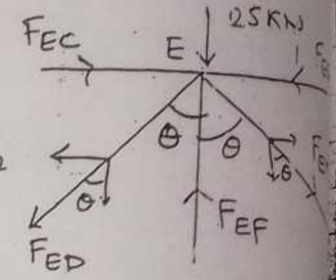
or, $F_{FG} = F_{FC} = 87.10\,KN$ (T)

$+\uparrow \Sigma F_y = 0$ ie. $F_{FC} \times \cos 61.92° - F_{FE} + F_{FG} \times \cos 61.92° = 0$

or, $2 \times 87.10 \cos 61.92° = F_{FE}$

or, $F_{FE} = 82\,KN$ (C)

At Joint E



$+\uparrow \Sigma F_y = 0$

ie. $F_{EF} - 25 - F_{EH} \cos 61.92 - F_{ED} \cos 61.92° = 0$

or, $82 - 25 - F_{EH} \cos 61.92° - 70.01 \times \cos 61$

or, $F_{EH} = 51\,KN$ (T)

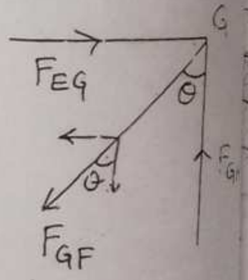
$\xrightarrow{+} \Sigma F_x = 0$

ie. $F_{EC} - F_{EG} - F_{ED} \sin 61.92° + F_{EH} \sin 61$

or, $93.69 - F_{EG} - 70.10 \times \sin 61.92° + 51 \times \sin 61$

or, $F_{EG} = 76.83\,KN$ (C)

At Joint G



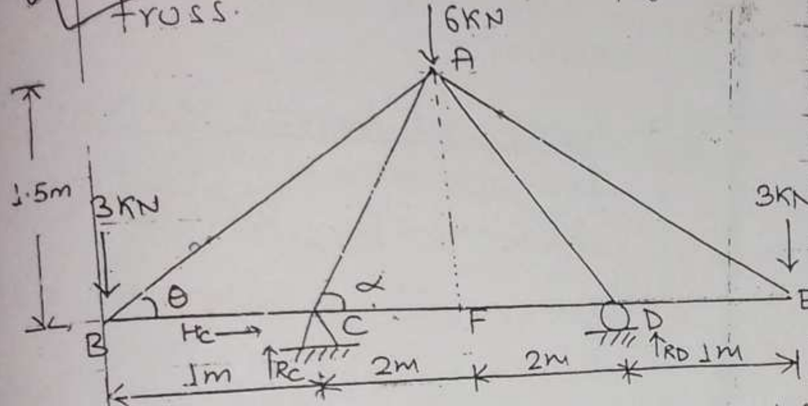
$+\uparrow \Sigma F_y = 0$

ie. $F_{GH} - F_{GF} \cos 61.92° = 0$

or, $F_{GH} = 87.10 \times \cos 61.92°$

or, $F_{GH} = 41\,KN$ (C)

(5) Using method of joint, find out the forces in the member of given truss.



⇒ The given truss is symmetric structural and in loading also. The member force calculated for half of truss can be applied for the rest.

In rt. △ABF

$\tan\theta = \frac{AF}{BF} = \frac{1.5}{3} \Rightarrow \theta = 26.56°$

In rt. △ACF

$\tan\alpha = \frac{AF}{CF} = \frac{1.5}{2} \Rightarrow \alpha = 36.87$

Calculation of support rxn

$\xrightarrow{+} \Sigma F_x = 0$ ie. $H_C = 0$

$+\uparrow \Sigma F_y = 0$ ie. $R_C + R_D = 12 \longrightarrow (1)$

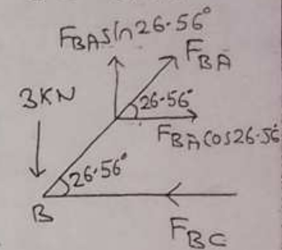
$\circlearrowleft \Sigma M_C = 0$

ie. $-3\times1 + 6\times2 - R_D\times4 + 3\times5 = 0$

or, $R_D = 6KN$

From eqn (1) $R_C = 12 - 6 = 6KN$

At Joint B



$+\uparrow \Sigma F_y = 0$

ie. $F_{BA}\sin 26.56° - 3 = 0$

or, $F_{BA} = 6.70\,KN\ (T)$

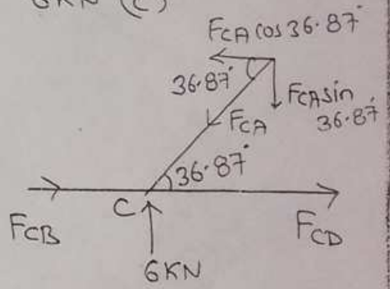
$\xrightarrow{+} \Sigma F_x = 0$

ie. $F_{BA}\cos 26.56° - F_{BC} = 0$

or, $F_{BC} = 6.70 \times \cos 25.56°$

or, $F_{BC} = 6KN\ (C)$

At Joint C



$+\uparrow \Sigma F_y = 0$

ie. $6 - F_{CA}\sin 36.87° = 0$

or, $F_{CA} = 10KN\ (C)$

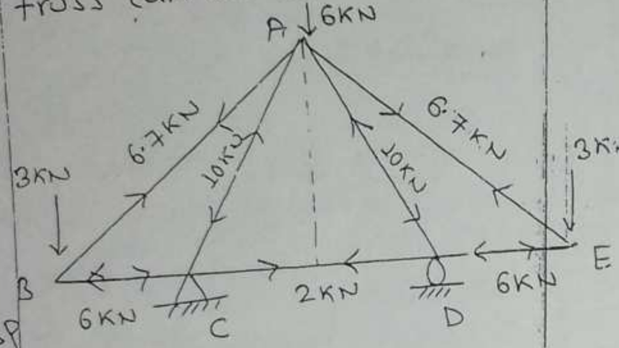
$\xrightarrow{+} \Sigma F_x = 0$

or, $F_{CB} + F_{CD} - F_{CA}\cos 36.87° = 0$

or, $6 + F_{CD} - 10\times\cos 36.87° = 0$

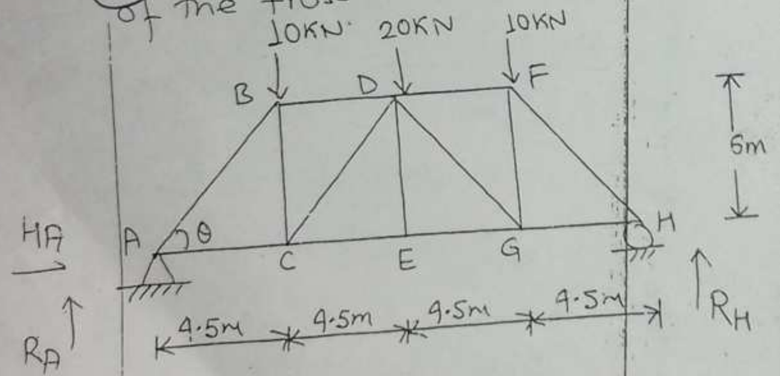
or, $F_{CD} = 2KN$ (T)

Now,
The forces on the member of given truss can be shown as



(2006) SP

6. Determine the forces in the member of the truss as shown in figure.



⇒ In rt. Δ ABC

$\tan\theta = \frac{BC}{AC} = \frac{6}{4.5} \Rightarrow \theta = 53.13°$

The given truss is symmetric structurally and in loading also. The member force calculated for half portion of truss can be applied the rest.

* Calculation of support reaction

$\xrightarrow{+} \Sigma F_x = 0$

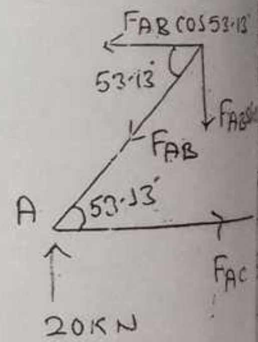
$+\uparrow \Sigma F_y = 0$ ie. $R_A + R_H = 40$ → (1)

$\circlearrowright \Sigma M_A = 0$ ie. $10 \times 4.5 + 20 \times 9 + 10 \times 13.5 - R_H \times 18 = 0$

or, $R_H = 20KN$

From eqn (1) $R_A = 40 - 20 = 20KN$

At Joint A



$+\uparrow \Sigma F_y = 0$

ie. $20 = F_{AB} \sin 53.13°$

or, $F_{AB} = 25KN$ (C)

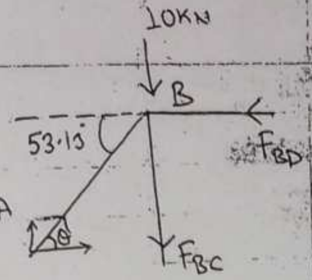
$\xrightarrow{+} \Sigma F_x = 0$

ie. $F_{AC} = F_{AB} \cos 53.13°$

or, $F_{AC} = 25 \times \cos 53.13°$

or, $F_{AC} = 15KN$ (T)

At Joint B



$+\uparrow \Sigma F_y = 0$

or, $F_{BA}\sin 53.13 - 10 - F_{BC} = 0$

or, $F_{BC} = 25 \times \sin 53.13 - 10 = 10KN$ (T)

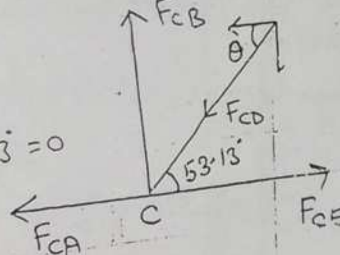
$\xrightarrow{+} \Sigma F_x = 0.$

ie. $F_{BA}\cos 53.13 - F_{BD} = 0$

or, $F_{BD} = 25 \times \cos 53.13$

or, $F_{BD} = 15KN$ (C)

At Joint C



$+\uparrow \Sigma F_y = 0$

ie. $F_{CB} - F_{CD}\sin 53.13 = 0$

or, $F_{CD} = \dfrac{10}{\sin 53.13}$

or, $F_{CD} = 12.5KN$ (C)

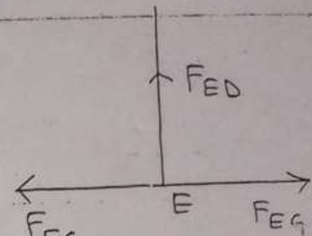
$\xrightarrow{+} \Sigma F_y = 0$

ie. $-F_{CA} + F_{CE} - F_{CD}\cos 53.13 = 0$

or, $-15 + F_{CE} - 12.5\cos 53.13 = 0$

or, $F_{CE} = 22.5 KN$ (T)

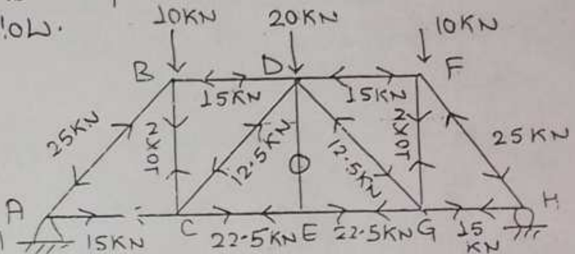
At Joint E



$+\uparrow \Sigma F_y = 0$

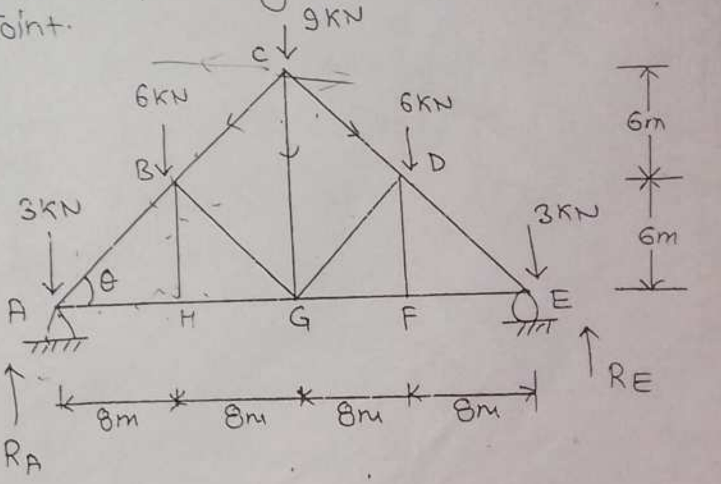
ie. $F_{ED} = 0$

The force in all the member of the truss is shown in fig below.



(2007) Fall

⑦ Solve the following truss by Method of Joint.



In rt. Δ ABH

$\tan\theta = \frac{BH}{AH} = \frac{6}{8} \Rightarrow \theta = 36.87°$

calculation of support reaction

$\xrightarrow{+} \Sigma F_x = 0$ ie. $H_A = 0$

$+\uparrow \Sigma F_y = 0$ ie. $R_A + R_E = 27 \longrightarrow (1)$

$\circlearrowright\!+ \; \Sigma M_A = 0$

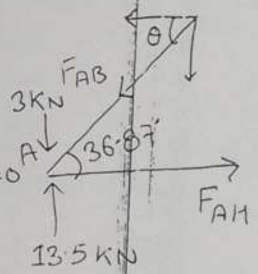
ie. $6\times8 + 9\times16 + 6\times24 + 3\times32 - R_E\times32 = 0$

or, $R_E = 13.5\ KN$

From eqn (1)

$R_A = 27 - 13.5 = 13.5\ KN$

At Joint A



$+\uparrow \Sigma F_y = 0$

or, $13.5 - 3 - F_{AB}\sin 36.87° = 0$

or, $F_{AB} = 17.5\ KN$ (C)

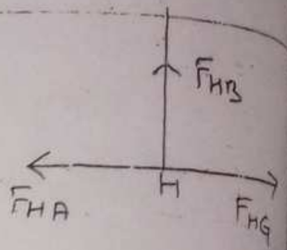
$\xrightarrow{+} \Sigma F_x = 0$

ie. $F_{AH} = F_{AB}\cos 36.87°$

or, $F_{AH} = 17.5 \times \cos 36.87°$

or, $F_{AH} = 14\ KN$ (T)

At Joint H

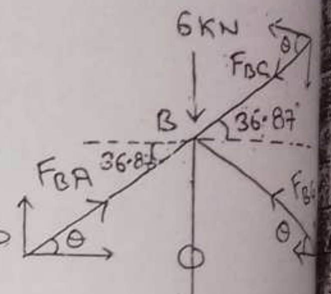


$+\uparrow \Sigma F_y = 0$ ie. $F_{HB} = 0$

$\xrightarrow{+} \Sigma F_x = 0$

ie. $F_{HG} = F_{HA} = 14\ KN$ (T)

At Joint B



$+\uparrow \Sigma F_y = 0$

ie. $F_{BA}\sin 36.87° - 6 + F_{BG}\sin 36.87° - F_{BC}\sin 36.87° = 0$

or, $17.5\sin 36.87 - 6 + F_{BG}\sin 36.87° - F_{BC}\sin 36.8$

or, $F_{BG} - F_{BC} = -7.5$

or, $F_{BC} - F_{BG} = 7.5 \longrightarrow (1)$

$\xrightarrow{+} \Sigma F_x = 0$

ie. $F_{BA}\cos 36.87° - F_{BG}\cos 36.87° - F_{BC}\cos 36.87° = 0$

or, $F_{BG} + F_{BC} = 17.5 \longrightarrow (ii)$

solving eqn (1) and (ii)

$F_{BC} = 12.5\ KN$ (C)

$F_{BG} = 5\ KN$ (C)

F_{HB}

F_{HG}

F_{BC}

$36.87°$

F_{BG}

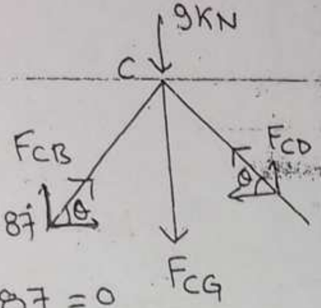
$_{BC} \sin 36.87 =$

$\cos 36.87 = 0$

At Joint C

$+\uparrow \Sigma F_y = 0$

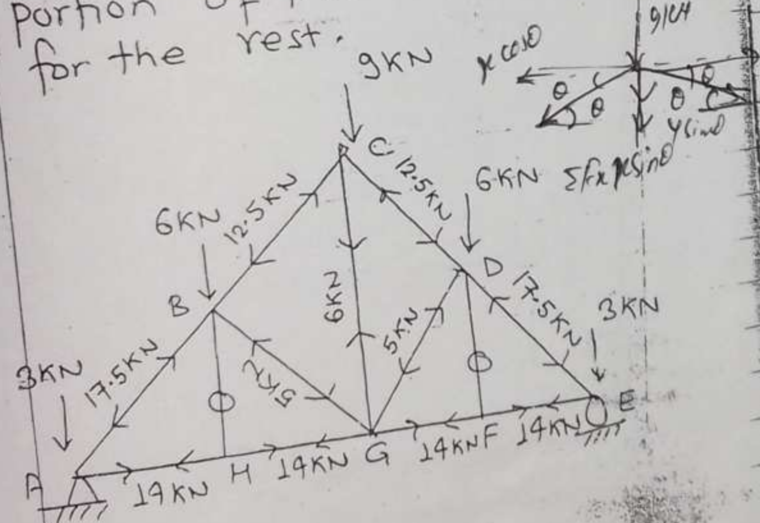
i.e. $-9 - F_{CG} + F_{CB} \sin 36.87 + F_{CD} \sin 36.87 = 0$



or, $F_{CG} = 2 \times 12.5 \times \sin 36.87 - 9$

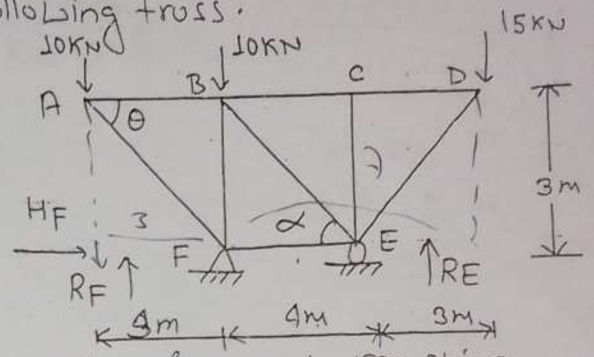
or, $F_{CG} = 6KN$ (T)

since, the given truss is symmetric in loading and structurally so member force calculated for half portion of truss can be applied for the rest.



(2013) SP

(8) Analyze the force in each member of following truss.



⇒ Calculation of support reaction

$\xrightarrow{+} \Sigma F_x = 0$ i.e. $H_F = 0$

$+\uparrow \Sigma F_y = 0$ i.e. $R_F + R_E = 35 \longrightarrow (1)$

$+\circlearrowright \Sigma M_F = 0$

i.e. $-10 \times 3 + 15 \times 7 - R_E \times 4 = 0$

or, $R_E = 18.75$ KN

From eqn (1)

$R_F = 35 - 18.75 = 16.25$ KN

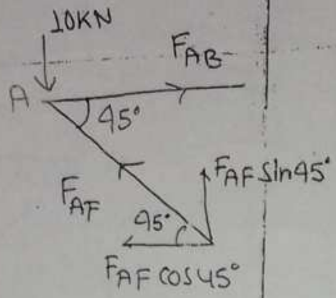
In rt. Δ ABF

$\tan\theta = \frac{BF}{AB} = 3/3 \Rightarrow \theta = 45°$

and In rt. Δ BFE

$\tan\alpha = \frac{BF}{FE} = \frac{3}{4} \Rightarrow \alpha = 36.87°$

At Joint A



$+\uparrow \Sigma F_y = 0$

ie. $F_{AF}\sin 45° = 10$

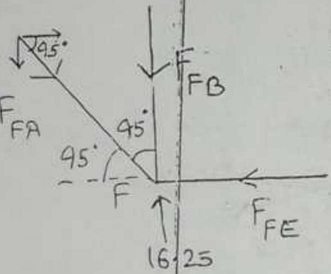
or, $F_{AF} = 14.14\ KN$ (C)

$\xrightarrow{+} \Sigma F_x = 0$

ie. $F_{AB} - F_{AF}\cos 45° = 0$

or, $F_{AB} = 14.14\cos 45° = 10KN$ (T)

At Joint F



$\xrightarrow{+} \Sigma F_x = 0$

ie. $F_{FA}\cos 45° - F_{FE} = 0$

or, $F_{FE} = 14.14\cos 45°$

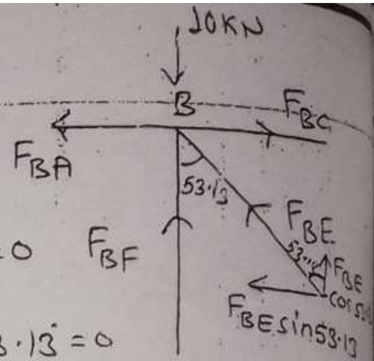
or, $F_{FE} = 10KN$ (C)

$+\uparrow \Sigma F_y = 0$ ie. $16.25 - F_{FA}\sin 45° - F_{FB} = 0$

or, $16.25 - 14.14\sin 45° = F_{FB}$

or, $F_{FB} = 6.25KN$ (C)

At Joint B



$+\uparrow \Sigma F_y = 0$

or, $-10 + F_{BF} + F_{BE}\cos 53.13° = 0$

or, $-10 + 6.25 + F_{BE}\cos 53.13° = 0$

or, $F_{BE} = 6.25\ KN$ (C)

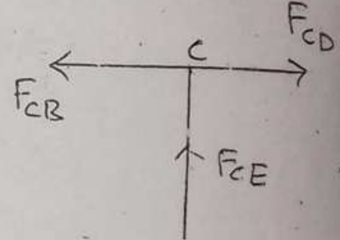
$\xrightarrow{+} \Sigma F_x = 0$

ie. $-F_{BA} + F_{BC} - F_{BE}\sin 53.13 = 0$

or, $-10 + F_{BC} - 6.25\sin 53.13° = 0$

or, $F_{BC} = 15KN$ (T)

At Joint C

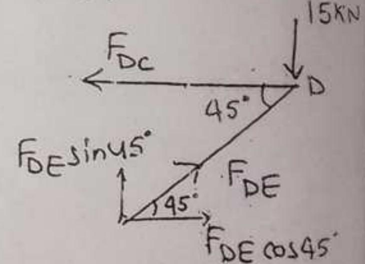


$+\uparrow \Sigma F_y = 0$ ie. $F_{CE} = 0$

$\xrightarrow{+} \Sigma F_x = 0$

ie. $F_{CD} = F_{CB} = 15KN$ (T)

At Joint D

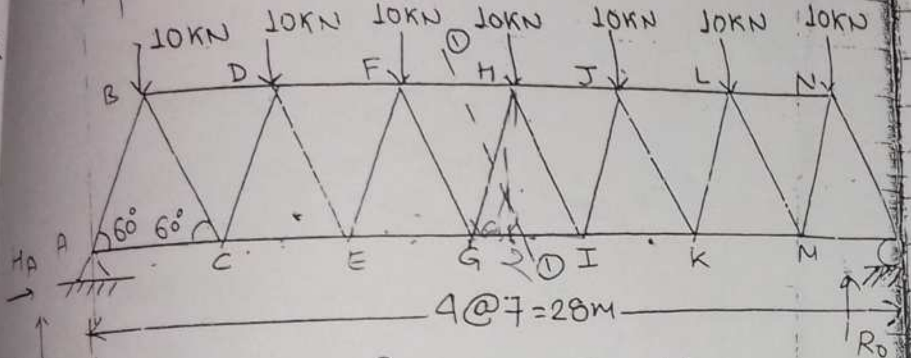


$+\uparrow \Sigma F_y = 0$

ie. $F_{DE}\sin 45° = 15$

or, $F_{DE} = 15/\sin 45 = 21.21\ KN$ (C)

(a) Determine the forces in the members FH, HG and GI in the truss shown below. Each load is 10KN and length of each member is 4m.



=> calculation of support Reaction

$\xrightarrow{+} \Sigma F_x = 0$

ie. $H_A = 0$

$+\uparrow \Sigma F_y = 0$ ie. $R_A + R_O = 70 \longrightarrow (1)$

$\circlearrowright^{+} \Sigma M_A = 0$

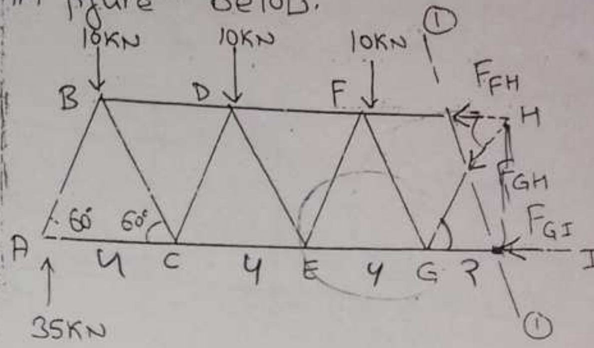
ie. $10\times2 + 10\times6 + 10\times10 + 10\times14 + 10\times18 + 10\times22 + 10\times26 - R_O\times28 = 0$

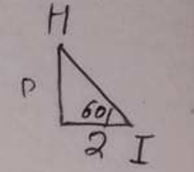
or, $R_O = 35KN$

From eq^n (1) $R_A = 70-35 = 35KN$

Here (1)-(1) section cut the member FH, GH and GI and separate the truss into two parts.

considering the left part of section as shown in figure below.





$\tan 60 = \frac{P}{2}$

$\therefore P = 2\tan 60$

$\circlearrowright^{+} \Sigma M_H = 0$ ie. $35\times14 - 10\times12 - 10\times8 - 10\times4 + F_{GI}\times 2\tan60° = 0$

or, $F_{GI} = -72.17KN = 72.17KN$ (T)

$\circlearrowright^{+} \Sigma M_G = 0$

ie. $35\times12 - 10\times10 - 10\times6 - 10\times2 - F_{FH}\times2\tan60° = 0$

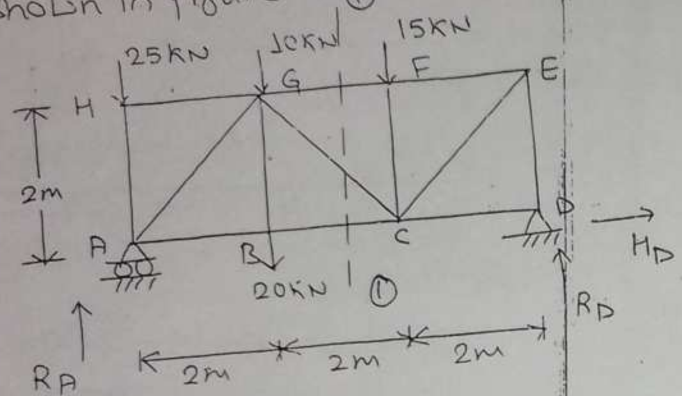
or, $F_{FH} = 69.28KN$ (C)

$+\uparrow \Sigma F_y = 0$ ie. $35 - 10 - 10 - 10 - F_{GH}\sin60° = 0$

or, $F_{GH} = 5.77KN$ (C)

(c) ~~Using~~ the method of section determine the force in the members BC, GC and

GF of the pin-jointed plane truss as shown in figure.



⇒ Calculation of support reaction

$\xrightarrow{+} \Sigma F_x = 0$

ie. $H_D = 0$

$+\uparrow \Sigma F_y = 0$ ie $R_A + R_D = 70 \longrightarrow ①$

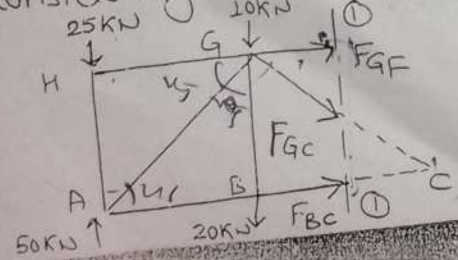
$\circlearrowright^{+} \Sigma M_A = 0$

ie. $10 \times 2 + 20 \times 2 + 15 \times 4 - R_D \times 6 = 0$

or, $R_D = 20KN$

From eqn ① $R_A = 70 - 20 = 50KN$

Considering the left part of section ①-①



Now,

$\circlearrowright^{+} \Sigma M_C = 0$ ie. $50 \times 4 - 25 \times 4 - 20 \times 2 - 10 \times 2 + F_{GF} \times 2 = 0$

or, $F_{GF} = -20KN = 20KN$ (C)

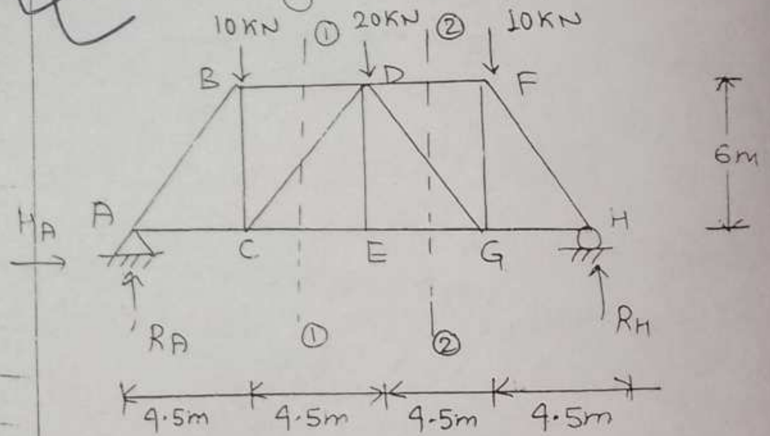
$\circlearrowright^{+} \Sigma M_G = 0$ ie. $50 \times 2 - 25 \times 2 - F_{BC} \times 2 = 0$

or, $F_{BC} = 25KN$ (T)

$+\uparrow \Sigma F_y = 0$ ie. $50 - 25 - 10 - 20 - F_{GC} \times \sin 45 = 0$

or, $F_{GC} = -7.07KN = 7.07KN$ (C)

(ii) Determine the forces in the members CD and DF using method of section.



