Mechanical Oscillation-

* Harmonic motion -

The motion which repeats after a regular interval of time is called periodic motion. In such a motion there is always an equilibrium position or mean position at which body will come to at rest. If the body is displaced from equilibrium position there exists a certain kind of force which tries to keep the body back to its mean position. such a force is called restoring force. The restoring force is the function of displacement. The displacement of the body can also be expressed interms of trigonometric function. Therefore the periodic motion is also called harmonic motion.

* Free oscillation -

Free oscillations are the oscillations that appear in a system as a result of single initial deviation of the system from its state of mean or equilibrium position. If there is no resistance in its motion then object oscillates freely at its own is called free oscillation and the frequency of oscillation is called natural frequency.

x Damped oscillation.

If an object is set into oscillation and observed for a certain time the amplitude of oscillation goes on decreasing and finally dies off. such oscillation is called damped oscillation. This is due to the frictional force i,e resistance which opposes the motion of the body. Therefore, energy given to the body is converted slowly and slowly into heat for doing work against friction. This is called dissipation of energy. Therefore, resultant force is the restoring force plus frictional force. The frictional force is the function of velocity.

* Forced oscillation-

It is known that energy of the damped oscillation decrease with time, but it is possible to compensate for the energy if mechanical force is applied to the system. The oscillation produced when an external oscillating force is applied to a body subject to an elastic force is called forced oscillation. In this case the body oscillates with frequency other than natural frequency. The force applied externally is of periodic type Therefore, forced oscillation is sum of restoring force, frictional force and periodic external force.

* simple harmonic motion -

simple harmonic motion is an especial type of periodic motion in which body oscillates in a straight line in such a way that restoring force is directly proportional to the displacement from mean position and alway acting towards the mean position.

If F be the restoring force,
x be the displacement then

$$F \propto x \quad \text{----(1)}$$

$\Rightarrow F = -kx \quad \ldots (2)$

Also, From Newton's law of motion.

$$F = \text{mass} \times \text{accl}^n \quad \ldots (3) \qquad [F = ma]$$

From (2) and (3)

$$ma = -kx$$

$$\Rightarrow a = -\frac{k}{m} x$$

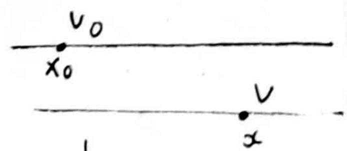
$$\Rightarrow a + \frac{k}{m} x$$

$$\Rightarrow \frac{d^2x}{dt^2} + \omega^2 x = 0 \quad \ldots (4) \qquad \left[\because \frac{k}{m} = \omega^2 \text{ is constant}; \frac{d^2x}{dt^2} = \text{accl}\right]$$

we can also observed that the accl^n is directly proportional to displacement and directed towards mean position.

Eq^n (4) is the standard differential form of S.H.M.

* Equation of simple harmonic motion -



consider a particle of mass 'm' moving towards positive 'x' direction. suppose at any instant of time ($t=0$) the position of the particle is x_0 and velocity 'v_0'. Let 'v' and 'x' be the velocity and position of particle at time after time 't'.

If F be the restoring force, then

$$F = -kx \quad \ldots (1)$$

If 'a' be the accl^n at time 't' then,

$$a = \frac{F}{m} \quad \ldots (2)$$

From (1) and (2) $\quad a = -\frac{k}{m} x$

$$\Rightarrow a = -\omega^2 x$$

$$\Rightarrow \frac{dv}{dt} = -\omega^2 x$$

$$\Rightarrow \frac{dv}{dx}\frac{dx}{dt} = -\omega^2 x \qquad \left[\because \frac{dx}{dt} = v\right]$$

$$\Rightarrow v\,dv = -\omega^2 x \, dx \quad \ldots (3)$$

As velocity of particle is v_0 at x_0 and becomes v at x, then,

$$\int_{v_0}^{v} v\,dv = \int_{x_0}^{x} -\omega^2 x \, dx$$

$$\Rightarrow \left(\frac{v^2}{2}\right)_{v_0}^{v} = -\omega^2 \left(\frac{x^2}{2}\right)_{x_0}^{x}$$

$$\Rightarrow v^2 - v_0^2 = -\omega^2 x^2 + \omega^2 x_0^2$$

$$\Rightarrow \quad v^2 = v_0^2 + \omega^2 x_0^2 - \omega^2 x^2$$

$$\Rightarrow \quad v^2 = \omega^2 \left(\frac{v_0^2}{\omega^2} + x_0^2 - x^2\right)$$

$$\Rightarrow \quad v = \omega\sqrt{\left(\frac{v_0^2}{\omega^2} + x_0^2 - x^2\right)}$$

$$\Rightarrow \quad v = \omega\sqrt{A^2 - x^2} \quad \text{....(4)} \qquad \left[\because \frac{v_0^2}{\omega^2} + x_0^2 \text{ is constant replaced by } A^2\right]$$