⇒ Calculation of support reaction

$\xrightarrow{+} \Sigma F_x = 0$ ie. $H_A = 0$

$-20\times2-10\times2$
$=0$
kN (C)

$F_{BC}\times2=0$

$F_{GC}\times\sin45=0$

7 kN (C)

embers CD

on.

$+\uparrow\Sigma F_y=0$ ie. $R_A+R_H=40 \longrightarrow (1)$

$\circlearrowright^{+}\ \Sigma M_A=0$

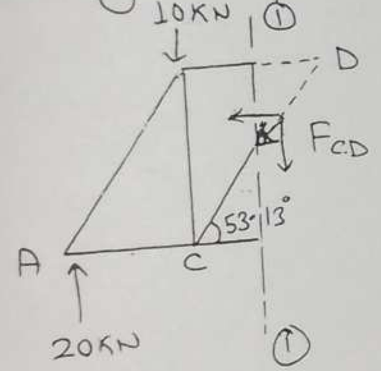
ie. $10\times4.5+20\times9+10\times13.5-R_H\times18=0$

or, $R_H=20\,KN$

From eqn (1) $R_A=40-20=20\,KN$

Here, section (1)-(1) and (2)-(2) cut the member BD, CD, CE and DF, DG, EG resp.

considering left part of section (1)-(1)



$+\uparrow\Sigma F_y=0$

ie. $20-10-F_{CD}\sin53.13^\circ=0$

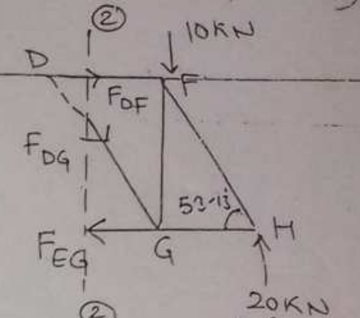
or, $F_{CD}=12.5\,KN\,(C)$

considering section (2)-(2) right part

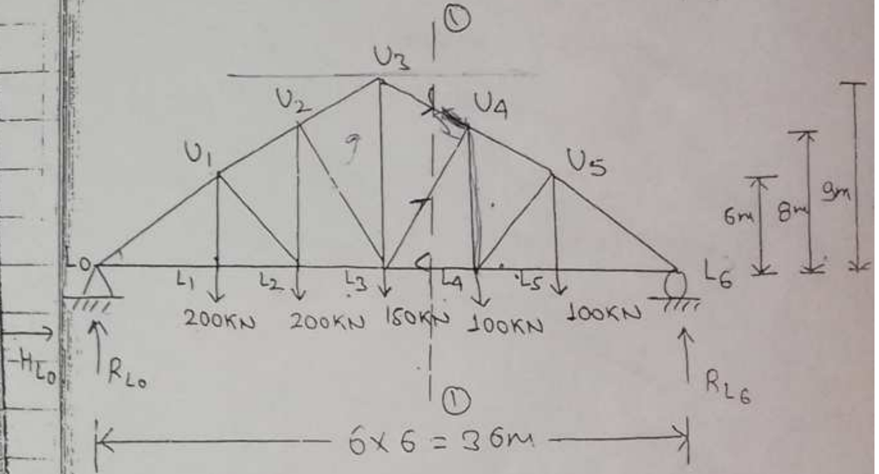
$\circlearrowright^{+}\ \Sigma M_G=0$

ie. $F_{DF}\times6-20\times4.5=0$

or, $F_{DF}=15\,KN\,(C)$



(12) Find the magnitude and nature of force in members U_3U_4, L_3L_4 and U_4L_3 of the loaded truss as shown in figure.



$\circlearrowright \Sigma M_{L0} = 0$

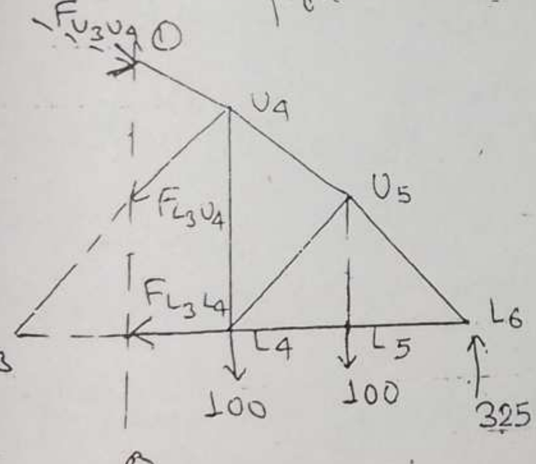
ie $200 \times 6 + 200 \times 12 + 150 \times 18 + 100 \times 24 + 100 \times 300 = 36 \times R_{L6}$

or, $R_{L6} = 325\,KN$

$\therefore R_{L0} = 750 - 325 = 425\,KN.$

Here, section ① - ① cut the members U_3U_4, L_3L_4, L_3U_4 and separate the truss into two parts.

considering right part of the truss as shown in figure.



Then,

$\circlearrowright \Sigma M_{U4} = 0$

ie. $F_{L_3L_4} \times 8 + 100 \times 6 - 325 \times 12 = 0$

or, $F_{L_3L_4} = 412.5\,KN$ (T)

$\circlearrowright \Sigma M_{L3} = 0$

ie. $F_{U_3U_4} \cos 9.46 \times 8 + F_{U_3U_4} \sin 9.46 \times 6 - 325 \times 18 + 100 \times 12 + 100 \times 6 = 0$

or, $F_{U_3U_4} = 456.2\,KN$ (C)

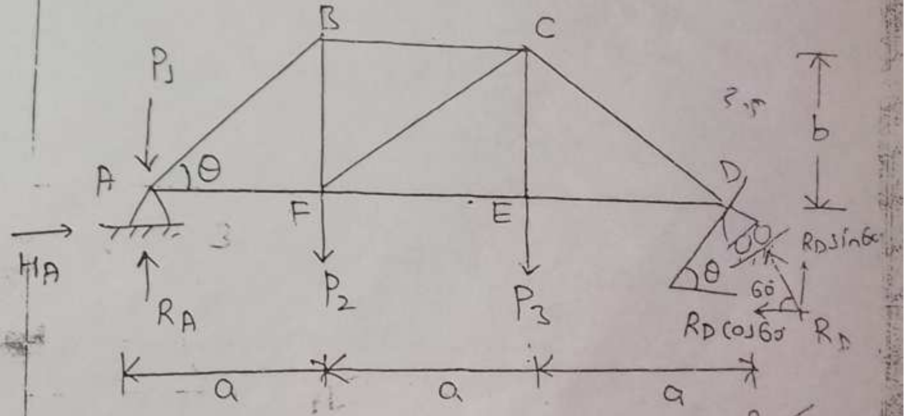
$\xrightarrow{+} \Sigma F_x = 0$

ie. $456.2 \times \cos 9.46 - F_{L_3U_4} \cos 53.13° - 412.5 = 0$

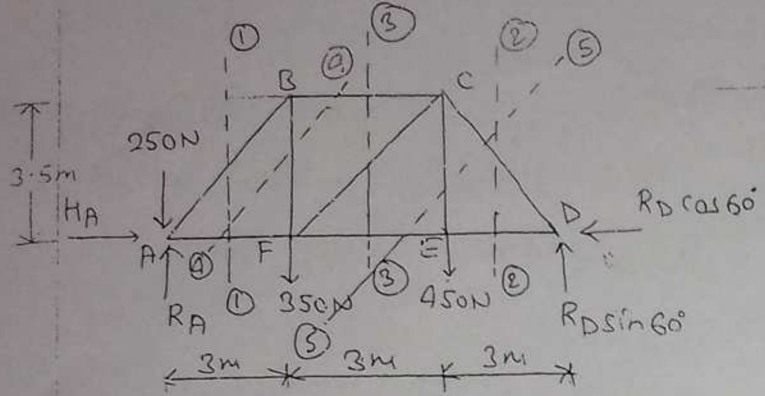
or, $F_{L_3U_4} = 62.5\,KN$ (T)

(2014) Fall (2011) Fall

13) Determine the force in each member of following truss by using section Method. where $P_1 = 250N$, $P_2 = 350N$, $P_3 = 450N$, $a = 3m$, $b = 3.5m$, $\theta = 30°$



$\tan\theta = \frac{3.5}{3} \Rightarrow \theta = 49.40°$



calculation of support reaction:

$\xrightarrow{+} \Sigma F_x = 0$ ie. $H_A = R_D \cos 60°$ ⟶ ①

$+\uparrow \Sigma F_y = 0$ ie. $R_A + R_D \sin 60° = 1050$ ⟶ ②

$\circlearrowright \Sigma M_A = 0$

ie. $350 \times 3 + 450 \times 6 - R_D \sin 60° \times 9 = 0$

or, $R_D = 481.13\,N$

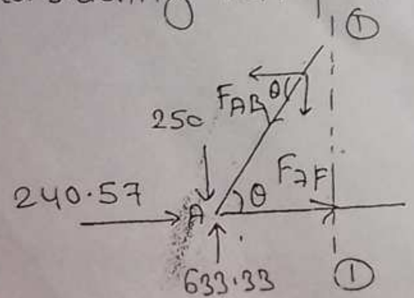
From eqn ①

$H_A = 240.57\,N$

and from eqn ②

$R_A = 633.33\,N$

considering left part of section ① - ①



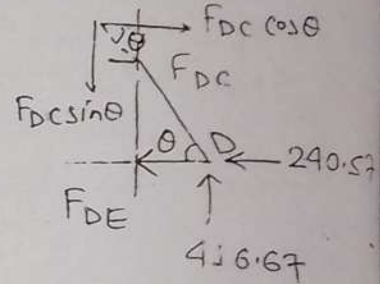
$+\uparrow \Sigma F_y = 0$ ie. $633.33 - 250 - F_{AB} \sin 49.4° = 0$

or, $F_{AB} = 504.87\,N$ (C)

$\xrightarrow{+} \Sigma F_x = 0$ ie. $240.57 + F_{AF} - F_{AB} \cos 49.4 = 0$

or, $F_{AF} = 88\,N$ (T)

Considering Right Part of section ② - ②



$+\uparrow \Sigma F_y = 0$

ie. $416.67 = F_{DC} \sin 49.4$

or, $F_{DC} = 548.78\,N$ (C)

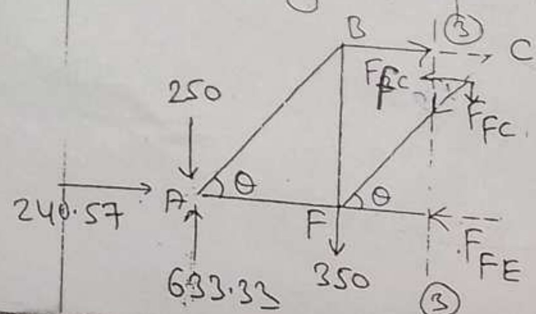
$\xrightarrow{+} \Sigma F_x = 0$

ie. $F_{DC} \cos 49.4 - 240.57 - F_{DE} = 0$

or, $F_{DE} = 548.78 \times \cos 49.4 - 240.57$

or, $F_{DE} = 116.56\,N$ (T)

Considering left part of section ③ - ③



$\circlearrowright \Sigma M_C = 0$

ie. $633.33 \times 6 - 250 \times 6 - 240.57 \times 3.5 - 350 \times 3 + F_{FE} \times 3.5 = 0$

or, $F_{FE} = -116.57\ N$

or, $F_{FE} = 116.57\ N\ (T)$

$\circlearrowright \Sigma M_F = 0$

ie. $-250 \times 3 + 633.33 \times 3 + F_{BC} \times 3.5 = 0$

or, $F_{BC} = -1150\ N$

or, $F_{BC} = 1150\ N\ (C)$

$\xrightarrow{+} \Sigma F_x = 0$

ie. $240.57 + F_{BC} - F_{FC} \times \cos 49.4' - F_{FE} = 0$

or, $240.57 - 1150 - F_{FC} \times \cos 49.4' - (-116.57)$

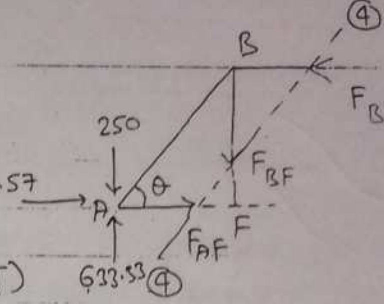
or, $F_{FC} = -1218.33\ N$

or, $F_{FC} = 1218.33\ N\ (T)$

considering left part of section (4) - (4)

$+\uparrow \Sigma F_y = 0$

ie. $633.33 - 250 - F_{BF} = 0$

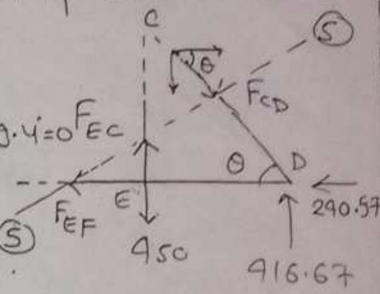


or, $F_{BF} = 383.33\ N\ (T)$

Considering right part of section (5) - (5)

$+\uparrow \Sigma F_y = 0$

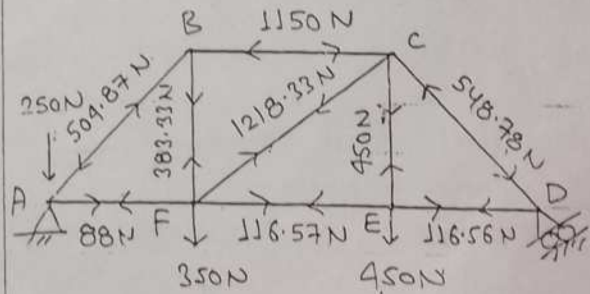
ie. $F_{EC} - 450 + 416.67 - F_{CD} \sin 49.4' = 0$



or, $F_{EC} = 450 - 416.67 + 548.78 \times \sin 49.4'$

or, $F_{EC} = 450\ N\ (T)$

Hence, the force in all members are shown in fig below.



chapter → 9 (6hrs)

kinematics of Particles and Rigid body

Trajectory

Trajectory is a path followed by a body during its motion. It may be a straight line or a curve.

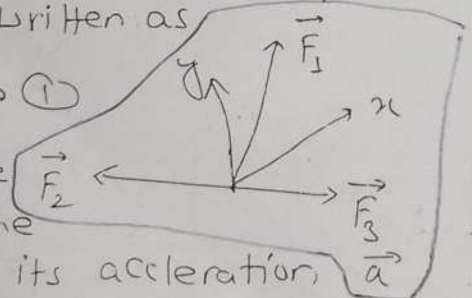
- → Rectilinear motion
- → Curvilinear motion

(2013) SP

Rectilinear Motion:-

If the motion of particle is in straight line then the motion of a particle is called rectilinear motion.

Consider a particle moving in the straight line under the action of coplanar forces, $\vec{F_1}, \vec{F_2}, \vec{F_3}$ etc. as shown in fig. Then the motion of a particle can be written as

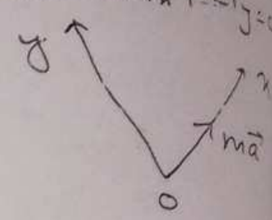


$$\Sigma \vec{F} = m\vec{a} \longrightarrow (1)$$

since the particle moves along the straight line, its accleration $\vec{a}$ must be directed along that line. (ie. x-axis) (say)

Hence eqn (1) becomes $\Sigma F_x = ma_x$, $\Sigma F_y = 0$

which is the eqn of a particle moving in a straight line.



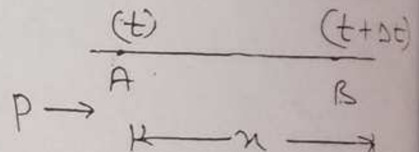
In rectilinear motion

$$\Sigma F_x = ma_x$$
$$\Sigma F_y = ma_y$$
and $$\Sigma F_z = ma_z$$

Position, velocity and accleration of a Particle and Rigid body.

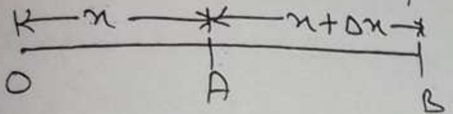
a) **Displacement:** The change of position of a particle or a body with respect to a certain fixed reference point is termed as displacement.

let us consider a particle 'p' moving in a straight line from point A to position B, then the displacement of particle $= x$.



(b) **velocity:** The rate of change of position of a body with respect to time is called velocity.

Suppose $(x+\Delta x)$ be the distance travelled by a body in time interval $(t+\Delta t)$ from reference point O.



Then,

average velocity $= \dfrac{(x+\Delta x)-x}{(t+\Delta t)-t}$

or, $V_{avg} = \dfrac{\Delta x}{\Delta t}$

Instantaneous velocity $(v) = \lim_{\Delta t \to 0} \dfrac{\Delta x}{\Delta t} = \dfrac{dx}{dt}$

② Accleration: The rate of change of velocity of a body with respect to time is called accleration. let 'v' be the velocity of body at any time 't' and let it become $(v+\Delta v)$ at a time $(t+\Delta t)$.

Avg. accl^n $(a_{avg}) = \dfrac{(v+\Delta v)-v}{(t+\Delta t)-t}$

or, $a_{avg} = \dfrac{\Delta v}{\Delta t}$

Instantaneous accl^n $(a) = \lim_{\Delta t \to 0} \dfrac{\Delta v}{\Delta t} = \dfrac{dv}{dt}$

Determination of Motion of particle and Rigid body (Equation of Rectilinear Motion)

a) Accleration is function of time $[a=f(t)]$

→ We have, $a = \dfrac{dv}{dt}$

or, $dv = a\,dt$

Integrating bothside,

$$\int_u^v dv = a\int_0^t dt$$

or, $v-u = at$

or, $v = u+at$ → ① This is the eq^n of motion for velocity.

Again, $v = \dfrac{dx}{dt}$

or, $dx = v\,dt = (u+at)\,dt$

Integrating bothside

$$\int_0^s dx = \int_0^t (u+at)\,dt$$

or, $s = ut + \frac{1}{2}at^2$ → ② This is the equation of motion for position.

b) If accleration is function of position $[a=f(x)]$

→ We have

$$a = \frac{dv}{dt} = \frac{dv}{dt} \times \frac{dx}{dx}$$

or, $a = \frac{dv}{dx} \times \frac{dx}{dt}$

or, $a = v\frac{dv}{dx}$

or, $a\,dx = v\,dv$

Integrating both side,

$$a\int_0^s dx = \int_u^v v\,dv$$

or, $as = \left[\frac{v^2}{2}\right]_u^v$

or, $v^2 = u^2 + 2as \longrightarrow (3)$ This is the equation of motion for velocity.

Note: Distance travelled in n^{th} sec

$$S_{nth} = u + \frac{a}{2}(2n-1).$$

⊛ Uniform Rectilinear Motion of Particle.

→ If a particle travells equal distance in equal interval of time then we can say that the motion of particle is uniform. In this motion the accⁿ a of a particle is zero for every value of 't'.

Then, velocity $(v) = \frac{dx}{dt}$ = constant.

⊛ Uniformly accelerated Rectilinear Motion of particle

→ If the accleration of a body is constant then it is called uniformly accelerated motion.

ie. $a = \frac{du}{dt}$ = constant.

⊛ Curvilinear Motion of a particle.

→ The motion of a particle along a curved path other than a straight line is known as curvilinear motion. For eg. projectile Motion of a bullet fired from gun.

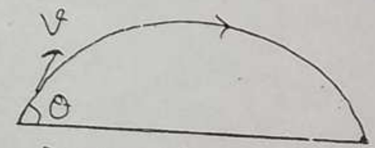


fig. projectile Motion.

⊛ Rectangular component of velocity & accⁿ

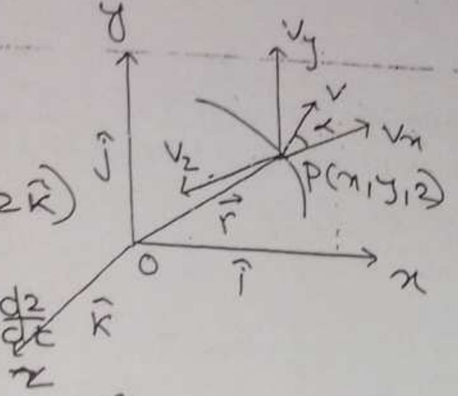
→ let the position of particle on the curved path at any instant is $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$.

Then,

$\vec{V} = \frac{d\vec{r}}{dt}$

or, $\vec{V} = \frac{d}{dt}(x\hat{i} + y\hat{j} + z\hat{k})$

or, $\vec{V} = \hat{i}\frac{dx}{dt} + \hat{j}\frac{dy}{dt} + \hat{k}\frac{dz}{dt}$



or, $\vec{V} = V_x\hat{i} + V_y\hat{j} + V_z\hat{k}$

Resultant velocity $(V) = \sqrt{(V_x)^2 + V_y^2 + V_z^2}$

The direction of velocity is tangential to the path of motion of particle. If 'α' is the angle made by resultant with x-axis (ie. In xy plane)

Then, $\tan\alpha = \frac{V_y}{V_x}$

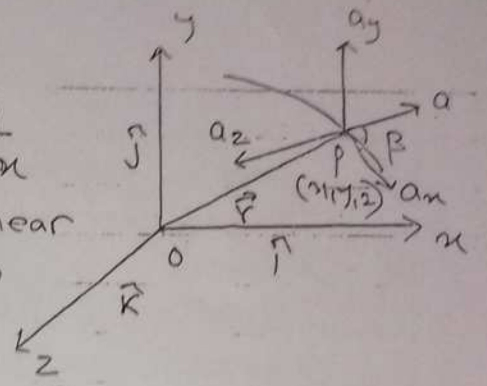
Again, $\vec{a} = \frac{d\vec{v}}{dt} = \frac{d}{dt}(V_x\hat{i} + V_y\hat{j} + V_z\hat{k})$

or, $\vec{a} = a_x\hat{i} + a_y\hat{j} + a_z\hat{k}$

Resultant acc^n $(a) = \sqrt{a_x^2 + a_y^2 + a_z^2}$

If 'β' be the angle made by the resultant acc^n with x-axis (ie. xy plane)

Then, $\tan\beta = \frac{a_y}{a_x}$



$\therefore$ For a curvilinear motion, we can write,

$\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$

$\vec{V} = V_x\hat{i} + V_y\hat{j} + V_z\hat{k}$ and

$\vec{a} = a_x\hat{i} + a_y\hat{j} + a_z\hat{k}$

(2013)SP

Tangential and Normal Components of Velocity and acc^n:

(Equation for Curvilinear Motion)

→ The velocity and accl^n of a moving particle are expressed in tangential and normal component. Consider a particle that moves along a curved path from point A to point B and traverse an infinitely small distance ΔS in a small interval of time Δt.

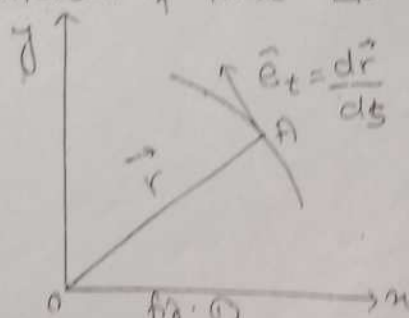


fig. (i)

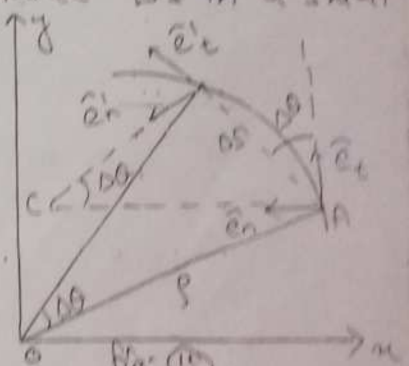


fig. (ii)

let $\hat{e}_t$ and $\hat{e}_n$ are unit vector directed along the tangent and along the normal of path resp.

In small interval of time

Chord AB = arc AB = Δs

Then, velocity $(v) = \frac{ds}{dt}$

or, $v = \lim_{\Delta t \to 0} \frac{\Delta s}{\Delta t}$

or, $v = \lim_{\Delta t \to 0} \frac{\rho \Delta\theta}{\Delta t}$ $\left[\theta = \frac{l}{r}\right]$

Where, ρ = radius of curvature

$v = \rho \frac{d\theta}{dt} = \rho\dot{\theta}$

or, $v = \rho\dot{\theta} \longrightarrow (1)$

Again,

$\vec{v} = \frac{d\vec{r}}{dt} = \frac{d\vec{r}}{ds} \times \frac{ds}{dt}$

or, $\vec{v} = \hat{e}_t \cdot v \longrightarrow (2)$

Again

acc^n $(\vec{a}) = \frac{d\vec{v}}{dt} = \frac{d}{dt}(\hat{e}_t v)$

or, $\vec{a} = v\frac{d\hat{e}_t}{dt} + \frac{dv}{dt}\hat{e}_t$

or, $\vec{a} = v \cdot \frac{d\hat{e}_t}{d\theta} \times \frac{d\theta}{ds} \times \frac{ds}{dt} + \dot{v}\hat{e}_t$

or, $\vec{a} = v \cdot \hat{e}_n \cdot \frac{1}{\rho} \cdot v + \dot{v}\hat{e}_t$ $\left[\because \frac{d\hat{e}_t}{d\theta} = \hat{e}_n, \ \frac{d\theta}{ds} = \frac{1}{\rho}\right]$

or, $\vec{a} = \frac{v^2}{\rho}\hat{e}_n + \dot{v}\hat{e}_t \longrightarrow (3)$

or, $\vec{a} = a_n\hat{e}_n + a_t\hat{e}_t$

comparing with eq^n (3)

normal component of acc^n $(a_n) = \frac{v^2}{\rho}$

tangential " " " $(a_t) = \dot{v} = \frac{dv}{dt}$

$\therefore$ tot. acc^n $(a) = \sqrt{a_n^2 + a_t^2}$

⊛ Radial and Transverse component of Velocity and accleration

→ The position of a particle is defined by the polar coordinates r and θ, where as the velocity and accleration of particle are expressed in the radial and transverse direction. In fig. (1) the Position of Particle 'A' is defined by co-ordinate 'r' and θ, Where 'r' is length and θ is angle

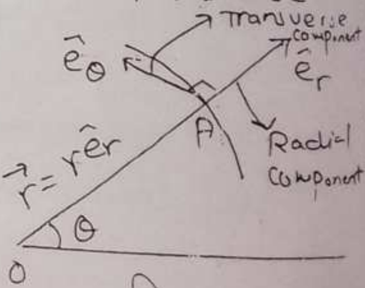


fig. (1)

in radian. The unit vector parallel and perp^n with radial vector at point P is called radial and transverse component. Suppose $\hat{e}_r$ is directed outward through OA & $\hat{e}_\theta$ is obtained by rotating $\hat{e}_r$ through 90° in anticlockwise direction.

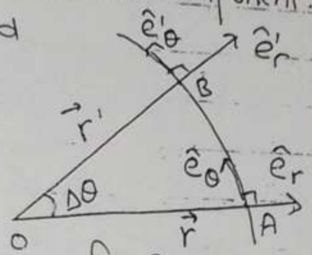


fig. (ii)

As the particle moves from A to B the unit vectors $\hat{e}_r$ and $\hat{e}_\theta$ changed to $\hat{e}'_r$ and $\hat{e}'_\theta$ by amounts $\Delta\hat{e}_r$ and $\Delta\hat{e}_\theta$ resp. as shown in fig.

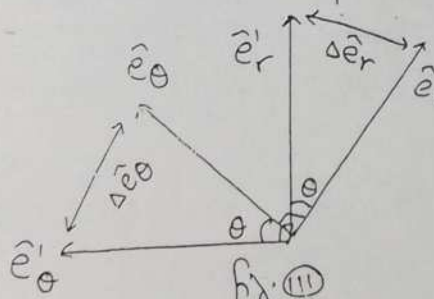


fig. (iii)

Then,

$$\frac{d\hat{e}_r}{d\theta} = \hat{e}_\theta \;\&$$

$$\frac{d\hat{e}_\theta}{d\theta} = -\hat{e}_r$$

Now,

$$\dot{\hat{e}}_r = \frac{d\hat{e}_r}{dt} = \frac{d\hat{e}_r}{d\theta} \times \frac{d\theta}{dt} = \dot{\theta}\hat{e}_\theta$$

$$\therefore \dot{\hat{e}}_r = \hat{e}_\theta \dot{\theta} \longrightarrow (1)$$

Similarly,

$$\dot{\hat{e}}_\theta = \frac{d\hat{e}_\theta}{dt} = \frac{d\hat{e}_\theta}{d\theta} \times \frac{d\theta}{dt}$$

$$\text{or, } \dot{\hat{e}}_\theta = -\hat{e}_r\dot{\theta} \longrightarrow (2)$$

Now,

Velocity is given by $(\vec{v}) = \frac{d\vec{r}}{dt} = \frac{d}{dt}(r\hat{e}_r)$

$$\text{or, } \vec{v} = \frac{dr}{dt}\hat{e}_r + r\frac{d\hat{e}_r}{dt}$$

$$\text{or, } \vec{v} = \dot{r}\hat{e}_r + r\dot{\hat{e}}_r$$

$$\text{or, } \vec{v} = \dot{r}\hat{e}_r + r\dot{\theta}\hat{e}_\theta \longrightarrow (3)$$

comparing eq^n (3) with $\vec{v} = v_r\hat{e}_r + v_\theta\hat{e}_\theta$

$\therefore$ Radial component of velocity $(v_r) = \dot{r} = \frac{dr}{dt}$

and

Transverse component of velocity $(v_\theta) = r\dot{\theta}$

or, $v_\theta = r\frac{d\theta}{dt}$

Similarly accl^n $(\vec{a}) = \frac{d\vec{v}}{dt}$

$$\text{Or, } \vec{a} = \frac{d}{dt}(\dot{r}\hat{e}_r + r\dot{\theta}\hat{e}_\theta)$$

$$\text{or } \vec{a} = \frac{d\dot{r}}{dt}\hat{e}_r + \dot{r}\frac{d\hat{e}_r}{dt} + \frac{dr}{dt}\dot{\theta}\hat{e}_\theta + r\frac{d\dot{\theta}}{dt}\hat{e}_\theta + r\dot{\theta}\frac{d\hat{e}_\theta}{dt}$$

$$\text{or } \vec{a} = \ddot{r}\hat{e}_r + \dot{r}\dot{\hat{e}}_r + \dot{r}\dot{\theta}\hat{e}_\theta + r\ddot{\theta}\hat{e}_\theta + r\dot{\theta}\dot{\hat{e}}_\theta$$

or, $\vec{a} = \ddot{r}\hat{e}_r + \dot{r}\dot{\theta}\hat{e}_\theta + \dot{r}\dot{\theta}\hat{e}_\theta + r\ddot{\theta}\hat{e}_\theta - r\dot{\theta}\dot{\theta}\hat{e}_r$

or, $\vec{a} = (\ddot{r} - r\dot{\theta}^2)\hat{e}_r + (2\dot{r}\dot{\theta} + r\ddot{\theta})\hat{e}_\theta \rightarrow (4)$

comparing eq^n (4) with $\vec{a} = a_r\hat{e}_r + a_\theta\hat{e}_\theta$ we get,

Radial component of accl^n $(a_r) = \ddot{r} - r\dot{\theta}^2$

and

Transverse component of accl^n $(a_\theta) = 2\dot{r}\dot{\theta} + r\ddot{\theta}$

Note:

In case of particle moving in a circular path with its centre at origin O, we have

$r =$ constant

$\dot{r} = 0$

$\ddot{r} = 0$

Then, eq^n (3) and eq^n (4) become

$\vec{v} = r\dot{\theta}\hat{e}_\theta$ and

$\vec{a} = r\ddot{\theta}\hat{e}_\theta - r\dot{\theta}^2\hat{e}_r$

Numericals

(1) The motion of particle is defined by the position vector $\vec{r} = 6t\hat{i} + 4t^2\hat{j} + \frac{t^3}{4}\hat{k}$ where, 'r' is in meter and 't' in sec. At the instant when $t = 3$ sec find the unit position vector, velocity and accl^n.

→ Here,

position is given by $\vec{r} = 6t\hat{i} + 4t^2\hat{j} + \frac{t^3}{4}\hat{k}$

When $t = 3$sec,

$\vec{r} = 6\times3\,\hat{i} + 4\times3^2\,\hat{j} + \frac{3^3}{4}\hat{k}$

or, $\vec{r} = 18\hat{i} + 36\hat{j} + 27/4\,\hat{k}$

$|\vec{r}| = \sqrt{(18)^2 + (36)^2 + (27/4)^2} = 40.81$m

Then,

unit position vector $(\hat{r}) = \frac{\vec{r}}{|\vec{r}|}$

or, $\hat{r} = \frac{18\hat{i} + 36\hat{j} + 27/4\,\hat{k}}{40.81}$

or, $\hat{r} = 0.44\hat{i} + 0.88\hat{j} + 0.165\hat{k}$

Velocity $(\vec{v}) = \frac{d\vec{r}}{dt} = \frac{d}{dt}\left(6t\hat{i} + 4t^2\hat{j} + \frac{t^3}{4}\hat{k}\right)$

or, $\vec{v} = 6\hat{i} + 8t\hat{j} + \frac{3}{4}t^2\hat{k}$

At $t = 3$sec,

$\vec{v} = 6\hat{i} + 24\hat{j} + 27/4\,\hat{k}$

by the

$\hat{j} + \frac{t^3}{4}\hat{k}$

in sec

nd the

nd acc^n.

$\hat{j} + \frac{t^3}{4}\hat{k}$

40.81m

$\hat{j} + \frac{t^3}{4}\hat{k}$

or, $v = \sqrt{v_x^2 + v_y^2 + v_z^2}$

or, $v = \sqrt{6^2 + 24^2 + (27/4)^2} = 25.64\ m/s$

Again,

acc^n $(\vec{a}) = \frac{d\vec{v}}{dt} = \frac{d}{dt}\left(6\hat{i} + 8t\hat{j} + \frac{3}{4}t^2\hat{k}\right)$

or, $\vec{a} = 8\hat{j} + \frac{3}{2}t\hat{k}$

At $t = 3$ sec,

$\vec{a} = 8\hat{j} + \frac{9}{2}\hat{k}$

But, $a = \sqrt{a_x^2 + a_y^2 + a_z^2}$

or, $a = \sqrt{8^2 + (9/2)^2} = 9.18\ m/s^2$

(2) The motion of particle is defined by the relation $x = t^3 - 6t^2 + 9t + 5$ where, x is expressed in meter and 't' in sec. Determine

- when velocity is zero $(t = ?)$
- the position and acc^n at $t = 5$ sec.

→ The position of particle is given by

$x = t^3 - 6t^2 + 9t + 5$

or, $v = \frac{dx}{dt} = 3t^2 + 9 - 12t$

$a = \frac{dv}{dt} = 6t - 12$

Now, time (t) = ? where velocity is zero

$\therefore 3t^2 - 12t + 9 = 0$

or, $t = 1, 3$

position (x) = ? , acc^n (a) = ? at $t = 5$ sec

$x_5 = (5)^3 - 6 \times 25 + 9 \times 5 + 5 = 25m$

$a = 6 \times 5 - 12 = 18\ m/s^2$

(2013) SP (3) The motion of a particle is defined by the relation $x = 3t^3 - 8t^2 + 15$, where x is expressed in meters and t in sec. Determine the time, position and acc^n of a particle when $v = 0$ (v is velocity of a particle).

→ The motion of particle $(x) = 3t^3 - 8t^2 + 15$

Velocity $(v) = \frac{dx}{dt}$

or, $v = \frac{d}{dt}(3t^3 - 8t^2 + 15)$

or, $v = 9t^2 - 16t$

time (t) = ? when $v = 0$

$9t^2 - 16t = 0$

or, $t = 0, \cancel{8/16}\ 16/9$ sec.

At $t = 16/9$ sec,

$x = 3 \times \left(\frac{16}{9}\right)^3 - 8 \times \left(\frac{16}{9}\right)^2 + 15$

or, $x = 6.57$ m

acc^n (a) $= \frac{dv}{dt} = \frac{d}{dt}(9t^2 - 16t)$

or, $a = 18t - 16$

At $t = 16/9$ sec

$a = 18 \times \frac{16}{9} - 16 = 16 m/s^2$

(4) The accleration of the particle is defined by the relation $a = -4 m/s^2$. If $v = 24 m/s$ and $x = 0$, when $t = 0$. Determine the velocity, position and total distance travelled when $t = 2$ sec

→ Given

$a = -4$

or, $\frac{dv}{dt} = -4$

or, $dv = -4\,dt$

$\int dv = -4 \int dt$

or, $V = -4t + C_1 \longrightarrow (1)$

A/c to question at $t = 0$, $V = 24 m/s$

$\therefore 24 = C_1$

$\therefore$ eq^n (1) becomes

$V = -4t + 24$

$\frac{dx}{dt} = -4t + 24$

or, $dx = (-4t + 24)dt$

Integrating both side

$\int dx = \int (-4t + 24)dt$

or, $x = -\frac{4t^2}{2} + 24t + C_2 \longrightarrow (2)$

A/c to question, at $t = 0$, $x = 0$

$\therefore C_2 = 0$

Hence, eq^n (2) can be written as

$x = -2t^2 + 24t$

When, $t = 2$ sec,

velocity $(V) = -4 \times 2 + 24 = 16 m/s$

Position $(x) = -2 \times 2^2 + 24 \times 2 = 40 m$

total distance travelled $(D) = ?$ In 2sec

$(D) = |x_2 - x_1| + |x_1 - x_0|$

or, $D = |40 - 22| + |22 - 0| = 40m$

(3) The motion of particle is defined by the relation, $a = 9t + 5$, where 'a' is expressed in m/s^2 and t is in sec. At $t = 0$, $V = 2m/s$ and $x = 5m$, Determine the velocity and acc^n at $t = 4$ sec.

→ The eq^n of acc^n (a) = $9t + 5$

or, $\frac{dv}{dt} = 9t + 5$

or, $dv = (9t + 5)dt$

Integrating both side,

$\int dv = \int (9t+5)dt$

or, $V = 9\frac{t^2}{2} + 5t + C_1 \longrightarrow ①$

At $t = 0$, $V = 2m/s$

$\therefore 2 = C_1$

Hence, eq^n ① becomes,

$V = 4.5t^2 + 5t + 2$

acc^n (a) = $\frac{dV}{dt} = \frac{d}{dt}(4.5t^2 + 5t + 2)$

or, $a = 9t + 5$

At $t = 4$sec

$V = 4.5 \times 4^2 + 5 \times 4 + 2 = 94$ m/s

and

$a = 9 \times 4 + 5 = 41$ m/s^2

(4) The accleration of a particle is directly proportional to the time. At $t = 0$ sec, the velocity of a particle $V = -16m/s$. Knowing that both the velocity and position are zero when $t = 4$sec. write the equation of motion of the particle.

→ A/c to the question,

accleration (a) $\propto$ time(t)

ie. $a = Kt \longrightarrow ①$

where, K = proportionality constant

we know that

$a = \frac{dv}{dt}$

or, $dv = a\,dt$

or, $dv = Kt\,dt$ $[\because a = Kt]$

Integrating bothside

$\int dv = K\int t\,dt$

or, $V = K\frac{t^2}{2} + C_1 \longrightarrow ①$

At $t=0$, $v = -16 m/s$

$\therefore C_1 = -16$

Hence, eqⁿ ① becomes,

$$V = K\frac{t^2}{2} - 16 \longrightarrow ②$$

At $t = 4sec$, $V=0$, $x=0$

$$\therefore 0 = K \times \frac{4^2}{2} - 16$$

or, $K = 2$

Hence, eqⁿ ① becomes $(a) = 2t$

and eqⁿ ② becomes

$$V = \frac{2t^2}{2} - 16$$

or, $V = t^2 - 16$

Also, $\frac{dx}{dt} = V$

$$\therefore \frac{dx}{dt} = t^2 - 16$$

or, $dx = (t^2-16)dt$

Integrating bothside

$$\int dx = \int (t^2-16)\, dt$$

or, $x = \frac{t^3}{3} - 16t + C_2 \longrightarrow ③$

At $t = 4sec$, $x = 0$

$$\therefore 0 = \frac{4^3}{3} - 16\times 4 + C_2$$

or, $C_2 = 128/3$

Hence, equation of Motion of Particle are

Ans

$$x = \frac{t^3}{3} - 16t + 128/3$$

$$V = t^2 - 16$$

$$a = 2t$$

⑦ The velocity of a particle is defined by the relation $V = 8 - 0.02x$. Knowing that $x=0$ at $t=0$. Determine the total distance travelled before the particle comes to rest and accⁿ at $t=0$.

TU

→ The velocity of a particle is defined by

$$V = 8 - 0.02x \longrightarrow ①$$

When the particle comes to rest $V=0$

$$\therefore 0 = 8 - 0.02x$$

or, $x = 400m$

So, total distance travelled by particle before coming to rest $= 400m$

Again,

$$\boxed{\text{We know } a = \frac{dv}{dt} = \frac{dv}{dx} \times \frac{dx}{dt}}$$

or, $a = v\frac{dv}{dx}$

2010

or, $a = (8-0.02x)\frac{d}{dx}(8-0.02x)$

or, $a = (8-0.02x) \times (-0.02)$

At $t=0, x=0$

$\therefore a = 8 \times (-0.02) = -0.16 m/s^2$

(8) The accl^n of a particle is defined by the relation $a = 25 - 3x^2$, where a is expressed in mm/sec^2 and x in mm. The particle starts with no initial velocity at position $x=0$. Determine

i) the velocity when $x = 2mm$

ii) the position where velocity is again zero.

iii) the position when the velocity is maximum.

→ The relation of accl^n $(a) = 25-3x^2$

or, $v\frac{dv}{dx} = 25-3x^2$

or, $v\,dv = (25-3x^2)\,dx$

Integrating both side

$$\int v\,dv = \int (25-3x^2)\,dx$$

$$25x - 3x^3/3 + C_1 \rightarrow (1)$$

or, $\frac{V^2}{2} =$

At no initial velocity $(V)=0$, and $x=0$

$\therefore C_1 = 0$

Hence eq^n (1) becomes,

$$V^2 = 50x - 2x^3 \longrightarrow (2)$$

At $x = 2mm$

$$V = \sqrt{50 \times 2 - 2 \times 2^3}$$

or, $V = 9.17$ mm/sec.

Position $(x) = ?$ when velocity is zero

$\therefore 0 = 50x - 2x^3$

or, $x = 0, \pm 5$ mm

Again, Position $(x) = ?$ When velocity is max^m.

For Max^m velocity

$$\frac{dV^2}{dx} = 0$$

i.e. $\frac{d}{dx}(50x-2x^3) = 0$

or, $50 - 6x^2 = 0$

or, $x = \pm 2.89$ mm

Also, $\frac{d^2v^2}{dx^2} = -12x$

At $x = 2.89$ mm

$$\frac{d^2v^2}{dx^2} = -12 \times 2.89 < 0$$

Hard

(2012) Fall

Hence, velocity is max^m at $x = 2.89$ mm

⑨ The acc^n of particle is given by a relation $a = -kx$, at the initial position velocity is 2m/sec. obtain the position as a function of a time. When position is 4m, velocity is 0m/s.

→ We have,

$$\text{accl}^n\,(a) = -kx \longrightarrow ①$$

$$\text{or, } v\frac{dv}{dx} = -kx$$

$$\text{or, } v\,dv = -k\,dx \cdot x$$

Integrating both side

$$\int v \cdot dv = -\int kx\,dx$$

$$\text{or, } \frac{v^2}{2} = -\frac{kx^2}{2} + C_1 \longrightarrow ②$$

At initial position $x = 0$, $v = 2$m/s

$$\therefore \frac{2^2}{2} = C_1$$

$$\text{or, } C_1 = 2$$

Hence, eq^n ① becomes

$$\frac{v^2}{2} = -\frac{kx^2}{2} + 2$$

$$\text{or, } v^2 = 4 - kx^2 \longrightarrow ③$$

At $x = 4$m, $v = 0$

$$\therefore 0 = 4 - k \times 4^2$$

$$\text{or, } k = 1/4$$

Hence, eq^n ③ becomes

$$v^2 = 4 - \frac{x^2}{4}$$

$$\text{or, } v = \sqrt{\frac{16 - x^2}{2}}$$

$$\text{or, } \frac{dx}{dt} = \sqrt{\frac{16 - x^2}{2}}$$

$$\text{or, } dt = \frac{2\,dx}{\sqrt{16 - x^2}}$$

Integrating both side

$$\int_0^t dt = \int_0^x \frac{2\,dx}{\sqrt{16 - x^2}}$$

or, put $x = 4\sin\theta$

$dx = 4\cos\theta\,d\theta$

At $x = 0$, $\theta = 0$

At $x = x$, $\theta = \sin^{-1}(x/4)$

$$\therefore \int_0^t dt = \int_0^{\sin^{-1}(x/4)} \frac{2 \times 4\cos\theta\,d\theta}{\sqrt{16 - (4\sin\theta)^2}}$$

$$\text{or, } \int_0^t dt = \int_0^{\sin^{-1}(x/4)} \frac{8\cos\theta\,d\theta}{4\cos\theta}$$

[signature]

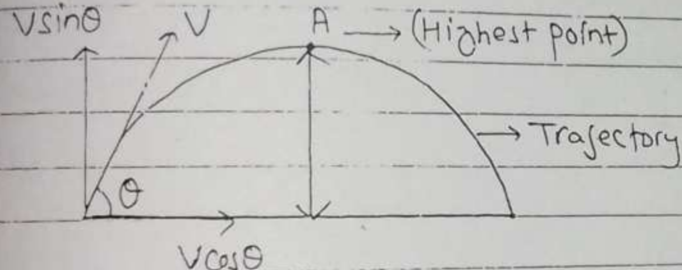
or, $t = 2\left[\theta\right]_0^{\sin^{-1}(x/4)}$

or, $\frac{t}{2} = \sin^{-1}(x/4)$

or, $\frac{x}{4} = \sin(t/2)$

or, $x = 4\sin(t/2)$ which is req. eqn

Projectile



Formula:-

$V_x = V\cos\theta \quad V_y = V\sin\theta$

a) Maxm height attained $(H) = \frac{V^2\sin^2\theta}{2g}$

b) Time of flight $(T) = \frac{2V\sin\theta}{g}$

c) time to reach maxm height $(t) = \frac{V\sin\theta}{g}$

d) Horizontal range $(R) = V_x \times T$

or, $R = V\cos\theta \times \frac{2V\sin\theta}{g}$

or, $R = \frac{V^2\sin 2\theta}{g}$

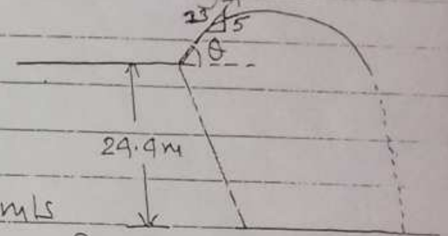
For Maxm Range,

$2\theta = 90^\circ \Rightarrow \theta = 45'$

$\therefore R = \frac{V^2}{g}$

Numericals

1) A projectile is projected as shown in figure with velocity 30.5 m/s. Determine the total time and range covered by it.



⇒ From figure,

$\sin\theta = \frac{5}{13}$

$\cos\theta = 12/13$

Velocity $(V) = 30.5$ m/s

Horizontal component of velocity $(V_x) = V\cos\theta$

or, $V_x = 30.5 \times \frac{12}{13}$

Vertical Component $(V_y) = V\sin\theta$

or, $V_y = 30.5 \times 5/13 = \frac{152.5}{13}$

In vertical motion,

$h = V_y t - \frac{1}{2}gt^2$

or, $-24.4 = \frac{152.5}{13}t - \frac{9.81}{2}t^2$

$\therefore t = 3.72$ sec.

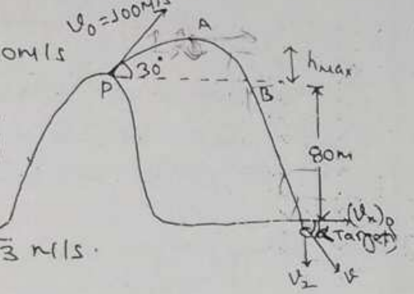
and

Range $(R) = V_x t = 30.5 \times \frac{12}{13} \times 3.72 = 104.7$ m

$\tan\alpha = \frac{-1}{(v_x)_0}$

② A projectile is fired upward at an angle of 30° to the horizontal from a point P on a hill and it strikes a target which is 80m lower than the label of projection as shown in figure. The initial velocity of the projectile is 100m/sec; calculate ⓐ The max^m height to which the bullet will rise above the horizontal:
ⓑ The actual velocity with which it will strike the target
ⓒ The total time required for the flight of bullet (Neglect the resistance due to air).

→ Here,
Initial velocity $(v_0) = 100 m/s$
Then,
horizontal component of velocity $(v_x)_0 = v_0 \cos 30°$



or, $(v_x)_0 = 100 \times \frac{\sqrt{3}}{2} = 50\sqrt{3}$ m/s.

vertical component of velocity $(v_y)_0 = v_0 \sin\theta$
or, $(v_y)_0 = 100 \times \frac{1}{2} = 50 m/s$.

a) For the max^m height,

$$h_{max} = \frac{(v_0)^2 \sin^2\theta}{2g} = \frac{(100)^2 \times \sin^2 30}{2 \times 9.8} = 127.55 m$$

b) At point A:
let, v_1 = vertical component of velocity = 0.
v_2 = " " " striking the target

H = vertical distance between point A and target.
Then, $H = h_{max} + 80 = 127.55 + 80 = 207.55$ m
We know,

$$v_2^2 - v_1^2 = 2gH$$

or, $v_2^2 = 2 \times 9.81 \times 207.55$
or, $v_2 = 63.81$ m/s.

The horizontal component of velocity from the point of projection and during its flight remains constant as the air resistance is neglected.

$\therefore (v_x)_0 = 50\sqrt{3} = 86.60$ m/s.

Hence,
Actual velocity with which the projectile will strike the target $v = \sqrt{(v_x)_0^2 + v_2^2}$

or, $v = \sqrt{(86.60)^2 + (63.81)^2} = 107.56$ m/s.

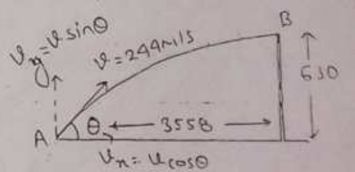
c) Again we have,

$$h = (v_y)_0 t - \frac{1}{2} g t^2$$

or, $-80 = 50t - \frac{9.81}{2} t^2$

or, $4.905t^2 - 50t - 80 = 0$
or, $t = 11.6$ sec.

③ A projectile is fired with an initial velocity of 244m/s at target B located 610m above the gun A and at a horizontal distance of 3558m. Neglecting the air resistance find the value of θ.
Here,
⇒ Initial velocity of the projectile $(v) = 244$ m/s.



Then, $v_x = v\cos\theta = 244\cos\theta$ &

$v_y = v\sin\theta = 244\sin\theta$.

considering horizontal motion,

Range = $v_x t$

or, $x = v_x t$

or, $t = \frac{x}{v_x} = \frac{3650}{244\cos\theta} = \frac{15}{\cos\theta}$ ---(i)

For vertical motion

$h = v_y t - \frac{1}{2} g t^2$

or, $610 = 244\sin\theta \times \frac{15}{\cos\theta} - \frac{9.81}{2} \times \left(\frac{15}{\cos\theta}\right)^2$

or, $610 = 3660\tan\theta - 1103.625(1+\tan^2\theta)$

or, $\tan\theta = 0.5638$, 2.75

$\therefore \theta = 29.4°$, $70°$.

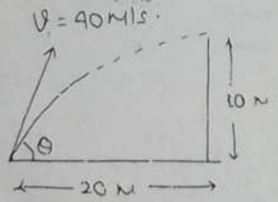
(4) A shot is fired with a bullet from a point 20m infront of vertical wall 10m height. Find the minimum angle of projection with horizontal to enable the shot to just clear the wall.

→ Here,

Velocity (v) = 40m/s.

$v_x = 40\cos\theta$

$v_y = 40\sin\theta$



For ~~vertic~~ horizontal motion

$x = v_x \cancel{\cos\theta}\, t$

or, $t = \frac{x}{v_x} = \frac{20}{40\cos\theta} = \frac{1}{2\cos\theta}$ ---(ii)

For vertical motion: $h = v_y t - \frac{1}{2} g t^2$

or, $10 = 40\sin\theta \times \frac{1}{2\cos\theta} - \frac{9.81}{2} \times \left(\frac{1}{2\cos\theta}\right)^2$

or, $10 = 20\tan\theta - \frac{4.905}{4}(1+\tan^2\theta)$

or, $10 = 20\tan\theta - 1.226 - 1.226\tan^2\theta$

or, $1.226\tan^2\theta - 20\tan\theta + 11.226 = 0$

or, $\tan\theta = \frac{20 \pm \sqrt{400 - (4\times 1.226\times 11.226)}}{2\times 1.226}$

or, $\tan\theta = \frac{20 \pm 18.57}{2.452}$

or, $\tan\theta = 15.73$, 0.583

or, $\theta = 86.36°$, $30.25°$

Hence minimum angle of projection is 30.25°

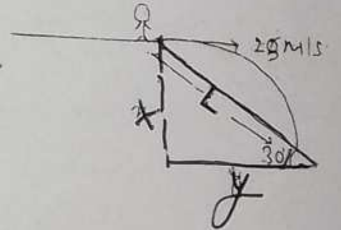
(2010)

(5) A sky jumper starts with a horizontal take off velocity 25m/s and lands on st. landing hill inclined at 30°. Determine

- time between take off and landing
- The length of jump.

Given,

$v_x = 25$ m/s, $v_y = 0$ m/s.



From fig.

$\sin 30 = \frac{x}{L}$

or, $x = L\sin 30°$

and $\cos 30° = \frac{y}{L}$

or, $y = L\cos 30°$

$\frac{1}{2}gt^2$

$\times \left(\frac{1}{2\cos\theta}\right)^2$

is 30.25°

take off hill inclined

25m/s

Range $= L\cos30° = v_x t$

or, $L\cos30° = 25t$ ⟶ (1)

In vertical motion,

$y = v_y t - \frac{1}{2}gt^2$

or, $L\sin30° = \frac{9.81}{2}t^2$ ⟶ (2)

Dividing eqn (2) by (1)

$\tan30° = \frac{4.905t^2}{25t}$

or, $t = \frac{25\times\tan30°}{4.905} = 2.94$ sec.

Putting the value of t in eqn (1)

$L = \frac{25\times2.94}{\cos30°} = 84.87$ m.

Hence the length of jump is 84.87 m.

2013 Fall

(6) A girl throws a ball with an initial velocity 18 m/s. Determine
the max^m height at which the ball can strike the wall
. The corresponding angle made with the horizontal.

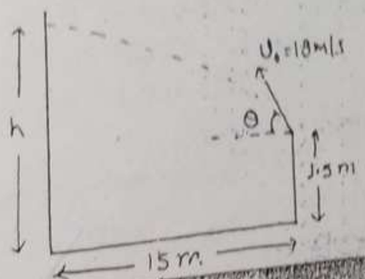
Here,

Initial velocity

$v_0 = 18$ m/s.

Then,

$(v_x)_0 = v_0\cos\theta$



2013 first

or, $(v_x)_0 = 18\cos\theta$ and $(v_y)_0 = 18\sin\theta$.

Suppose θ be the angle of projection.

Then,

horizontal distance (range) = 15m

i.e. $(v_x)_0 t = 15$

or, $t = \frac{15}{(v_x)_0} = \frac{15}{18\cos\theta} = \frac{5}{6\cos\theta}$ ⟶ (1)

In vertical motion,

$h = 1.5 + (v_y)_0 t - \frac{1}{2}gt^2$

or, $h = 1.5 + 18\sin\theta \times \frac{5}{6\cos\theta} - \frac{9.81}{2}t^2$

or, $h = 1.5 + 15\tan\theta - 4.905t^2$ ⟶ (2)

or, $h = 1.5 + 15\tan\theta - 4.905\times\left(\frac{5}{6\cos\theta}\right)^2$

or, $h = 1.5 + 15\tan\theta - 3.406\sec^2\theta$ ⟶ (3)

For Maxima and minima

$\frac{dh}{d\theta} = 0$

or, $15\sec^2\theta - 2\times3.406\sec\theta\cdot\sec\theta\cdot\tan\theta = 0$

or, $\sec^2\theta(15 - 6.813\tan\theta) = 0$

or, $\tan\theta = \frac{15}{6.813}$ ⇒ $\theta = 65.57°$

Putting the value of θ in eqn (3)

$h = 1.5 + 15\times\tan65.57° - 3.406\sec^2(65.57°)$

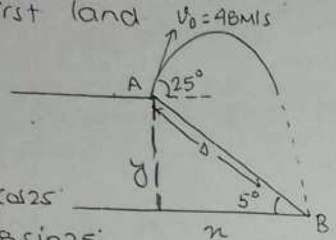
or, $h = 14.60$ m.