Again, from (4)

As $dx/dt = v$ ...(5)

$\therefore$ From (4) and (5)

$$\frac{dx}{dt} = \omega\sqrt{A^2 - x^2}$$

$$\Rightarrow \frac{dx}{\sqrt{A^2 - x^2}} = \omega\, dt$$

Integrating we get,
$$\int_{x_0}^{x} \frac{dx}{\sqrt{A^2 - x^2}} = \int_0^t \omega\, dt$$

$$\Rightarrow \left(\sin^{-1}\frac{x}{A}\right)_{x_0}^{x} = \omega t$$

$$\Rightarrow \sin^{-1}\frac{x}{A} - \sin^{-1}\frac{x_0}{A} = \omega t$$

$$\Rightarrow \sin^{-1}\frac{x}{A} = \sin^{-1}\frac{x_0}{A} + \omega t$$

$$\Rightarrow \sin^{-1}\frac{x}{A} = \omega t + \delta \qquad \left[\sin^{-1}\frac{x_0}{A} = \delta, \text{ phase constant}\right]$$

$$\Rightarrow \frac{x}{A} = \sin(\omega t + \delta)$$

$$\Rightarrow x = A\sin(\omega t + \delta) \quad \text{....(6)}$$

Also, $\frac{dx}{dt} = v$, $\Rightarrow$ $v = A\omega\cos(\omega t + \delta)$ ....(7)

From (6) and (7) we conclude that displacement and velocity of particle which executes periodic motion can be expressed in terms of harmonic or trigonometric function. Therefore, periodic motion is also called harmonic motion.

x Some terms associated with simple harmonic motion -

(1) Amplitude:

The maximum displacement of the particle in simple harmonic motion, from mean position is called amplitude of simple harmonic motion.

As we have,

displacement, $x = A\sin(\omega t + \delta)$

and $\sin(\omega t + \delta)$ can take values betn -1 to $+1$ then maximum displacement of particle will be $x = \pm A$

Therefore, A is amplitude of oscillation.

(2) Time period -

The time taken to complete one oscillation is called time period and it is denoted by T.

we have displacement of particle executing simple harmonic motion.

$x = A\sin(\omega t + \delta)$

If T be the time period, the displacements have same value at 't' and 't+T'

$\therefore \quad A\sin(\omega t+\delta) = A\sin[\omega(t+T)+\delta]$

since $(\omega t+\delta)$ repeats its value if the angle $(\omega t+\delta)$ is increased by 2π or its multiple

$\therefore \omega(t+T)+\delta = (\omega t+\delta)+2\pi$

$\Rightarrow \omega t + \omega T + \delta = \omega t + \delta + 2\pi$

$\Rightarrow \omega T = 2\pi$

$\Rightarrow T = \dfrac{2\pi}{\omega} \quad \cdots (*)$

$\Rightarrow T = 2\pi\sqrt{\dfrac{m}{k}} \quad \cdots (*)(*) \quad [\because \omega^2 = k/m]$

(3) Frequency -

Frequency is the number of oscillations per sec. It is the reciprocal of time period and ~~depa~~ denoted by f and given as

$f = \dfrac{1}{T}$

$\Rightarrow f = \dfrac{1}{2\pi}\sqrt{\dfrac{k}{m}}$

(4) Angular frequency (Angular velocity)

Angular velocity is and angular frequency. It is denoted by ω and given by $\omega = \dfrac{2\pi}{T}$ [From (*)]

$\Rightarrow \omega = 2\pi f$

(5) phase and phase constant -

phase is the status of the particle in simple harmonic motion. It is denoted by ϕ.