∴ Max^m height at which ball can strike = 14.60m

(7) A golfer hits a ball with an initial velocity of 48m/s at an angle of 25° with the horizontal knowing that the fairway slopes downward at an average angle of 5°. Determine the distance 'D' between the golfer and point B where the ball first land.

Here,

→ Initial velocity $(v_0) = 48 m/s$.



Then,

$(v_x)_0 = v_0 \cos 25^\circ = 48 \cos 25^\circ$

$(v_y)_0 = v_0 \sin 25^\circ = 48 \sin 25^\circ$

Suppose D be the distance between the golfer and the point of strike of ball.

from fig,

$x = D \cos 5^\circ$ & $y = D \sin 5^\circ$.

For horizontal motion,

$x = (v_x)_0 t$

or, $D \cos 5^\circ = 48 \cos 25^\circ \, t \longrightarrow (1)$

or, $t = \dfrac{D \cos 5^\circ}{48 \cos 25^\circ} \longrightarrow (ii)$

For vertical motion

$y = (v_y)_0 t - \frac{1}{2} g t^2$

or, $-D \sin 5^\circ = 48 \sin 25^\circ \times \left(\dfrac{D \cos 5^\circ}{48 \cos 25^\circ}\right) - \dfrac{9.81}{2}\left(\dfrac{D \cos 5^\circ}{48 \cos 25^\circ}\right)^2$

or, $-D \sin 5^\circ = D \tan 25^\circ \times \cos 5^\circ - 4.905 D^2 \left(\dfrac{\cos 5^\circ}{48 \cos 25^\circ}\right)^2$

or, $-D \sin 5^\circ = 0.465 D - 0.003 D^2$

or, $0.003 D^2 - 0.087 D - 0.465 D = 0$

or, $D = \dfrac{0.087 \pm \sqrt{(0.087)^2 + 4 \times 0.003 \times 0.465}}{2 \times 0.003}$

or, $D = \dfrac{0.087 \pm 0.115}{0.006}$

or, $-D \sin 5^\circ = 48 \sin 25^\circ t - 4.905 t^2 \longrightarrow (3)$

Dividing eqn (3) by eqn (1)

$-\tan 5^\circ = \dfrac{48 \sin 25^\circ t - 4.905 t^2}{48 \cos 25^\circ t}$

or, $-\tan 5^\circ = \tan 25^\circ - \dfrac{4.905}{48 \times \cos 25^\circ} t$

or, $t = (\tan 25^\circ + \tan 5^\circ) \times \dfrac{48 \cos 25^\circ}{4.905}$

or, $t = 4.89$ sec.

Putting the value of t in eqn (2)

$4.89 = \dfrac{D \cos 5}{48 \cos 25^\circ}$

$\therefore D = \dfrac{4.89 \times 48 \cos 25^\circ}{\cos 5} = 213.54$ m.

(2010)

(8) A basket ball player shoots when she is 5m from the backboard. Knowing that the ball has an initial velocity v at an angle of 30° with the horizontal. Determine the value of v when $d = 228$ mm & 930 mm

Here,

→ Initial velocity $= v$.

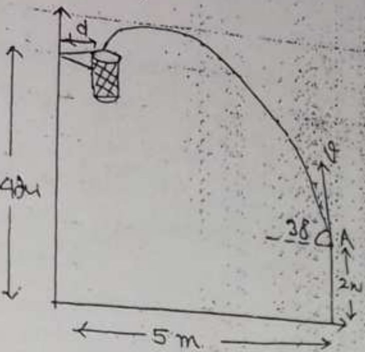
Then, $(v_x)_0 = v \cos 30^\circ$ and $(v_y)_0 = v \sin 30^\circ$

f...
initial
zontal
& 430mm

Horizontal distance

range $= 5-d$.

or, $(v_x)_0 t = 5-d$



or, $v\cos30° \, t = 5-d$

or, $t = \dfrac{5-d}{v\cos30°}$

In vertical motion,

$h = 2 + (v_y)_0 t - \frac{1}{2}gt^2$

or, $3.048 = 2 + v\sin30° \times \left(\dfrac{5-d}{v\cos30°}\right) - 4.905\left(\dfrac{5-d}{v\cos30°}\right)^2$

When $d = 228\,mm = 0.228\,m$

above eqn becomes,

$3.048 = 2 + \tan30° \times (5-0.228) - 4.905\left(\dfrac{5-0.228}{v\cos30°}\right)^2$

or, $\left(\dfrac{5-0.228}{v\cos30°}\right)^2 4.905 = 1.707$

or, $\dfrac{4.772}{v\cos30°} = 0.590$

or, $v = 9.341$ m/s.

Again, when $d = 430\,mm = 0.43\,m$

$3.048 = 2 + v\sin30° \times \left(\dfrac{5-0.43}{v\cos30°}\right) - 4.905\left(\dfrac{5-0.43}{v\cos30°}\right)^2$

or, $3.048 = 2 + \tan30° \times 4.57 - 4.905\left(\dfrac{4.57}{v\cos30°}\right)^2$

or, $v = 9.267$ m/s.

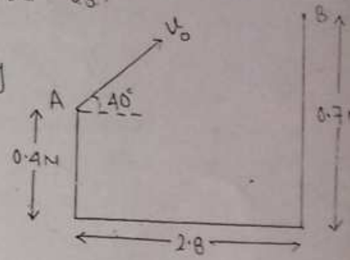
(3) A home owner uses a snow blower to clear his drive way knowing that the snow is discharged at an average angle of 40° with the horizontal. Determine the initial velocity v_0 of the snow.

let initial velocity of the snow be v_0.

Then,

horizontal component of velocity $(v_x)_0 = v_0\cos40°$.

Similarly, vertical component of velocity $(v_y)_0 = v_0\sin40°$.



Considering horizontal motion,

horizontal range $= 2.8$

or, $(v_x)_0 t = 2.8$

or, $t = \dfrac{2.8}{v_0\cos40°}$ ⟶ ①

Considering vertical motion,

$h = 0.4 + (v_y)_0 t - \frac{1}{2}gt^2$

or, $0.7 = 0.4 + v_0\sin40° \times \dfrac{2.8}{v_0\cos40°} - \frac{1}{2}\times 9.81 \times \left(\dfrac{2.8}{v_0\cos40°}\right)^2$

or, $0.3 = \tan40° \times 2.8 - 4.905 \times \left(\dfrac{2.8}{v_0\cos40°}\right)^2$

or, $\dfrac{2.8}{v_0\cos40°} = 0.646$

or, $v_0 = \dfrac{2.8}{0.646 \times \cos40°} = 5.655$ m/s.

Hence, the initial velocity v_0 of the snow is 5.655 m/s.

(signature)

(2010) SP

(10) The speed of racing car is increased at a constant rate from 100KM/hr to 120 KM/hr over a distance of 180m along a curve of radius 240m. Determine the magnitude of the total acc^n of the car after it has travelled 120m along the curve.

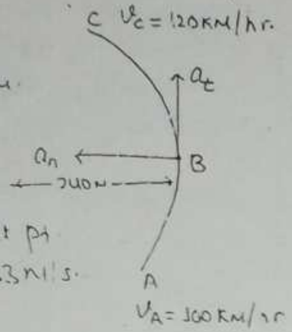
Here,

→ Radius of curve path (ρ) = 240m

Velocity of car at pt A (v_A) = 100KM/hr

or, v_A = 27.778 m/s.

Similarly, Velocity of car at pt C (v_C) = 120KM/hr = 33.333 m/s.



Distance AB = 120m

Distance ABC = 180m

accleration of car at point B (a) = ?

At point B, tangential and normal acc^n acts as shown in figure.

$$\therefore a = \sqrt{a_n^2 + a_t^2} \longrightarrow (1).$$

For a_t.

We know, $V_C^2 = V_A^2 + 2a_t S_{ABC}$

or, $a_t = \dfrac{V_C^2 - V_A^2}{2 S_{ABC}}$

or, $a_t = \dfrac{(33.333)^2 - (27.778)^2}{2 \times 180}$

or, $a_t = 0.943$ m/s^2.

Similarly

After passing 120m distance

$v_B^2 = v_A^2 + 2a_t S_{AB}$

or, $v_B^2 = (27.778)^2 + 2 \times 0.943 \times 120$

or, v_B = 31.59 m/s.

Then,

normal component of accl^n (a_n) = $\dfrac{v_B^2}{\rho}$

or, $a_n = \dfrac{(31.59)^2}{240} = 4.158$ m/s^2.

Putting value of a_t and a_n in eq^n (1)

$a = \sqrt{(0.943)^2 + (4.158)^2} = 4.264$ m/s^2

In case of 120m if given after 5 sec, Then

$v_B = v_A + a_t t$

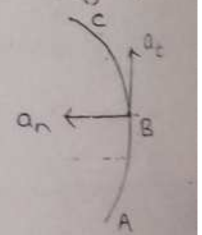
or, $v_B = 27.778 + 0.943 \times 5 = 32.49$ m/s.

and, $a_n = \dfrac{v_B^2}{\rho} = \dfrac{(32.49)^2}{240} = 4.39$ m/s^2.

again, $a = \sqrt{(0.943)^2 + (4.39)^2} = 4.49$ m/s^2.

(11) ~~An~~ outdoor track is 130m in diameter. A runner increases his speed at a constant rate from 4m/s to 7m/s over a distance of 30m. Determine the total accl^n of the runner 2 sec after he begins to increase his speed.

→ Radius of track (ρ) = $\dfrac{130}{2}$ = 65m.



Initial velocity (u) = 4m/s.

Final velocity (v) = 7m/s.

Distance travelled (AC) = 30 m.
total acceleration (a) = ?
we know,

$v^2 = u^2 + 2a_t S_{AC}$

or, $(7)^2 = (4)^2 + 2a_t \times 30$

or, $a_t = \frac{49-16}{60} = 0.55 \, m/s^2$.

Again, after t = 2 sec

$v_B = u + a_t t$

or, $v_B = 4 + 0.55 \times 2 = 5.1 \, m/s$.

normal component of accl^n $(a_n) = \frac{v_B^2}{\rho}$

or, $a_n = \frac{(5.1)^2}{65} = 0.4 \, m/s^2$

So, tot accl^n $(a) = \sqrt{(0.55)^2 + (0.4)^2}$

or, $a = 0.68 \, m/s^2$.

(12) A projectile is fired from the edge of a 150 m cliff with an initial velocity of 180 m/s at an angle of 30° with the horizontal. Neglecting air resistance, find

- horizontal distance from ground to the point of strike of the projectile.
- the greatest elevation above the ground reached by projectile.

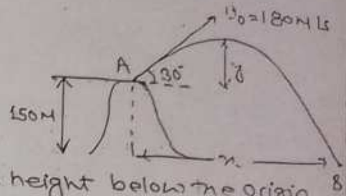
→ Here,
Initial velocity of projectile $(v_0) = 180 \, m/s$.
horizontal component $(u_x)_0 = v_0 \cos 30°$

or, $(u_x)_0 = 155.88 \, m/s$

Vertical component of velocity $(u_y)_0 = v_0 \sin 30°$

or, $(u_y)_0 = 180 \times 0.5 = 90 \, m/s$

considering vertical motion,



$h = (u_y)_0 t - \frac{1}{2} g t^2$

or, $-150 = 90t - 4.905t^2$

or, $4.905t^2 - 90t - 150 = 0$

or, $t = 19.89$ sec.

[The height below the origin is taken (-ve) since the pt. of firing is fixed at origin].

a) Horizontal range $(x) = (u_x)_0 t$

or, $x = 155.88 \times 19.89 = 3100.45$ m.

b) When the projectile reaches at greatest elevation $v_y = 0$.

Then, $(v_y)^2 - (u_y)_0^2 = -2gy$

or, $y = \frac{(90)^2}{2 \times 9.81} = 412.84$ m.

$H = \frac{180^2 \times \sin^2 30}{2 \times 9.81} = 412.84$

$H = 412.84 + 150 = 562.84$ m

Hence, greatest elevation above the ground

$H = 150 + 412.84 = 562.844$ m.

(13) A bus starts from rest on a curve of 300 m radius and accelerates at the constant rate 0.75 m/s². Determine the distance and the time that the bus will travel before the magnitude of its total accl^n is 0.9 m/s².

→ Here, Radius of curve $(\rho) = 300$ m.

Tangential component of accl^n $(a_t) = 0.75 \, m/s^2$

total accl^n $(a) = 0.9 \, m/s^2$.

We know,

$$a = \sqrt{a_t^2 + a_n^2}$$

or, $a_n = \sqrt{a^2 - a_t^2}$

or, $a_n = \sqrt{(0.9)^2 - (0.75)^2} = 0.497 \text{ m/s}^2$.

We know,

$$a_n = \frac{v^2}{\rho}$$

or, $v = \sqrt{a_n \rho} = \sqrt{0.497 \times 300} = 12.21 \text{ m/s}$.

Again,

$$v = u + a_t t$$

or, $12.21 = 0 + 0.75 \times t$

or, $t = \dfrac{12.21}{0.75} = 16.28 \text{ sec}$.

acc^n constant

ইহা velocity $= 0$

[illegible] $u = 0$

Also,

$$v^2 = u^2 + 2a_t s$$

or, $s = \dfrac{v^2}{2a_t} = \dfrac{(12.21)^2}{2 \times 0.75} = 99.38 \text{ m}$.

Hard

(14) The motion of the particle is defined by the position vector $\vec{r} = 3t^2\hat{i} + 4t^3\hat{j} + 5t^4\hat{k}$ where, $\vec{r}$ is in meter and t in sec. At instant when $t = 4$ sec find the normal and tangential component of the acc^n and principle radius of curvature.

→ The motion of particle is defined by

$$\vec{r} = 3t^2\hat{i} + 4t^3\hat{j} + 5t^4\hat{k} \quad \text{--- (1)}$$

Then,

$$\vec{v} = \frac{d\vec{r}}{dt} = 6t\hat{i} + 12t^2\hat{j} + 20t^3\hat{k}$$

$$|\vec{v}| = \sqrt{(6t)^2 + (12t^2)^2 + (20t^3)^2}$$

When $t = 4$ sec,

$$|\vec{v}_4| = \sqrt{(24)^2 + (12 \times 16)^2 + (20 \times 4^3)^2} = 1294.54 \text{ m/s}$$

$$a = \frac{d\vec{v}}{dt} = 6\hat{i} + 24t\hat{j} + 60t^2\hat{k}$$

$$|\vec{a}| = \sqrt{(6)^2 + (24t)^2 + (60t^2)^2}$$

At $t = 4$ sec

$$|\vec{a}_4| = \sqrt{36 + (24 \times 4)^2 + (60 \times 16)^2} = 964.80 \text{ m/s}$$

We know,

$$a_t = \frac{dv}{dt} = \frac{d}{dt}\sqrt{(6t)^2 + (12t^2)^2 + (20t^3)^2}$$

or, $a_t = \dfrac{1}{2\sqrt{(6t)^2 + (12t^2)^2 + (20t^3)^2}} \times (36 \times 2t + 144 \times 4t^3 + 400$

At $t = 4$ sec,

$$a_t = \frac{1}{2\sqrt{(6 \times 4)^2 + (12 \times 16)^2 + (20 \times 4^3)^2}} \times (72 \times 4 + 144 \times 4^4 + 400 \times 6$$

or, $a_t = 963.56 \text{ m/s}^2$.

We know, $a = \sqrt{a_n^2 + a_t^2}$ $\quad \therefore a_n = \sqrt{a^2 - a_t^2}$

or, $a_n = \sqrt{(964.80)^2 - (963.56)^2} = 48.90 \text{ m/s}$.

Again, $a_n = \dfrac{u^2}{\rho}$

or, $\rho = \dfrac{u^2}{a_n} = \dfrac{(1294.54)^2}{48.90} = 34270.63 \text{ m}$.

$= 1294.54$ m/s

964.80 m/s

$2t + 144 \times 4t^3 + 400\times$

$144 \times 4^4 + 400 \times 6 \times 4$

$= \sqrt{a^2 - a_t^2}$

18.50 m/s.

4270.63 m.

(15) A cam A rotates, follower wheel B rolls without slipping on the force of cam. Knowing that the normal component of the accⁿ of the points of contact at C of cam A and the wheel B are $0.66\,m/s^2$ and $6.8\,m/s^2$ resp. Determine the diameter of the follower wheel.

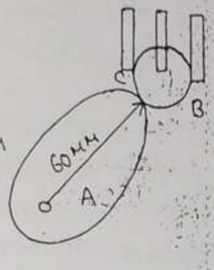
Here,

$(a_n)_A = 0.66\,m/s^2$

$(a_n)_B = 6.8\,m/s^2$.

radius of cam A $(\rho_A) = 60$ mm

Diameter of follower wheel B = ?



Then, $(a_n)_A = \dfrac{V_c^2}{\rho_A}$

or, $V_c^2 = 0.66 \times 0.06 = 0.0396$

or, $V_c = 0.198$ m/s.

Since the cam and the wheel are rotating without slipping they have same velocity at point C.

Again,

$$(a_n)_B = \frac{V_c^2}{\rho_B}$$

or, $6.8 = \dfrac{(0.198)^2}{\rho_B}$

or, $\rho_B = \dfrac{(0.198)^2}{6.8} = 0.00576$ m.

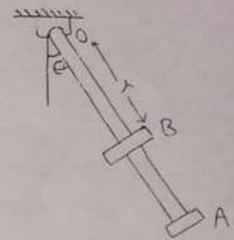
$\therefore$ Diameter of follower wheel $(D_B) = 2\rho_B$

or, $D_B = 2 \times 0.00576 = 0.01153$

$\therefore D_B = 11.53$ mm.

(16) The rotation of rod OA about O is defined by the relation $\theta = 2t^2$ where θ is expressed in radian & 't' in sec. Collar B slides along the rod in such a way that its distance from O is $r = 60t^2 - 20t^3$ where 'r' is in mm and 't' is in sec. When t = 1 sec, Determine • Velocity of collar and • tot accⁿ of the collar.

Here,



We know, radial and transverse component of velocity and accⁿ

$v = v_\theta e_\theta + v_r e_r$

or, $v = r\dot{\theta} e_\theta + \dot{r} e_r \longrightarrow (1)$

and,

$a = (\ddot{r} - r\dot{\theta}^2) e_r + (r\ddot{\theta} + 2\dot{r}\dot{\theta}) e_\theta \longrightarrow (2)$

Distance of collar B from the origin O is given by

$r = 60t^2 - 20t^3$

then, $\dot{r} = 120t - 60t^2$

$\ddot{r} = 120 - 120t$.

Also,

$\theta = 2t^2$

$\dot{\theta} = 4t$

$\ddot{\theta} = 4$

At t = 1 sec

$r = 60 - 20 = 40$ m

$\dot{r} = 120 - 60 = 60$ m/s

$\ddot{r} = 120 - 120 = 0\,m/s^2$

$\theta = 2$ rad

$\dot{\theta} = 4$ rad/s

$\ddot{\theta} = 4$ rad/s²

Note :- If time is not given then value of θ will be given. ie. $\theta = 0.15t^2$ (a) and $\theta = 30°$ Then, $30° = 0.524$ rad. from (a) $0.524 = 0.15t^2$ or, $t = 1.869$ sec

We know, $v_\theta = r\dot{\theta} = 40 \times 4 = 160$

$v_r = \dot{r} = 60$

substituting these in eqⁿ (1) & (2).

$v = 160\, e_\theta + 60\, e_r$

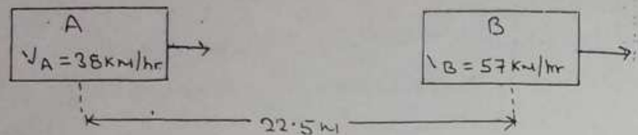
and

$a = (0 - 40 \times 4^2) e_r + (40 \times 4 + 2 \times 60 \times 4) e_\theta$

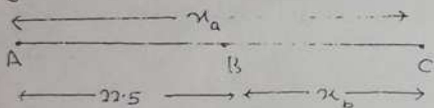
or, $a = -640\, e_r + 640\, e_\theta$

(2008)

(17) Car A and car B travelling in adjacent highway lanes, at $t=0$ have the position and speed shown. Knowing that car A has constant accⁿ of $0.6 m/s^2$ and that of B has a constant deceleration of $0.4 m/s^2$. Determine
- when and where A will overtake B.
- speed of each car at that time.



Here,
→ acceleration of car A $(a_A) = 0.6 m/s^2$
acceleration of car B $(a_B) = -0.4 m/s^2$.
velocity of car A $(V_A) = 38 km/hr = 10.556 m/s$
velocity of car B $(V_B) = 57 km/hr = 15.833 m/s$.



Suppose the car A and B meet at point C after time 't' covering distance x_a & x_b resp. Then,

$$x_a = x_b + 22.5$$

or, $V_A t + \frac{1}{2} a_A t^2 = V_B t + \frac{1}{2} a_B t^2 + 22.5$

or, $10.556t + \frac{1}{2} 0.6t^2 = 15.833t - \frac{1}{2} \times 0.4 t^2 + 22.5$

or, $10.556t + 0.3t^2 = 15.833t - 0.2t^2 + 22.5$

or, $0.5t^2 - 5.277t - 22.5 = 0$

or, $t = \frac{5.277 \pm \sqrt{(5.277)^2 + (4 \times 0.5 \times 22.5)}}{2 \times 0.5}$

or, $t = \frac{5.277 \pm 8.535}{1}$

or, $t = 13.812$ sec.

Again, we know,
$$x_a = V_A t + \frac{1}{2} a_A t^2$$

or, $x_a = 10.556 \times 13.812 + \frac{1}{2} \times 0.6 \times (13.812)^2$

or, $x_a = 203.03$ m.

Hence car A will overtake car B after 13.812 sec at distance 203.03 m from car A.

Similarly we know, after 13.812 sec.

$(V_A)_c = V_A + a_A t$ where, $(V_A)_c$ is velocity at pt c

or, $(V_A)_c = 10.556 + 0.6 \times 13.812 = 18.843 m/s$

and

$(V_B)_c = V_B + a_B t$ where, $(V_B)_c$ is velocity at pt c

or, $(V_B)_c = 15.833 + (-0.4) \times 13.812 = 10.308 m/s$.

(18) A stone is dropped from the top of tower of 40m high. At the same instant, another ~~instant~~ stone is thrown upward from the foot of tower with an initial velocity of 20m/s. At what distance from the top and after how much time the two stone cross each other? Also calculate the relative velocity with which the two stone cross.

→ Suppose two stones cross each other at distance h from the top of tower after 't' second

$(13.812)^2$

after 13.812 se

is velocity at pt c

18.843 m/s

velocity at pt c

0.308 m/s.

r of 40m high

tone is thrown

nitial velocity

p and after

ach other?

ith which the

ner at distance

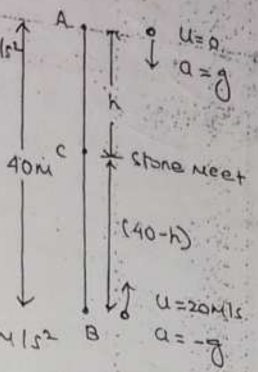
't' second

For stone dropped from pt. A

$u = 0$, $s = h$ and $a = g = 9.81\,m/s^2$

$h = 0 + \frac{1}{2}\, 9.81\, t^2$

or, $h = 4.905 t^2 \longrightarrow (1)$



similarly,

for upward motion of second stone,

$u = 20\,m/s$, $a = -g = -9.81\,m/s^2$

$s = 40 - h$

$40 - h = 20t - \frac{1}{2}\, 9.81\, t^2$

or, $40 - h = 20t - 4.905t^2 \longrightarrow (2)$

Adding eq^n (1) & (2)

$40 = 20t \Rightarrow t = 2\,sec.$

putting the value of t in eq^n(1)

$h = 4.905 \times 2^2$

or, $h = 19.62\,m.$

The stones thus cross each other at a distance of 19.62 m from the top of the tower after 2 sec.

Again,

velocity of first stone at the crossing point

$v = u + gt = 0 + 9.81 \times 2 = 19.62\,m/s (\downarrow)$

similarly,

velocity of second stone at the crossing pt

$v = u - gt = 20 - 9.81 \times 2 = 0.38\,m/s (\uparrow)$

Then,

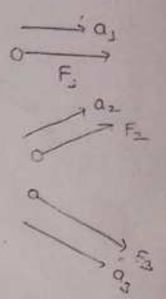
Relative velocity $= 19.62 - (-0.38)$

$= 20\,m/s.$

Newton's second law of motion:

→ Newton's second law of motion states that " If the resultant force acting on particle is not zero, The particle will have an accleration proportional to the magnitude of the resultant & in the direction of the resultant force.

If a particle is subjected to the forces $F_1, F_2, F_3, \ldots$ having different magnitude or direction we found that the particle moves in the direction of force acting on it and that the magnitudes $a_1, a_2, a_3, \ldots$ of the acclerations are proportional to the magnitudes $F_1, F_2, F_3 \ldots$ of the corresponding forces.



A/c to newton second law,

$a \propto F$

or, $\frac{F}{a} = \text{constant}$.

i.e. $\frac{F_1}{a_1} = \frac{F_2}{a_2} = \frac{F_3}{a_3} \cdots\cdots = \text{constant}.$

The constant value is obtained by the ratio of the magnitude of forces and acclerations of the particle under accleration and is called the mass of particle denoted by m.

or, $\frac{F}{a} = m$

or, $\boxed{F = ma}$

④ Linear momentum of particle:

The product of mass and velocity is called linear momentum. It has the same direction as the velocity of the particle so, it is a vector quantity. It is denoted by L & given by $L = mv$.

A/c to newton 2^{nd} law of motion

$$F = ma$$

or, $$F = m\frac{dv}{dt}$$

or, $$F dt = m\, dv$$

Integrating both side,

$$\int_{t_1}^{t_2} F dt = \int_{v}^{v_2} m\, dv$$

Impulse = change in momentum.

$$I_{1\to 2} = mv_2 - mv_1$$

Hence, the resultant of the forces acting on the particle is equal to the rate of change of the linear momentum of the particle.

If the resultant force acting on a particle is zero, the magnitude and direction of the linear momentum of the particle remain constant and is known as the principle of conservation of linear momentum.

Note:-- The product of force and the time during which it acts is called Impulse of force. ie. $I = F \times t$

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⑤ Angular momentum of particle: (2008)

Consider a particle having mass 'm' moving in x-y plane as shown in fig. Then the momentum or linear momentum of the particle is equal to the $m\vec{v}$.

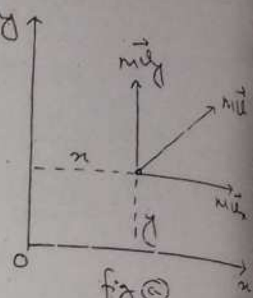


fig ②

The moment about O of the vector $m\vec{v}$ is called angular momentum of the particle about O and is denoted by $\vec{H_o}$.

Suppose $m\vec{v_x}$ and $m\vec{v_y}$ be the components of $m\vec{v}$ in x and y-direction.

Then,

$$H_o = x(mv_y) - y(mv_x)$$

$$\therefore H_o = m(xv_y - yv_x) \longrightarrow ①$$

Again,

Rate of change of angular momentum $\dot{H}_o = \frac{dH_o}{dt}$

or, $$\dot{H}_o = \frac{d}{dt} m(xv_y - yv_x)$$

or, $$\dot{H}_o = m\left\{\frac{d(xv_y)}{dt} - \frac{d(yv_x)}{dt}\right\}$$

$$= m\{\dot{x}v_y + x\dot{v}_y - \dot{y}v_x - y\dot{v}_x\}$$

$$= m\{\cancel{v_x v_y} + x a_y - \cancel{v_y v_x} - y a_x\}$$

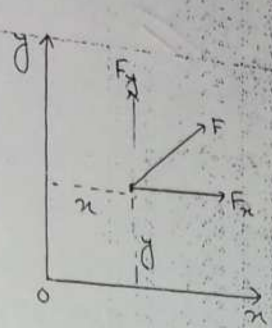
$$= m(x a_y - y a_x)$$

$$= (x m a_y - y m a_x)$$

or, $$\dot{H}_o = (x F_y - y F_x) = \text{Moment of force } \bar{F} \text{ about O as in fig ②}$$

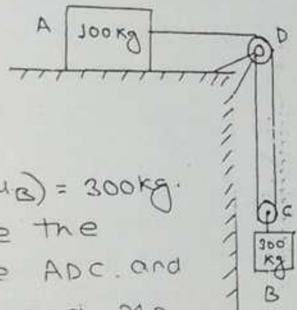
74

Thus, the rate of change of angular momentum of the particle about any point at any instant is equal to the moment of force $\vec{F}$ acting on that particle about the same point.



Numericals:

1. The two blocks shown in fig start from rest. The horizontal plane and the pullys are frictionless and the pully is assumed to be negligible mass. Determine the accl^n of each block & the tension of each cord.



Given,

→ mass of block A $(m_A) = 100kg$

mass of block B $(m_B) = 300kg$.

let T_1 and T_2 be the tension in the cable ADC and BC resp. And x_A and x_B be the distance moved by block A and B resp.

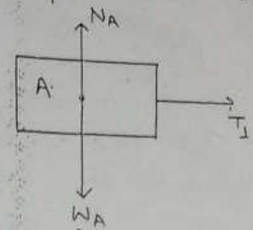
Then, from figure,

$x_A = 2x_B$

Diff. twice w.r. to t

$a_A = 2a_B \longrightarrow ①$

FBD of block A



(external force diagram).

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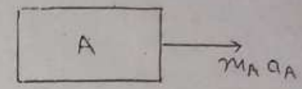
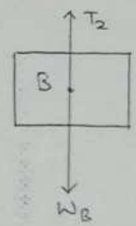


(effective diagram)

$\xrightarrow{+} \Sigma F_x = m_A a_A$

or, $T_1 = 100 a_A \longrightarrow ②$

FBD of block B



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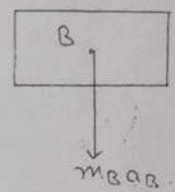


$\downarrow + \ \Sigma F_y = m_B a_B$

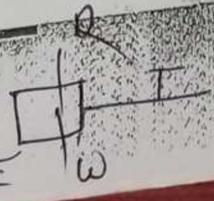
or $W_B - T_2 = m_B a_B$

or, $T_2 = 300 \times 9.81 - 300 a_B$

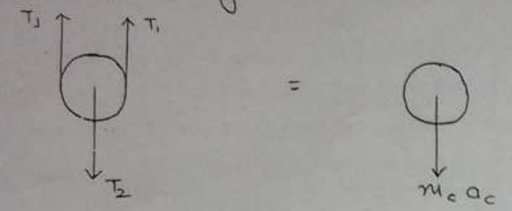
or, $T_2 = 2943 - 300 a_B$

or $T_2 = 2943 - 300 \frac{a_A}{2}$ $\left[\because \text{from eq}^n ① \ a_B = \frac{a_A}{2}\right]$

or, $T_2 = 2943 - 150 a_A \longrightarrow ③$



FBD for pully c



$+\downarrow \Sigma F_y = m_c a_c$

or, $T_2 - 2T_1 = 0$ [∵ m_c is negligible i.e. $m_c = 0$]

or, $T_2 = 2T_1$.

Then eqⁿ ③ becomes,

$2T_1 = 2943 - 150 a_A$

or, $T_1 = 1471.5 - 75 a_A \longrightarrow$ ④

solving eqⁿ ② and ④

$100 a_A = 1471.5 - 75 a_A$

or, $a_A = 8.40 \text{ m/s}^2$.

from eqⁿ ① $a_B = \frac{a_A}{2} = \frac{8.40}{2} = 4.20 \text{ m/s}^2$.

from eqⁿ ②, $T_1 = 100 a_A$

or, $T_1 = 100 \times 8.40$

or, $T_1 = 840$ N.

from eqⁿ ③

$T_2 = 2943 - 150 a_A$

or, $T_2 = 2943 - 150 \times 8.40$

or, $T_2 = 1683$ N.

② A 90.7 kg block rest on a horizontal plane. Find the magnitude of the force P required to give the block an accelⁿ of 3 m/s² to right. The coeff. of kinetic friction between the block and the plane is $\mu_k = 0.2$

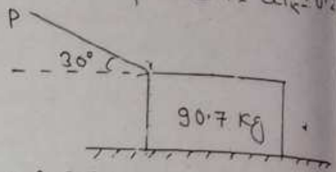
Here,

Mass of block (M) = 90.7 kg.

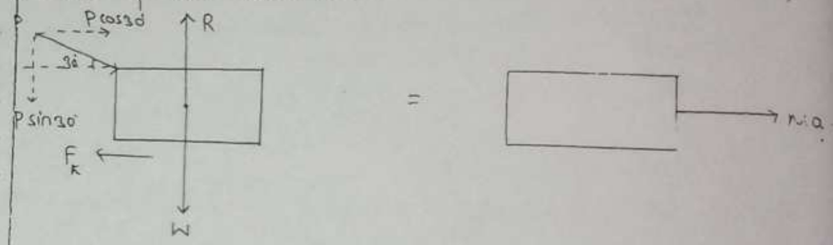
accleration (a) = 3 m/s².

Magnitude of force P = ?.

coeff. of kinetic friction (μ_k) = 0.25.



FBD of the block:



$\xrightarrow{+} \Sigma F_x = Ma$

or, $P\cos30° - F_k = Ma$

or, $P\cos30° - 0.25R = 90.7 \times 3$

or, $P\cos30° - 0.25R = 272.1 \longrightarrow$ ①

$+\uparrow \Sigma F_y = 0$

or, $R - W - P\sin30° = 0$

or, $R = W + P\sin30°$

or, $R = 90.7 \times 9.81 + P\sin30°$

or, $R = P\sin30° + 890 \longrightarrow$ ②

from eqⁿ ① and ②

$P\cos30° - 0.25(P\sin30° + 890) = 272.1$

or, $P(\cos30° - 0.125) = 494.6$

or, $P = \frac{494.6}{(\cos30° - 0.125)}$

or, $P = 667.45$ N.

...izontal plane. Find
...ired to give the b...
...coeff. of kinetic
... the plane is $\mu_K =$

... $30°$ ...

90.7 kg

... 25.

... N_a

...q^n ① and ②

...$25(P\sin 30° + 890) = 2$...

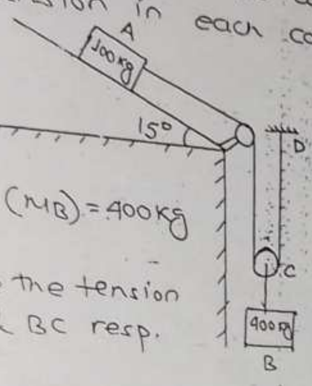
...$-0.125) = \cancel{273.71}$ 494.6

~~...~~ 494.6

...$\cos 30° - 0.125)$

...667.45 N.

Two blocks shown starts from rest. The polly are frictionless and have no mass. The kinetic coeff. of friction between the block A and the inclined plane is 0.4. Determine acc^n of each block and tension in each cord.



→ Here,

Mass of block A is $(M_A) = 100$ kg

Mass of block B $(M_B) = 400$ kg

$\mu_K = 0.4$

let T_1 and T_2 be the tension on the cord ADC & BC resp.

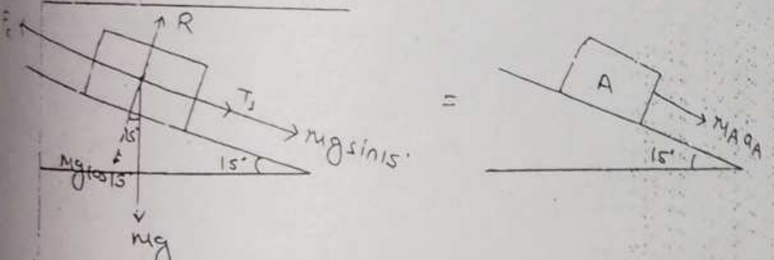
from fig.

$x_A = 2x_B$

Diff. twice w.r. to t

$a_A = 2a_B$ ⟶ ①

FBD for block A :



$\xrightarrow{+} \Sigma F_x = m_A a_A$

or, $T_1 + mg\sin 15° - F_K = m_A a_A$

or, $T_1 = -100 \times 9.81 \times \sin 15° + F_K + 100 a_A$

or, $T_1 = -253.90 + F_K + 100 a_A$ ⟶ ①

75

~~$+\uparrow \Sigma F_y = 0$~~

~~or, $R = Mg + Mg\cos 15°$~~

~~or, $F_2 = 100 \times 9.81 + 100 \times 9.81 \times \cos 15° = 1928.57$~~

~~Then eq^n ① becomes,~~

~~$T_1 = 253.90 + (1928.57)^{\times 0.4} + 100 a_A$~~

~~or, $T_1 = 2182.47 + 100 a_A$~~

$+\uparrow \Sigma F_y = 0$

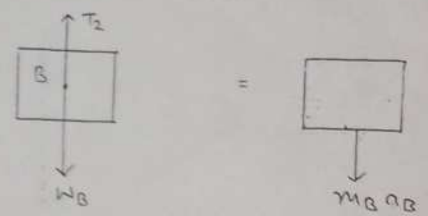
i.e. $R = Mg\cos 15° = 100 \times 9.81 \times \cos 15° = 947.57$

Then eq^n ① becomes,

$T_1 = -253.9 + 0.4 \times 947.57 + 100 a_A$ $[\because F_K = \mu_K R]$

or, $T_1 = 125.13 + 100 a_A$ ⟶ ②

FBD for block B :



$+\downarrow \Sigma F_y = M_B a_B$

or, $W_B - T_2 = M_B a_B$

or, $T_2 = W_B - M_B a_B$

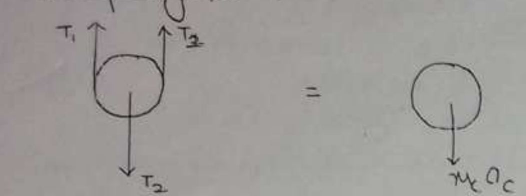
or, $T_2 = 400 \times 9.81 - 400 a_B$

or, $T_2 = 3924 - 400 a_B$

or, $T_2 = 3924 - 400 \times \frac{a_A}{2}$ [from eq^n ①]

or, $T_2 = 3924 - 200 a_A$ ⟶ ③

For pully C



$+\downarrow \Sigma F_y = 0$ ie. $T_2 = 2T_1$ → ④

Then eqn ③ becomes,

$2T_1 = 3924 - 200a_A$

or, $T_1 = 1962 - 100a_A$ → ⑤

solving eqn ② and ⑤

$1962 - 100a_A = 125.13 + 100a_A$

or, $a_A = 9.18\,m/s^2$.

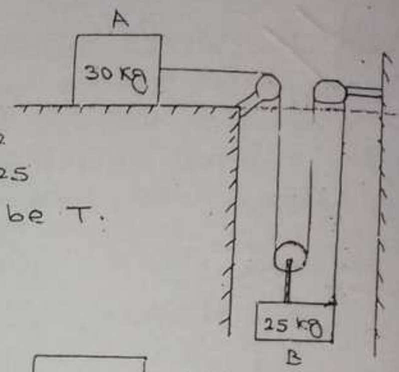
from eqn ① $a_B = \frac{a_A}{2} = \frac{9.18}{2} = 4.59\,m/s^2$.

from eqn ② $T_1 = 125.13 + 100 \times 9.18 = 1043.57\,N$

from eqn ④ $T_2 = 2 \times 1043.57 = 2087.13\,N$.

④ Two blocks shown are originally at rest neglecting the masses of pully and effect of friction in the pullys and assuming that coeff. of the friction between block A and the horizontal surface are $\mu_s = 0.25$ and $\mu_k = 0.2$ Determine

- accln of each block
- tension in the cable.



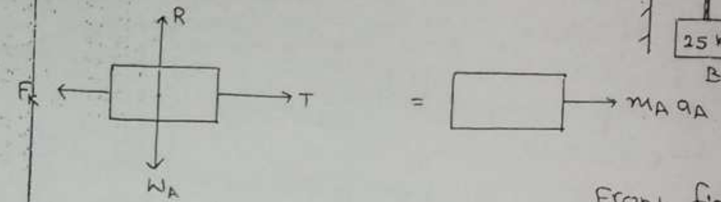
Given,

$m_A = 30\,kg$ $\mu_k = 0.2$

$m_B = 25\,kg$ $\mu_s = 0.25$

let tension in cable be T.

FBD for block A :



$\xrightarrow{+} \Sigma F_x = m_A a_A$

or, $T - F_K = m_A a_A$

or, $T = \mu_K R + m_A a_A$

or, $T = 0.2 \times 30 \times 9.81 + \cancel{9.81}\,30\, a_A$.

or, $T = 58.86 + 30a_A$ → ①

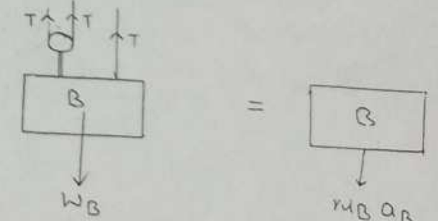
From fig. we can write,

$x_A = 3x_B$

Diff. w.r. to t twice

$a_A = 3a_B$ → ②

FBD for block B :



$+\downarrow \Sigma F_y = m_B a_B$

ie. $W_B = 3T + m_B a_B$

or, $3T = 25 \times 9.81 - 25a_B$

or, $3T = 245.25 - 25 \times a_A/3 = 245.25 - 8.33a_A$ → ②

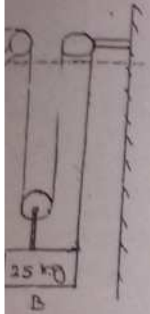


fig. we can

$a_A = 3x_B$

w.r.to t twice

$a_A = 3a_B \longrightarrow (2)$

$- 8.33a_A \longrightarrow (3)$

solving eqn (1) & (3)

$3(58.86 + 30a_A) = 245.25 - 8.33a_A$

or, $176.58 + 90a_A = 245.25 - 8.33a_A$

or, $a_A = 0.69\ m/s^2$

from eqn (2)

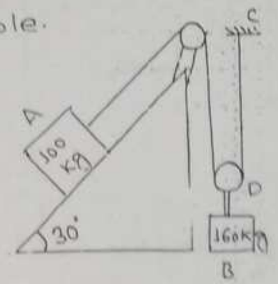
$$a_B = \frac{a_A}{3} = \frac{0.69}{3} = 0.232\ m/s^2$$

from eqn (1)

$T = 58.86 + 30 \times 0.69 = 79.56\ N$

(5) The two blocks shown are originally at rest. Neglecting the masses of pully and effect of friction in the pully and assuming that there is no coeff. of friction between block A and the inclined surface. Determine the

- accl^n of each block
- Tension in the cable.



Here,

$m_A = 100\ kg,\ m_B = 160\ kg$

$\mu_k = 0$

let T be the tension on the cable ADC.

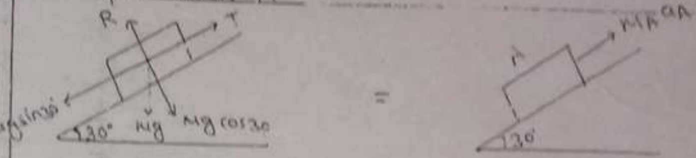
from fig.

$x_A = 2x_B$

Diff twice w.r.to t

$a_A = 2a_B \longrightarrow (1)$

FBD for block A



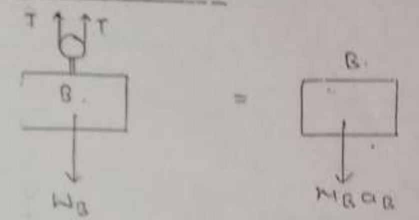
$\overset{+}{\rightarrow} \Sigma F_x = m_A a_A$

i.e. $T - mg\sin 30° = m_A a_A$

or, $T = 100 \times 9.81 \times \sin 30° + 100a_A$

or, $T = 490.5 + 100a_A \longrightarrow (2)$

FBD for block B



$\downarrow \Sigma F_y = m_B a_B$

or $W_B = 2T + m_B a_B$

or, $2T = 160 \times 9.81 - 160a_B$

or, $2T = 160 \times 9.81 - \frac{160}{2} a_A$ [from eqn (1)]

or, $T = 784.8 - 40a_A \longrightarrow (3)$

Solving eqn (2) & (3)

$490.5 + 100a_A = 784.8 - 40a_A$

or, $a_A = 2.10\ m/s^2$

from eqn (1) $a_B = \frac{a_A}{2} = \frac{2.10}{2} = 1.05\ m/s^2$

From eq^n ①

$T = 490.5 + 100 \times 2.10 = 700.5$ N.

2019 Fall ⑥ A 5 kg block is at rest on an inclined when a constant horizontal force P is applied to it. The μ_s & μ_k between the block & surface are 0.3 and 0.25 resp. knowing that the speed of the block is 12 m/sec after 6 sec. Determine magnitude of P.

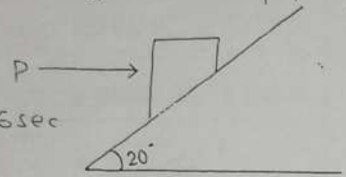
Here,

→ Mass of block (m) = 5 kg

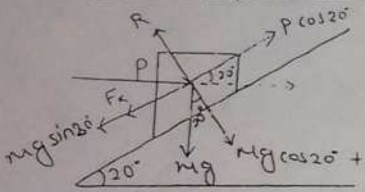
$\mu_k = 0.25$

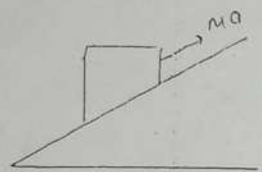
$u = 0$, $v = 12$ m/s, $t = 6$ sec

Magnitude of P = ?



FBD of the block





acc^n of block (a) $= \frac{v-u}{t} = \frac{12-0}{6} = 2$ m/s^2.

$+\uparrow \Sigma F_y = 0$

ie. $R = P\sin20° + mg\cos20°$ ——— ①

$\xrightarrow{+} \Sigma F_x = ma$

ie. $P\cos20° - F_k - mg\sin20° = 5 \times 2$

or, $P\cos20° - 0.25(P\sin20° + mg\cos20°) - mg\sin20° = 10$

or, $P\cos20° - (P\,0.086 + 11.523) - 16.776 = 10$

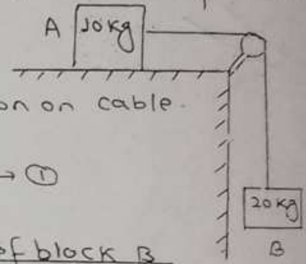
or, $P(\cos20° - 0.086) = 38.299$

or, $P = 44.86$ N.

⑦ A 10 kg block is rest on a horizontal surface as shown in fig. The polly is frictionless and weightless. The coeff. of friction between block and surface is 0.18. Determine tension and acc^n of each block.

→ Here, $m_A = 10$ kg, $m_B = 20$ kg

$\mu_k = 0.18$.

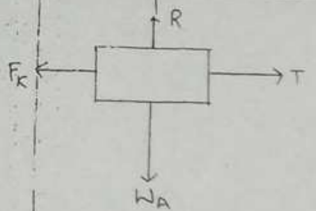


suppose T be the tension on cable.

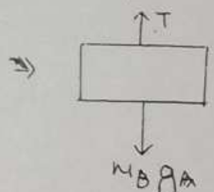
from fig. $x_A = x_B$

$\therefore a_A = a_B$ ——→ ①

FBD of block A



FBD of block B



$\xrightarrow{+} \Sigma F_x = m_A a_A$

or, $T - F_k = 10 a_A$

or, $T = 0.18 \times 10 \times 9.81 + 10 a_A$

or, $T = 17.658 + 10 a_A$ ——→ ⑪

$+\downarrow \Sigma F_y = m_B a_B$

or, $W_B - T = 20 a_B$

or, $T = 20 \times 9.81 - 20 a_A$

or, $T = 196.2 - 20 a_A$ ——③ $[\because a_A = a_B]$

solving eq^n ⑪ & ③

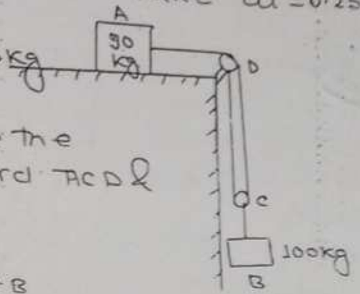
or, $17.658 + 10 a_A = 196.2 - 20 a_A \Rightarrow a_A = a_B = 5.95$ m/s^2

and $T = 17.658 + 10 \times 5.95 = 77.15$ N.

(2007) (2013) Fall (2013) sp

8. The two blocks shown start from rest. Where the polly is assumed to be of negligible mass. Determine the accln of each block and the tension in each cord. Take $\mu = 0.25$

→ Here,



$m_A = 90kg$, $M_B = 100kg$

$\mu_K = 0.25$.

let T_1 and T_2 be the tension on the cord ACD & BC resp.

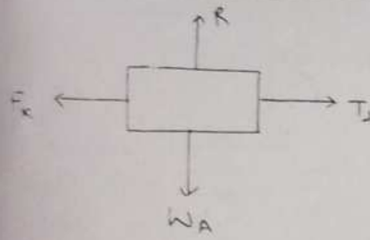
from fig

$x_A = 2x_B$

Diff twice w.r.to t

$a_A = 2a_B \longrightarrow (1)$

FBD for block A



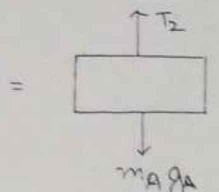
ie. $\xrightarrow{+} \Sigma F_x = M_A a_A$

$T_1 - F_K = 90\, a_A$

or, $T_1 = 0.25 \times 90 \times 9.81 + 90 a_A$

or $T_1 = 220.725 + 90 a_A \longrightarrow (11)$

FBD for block B



$+\downarrow \Sigma F_y = 0$

ie. $W_B - T_2 = M_B a_B$

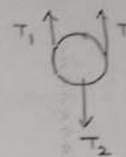
or, $T_2 = 100 \times 9.81 - 100 a_B$

or, $T_2 = 981 - 50 a_A \longrightarrow (111)$

$[\because a_B = a_A/2]$

77

FBD for polly



$+\downarrow \Sigma F_y = 0$

i.e. $T_2 = 2T_1 \longrightarrow (1V)$

Then eqn (111) becomes,

$2T_1 = 981 - 50 a_A$

or, $T_1 = 490.5 - 25 a_A \longrightarrow (V)$

solving eqn (11) and (V)

$220.75 + 90 a_A = 490.5 - 25 a_A$

or, $a_A = 2.346$ m/s².

from eqn (1), $a_B = \frac{a_A}{2} = \frac{2.346}{2} = 1.173$ m/s²

from eqn (11) $T_1 = 220.725 + 90 \times 2.346 = 431.83$ N

from eqn (1V) $T_2 = 2T_1 = 2 \times 431.83 = 863.66$ N.

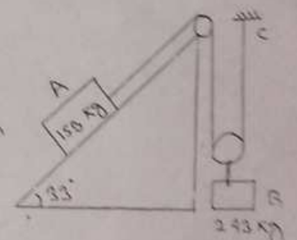
(2008) 9. Two blocks are originally at rest. Neglecting the mass of the pollys and effect of friction in the pollys and between block A and the inclined is ($\mu = 0.18$) Determine the acceleration of each block & tension in the cable.

→ Here,

$M_A = 158$ kg, $m_B = 243$ kg.

$\mu_K = 0.18$

suppose T be the tension on the cable

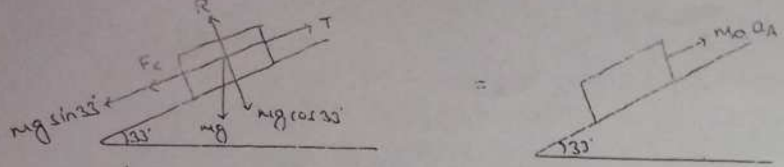


from fig.

$x_A = 2x_B$

$\therefore a_A = 2a_B \longrightarrow (1)$

FBD for block A



$\xrightarrow{+} \Sigma F_x = m_A a_A$

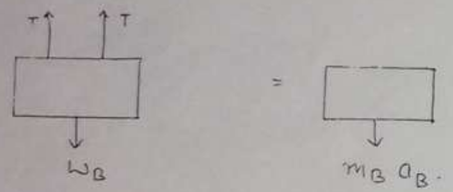
ie. $T - F_K - mg\sin 33^\circ = m_A a_A$

or, $T - 0.18(158\times 9.81\times\cos 33^\circ) - 158\times 9.81\times\sin 33^\circ = 158\, a_A$

or, $T - 233.986 - 844.180 = 158\, a_A$

or, $T = 1078.16 + 158\, a_A \longrightarrow (1)$.

FBD for block B



$+\downarrow \Sigma F_y = m_B a_B$

ie. $W_B - 2T = m_B a_B$

or, $2T = 243\times 9.81 - 243\, a_B$

or, $T = 1191.91 - 121.5\, a_B$

or, $T = 1191.91 - 60.750\, a_A \rightarrow (2) \left[\because a_B = a_A/2\right]$

from eqn (1) & (2)

$1078.16 + 158\, a_A = 1191.91 - 60.75\, a_A$

or $a_A = 0.52\ m/s^2$

Then, $a_B = \frac{a_A}{2} = \frac{0.52}{2} = 0.26\ m/s^2$.

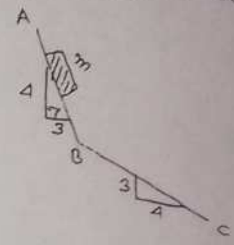
from eqn (1) $T = 1078.16 + 158\times 0.52 = 1160.22\ N$

Q. The acceleration of package sliding down section AB of inclined AB is $5\ m/s^2$. Assuming that the coeff. of kinetic friction is same for each section. Determine the accln of package on section BC of inclined.

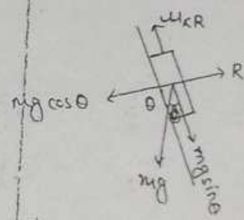
→ Given:

$a_{AB} = 5\ m/s^2$.

$a_{BC} = ?$



For section AB:



$\sin\theta = \frac{4}{5}$

and $\cos\theta = \frac{3}{5}$

$\xrightarrow{+} \Sigma F_x = m\, a_{AB}$

or, $mg\sin\theta - \mu_K R = m\, a_{AB}$

or, $\cancel{m}g\sin\theta - \mu_K \cancel{m}g\cos\theta = \cancel{m}\, a_{AB}$

or, $9.81\times\frac{4}{5} - 9.81\mu_K\times\frac{3}{5} = 5$

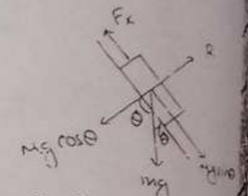
or, $\mu_K = 0.48$.

For section BC:

Here,

$\sin\theta = 3/5$

$\cos\theta = 4/5$.



$\xrightarrow{+} \Sigma F_x = m\, a_{BC}$

or, $mg\sin\theta - F_K = m\, a_{BC}$

or, $\cancel{m}g\sin\theta - \mu_K \cancel{m}g\cos\theta = \cancel{m}\, a_{BC}$

or, $9.81\times\frac{3}{5} - 0.48\times 9.81\times\frac{4}{5} = a_{BC}$

or, $a_{BC} = 2.11\ m/s^2$.

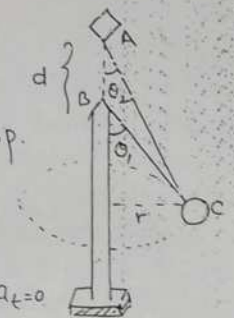
Q14) A single wire ACB passes through a ring at C is attached to a 5kg sphere which revolves at constant speed v in the horizontal circle as shown. Knowing that $\theta_1 = 50°$ and $d = 0.8m$ & that the tension in both portions of wire is 34N. Determine a) the angle θ_2 (b) The speed v.

→ Given:

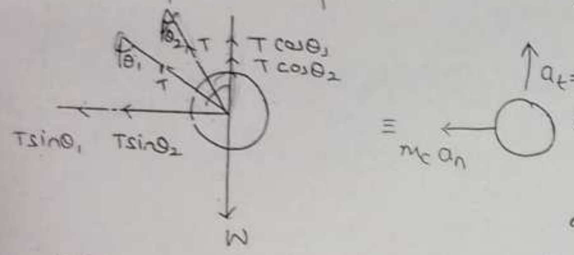
Mass of sphere (M) = 5kg

Tension in wire ACB

(i.e. AC and BC) = 34N resp.