As the displacement and velocity of particle in S.H.M. is given as

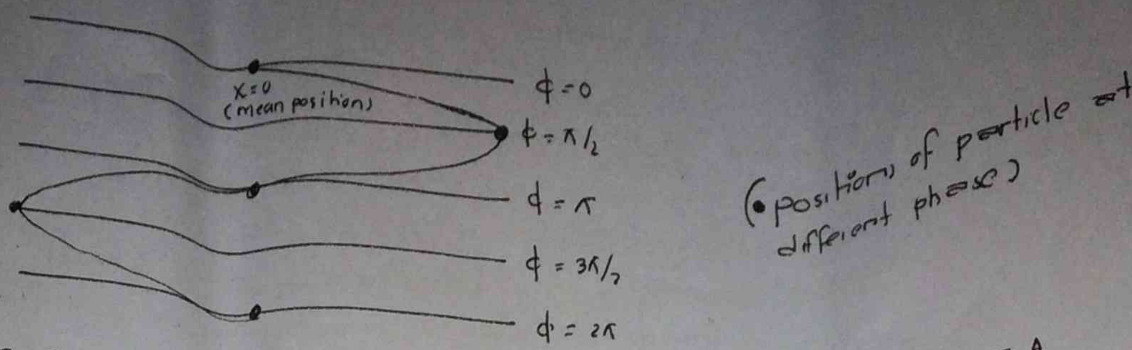
$x = A\sin(\omega t+\delta)$ and $v = A\omega\cos(\omega t+\delta)$

then $\phi = \omega t + \delta$

As the time increases, phase increases and δ is called phase constant. The phase constant depends upon the choice of instant time, t=0. If we choose the instant time at mean (original) position, then

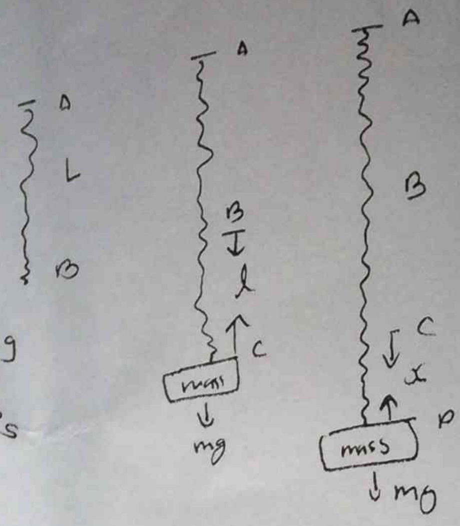
$\phi = \omega t+\delta$ is zero that means,

$\delta = 0$



* spring mass system-

consider a spring of length $AB = l$, of negligible weight suspended from rigid support. Now, Let a mass 'm' is attached to the free end and then the spring elongates. Let l be the elongation produced on the spring because of mass 'm'.



According to Hook's law, extension produced is directly proportional to the force applied.
If F_1 be the force applied for extension then,

$$F_1 = -Cl \quad \text{------} (1)$$

where c is force constant.

Now, pull the mass through a distance 'x' then the mass start to oscillate i,e periodic motion.
If F_2 be the force on the spring then

$$F_2 = -C(l+x) \quad \text{---} (2)$$

The resultant force on spring due to which mass sets into oscil is $F = F_2 - F_1$

$$\Rightarrow F = -C(l+x) + Cl$$

$$\Rightarrow F = -Cx \quad \text{---} (3)$$

Also, According to Newton's Law,

$$F = \text{mass} \times \text{accl}^n$$

$$F = m\frac{d^2x}{dt^2} \quad \text{---} (4)$$

From (3) and (4) $m\dfrac{d^2x}{dt^2} = -Cx$

$$\Rightarrow \frac{d^2x}{dt^2} + \frac{c}{m}x = 0$$

$$\Rightarrow \frac{d^2x}{dt^2} + \omega^2 x = 0 \quad \text{---} (5)$$

∴ This is analogous to simple harmonic motion.
I,e the spring mass system executes simple harmonic motion.
∴ The time period of spring mass system is

$$T = 2\pi/\omega$$

$\Rightarrow T = 2\pi\sqrt{m/c}$

or, frequency, $f = \frac{1}{2\pi}\sqrt{\frac{c}{m}}$

* Angular harmonic motion.