FBD for the sphere.



at = tangential comp

an = Normal component

$a_n = \frac{v^2}{r}$

(i.e. $a_t = 0$)

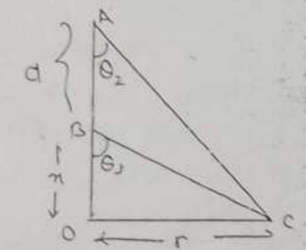
$(\uparrow +)$ $\Sigma F_y = M_C a_t$

or, $T\cos\theta_1 + T\cos\theta_2 - W = 0$

or, $34 \times \cos 50 + 34 \cdot \cos\theta_2 = 5 \times 9.81$

or, $\cos\theta_2 = 0.799$

or, $\theta_2 = 36.96°$.



From fig.

$\tan\theta_1 = \frac{r}{x}$

or, $\tan 50° = \frac{r}{x}$

or, $r = x\tan 50°$ ⟶ ①

and,

$\tan\theta_2 = \frac{r}{x+d}$

or, $r = (x + 0.8)\tan 36.96°$

or, $x\tan 50° = x\tan 36.96° + 0.8\tan 36.96°$ [from eqⁿ ①]

or, $x = \frac{0.8 \cdot \tan 36.96°}{(\tan 50° - \tan 36.96°)} = 1.37m$

Then, $r = 1.63m$.

$(\overset{+}{\leftarrow})$ $\Sigma F_n = M_C a_n$

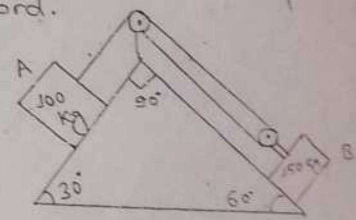
or, $T\sin\theta_1 + T\sin\theta_2 = M_C a_n$

or, $a_n = \frac{34(\sin 50° + \sin 36.96°)}{5} = 9.29 m/s^2$

We know, $a_n = \frac{v^2}{r}$

or, $v = \sqrt{a_n r} = \sqrt{9.29 \times 1.63} = 3.89 m/s$

Q15) The two blocks having masses 100kg and 150kg shown are originally at rest. Assuming that the coeff. of friction between the blocks and the surface $\mu_s = 0.25$ and $\mu_k = 0.2$. Determine (a) accⁿ of block A: (b) tension in the cord.



→ Here,

Mass of block A $(M_A) = 100kg$

Mass of block B $(M_B) = 150kg$

Coff. of kinetic friction

$(\mu_k) = 0.2$

accⁿ of block A $(a_A) = ?$

tension in the cord $(T) = ?$

98

But,

$v_B = u_B + a_B t$

or, $t = \dfrac{v_B - u_B}{a_B} = \dfrac{v_A - u_B}{a_B}$ $(\because v_A = v_B)$

or, $t = \dfrac{v_A - 20}{0.75}$ ⟶ ①

Again, $s_A - s_B = 60 - 6$

or, $v_A t - (u_B t + \frac{1}{2} a_B t^2) = 54$

or, $v_A\left(\dfrac{v_A-20}{0.75}\right) - \left\{20 \times \dfrac{(v_A-20)}{0.75} + \frac{1}{2} \times 0.75 \times \left(\dfrac{v_A-20}{0.75}\right)^2\right\} = 54$

or, $v_A^2 - 20v_A - 20v_A + 400 - 0.5(v_A - 20)^2 = 40.5$

or, $0.5v_A^2 - 20v_A + 159.5 = 0$

or, $v_A = \dfrac{40 \pm 18}{2}$

$v_A = 29$ m/s, 11 m/s.

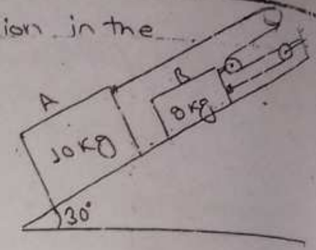
Since $v_A = 11$ m/s cannot be possible so,

$v_A = 29$ m/s.

13. The two blocks shown are originally at rest. Neglecting the masses of pollys and the effect of friction in the pulleys and assuming that the coeff. of friction between the both blocks and the inclined plane are $\mu_s = 0.25$ and $\mu_k = 0.20$. Determine

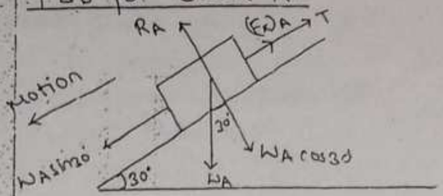
ⓐ accl^n of each block

ⓑ Tension in the cable.



Here, suppose T be the tension in the cable.

FBD for block A.



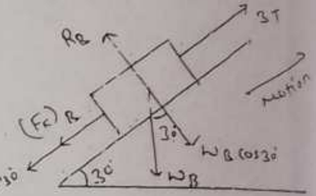
$\overleftarrow{+}$ $\Sigma F_x = m_A a_A$

i.e. $W_A \sin 30° - T - (F_k)_A = m_A a_A$

or, $10 \times 9.81 \times 0.5 - T - (0.2 \times 10 \times 9.81 \times \cos 30°) = m_A a_A$

or, $T = 32.06 - 10 a_A$ ⟶ ①

FBD for block B.



$\overrightarrow{+}$ $\Sigma F_x = m_B a_A$

i.e. $3T - W_B \sin 30° - (F_k)_B = m_B a_B$

or, $3T - (8 \times 9.81 \times 0.5) - (0.2 \times 8 \times 9.81 \times \cos 30°) = 8 a_B$

or, $3T - 52.83 = 8a_B$

or, $T = \dfrac{8a_B + 52.83}{3}$ ⟶ ②

From eq^n ① and ②

$3(32.06 - 10a_A) = 8a_B + 52.83$

or, $8a_B + 30a_A = 43.35$ ⟶ ③

Also, we have, $\boxed{a_A = 3a_B.}$

So, eq^n ③ becomes,

$8a_B + 90a_B = 43.35$ $\Rightarrow a_B = 0.442$ m/s²

and $a_A = 3a_B = 3 \times 0.442 = 1.327$

Similarly $T = 18.78$ N. (from eq^n ①)

Applied Mechanics I

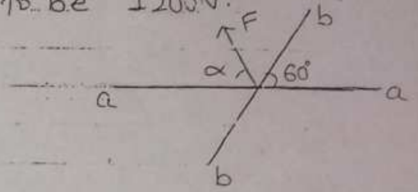
Prepared by: Er. Ravi Ghimire

B.E Civil and Elx.

2nd Sem

Prepared By:- Er. Ravi Ghimire

① The force 'F' of magnitude 1500N is to be resolved into two components along the line a-a and b-b. Determine the angle 'α', knowing that the components of F along line a-a to be 1200N.

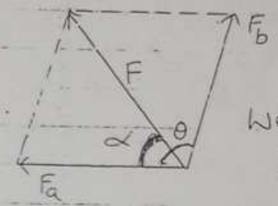


⇒ Given:

$F_a = 1200N$

$F = 1500N$

angle $(\alpha) = ?$



From figure $\theta = 180° - 60°$

or, $\theta = 120°$

We know that,

$$F^2 = F_a^2 + F_b^2 + 2F_a F_b \cos\theta$$

or, $1500^2 = 1200^2 + F_b^2 + 2 \times 1200 \times F_b \times \cos 120°$

Solving we get,

$F_b = 1681.67$ N

Again,

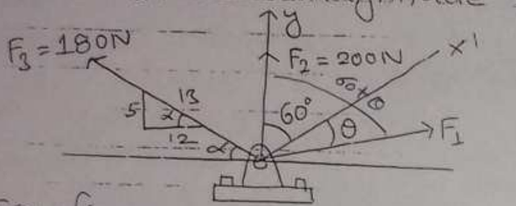
$$\tan\alpha = \frac{F_b \sin\theta}{F_a + F_b \cos\theta}$$

or, $\alpha = \tan^{-1}\left(\frac{1681.67 \times \sin 120}{1200 + 1681.67 \cos 120}\right) = 76.15°$

② Three forces act on the bracket. Determine the magnitude and direction θ of F_1 so that the resultant force is directed along the positive x'

Ravi Ghimire

axis and has a magnitude of 800N.



angle [illegible] value [illegible] x-axis [illegible] $\sin\theta$ component [illegible]

$\Rightarrow$ From figure,

$\tan\alpha = 5/12 \Rightarrow \alpha = 22.62°$

Resolving the force in horizontal and vertical direction.

$\xrightarrow{+} \Sigma F_x = F_1 \sin(60+\theta) - 180 \cos 22.62°$
$= -166.154 + F_1 \sin(60+\theta)$

$+\uparrow \Sigma F_y = F_1 \cos(60+\theta) + 200 + 180 \cdot \sin 22.62°$
$= F_1 \cos(60+\theta) + 269.231$

A/c to question,

Resultant force $(R) = 800 \sin 60 \hat{i} + 800 \cos 60 \hat{j}$

Now equating the coeff of $\hat{i}$ and $\hat{j}$

$-166.154 + F_1 \sin(60+\theta) = 800 \sin 60°$

or, $F_1 \sin(60+\theta) = 858.974 \longrightarrow$ (I)

and

$F_1 \cos(60+\theta) + 269.231 = 800 \cos 60°$

or, $F_1 \cos(60+\theta) = 130.769 \longrightarrow$ (II)

Dividing eqn. (I) by (II)

$$\frac{F_1 \sin(60+\theta)}{F_1 \cos(60+\theta)} = \frac{858.974}{130.769}$$

or, $\tan(60+\theta) = 6.569$

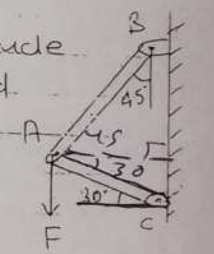
or, $60+\theta = 81.344$

or, $\theta = 21.344°$

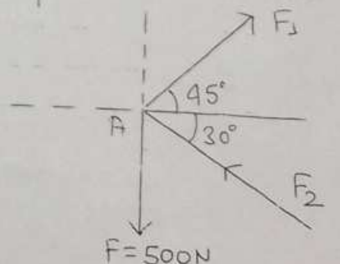
From eqn (I) $F_1 \sin(60+21.344) = 858.974$

or, $F_1 = \dfrac{858.974}{\sin 81.344} = 868.87$ N

(3) The vertical force $F = 500N$ acts downward at A on the two membered frame. Determine the magnitude of two components of F directed along the axes of AB and AC.



$\Rightarrow$ Suppose F_1 and F_2 be the magnitude of force on the member AB and AC resp.



(4)

$\Rightarrow$

Resolving the forces in x and y direction

$\xrightarrow{+} \Sigma F_x = 0$

ie. $F_1 \cos 45^\circ - F_2 \cos 30^\circ = 0$

or, $0.707 F_1 - 0.866 F_2 = 0 \longrightarrow (1)$

and

$+\uparrow \Sigma F_y = 0$

ie. $F_1 \sin 45^\circ + F_2 \sin 30^\circ - 500 = 0$

or, $0.707 F_1 + 0.5 F_2 = 500 \longrightarrow (2)$

Solving eqn (1) and (2)

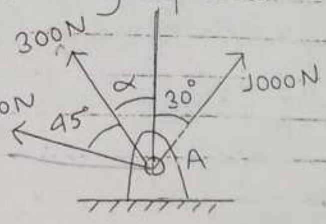
$F_1 = 448.351 \text{ N}$

and

$F_2 = 366.032 \text{ N}$

(4) The directions of the 300N forces may vary, but the angle between the forces is always is 45°. Determine the value of α for which resultant of force acting at A is directed vertically upward.

⇒ Here,



The resultant is vertically upward so, the x-component of resultant force must be zero.

$\xrightarrow{+} \Sigma F_x = 0$

ie. $1000 \sin 30^\circ - 300 \sin\alpha - 300 \sin(45+\alpha) = 0$

or, $1000 \sin 30^\circ - 300 \{ \sin\alpha + \sin(45+\alpha) \} = 0$

or, $500 - 300 \left\{ 2 \times \sin\left(\frac{\alpha + 45 + \alpha}{2}\right) \cdot \cos\left(\frac{\alpha - 45 - \alpha}{2}\right) \right\} = 0$

$\left[\because \sin A + \sin B = 2 \sin \frac{A+B}{2} \cos \frac{A-B}{2} \right]$

or, $500 - 600 \sin(\alpha + 22.5) \cdot \cos(-22.5) = 0$

or, $\sin(\alpha + 22.5) = \frac{500}{554.33}$

or, $\alpha + 22.5 = \sin^{-1}\left(\frac{500}{554.33}\right)$

or, $\alpha + 22.5 = 64.42^\circ$

or, $\alpha = 41.92^\circ$

(5) A block of weight 'W' is suspended form a 25cm long cord and two springs of which the unstretched length are 22.5cm. Knowing that the constants of springs are $K_{AB} = 9 \text{ N/cm}$ & $K_{AD} = 3 \text{ N/cm}$, Determine

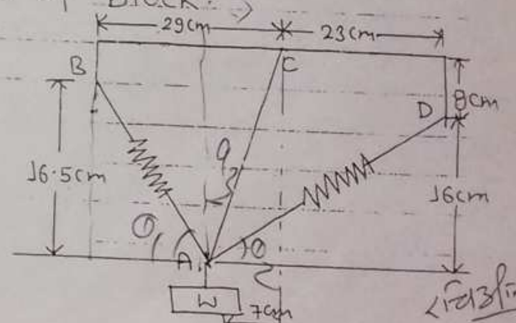
i) Tension in the cord

ii) Weight of block.

⇒ Here,

Unstretched length $(x_0) = 22.5 \text{ cm}$

Then,



From figure

$x_{AB} = \sqrt{(16.5)^2 + (29-7)^2} = 27.5 cm$

$x_{AD} = \sqrt{16^2 + (23+7)^2} = 34 cm$

$\therefore \Delta x_{AB} = (27.5 - 22.5) = 5 cm$

and $\Delta x_{AD} = 34 - 22.5 = 11.5 cm$

Force on AB $(F_1) = \Delta x_{AB} \cdot K_{AB}$

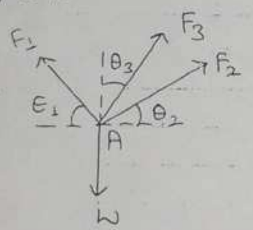
or, $F_1 = 5 \times 9 = 45 N$

and

Force on AD $(F_2) = \Delta x_{AD} \cdot K_{AD} = 11.5 \times 3$

or, $F_2 = 34.5 N$.

From figure,



$\theta_1 = \tan^{-1}\left(\frac{16.5}{29-7}\right) = 36.87^\circ$

$\theta_2 = \tan^{-1}\left(\frac{16}{23+7}\right) = 28.07^\circ$

$\theta_3 = \tan^{-1}\left(\frac{7}{16+8}\right) = 16.26^\circ$

Considering equilibrium at point A,

$\xrightarrow{+} \Sigma F_x = 0$

ie. $F_2 \cos\theta_2 + F_3 \sin\theta_3 - F_1 \cos\theta_1 = 0$

or, $34.5 \cos 28.07^\circ + F_3 \sin 16.26^\circ - 45 \cos 36.87 = 0$

or, $F_3 = 19.85 N$

$+\uparrow \Sigma F_y = 0$

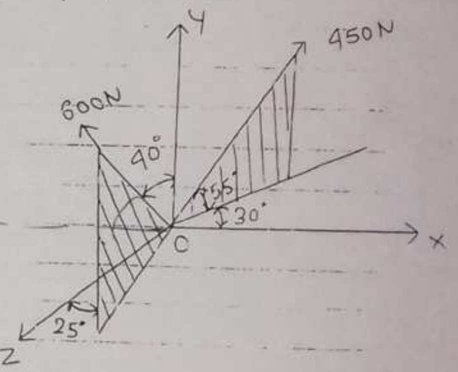
$F_1 \sin\theta_1 + F_3 \cos\theta_3 + F_2 \sin\theta_2 - W = 0$

or, $45 \sin 36.87 + 19.85 \cos 16.26 + 34.5 \sin 28.07 - W = 0$

or, $W = 62.29 N$.

TU (2010) Fall

6. Determine the resultant of force system as shown in the figure.



$\Rightarrow$ Here,

$F_1 = 450N$, $F_2 = 600N$

Resolving the forces in x, y and z direction

$F_{1x} = (450 \times \cos 55) \times \cos 30$

or, $F_{1x} = 223.53 N$

$F_{1y} = 450 \times \sin 55$

or, $F_{1y} = 368.62 N$

$F_{1z} = (-450 \times \cos 55) \times \sin 30 = -129.05 N$

Again,

$F_{2x} = 600 \times \sin 40 \times \sin 25 = 162.99 N$

$F_{2y} = 600 \times \cos 40 = 459.63 N$

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$F_{z2} = 600 \times \sin 40 \times \cos 25 = 349.54\ N$

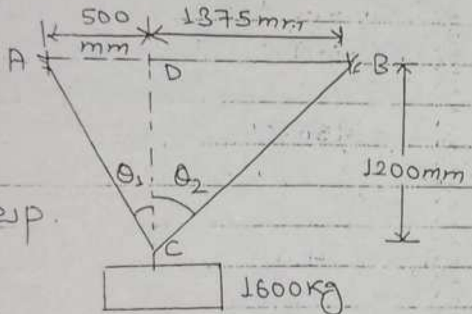
Resultant force (R) = $\Sigma F_x \hat{i} + \Sigma F_y \hat{j} + \Sigma F_z \hat{k}$

or, $\vec{R} = (223.53 + 162.59)\hat{i} + (368.62 + 459.63)\hat{j} + (-129.05 + 349.54)\hat{k}$

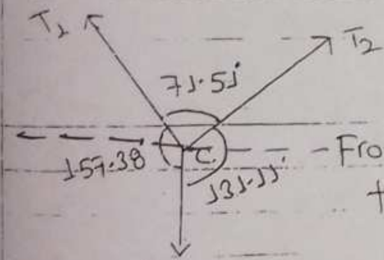
or, $\vec{R} = 386.52\hat{i} + 828.25\hat{j} + 220.49\hat{k}$

3) Fall

7. Two cables are tied together at C and loaded as shown. Determine the tension in AC and BC.



=> Here,
Suppose T_1 and T_2 be the tension in cable AC and BC resp.



- From figure In rt. ΔACD

$\tan\theta_1 = \frac{AD}{CD} = \frac{500}{1200}$

or, $\theta_1 = 22.62°$

and In rt Δ BCD

$\tan\theta_2 = \frac{BD}{CD} = \frac{1375}{1200} \Rightarrow \theta_2 = 48.89°$

At point C, using Lami's theorem

$$\frac{1600}{\sin 71.51°} = \frac{T_1}{\sin 131.11°} = \frac{T_2}{\sin 157.38°}$$

Taking 1st and 2nd ratio

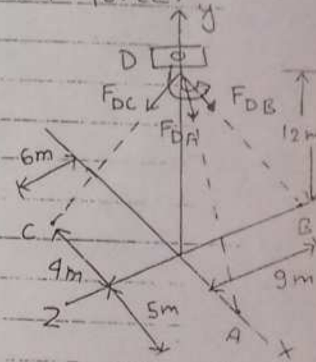
$$T_1 = \frac{1600 \times \sin 131.11}{\sin 71.51°} = 1271.14\ kg$$

Taking 1st and 3rd ratio

$$T_2 = \frac{1600 \times \sin 157.38}{\sin 71.51°} = 648.88\ kg$$

8. Three forces act at hook at D as shown. If their resultant force at D is $R = (-370j)$ N, Determine the magnitude of each force.

=> Here,



Co-ordinate of O = (0,0,0)
" " D = (0, 12, 0)
" " A = (5,0,0)
" " B = (0,0,-9)
" " C = (-4,0,6)

Resultant = $(-370\hat{j})$ N

Now,

$\vec{DA} = (5-0)\hat{i} + (0-12)\hat{j} + (0-0)\hat{k} = 5\hat{i} - 12\hat{j}$

$\vec{DB} = (0-0)\hat{i} + (0-12)\hat{j} + (-9-0)\hat{k} = -12\hat{j} - 9\hat{k}$

$\vec{DC} = (-4-0)\hat{i} + (0-12)\hat{j} + (6-0)\hat{k} = -4\hat{i}-12\hat{j}+6\hat{k}$

Then,

Force along DA $(\vec{F}_{DA}) = F_{DA} \cdot \hat{DA}$

or, $\vec{F}_{DA} = F_{DA} \cdot \dfrac{\vec{DA}}{|\vec{DA}|}$

or, $\vec{F}_{DA} = F_{DA} \cdot \dfrac{5\hat{i}-12\hat{j}}{\sqrt{(5)^2+(-12)^2}} = \left(\dfrac{5\hat{i}-12\hat{j}}{13}\right)F_{DA}$

Similarly,

$\vec{F}_{DB} = F_{DB} \cdot \hat{DB} = F_{DB} \cdot \dfrac{\vec{DB}}{|\vec{DB}|}$

or, $\vec{F}_{DB} = F_{DB} \cdot \dfrac{-12\hat{j}-9\hat{k}}{\sqrt{(-12)^2+(-9)^2}} = \left(\dfrac{-12\hat{j}-9\hat{k}}{15}\right)F_{DB}$

and

$\vec{F}_{DC} = F_{DC} \cdot \hat{DC} = F_{DC} \cdot \dfrac{\vec{DC}}{|\vec{DC}|}$

or, $\vec{F}_{DC} = F_{DC} \cdot \dfrac{-4\hat{i}-12\hat{j}+6\hat{k}}{\sqrt{(-4)^2+(-12)^2+6^2}} = \dfrac{-4\hat{i}-12\hat{j}+6\hat{k}}{14} F_{DC}$

Then,

Resultant $(\vec{R}) = \vec{F}_{DA} + \vec{F}_{DB} + \vec{F}_{DC}$

or, $\vec{R} = \left(\dfrac{5\hat{i}-12\hat{j}}{13}\right)F_{DA} + \left(\dfrac{-12\hat{j}-9\hat{k}}{15}\right)F_{DB} + \left(\dfrac{-4\hat{i}-12\hat{j}+6\hat{k}}{14}\right)F_{DC}$

or, $-370\hat{j} = \left(\dfrac{5\hat{i}-12\hat{j}}{13}\right)F_{DA} + \left(\dfrac{-12\hat{j}-9\hat{k}}{15}\right)F_{DB} + \left(\dfrac{-4\hat{i}-12\hat{j}+6\hat{k}}{14}\right)F_{DC}$

Equating the coeff. of $\hat{i}$, $\hat{j}$ and $\hat{k}$

$\dfrac{5}{13}F_{DA} - \dfrac{4}{14}F_{DC} = 0 \longrightarrow (1)$

$-\dfrac{12}{13}F_{DA} - \dfrac{12}{15}F_{DB} - \dfrac{12}{14}F_{DC} = -370 \longrightarrow (2)$

$-\dfrac{9}{15}F_{DB} + \dfrac{6}{14}F_{DC} = 0 \longrightarrow (3)$

solving eq^n (1), (2) and (3)

$F_{DA} = 130N$, $F_{DB} = 130N$ and $F_{DC} = 175N$

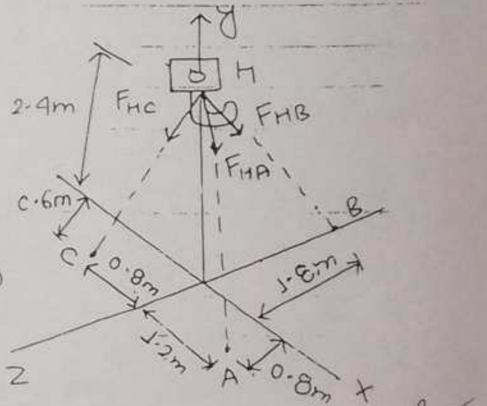
(9) Three forces F_{HA}, F_{HB} and F_{HC} act on hook at H as shown. If the magnitude of these forces are $F_{HA} = 420N$, $F_{HB} = 500N$ and $F_{HC} = 390N$. Determine the resultant force 'R' acting on the hook.

$\Rightarrow$ Here,

$F_{HA} = 420N$

$F_{HB} = 500N$

$F_{HC} = 390N$



co-ordinate of O = (0,0,0)

" " H = (0, 2.4, 0)

" " A = (1.2, 0, 0.8)

" " B = (0, 0, -1.8)

" " C = (-0.8, 0, 0.6)

Resultant force $(\vec{R}) = ?$

Now,

$\vec{HA} = 1.2\hat{i} - 2.4\hat{j} + 0.8\hat{k}$

$\vec{HB} = -2.4\hat{j} - 1.8\hat{k}$

$\vec{HC} = -0.8\hat{i} - 2.4\hat{j} + 0.6\hat{k}$

Then,

$$\vec{F}_{HA} = F_{HA} \cdot \hat{HA} = F_{HA} \cdot \frac{\vec{HA}}{|\vec{HA}|}$$

or, $\vec{F}_{HA} = 420 \cdot \left(\frac{1.2\hat{i} - 2.4\hat{j} + 0.8\hat{k}}{\sqrt{1.2^2 + (-2.4)^2 + 0.8^2}}\right)$

or, $\vec{F}_{HA} = 180\hat{i} - 360\hat{j} + 120\hat{k}$

Similarly,

$$\vec{F}_{HB} = F_{HB} \cdot \hat{HB} = F_{HB} \cdot \frac{\vec{HB}}{|\vec{HB}|}$$

or, $\vec{F}_{HB} = 500 \cdot \frac{-2.4\hat{j} - 1.8\hat{k}}{\sqrt{(-2.4)^2 + (-1.8)^2}}$

or, $\vec{F}_{HB} = -400\hat{j} - 300\hat{k}$

and

$$\vec{F}_{HC} = F_{HC} \cdot \hat{HC} = F_{HC} \cdot \frac{\vec{HC}}{|\vec{HC}|}$$

or, $\vec{F}_{HC} = 390 \cdot \frac{-0.8\hat{i} - 2.4\hat{j} + 0.6\hat{k}}{\sqrt{(-0.8)^2 + (-2.4)^2 + (0.6)^2}}$

or, $\vec{F}_{HC} = -120\hat{i} - 360\hat{j} + 90\hat{k}$

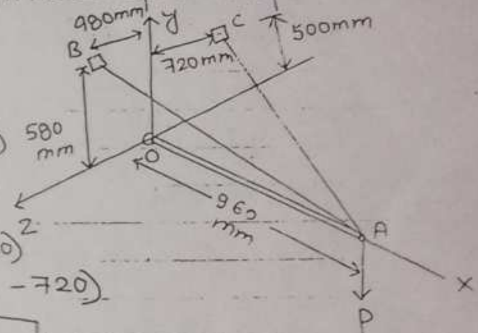
$\therefore$ Resultant $(\vec{R}) = \vec{F}_{HA} + \vec{F}_{HB} + \vec{F}_{HC}$

or, $\vec{R} = 180\hat{i} - 360\hat{j} + 120\hat{k} - 400\hat{j} - 300\hat{k} - 120\hat{i} - 360\hat{j} + 90\hat{k}$

or, $\vec{R} = 60\hat{i} - 1120\hat{j} - 90\hat{k}$

or, $|\vec{R}| = \sqrt{(60)^2 + (-1120)^2 + (-90)^2} = 1125.21$ N.

(10) The boom OA carries a load 'P' and is supported by two cables as shown. Knowing that the tension in cable AB is 372 N and that the resultant of the load P and of the force exerted at A by the two cables must be directed along OA. Determine the magnitude of load P.



$\Rightarrow$ Here,

Tension in cable $(T_{AB}) = 372$ N

Co-ordinate of $O = (0, 0, 0)$

" " $A = (960, 0, 0)$

" " $B = (0, 580, 480)$

" " $C = (0, 500, -720)$

$$\boxed{\begin{array}{l} \text{Resultant} = R\hat{i} \\ \text{load (P)} = ? \quad -P\hat{j} \end{array}}$$

(since it is directed along OA i.e. x-axis)

Then,

$\vec{AB} = (0-960)\hat{i} + (580-0)\hat{j} + (480-0)\hat{k}$

or, $\vec{AB} = -960\hat{i} + 580\hat{j} + 480\hat{k}$

and

$\vec{AC} = (0-960)\hat{i} + (500-0)\hat{j} + (-720-0)\hat{k}$

or, $\vec{AC} = -960\hat{i} + 500\hat{j} - 720\hat{k}$

Tension along AB ($\vec{T}_{AB}$) = $T_{AB} \cdot \widehat{AB}$

or, $\vec{T}_{AB} = 372 \dfrac{(-960\hat{i} + 580\hat{j} + 480\hat{k})}{\sqrt{(-960)^2 + (580)^2 + (480)^2}}$

or, $\vec{T}_{AB} = -\dfrac{17856}{61}\hat{i} + \dfrac{10788}{61}\hat{j} + \dfrac{8928}{61}\hat{k}$

similarly,

Tension along AC ($\vec{T}_{AC}$) = $T_{AC} \cdot \widehat{AC}$

or, $\vec{T}_{AC} = T_{AC} \cdot \dfrac{(-960\hat{i} + 500\hat{j} - 720\hat{k})}{\sqrt{(-960)^2 + 500^2 + (-720)^2}}$

or, $\vec{T}_{AC} = \left(-\dfrac{48}{65}\hat{i} + \dfrac{5}{13}\hat{j} - \dfrac{36}{65}\hat{k}\right) T_{AC}$

At node A,

$\vec{T}_{AB} + \vec{T}_{AC} + \vec{R} + \vec{P} = 0$

or $-\dfrac{17856}{61}\hat{i} + \dfrac{10788}{61}\hat{j} + \dfrac{8928}{61}\hat{k} + T_{AC}\left(-\dfrac{48}{65}\hat{i} + \dfrac{5}{13}\hat{j} - \dfrac{36}{65}\hat{k}\right) + R\hat{i} - P\hat{j} = 0$

Equating the coeff. of $\hat{i}$, $\hat{j}$ and $\hat{k}$

$-\dfrac{17856}{61} - \dfrac{48}{65}T_{AC} + R = 0 \longrightarrow (1)$

$\dfrac{10788}{61} + \dfrac{5}{13}T_{AC} - P = 0 \longrightarrow (2)$

$\dfrac{8928}{61} - \dfrac{36}{65}T_{AC} = 0 \longrightarrow (3)$

From eq^n (1), (2) and (3)

$T_{AC} = 264.26$ N

$P = 278.50$ N

$R = 487.87$ N

(11) A container of weight 'W' is suspended from ring A. Cable BAC passes through the ring and is attached to fixed supports at B and C. Two forces $\vec{P} = P\hat{i}$ and $\vec{Q} = Q\hat{k}$ are applied to the ring to maintain the container in the position shown. Knowing that W = 1200N, Determine P & Q.

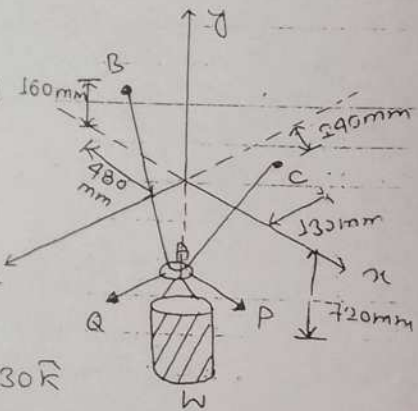
⇒ Here, W = 1200 N

From figure, the co-ordinate of O = (0,0,0)

A = (0, -720, 0)

B = (-480, 160, 0)

C = (240, 0, 130)



$\vec{AB} = -480\hat{i} + 880\hat{j}$

$\vec{AC} = 240\hat{i} + 720\hat{j} + 130\hat{k}$

Force along AB ($\vec{F}_{AB}$) = $F_{AB} \cdot \widehat{AB} = F_{AB} \cdot \dfrac{\vec{AB}}{|\vec{AB}|}$

or, $\vec{F}_{AB} = F_{AB} \cdot \dfrac{-480\hat{i} + 880\hat{j}}{\sqrt{(480)^2 + (880)^2}}$

or, $\vec{F}_{AB} = F_{AB}(-0.48\hat{i} + 0.88\hat{j})$

Similarly, Force along AC $(\vec{F}_{AC}) = F_{AC}\cdot\hat{AC}$

or, $\vec{F}_{AC} = F_{AC}\cdot\dfrac{290\hat{i} + 720\hat{j} + 130\hat{k}}{\sqrt{290^2 + 720^2 + 130^2}}$

or, $\vec{F}_{AC} = F_{AC}\cdot(0.31\hat{i} + 0.94\hat{j} + 0.17\hat{k})$

For equilibrium,

$$\vec{F}_{AB} + \vec{F}_{AC} + \vec{P} + \vec{Q} + \vec{W} = 0$$

or, $F_{AB}(-0.48\hat{i} + 0.88\hat{j}) + F_{AC}(0.31\hat{i} + 0.94\hat{j} + 0.17\hat{k}) + P\hat{i} + Q\hat{k} - W\hat{j} = 0$

Since cable passes through ring at A

$$F_{AB} = F_{AC} = T$$

$$T(-0.48\hat{i} + 0.88\hat{j}) + T(0.31\hat{i} + 0.94\hat{j} + 0.17\hat{k}) + P\hat{i} + Q\hat{k} - W\hat{j} = 0$$

Equating the coeff. of like vectors,

$-0.48T + 0.31T + P = 0$

or, $-0.17T + P = 0 \longrightarrow (1)$

$0.88T + 0.94T - W = 0$

or, $1.82T = W \longrightarrow (2)$

and,

$0.17T + Q = 0 \longrightarrow (3)$

From eqn (2)

$T = \dfrac{W}{1.82} = \dfrac{1200}{1.82} = 659.34\ N$

From eqn (3)

$Q = -0.17 \times 659.34 = -112.09\ N$

From eqn (1)

$-0.17T + P = 0$

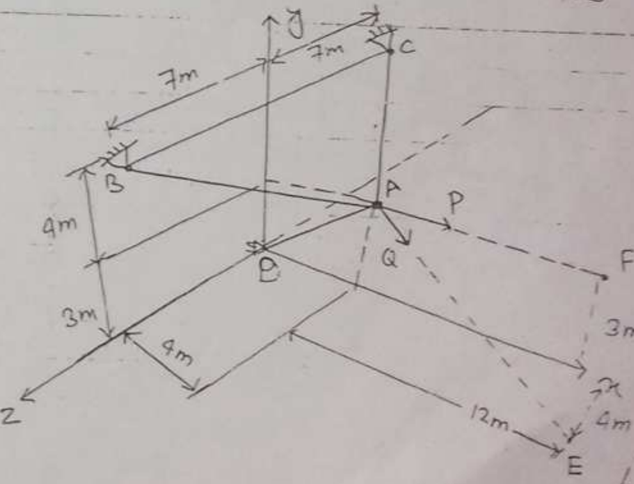
or, $P = 0.17 \times 659.34 = 112.09\ N$

12) Three cables are connected at A, where the forces P and Q are applied as shown. Knowing that Q = 3.6 kN and that the tension in cable AD is zero. Determine

Mon 4

i) Magnitude and sense of P

ii) The tension in cable AB and AC



→ From figure,
co-ordinate of $D = (0,0,0)$
" " $A = (4,3,0)$
" " $B = (0,7,7)$
" " $C = (0,7,-7)$
" " $E = (16,0,4)$
" " $F = (16,3,0)$

$F_{AE} = Q = 3.6\,kN$ Tension in cable AD $(T_{AD}) = 0$

Then,

$\vec{AB} = -4\hat{i} + 4\hat{j} + 7\hat{k}$

$\vec{AC} = -4\hat{i} + 4\hat{j} - 7\hat{k}$

$\vec{AF} = 12\hat{i} + 0\hat{j} + 0\hat{k}$

$\vec{AE} = 12\hat{i} - 3\hat{j} + 4\hat{k}$

$\vec{AD} = -4\hat{i} - 3\hat{j} + 0\cdot\hat{k}$

Force along AB $(\vec{F}_{AB}) = T_{AB}\cdot\hat{AB}$

or, $\vec{T}_{AB} = T_{AB}\cdot\dfrac{-4\hat{i}+4\hat{j}+7\hat{k}}{\sqrt{(-4)^2+4^2+7^2}}$

or, $\vec{T}_{AB} = \left(-\frac{4}{9}\hat{i} + \frac{4}{9}\hat{j} + \frac{7}{9}\hat{k}\right)T_{AB}$

Force along AC $(\vec{T}_{AC}) = T_{AC}\cdot\hat{AC}$

$\vec{T}_{AC} = T_{AC}\cdot\dfrac{-4\hat{i}+4\hat{j}-7\hat{k}}{\sqrt{(-4)^2+4^2+(-7)^2}}$

or, $\vec{T}_{AC} = \left(-\frac{4}{9}\hat{i} + \frac{4}{9}\hat{j} - \frac{7}{9}\hat{k}\right)T_{AC}$

Force along AD $(\vec{T}_{AD}) = T_{AD}\cdot\vec{AD} = 0$ $[\because T_{AD} = 0]$

Force along AE $(\vec{F}_{AE}) = F_{AE}\cdot\hat{AE}$

or, $\vec{F}_{AE} = 3.6\,\dfrac{(12\hat{i}-3\hat{j}+4\hat{k})}{\sqrt{12^2+(-3)^2+4^2}}$

or, $\vec{F}_{AE} = \frac{43.2}{13}\hat{i} - \frac{10.8}{13}\hat{j} + \frac{14.4}{13}\hat{k}$

Force along AF $(\vec{F}_{AF}) = F_{AF}\cdot\hat{AF}$

or, $\vec{F}_{AF} = P\cdot\dfrac{12\hat{i}}{\sqrt{12^2}} = P\hat{i}$ $[\because F_{AF} = P]$

For equilibrium $\Sigma F = 0$

ie. $\vec{T}_{AB} + \vec{T}_{AC} + \vec{T}_{AD} + \vec{F}_{AE} + \vec{F}_{AF} = 0$

or, $\left(-\frac{4}{9}\hat{i} + \frac{4}{9}\hat{j} + \frac{7}{9}\hat{k}\right)T_{AB} + \left(-\frac{4}{9}\hat{i} + \frac{4}{9}\hat{j} - \frac{7}{9}\hat{k}\right)T_{AC} + 0$
$+ \frac{43.2}{13}\hat{i} - \frac{10.8}{13}\hat{j} + \frac{14.4}{13}\hat{k} + P\hat{i} = 0$

comparing the coeff. of like vectors,

$-\frac{4}{9}T_{AB} - \frac{4}{9}T_{AC} + \frac{43.2}{13} + P = 0 \longrightarrow (1)$

$\frac{4}{9}T_{AB} + \frac{4}{9}T_{AB} - \frac{10.8}{13} = 0 \longrightarrow (2)$

$\frac{7}{9}T_{AB} - \frac{7}{9}T_{AC} + \frac{14.4}{13} = 0 \longrightarrow (3)$

solving eqn (1), (2) and (3)

$T_{AB} = 0.22\,kN$ and $P = -2.49\,kN$

$T_{AC} = 1.64\,kN$

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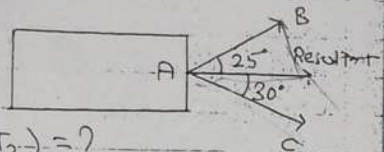
$\therefore$ The sense of 'p' is in opposite direction.

(2006 SP)

(13) A disabled automobile is pulled by means of two ropes as shown. The tension in AB is 1.2 KN and angle α is 25°. Knowing that the resultant of two forces applied at A is directed along the axis of the automobile. Determine (i) the tension in the rope AC ii) Magnitude of resultant of two force at A.

$\Rightarrow$ Given:

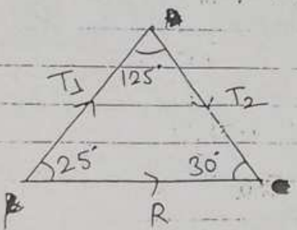


Tension in rope AB $(T_1) = 1.2$ KN

Tension in rope AC $(T_2) = ?$

Magnitude of Resultant $(R) = ?$

From the reference, to force diagram



Applying sine law

$$\frac{T_1}{\sin 30°} = \frac{T_2}{\sin 25°} = \frac{R}{\sin 125°}$$

From 1st and 2nd ratio

$$\frac{T_1}{\sin 30°} = \frac{T_2}{\sin 25°} \Rightarrow T_2 = \frac{1.2 \times \sin 25}{\sin 30°}$$

or, $T_2 = 1.01$ KN

$\therefore$ Tension in rope AC $(T_2) = 1.01$ KN

From 1st and 3rd ratio,

$$\frac{T_1}{\sin 30°} = \frac{R}{\sin 125°}$$

or, $R = \frac{1.2 \times \sin 125°}{\sin 30°} = 1.97$ KN.

$\therefore$ Resultant of two force $(R) = 1.97$ KN

(2014 Fall)
(2018 Fall)

(14) A square foundation mat supports the four column with the weight as shown. Determine the magnitude and point of application of resultant of four loads.

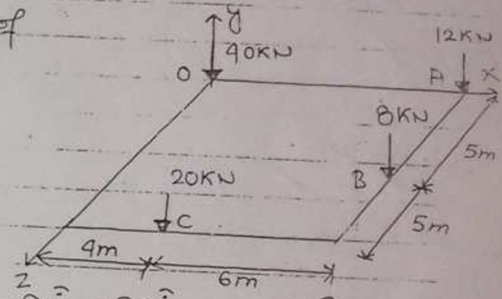
$\rightarrow$ Here, Co-ordinate of

$O = (0, 0, 0)$

$A = (10, 0, 0)$

$B = (10, 0, 5)$

$C = (4, 0, 10)$



Resultant of force

$(\vec{F_R}) = -40\hat{j} - 12\hat{j} - 8\hat{j} - 20\hat{j} = -80\hat{j}$

Let the resultant passes through the point having position vector $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$

Taking Moment about origin,

Moment due to resultant = Sum of Moment due to individual force.

$$\vec{r} \times \vec{F_R} = \vec{r_1} \times \vec{F_1} + \vec{r_2} \times \vec{F_2} + \vec{r_3} \times \vec{F_3} + \vec{r_4} \times \vec{F_4}$$

or, $(x\hat{i} + y\hat{j} + z\hat{k}) \times -80\hat{j} = 0 \times -40\hat{j} + 10\hat{i} \times -12\hat{j}$
$+ (10\hat{i} + 5\hat{k}) \times -8\hat{j} + (4\hat{i} + 10\hat{k}) \times 20\hat{j}$

or, $-80x\hat{k} + 80z\hat{i} = -120\hat{k} - 80\hat{k} + 40\hat{i} - 80\hat{k} + 200\hat{i}$

comparing the coeff. of like vector.

$-80x = -120 - 80 - 80$

or, $x = 3.5m$

and

$80z = 40 + 200$

or, $z = 3m$

$\therefore x = 3.5m$

$y = 0$ and

$z = 3m$.

$\hat{i}.\hat{j} = \hat{k}$

$\hat{j}.\hat{k} = \hat{i}$

$\hat{k}.\hat{i} = \hat{j}$

$\hat{j}.\hat{i} = -\hat{k}$

$\hat{k}.\hat{j} = -\hat{i}$

$\hat{i}.\hat{k} = -\hat{j}$

(2010) Fall

15) Add the couples whose force act along the diagonals of the sides of the rectangular parallelopiped.

$\Rightarrow$ Here,

co-ordinate of

$A = (10, 0, 5)$

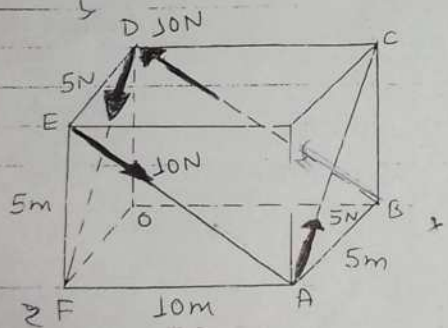
$C = (10, 5, 0)$

$D = (0, 5, 0)$

$E = (0, 5, 5)$

$F = (0, 0, 5)$

$B = (10, 0, 0)$



Force along BD $(\vec{F}_{BD}) = F_{BD} \cdot \widehat{BD}$

or, $\vec{F}_{BD} = 10 \cdot \dfrac{-10\hat{i} + 5\hat{j} + 0\cdot\hat{k}}{\sqrt{(-10)^2 + (5)^2}}$

or, $\vec{F}_{BD} = 10 \cdot \dfrac{-10\hat{i} + 5\hat{j}}{\sqrt{125}}$

also,

$\vec{AB} = \vec{r}_1 = -5\hat{k}$

$\therefore$ couple due to 10N force, $(\vec{C}_1) = \vec{r}_1 \times \vec{F}_{BD}$

or, $\vec{C}_1 = -5\hat{k} \times \left(\dfrac{-100\hat{i}}{\sqrt{125}} + \dfrac{50}{\sqrt{125}}\hat{j}\right)$

or, $\vec{C}_1 = 44.72\hat{j} + 22.36\hat{i}$

Again,

Force along AC $(\vec{F}_{AC}) = F_{AC} \cdot \widehat{AC}$

or, $\vec{F}_{AC} = 5 \cdot \dfrac{5\hat{j} - 5\hat{k}}{\sqrt{5^2 + (-5)^2}}$

or, $\vec{F}_{AC} = \dfrac{25\hat{j}}{\sqrt{50}} - \dfrac{25\hat{k}}{\sqrt{50}}$

also, $\vec{FA} = \vec{r}_2 = 10\hat{i}$

$\therefore$ couple due to 5N force, $(\vec{C}_2) = \vec{r}_2 \times \vec{F}_{AC}$

or, $\vec{C}_2 = 10\hat{i} \times \left(\dfrac{25\hat{j}}{\sqrt{50}} - \dfrac{25\hat{k}}{\sqrt{50}}\right)$

or, $\vec{C}_2 = 35.36\hat{k} + 35.36\hat{j}$

$\therefore$ Resultant couple $(\vec{C}) = \vec{C}_1 + \vec{C}_2$

or, $\vec{C} = 44.72\hat{j} + 22.36\hat{i} + 35.36\hat{k} + 35.36\hat{j}$

or, $\vec{C} = 22.36\hat{i} + 80.08\hat{j} + 35.36\hat{k}$

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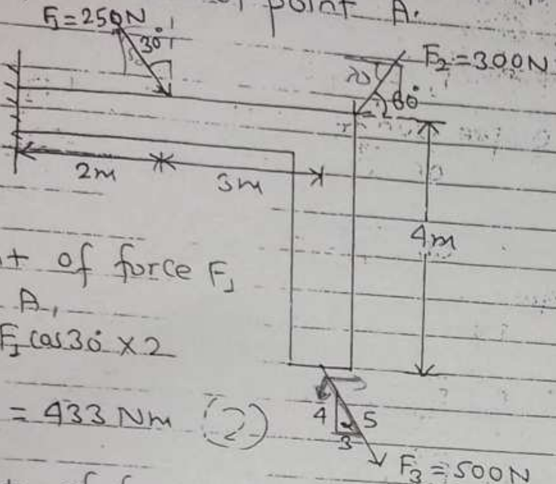
(16) Determine the moment of each of the three force about point A.



=) Here,

Moment of force F_1 about A,

$M_{A_1} = F_1 \cos 30° \times 2$

or, $M_{A_1} = 433$ Nm

Moment of force F_2 about A,

$M_{A_2} = F_2 \sin 60° \times 5 = 1299$ Nm

Moment of force F_3 about A,

$M_{A_3} = -F_3 \sin\alpha \times 5 + F_3 \cos\alpha \times 4$

or, $M_{A_3} = -500 \times \frac{4}{5} \times 5 + 500 \times \frac{3}{5} \times 4$

or, $M_{A_3} = -800$ Nm

(17) The 60N force 'p' is applied at point C of the bent bar. If $\theta = 45°$, determine the moment of 'p' about point A. For what value of angle 'θ' will be the moment about point 'A' be maximum? Determine the max^m value $(M_A)_{max}$.

=) Here, From figure (1)

Moment due to 60N force about A,

$M_A = p\cos 45° \times 1.2\sin 45° + p\sin 45° \times (1.2 + 1.2\cos 45°)$

or, $M_A = 60\cos 45° \times 1.2\sin 45° + 60\sin 45° \times (1.2 + 1.2\cos 45°)$

or, $M_A = 122.91$ Nm

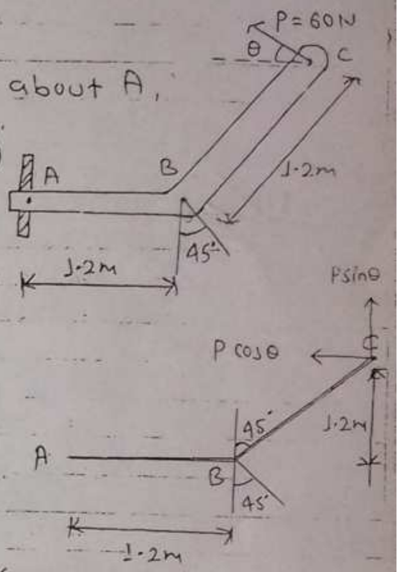


figure (1)

Again,

Moment due to 60N force about point A,

$M_A = 60\cos\theta \times 1.2\sin 45° + 60\sin\theta \times (1.2 + 1.2\cos 45°)$

or, $M_A = 50.91\cos\theta + 122.91\sin\theta$

Diff. both side w.r.to θ.

$$\frac{dM_A}{d\theta} = -50.91\sin\theta + 122.91\cos\theta$$

For max^m, $\frac{dM}{d\theta} = 0$

$-50.91\sin\theta + 122.91\cos\theta = 0$

or, $\tan\theta = 2.414 \Rightarrow \theta = 67.5^\circ$

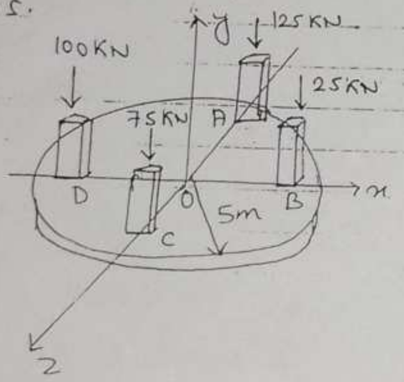
Hence, at $\theta = 67.5^\circ$ the moment about point A will be max^m.

$$\therefore (M_A)_{max} = 50.91 \times \cos 67.5 + 122.91 \sin 67.5^\circ$$

$$\text{or, } (M_A)_{max} = 133.036 \text{ Nm } (\circlearrowleft)$$

(2009)SP

18) A concrete foundation mat of 5m radius supports four equally spaced columns, each of which is loaded 4m from the centre of mat. Determine the magnitude and the point of application of the resultant of the four loads.



$\Rightarrow$ Here, Radius of concrete mat = 5m

co-ordinate of

$O = (0,0,0)$

$A = (0,0,-4)$

$B = (4,0,0)$

$C = (0,0,4)$

$D = (-4,0,0)$

Resultant of load $(\vec{R}) = -100\hat{j} - 75\hat{j} - 125\hat{j} - 25\hat{j}$

or, $\vec{R} = -325\hat{j}$

Suppose,

The resultant passes through the point $P = (x, y, z)$

Then, $\vec{OP} = \vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$.

Taking Moment about origin,

Moment due to resultant = Sum of Moment due to individual force.

$$\text{or, } \vec{r} \times \vec{F_R} = \vec{r_1} \times \vec{F_1} + \vec{r_2} \times \vec{F_2} + \vec{r_3} \times \vec{F_3} + \vec{r_4} \times \vec{F_4}$$

where, $\vec{r_1} = -4\hat{k}$, $\vec{F_1} = -125\hat{j}$

$\vec{r_2} = 4\hat{i}$, $\vec{F_2} = -25\hat{j}$

$\vec{r_3} = 4\hat{k}$, $\vec{F_3} = -75\hat{j}$

and $\vec{r_4} = -4\hat{i}$, $\vec{F_4} = -100\hat{j}$

Hence,

$$(x\hat{i} + y\hat{j} + z\hat{k}) \times (-325\hat{j}) = -4\hat{k} \times -125\hat{j} + 4\hat{i} \times -25\hat{j} + 4\hat{k} \times -75\hat{j} + (-4\hat{i}) \times (-100\hat{j})$$

$$\text{or, } -325x\hat{k} + 325z\hat{i} = -500\hat{i} - 100\hat{k} + 300\hat{i} + 400\hat{k}$$

comparing the coeff. of like vector,

$-325x = -100 + 400 \Rightarrow x = -0.92 \text{ m}$

and,

$325z = -500 + 300 \Rightarrow z = -0.61 \text{ m}$

Hence, the resultant passes through the point $P(-0.92, 0, -0.61)$.

(19) The rectangular platform is hinged at A and B and supported by a cable which passes over frictionless hook at E as shown in the figure. If the weight of platform is 1000 Determine the tension in the cable.

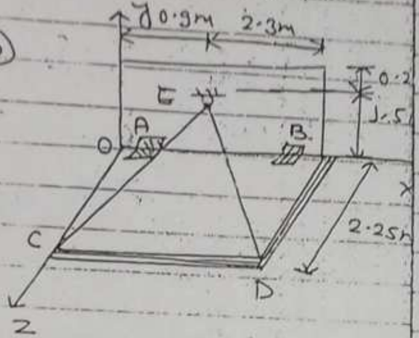
⇒ Here,

co-ordinate of $O = (0,0,0)$

$E = (0.9, 1.5, 0)$

$C = (0, 0, 2.25)$

$D = (3.2, 0, 2.25)$



Now,

Tension along the cable EC ($\vec{T}_{EC}$) $= T_{EC} \cdot \hat{EC}$

or, $\vec{T}_{EC} = T_{EC} \cdot \dfrac{-0.9\hat{i} - 1.5\hat{j} + 2.25\hat{k}}{\sqrt{(-0.9)^2 + (-1.5)^2 + (2.25)^2}}$

or, $\vec{T}_{EC} = T_{EC} \cdot (-0.32\hat{i} - 0.53\hat{j} + 0.79\hat{k})$

Similarly, tension along the cable ED ($\vec{T}_{ED}$)

$= T_{ED} \cdot \hat{ED}$

or, $\vec{T}_{ED} = T_{ED} \cdot \dfrac{2.3\hat{i} - 1.5\hat{j} + 2.25\hat{k}}{\sqrt{(2.3)^2 + (-1.5)^2 + 2.25^2}}$

or, $\vec{T}_{ED} = T_{ED} \cdot (0.65\hat{i} - 0.42\hat{j} + 0.63\hat{k})$

weight $(\vec{W}) = -1000\hat{j}$

We know that, For equilibrium $\Sigma\vec{F} = 0$

ie. $\vec{T}_{EC} + \vec{T}_{ED} + \vec{W} = 0$

or, $T_{EC}(-0.32\hat{i} - 0.53\hat{j} + 0.79\hat{k}) + T_{ED}(0.65\hat{i} - 0.42\hat{j} + 0.63\hat{k})$
$+ (-1000\hat{j}) = 0$

comparing the coeff. of like vectors,

$-0.32T_{EC} + 0.65T_{ED} = 0 \longrightarrow (1)$

$-T_{EC}\,0.53 - 0.42T_{ED} - 1000 = 0 \longrightarrow (2)$

$0.79T_{EC} + 0.63T_{ED} = 0 \longrightarrow (3)$

Since, the hook 'E' is frictionless so, tension T_{EC} must be equal to tension T_{ED}.

ie. $T_{EC} = T_{ED} \longrightarrow (4)$

From eqn (2) and eqn (4)

$-T_{EC}\,0.53 - 0.42T_{EC} - 1000 = 0$

or, $T_{EC} = -1052.63$ N.

$\therefore T_{EC} = T_{ED} = 1052.63$ N. (Neglecting -ve sign)

(2007) Fall

(20) The 0.732m × 1.2m lid ABCD of a storage bin is hinged along side AB and is held open by looping cord DEC over a frictionless hook at E. If the tension in the cord is

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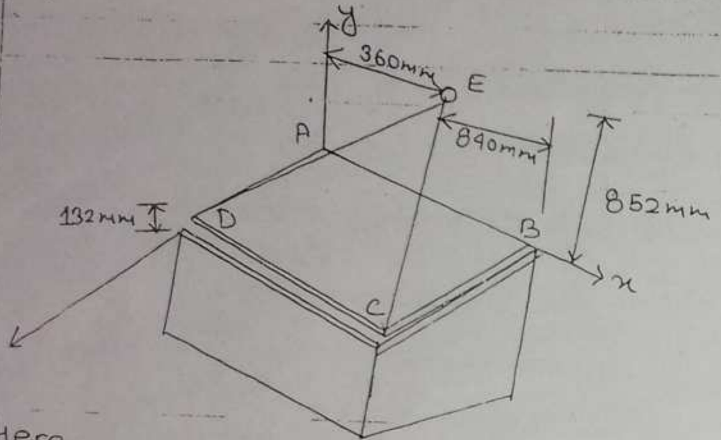
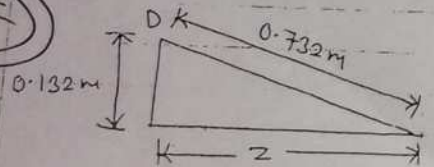
54N, determine the moment about each of the co-ordinate axes of the force exerted by the cord at C.



⇒ Here,

Tension in the cord = 54N.

Size of lid ABCD = 0.732 m × 1.2m



From fig.

$z = \sqrt{0.732^2 - 0.132^2}$

or, $z = 0.72$m.

co-ordinate of C = (1.2, 0.132, 0.72)

" " E = (0.36, 0.852, 0)

" " A = (0, 0, 0)

$\vec{CE} = (0.36-1.2)\hat{i} + (0.852-0.132)\hat{j} + (0-0.72)\hat{k}$

or, $\vec{CE} = -0.84\hat{i} + 0.72\hat{j} - 0.72\hat{k}$

$|\vec{CE}| = \sqrt{(-0.84)^2 + (0.72)^2 + (-0.72)^2} = 1.32$m.

Tension along CE $(\vec{T}_{CE}) = T_{CE} \cdot \hat{CE}$

or, $\vec{T}_{CE} = \frac{(-0.84\hat{i} + 0.72\hat{j} - 0.72\hat{k})}{1.32} \cdot 54$

or, $\vec{T}_{CE} = -36.363\hat{i} + 29.454\hat{j} - 29.454\hat{k}$

Again,

$\vec{r}_{AE} = 0.36\hat{i} + 0.852\hat{j}$

Moment about A $(\vec{M}_A) = \vec{r}_{AE} \times \vec{T}_{CE}$

or, $\vec{M}_A = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0.36 & 0.852 & 0 \\ -36.363 & 29.454 & -29.454 \end{vmatrix}$

or, $\vec{M}_A = (-25.095\hat{i} + 10.603\hat{j} + 39.881\hat{k})$ Nm.

∴ Moment about -x $(M_x) = -25.095$ Nm

Moment about -y $(M_y) = 10.603$ Nm

Moment about -z $(M_z) = 39.881$ Nm

(2010) Fall

(21) Determine the direction θ of the force F = 40N so that it produces (a) the max^m moment about point A. and (b) the minimum moment about point

<2013 fall>

1·32m.

$\hat{k}) \cdot 54$

$29{\cdot}454\hat{k}$

Nm.

o95 Nm

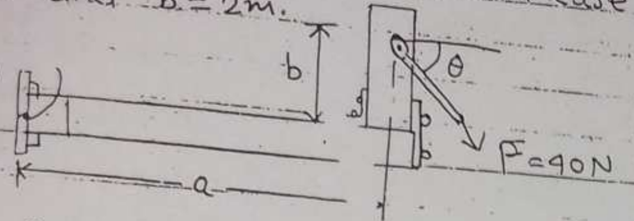
03 Nm

Nm

F = 40N about

bout Point

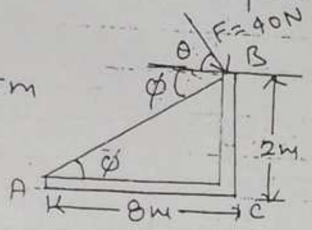
A. Compute the moment in each case. Take $a = 8m$ and $b = 2m$.



⇒ Here, force $(F) = 40N$.

Case I For Max^m moment about point A.

Then from fig.



$AB = \sqrt{8^2 + 2^2} = 8.25m$

$(M_A)_{max} = 40 \times 8.25$

or, $(M_A)_{max} = 330\ Nm$.

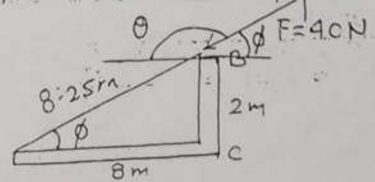
In rt. △ABC, $\tan\phi = \frac{BC}{AC} = \frac{2}{8}$

or, $\phi = 14.04°$

$\therefore \theta = 90° - 14.04° = 75.96°$

Hence, for max^m moment at point A direction $\theta = 75.96°$

Case II For Minimum moment about point A:



$(\Sigma M_A)_{min} = 0$

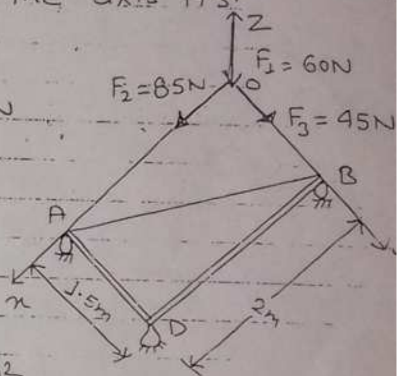
Since F = 40N, passes through point A. In rt. △ABC

$\phi = \tan^{-1}(2/8) = 14.04°$

$\therefore \theta = 180 - 14.04° = 165.96°$

Hence, for minimum moment at point A, direction $\theta = 165.96°$.

(22) Determine the magnitude of moment of each of the three forces about the axis AB.



⇒ Here,

Force, $F_1 = 60N$, $F_2 = 85N$, $F_3 = 45N$

Co-ordinate of $O = (0,0,0)$

$A = (2,0,0)$

$B = (0,1.5,0)$

$D = (2,1.5,0)$

$\vec{AB} = -2\hat{i} + 1.5\hat{j}$

$\hat{AB} = \frac{\vec{AB}}{|\vec{AB}|} = \frac{-2\hat{i}+1.5\hat{j}}{\sqrt{(-2)^2+(1.5)^2}}$

or, $\hat{AB} = \frac{-2\hat{i}+1.5\hat{j}}{2.5} = -0.8\hat{i} + 0.6\hat{j}$

$\vec{F_1} = -60\hat{k}$, $\vec{F_2} = 85\hat{i}$ and $\vec{F_3} = 45\hat{j}$

Moment of force $F_1 = 60N$ about AB,

$(M_{AB})_1 = (\vec{r_1} \times \vec{F_1}) \cdot \hat{AB}$. where $\vec{r_1} = \vec{OA} = 2\hat{i}$

or, $(M_{AB})_1 = (2\hat{i} \times -60\hat{k}) \cdot (-0.8\hat{i} + 0.6\hat{j})$

or, $(M_{AB})_1 = 120\hat{j} \cdot (-0.8\hat{i} + 0.6\hat{j})$

or, $(M_{AB})_1 = 72$ Nm

Again, Moment of force $F_2 = 85$N about AB

$(M_{AB})_2 = (\vec{r_2} \times \vec{F_2}) \cdot \hat{AB}$ where, $\vec{r_2} = 0$

$\therefore (M_{AB})_2 = 0$

and

Moment of force $F_3 = 45$N about AB

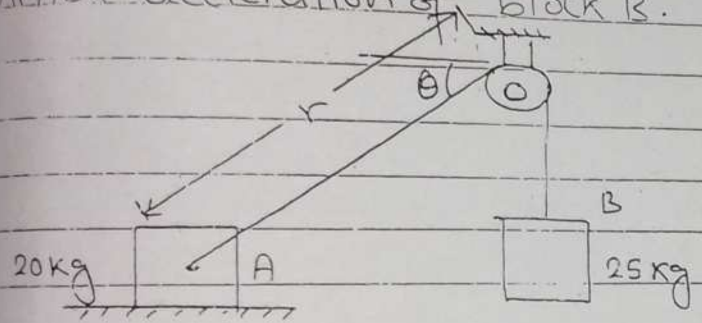
$(M_{AB})_3 = (\vec{r_3} \times \vec{F_3}) \cdot \hat{AB}$, where $\vec{r_3} = 0$

$\therefore (M_{AB})_3 = 0$ [$\vec{r}$ and $\vec{r_3}$ are zero because their force passes through point A and B.]

23

4

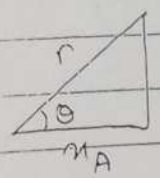
The two blocks are release from rest when $r = 0.9\,m$ and $\theta = 35°$. Neglecting the mass of the pully and the effect of friction in the pully and between A and the horizontal surface, Determine the initial tension in the cable, the initial accleration of block A and the initial accleration of block B.



Suppose 'T' be the tension in the cable. x_A and x_B be the distance moved by the block A and B resp.

From figure,

$$\cos\theta = \frac{x_A}{r}$$



or, $r\cos\theta = x_A$

But $x_A = x_B$

$\therefore\ r\cos\theta = x_B$

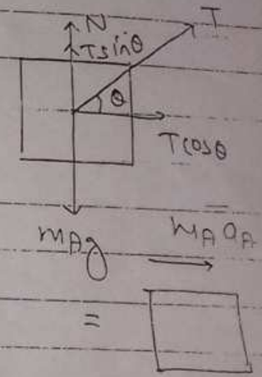
Diff w.r.to t

$V_A \cos\theta = V_B$

Diff w.r.to t

$a_A \cos\theta = a_B \longrightarrow ①$

For block A

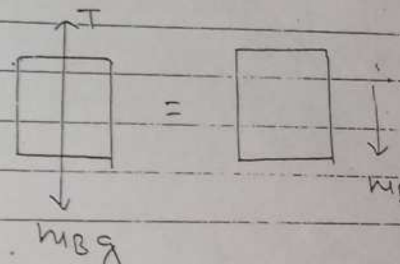


$\xrightarrow{+} \Sigma F_x = m_A a_A$

or, $T\cos\theta = m_A a_A$

or, $T = \dfrac{20\,a_A}{\cos 35°} \longrightarrow ②$

For block B



$+\downarrow \Sigma F_y = m_B a_B$

or, $m_B g - T = m_B a_B$

or, $T = 25 \times 9.81 - 25\,a_B \longrightarrow ③$

Equating eqn ② and ③

$$\frac{20\,a_A}{\cos 35°} = 25 \times 9.81 - 25\,a_B$$

But from eqn ①, $a_B = a_A \cos\theta$

∴ $\frac{20a_A}{\cos 35} = 25\times 9.81 - 25\, a_A \cos 35$

or, $a_A = 5.46\, m/s^2$

∴ From eqn ①

$$a_B = 5.46 \cos 35 = 4.47\, m/s^2$$

and putting the value of a_A in eqn ②

$$T = \frac{20\times 5.46}{\cos 35} = 133.31\, N.$$

Chapter :→ 11 (2hr)

Kinetic Energy of Rigid Particle:

→ The kinetic energy of a rigid particle or a body is a scalar quantity. The K.E of rigid body is a arthematical sum of the K.E of all the particles of the body. In motion of translation, the K.E of a body is given by

$$K.E = \Sigma\left(\tfrac{1}{2} m v^2\right) \longrightarrow ①$$

Where, m = mass of a particle

V = velocity of a particle

When a body rotates about an axis with an angular velocity ω, then the linear velocity can be expressed as

$$V = r\omega$$

Where, r = radius

ω = angular velocity.

∴ eqn ① becomes,

$$K.E = \Sigma\left(\frac{1}{2} m r^2 \omega^2\right)$$

or, $K.E = \Sigma \frac{1}{2} I \omega^2$

Where, $I = mr^2$, moment of Inertia.

S.N.	Linear Motion	Rotary Motion
1.	Mass (m)	Moment of inertia (I)
2.	Force (F)	Torque (T)
3.	Linear velocity (V)	Angular velocity (ω)
4.	Linear acc^n (a)	Angular acc^n (α)
5.	Linear momentum (mv)	Angular Momentum ($I\omega$)
6.	Force eq^n (F) = ma	Torque eq^n (T) = $I\alpha$
7.	Work done = F·S	Work done = $T\theta$
8.	Linear K.E = $\frac{1}{2}mv^2$	Rotational K.E = $\frac{1}{2}I\omega^2$
9.	Distance travell = S (Linear Displacement)	Angle travelled = θ (Angular displacement)

Equations of Circular Motion.

$$\omega = \omega_0 + \alpha t$$
$$\omega^2 = \omega_0^2 + 2\alpha\theta$$
$$\theta = \omega_0 t + \frac{1}{2}\alpha t^2$$
$$v = \omega r$$
$$a = r\alpha$$
$$S = r\theta$$

tangential acc^n (a_t) = α

Normal acc^n (a_n): $\frac{v^2}{r} = \omega^2 r$

$$\therefore a = \sqrt{a_t^2 + a_n^2}$$

h) $\omega = \frac{2\pi}{T} = 2\pi f$ where, f = frequency, T = Time period

i) No. of revolution (N) = $\frac{\theta}{2\pi}$

j) Total energy = $\frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$

Relation between v and ω:

$$v = r\omega$$
$$a = \sqrt{a_n^2 + a_t^2}$$
$$a_t = \frac{dv}{dt} = \frac{d}{dt}(r\omega) = r\frac{d\omega}{dt} = r\alpha$$
$$a_n = \frac{V^2}{r} = \frac{(r\omega)^2}{r} = r\omega^2$$
$$\tan\theta = \frac{a_n}{a_t} = \frac{r\omega^2}{r\alpha}$$
$$\therefore \theta = \tan^{-1}\left(\frac{\omega^2}{\alpha}\right)$$

Radial component of force (F_r) = ma_r

or, $F_r = m(\ddot{r} - r\dot{\theta}^2)$

Transverse component of force (F_θ) = ma_θ

or, $F_\theta = m(r\ddot{\theta} + 2\dot{r}\dot{\theta})$

(a) ## Numericals

① A body rotates according to the relation $\alpha = 3t^2 + 4$, angular displacement being measured in radian and time in seconds. If its initial angular velocity is 4 rad/sec and the initial angular displacement is zero, compute the value of θ and ω, for the instant when $t = 3sec$.

→ Given,

$$\alpha = 3t^2 + 4 \longrightarrow ①$$

Initial angular velocity $(\omega_0) = 4$ rad/sec.

Initial " displacement $(\theta_0) = 0$

Angular displacement $(\theta) = ?$ } At
Angular velocity $(\omega) = ?$ } $t = 3sec$

We know that

$$\alpha = \frac{d\omega}{dt}$$

or, $d\omega = \alpha \cdot dt$

or, $d\omega = (3t^2+4)\,dt$

Integrating bothside

$$\int d\omega = \int (3t^2+4)\,dt$$

or, $\omega = \frac{3t^3}{3} + 4t + C_1 \longrightarrow ①$

Initially, $t=0$, $\omega = 4$ rad/sec

$\therefore 4 = C_1$

From eqn ①

$$\omega = t^3 + 4t + 4 \longrightarrow ②$$

At $t = 3sec$,

$$\omega = 3^3 + 4\times3 + 4 = 43 \text{ rad/sec.}$$

Again,

$$\omega = \frac{d\theta}{dt}$$

or, $\frac{d\theta}{dt} = t^3 + 4t + 4$

or, $\int d\theta = \int (t^3 + 4t + 4)\,dt$

or, $\theta = \frac{t^4}{4} + \frac{4t^2}{2} + 4t + C_2 \longrightarrow ③$

Initially, $t = 0$, $\theta = 0$

$\therefore C_2 = 0$

So, eqn ③ becomes,

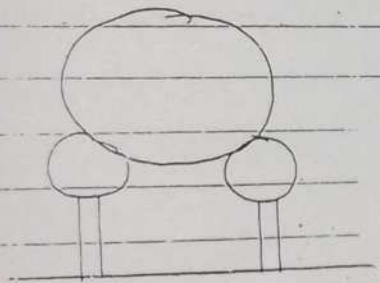
$$\theta = \frac{t^4}{4} + 2t^2 + 4t$$

At $t = 3sec$, $\theta = \frac{3^4}{4} + 2\times3^2 + 4\times3 = 50.25$

A mixing drum of 150mm outside radius rests on two casters each of 30mm radius. The drum executed 20 revolution during the time interval 't' while it's angular velocity is being increased uniformly from 25 to 55 rpm. Knowing that no slipping occurs between the drum and casters. Determine.

Angular accl^n of the casters

Time interval (t).



Given:

Radius of drum (r_D) = mm = 0.15m

Radius of caster (r_c) = mm = 0.03m

Revolution of drum (N) =

Then, $N = \frac{\theta}{2\pi}$

or, $\theta = 2\pi N = 2\pi \times 20$

or, $\theta = 125.66$ radian.

Initial angular velocity (ω_0) = 25 rpm

or, $\omega_0 = \frac{2\pi}{60} \times 25$

or, $\omega_0 = 2.62$ rad/sec

Final angular velocity (ω) = 55 rpm

or, $\omega = \frac{2\pi}{60} \times 55$

or, $\omega = 5.76$ rad/sec

let α_d be the angular accl^n of drum

Then,

$$\omega^2 = \omega_0^2 + 2\alpha_d \theta$$

or, $5.76^2 = 2.62^2 + 2 \times \alpha_d \times 125.66$

or, $\alpha_d = 0.105$ rad/sec^2

At contact point of Drum and caster

$$(a_t)_d = (a_t)_c$$

or, $(r\alpha)_d = (r\alpha)_c$

or, $0.15 \times 0.105 = 0.03\,\alpha_c$

or, $\alpha_c = 0.525$ rad/sec^2

<क्रमशः>

$\therefore$ Angular accl^n of casters $(\alpha_c) = 0.525 \, rad/s$

Again,

$$\omega = \omega_0 + \alpha_d t$$

or, $5.76 = 2.62 + 0.105 \, t$

or, $t = 29.9 \, sec$

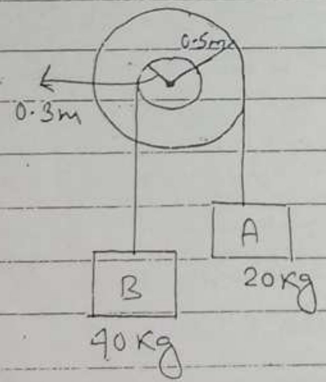
$\therefore$ time interval $(t) = 29.9 \, sec.$

Fall

3) A pully weighing 50kg having a radius of gyration of 0.4m is connected to two blocks as shown. Assuming no axle friction, determine the angular accleration of the pully and accleration of each block.

$\Rightarrow$ Given:

Mass of pully $(m_p) = 50kg$



Radius of gyration $(K) = 0.4m$

Moment of inertia of pully $(I) = m_p K^2$

or, $I = 50 \times 0.4^2$

Moment due to block B $= W_B \times 0.3$

$= m_B g \times 0.3$

$= 40 \times 9.81 \times 0.3$

$= 117.72 \, Nm$

Moment due to block A $= W_A \times 0.5$

$= m_A g \times 0.5$

$= 20 \times 9.81 \times 0.5$

$= 98.10 \, Nm$

Here, Moment due to block B is greater than moment due to block A, so block B moves downward and block A moves upward.

Resultant Moment = Moment due to $m_A a_A$ + Moment due to $m_B a_B$ + $I\alpha$

or, $(117.72 - 98.10) = m_A a_A \times 0.5 + m_B a_B \times 0.3 + 8\alpha$

or, $19.62 = 20 \times 0.5 \times r_A \alpha + 40 \times 0.3 \times r_B \alpha + 8\alpha$

or, $19.62 = \alpha [20 \times 0.5 \times 0.5 + 40 \times 0.3 \times 0.3 + 8]$

or, $\alpha = 1.18 \, rad/sec^2$

Angular accl^n of pully $\alpha) = 1.18$ rad/s^2

Again,

accl^n of block A $(a_A) = r_A \alpha$

or, $a_A = 0.5 \times 1.18$

or, $a_A = 0.59$ ~~rad~~ m/sec2

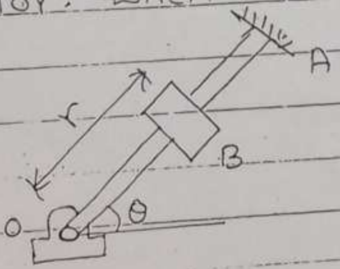
and

accl^n of block B $(a_B) = r_B \alpha$

or, $a_B = 0.3 \times 1.18$

or, $a_B = 0.35$ m/sec^2

Rod OA rotates about 'O' in a horizontal plane. The motion of the 400 gm collar B is defined by the relation $r = 500 + 300\sin\pi t$ and $\theta = 2\pi(t^2 - 2t)$, where 'r' is in millimeter, t in second and 'θ' in radians. Determine the radial and transverse component of force exerted on the collor. When $t=0$ and $t = 0.8$ sec.



Given:

Mass of collar B

$(m) = 0.4$ kg

$r = 500 + 300\sin\pi t$

Then,

$\dot{r} = 300\pi \cos\pi t$

$\ddot{r} = -300\pi^2 \sin\pi t$

At $t=0$	At $t = 0.8$ sec
$r = 0.5$	$r = 0.676$ m
$\dot{r} = 0.$ m/s	$\dot{r} = -0.762$ m/s
$\ddot{r} = 0$	$\ddot{r} = -1.740$ m/s^2

Also, $\theta = 2\pi(t^2 - 2t)$

$\dot{\theta} = 2\pi(2t-2)$

$\ddot{\theta} = 4\pi$

At $t=0$	At $t = 0.8$ sec
$\dot{\theta} = -4\pi$	$\dot{\theta} = -0.8\pi$
$\ddot{\theta} = 4\pi$	$\ddot{\theta} = 4\pi$

We have,

Radial component of force and transverse component of force are given by

$$F_r = m(\ddot{r} - r\dot{\theta}^2)$$

$$F_\theta = m(r\ddot{\theta} + 2\dot{r}\dot{\theta})$$

Case I, At $t=0$

$$F_r = 0.4(0 - 0.5 \times 16\pi^2) = -31.85 \text{ N}$$

$F_\theta = 0.4(0.5 \times 4\pi - 2 \times 0.3\pi \times 4\pi)$

or, $F_\theta = -6.96$ N

case II At $t = 0.8$ sec

$F_r = 0.4(-1.740 - 0.676 \times (-0.8\pi)^2)$

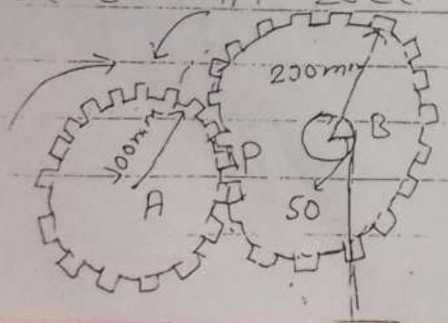
or, $F_r = -2.C$ N

and

$F_\theta = 0.4(0.676 \times 4\pi + 2 \times 0.762 \times 0.8\pi)$

$F_\theta = 4.93$ N

⑤ If a hoisting gear A has an initial angular velocity 8 rad/sec clockwise and an angular accl^n 1.5 rad/sec^2 anticlockwise. Determine the velocity and accleration of block C in 2 sec.



⇒ Here,

Angular velocity of gear A $(\omega_0)_A = 8$ rad/s (↻)

Angular accl^n of gear A $(\alpha_A) = 1.5$ rad/sec^2 (↺)

Velocity of block C $(v_C) = ?$

Accl^n of block C $(a_C) = ?$

time $(t) = 2$ sec.

At point of contact of gear (ie. at P)

$(a_t)_A = (a_t)_B$

or, $(\alpha r)_A = (\alpha r)_B$

or, $1.5 \times 0.1 = \alpha_B \times 0.2$

or, $\alpha_B = 0.75$ rad/sec^2 (↺)

Again,

At point of contact P,

$(v_0)_A = (v_0)_B$

or, $(\omega_0)_A r_A = (\omega_0)_B r_B$ [$\because v = \omega r$]

or, $(\omega_0)_B = \frac{8 \times 0.1}{0.2} = 4$ rad/sec (↺)

Since B and C are attached together

$\alpha_C = \alpha_B = 0.75$ rad/sec^2 (↺)

$\therefore a_C = \alpha_C r_C$ [$\because a = r\alpha$]

or, $a_C = 0.75 \times 0.05 \times 1000 = $ [illegible] $5\ mm/sec^2\ (\downarrow)$

Also, $(\omega_0)_B = (\omega_0)_C$

$\therefore (\omega_0)_C = 4$ rad/sec.

After 2sec,

$\omega_C = (\omega_0)_C + \alpha_B t$

or, $\omega_C = 4 + (-0.75) \times 2$

or, $\omega_C = 2.5$ rad/sec.

$\therefore$ Velocity of block C, $(V_C) = \omega_C r_C$

or, $V_C = 2.5 \times$ [illegible] $\times 1000$

or, $V = 125$ mm/[illegible]c.

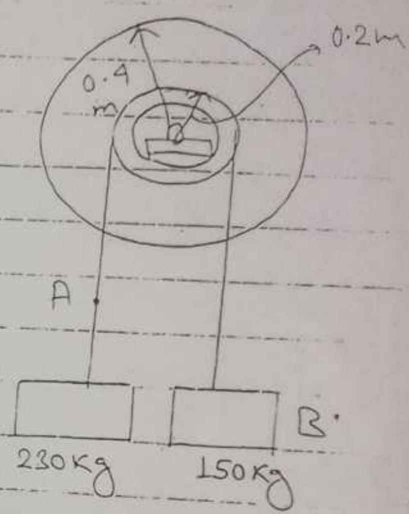
A pully as shown has a [illegible]ss moment of Inertia of 20kgm[illegible] [illegible]d is initially at rest. [illegible] outside radius is 0.4m and the inner radius is 0.2m, Determ[illegible] the angular accl^n of [illegible]lly [illegible] angular velocity of [illegible] after point A on the [illegible] has moved 3m.

⇒ Given:

Moment of inertia of pully $(I) = 20\ kgm^2$.

Outside radius $= 0.4m$

Inner radius $= 0.2m$



Now,

Moment due to block A $= W_A \times 0.2$

$= m_A g \times 0.2$

$= 230 \times 9.81 \times 0.2$

$= 451.26$ Nm

and

Moment due to block B $= W_B \times 0.2$

$= m_B g \times 0.2$

$= 150 \times 9.81 \times 0.2$

$= 294.3$ Nm

Here, Moment due to block A is greater than moment due to block B so block A moves downward and block B moves upward.

Then,

$F_R =$ Moment due to $m_A a_A$ + Moment due to $m_B a_B + I\alpha$

or, $(951.26 - 294.3) = w_A a_A \times 0.2 + w_B a_B \times 0.2 + I\alpha$

or, $156.96 = w_A r_A \alpha \times 0.2 + w_B r_B \alpha \times 0.2 + I\alpha$

or, $156.96 = (230 \times 0.2^2 + 150 \times 0.2^2 + 20)\alpha$

or, $156.96 = 35.2\alpha$

$\therefore \alpha = 4.45$ rad/sec

Angular accⁿ of pully $(\alpha) = 4.45$ rad/sec.

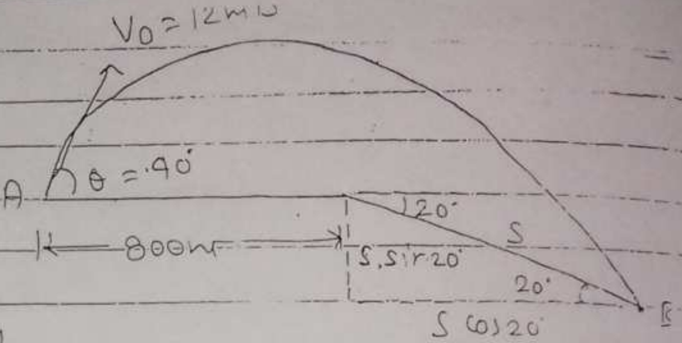
Again,

$$\omega_A^2 = (\omega_0)_A^2 + 2\alpha\theta$$

or, $\omega_A^2 = 0 + 2 \times 4.45 \times \frac{3}{0.2}$ $\left[\theta = \frac{l}{r}\right]$

or, $\omega_A = 11.55$ rad/sec.

(*) A projectile is launched from point A with the initial velocity $V_0 = 120$ m/s as shown in figure. Determine the slant 's' which locates the point B of impact. Calculate the time of flight.



Given

Initial velocity $(v_0) = 120$ m/s

It can be resolved into horizontal and vertical direction

$(v_0)_x = v_0 \cos 40° = 120 \cos 40$

$(v_0)_y = v_0 \sin 40° = 120 \sin 40°$

Horizontal distance travelled by projectile $R = 800 + S\cos 20$ and vertical distance travelled $(h) = -S\sin 20$

We know,

$$R = (v_0)_x \times t$$

or, $800 + S\cos 20 = 120\cos 40 \times t$

or, $t = \frac{800 + S\cos 20}{120\cos 40} = 8.7 + 0.01S$ → (1)

Again, In vertical motion,

$$h = (v_0)_y t - \frac{1}{2} g t^2$$

or, $-S\sin 20 = 120\sin 40° (8.7 + 0.01S) -$

$0.342S = 671.07 + 0.771S - 4.905(75.69 + 0.174S + 0.0001S^2)$

$0.342S = 671.07 + 0.771S - 371.25 - 0.853S - 0.00049S^2$

$0.00049S^2 - 0.26S - 99.82 = 0$

$$S = \frac{0.26 \pm \sqrt{(-0.26)^2 - 4 \times 0.00049 \times (-299.82)}}{2 \times 0.00049}$$

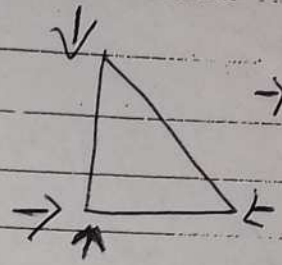
$$S = \frac{0.26 \pm 0.8099}{0.00098}$$

$S = 1091.22\,m.$

From eqⁿ ①

time taken $(t) = 8.7 + 0.01 \times 1091.22$

or, $t = 19.61$ sec.



	$\bar{X}$	$\bar{y}$
	$\to \frac{b}{3}$	$\uparrow \frac{h}{3}$
	$\frac{2b}{3}$	$\downarrow \frac{2h}{3}$