The motion in which the angular acc^n is directly proportional to the angular displacement is called angular harmonic motion. If θ be the angular displacement and α be the angular acceleration then,

$$\alpha \propto \theta$$
$$\Rightarrow \alpha = -\omega^2 \theta$$
$$\Rightarrow \frac{d^2\theta}{dt^2} + \omega^2\theta = 0$$

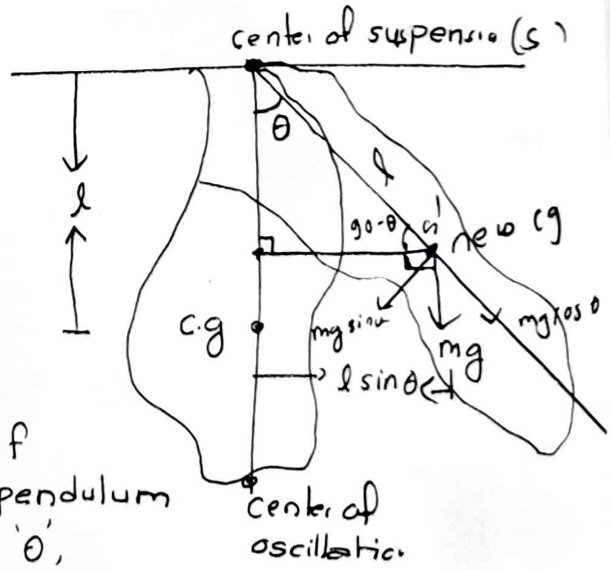
This is the equation of angular harmonic motion.

* Limitations of simple pendulum.

1. The heavy point mass bob is impossible.
2. The weightless, inextensible string is impossible.
3. The string has finite mass and hence it has finite moment of inertia but this inertia is not considered for time period of pendulum.
4. In simple pendulum, the center of oscillation and center of gravity lies at same point but in actual the center of oscillation always lies beyond the center of gravity.

Due to these limitations compound pendulum is prefered than simple pendulum.

* Compound pendulum
(Physical pendulum)



compound pendulum is the rigid body of any shaped capable of oscillating in a horizontal axis in vertical plane not passing through the center of gravity.

Let mg be the weight acting downward through the center of gravity. The distance of center of gravity from axis of rotation (center of suspension) is l. The pendulum is displaced through ~~cert~~ certain angle 'θ', then the pendulum starts oscillating. If a' be the position of new center of gravity then center of gravity a' and center of suspension 's' constitute a couple i.e weight mg acting vertically downward at a' and its reaction at 's' constitute torque which tend the pendulum back into its original position.

$\therefore$ Torque = force $\times$ perpendicular destance from axis of rotation to a'

$\Rightarrow \tau = mg \times l \sin\theta$

$\Rightarrow \tau = mgl\theta$ ---(1) [For small displacement, $\sin\theta \approx \theta$]

This torque provides the restoring force i,e

restoring torque, $\tau = -mgl\theta$ --(2)

If I be the moment of inertia of given axis and α be the angular accl^n then,

$$\tau = I\alpha \quad \text{---(3)}$$

From (2) and (3)

$$I\alpha = -mgl\theta$$

$$\Rightarrow \alpha = -\frac{mgl}{I}\theta \quad \text{---(4)}$$

i,e angular accl^n is directly proportional to angular displacemen and acting towards mean position. This means pendulum execute angular harmonic motion.

$\therefore$ comparing eq^n (4) to $\alpha = -\omega^2\theta$ we get,

$$\Rightarrow \omega^2 = \frac{mgl}{I}$$

$$\Rightarrow \omega = \sqrt{\frac{mgl}{I}}$$

$$\Rightarrow 2\pi f = \sqrt{\frac{mgl}{I}}$$

$$\Rightarrow T = 2\pi\sqrt{\frac{I}{mgl}} \quad \text{----(5)}$$

If I_0 be the moment of inertia about an axis passing througt center of gravity, then,
moment of inertia interms of radius of gyration k is

$$I_0 = mk^2 \quad \text{---(6)}$$

but using parallel axis theorem,
moment of inertia about an axis passing through center of suspension will be

$$I = I_0 + ml^2$$

$$\Rightarrow I = mk^2 + ml^2 \quad \text{---(7)}$$

From (5) and (7)

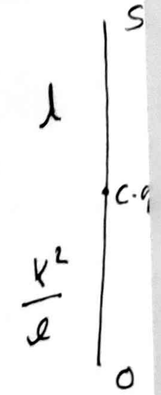
$$T = 2\pi\sqrt{\frac{(mk^2 + ml^2)}{mgl}}$$

$$T = 2\pi\sqrt{\frac{k^2 + l^2}{gl}}$$

$$\Rightarrow T = 2\pi\sqrt{\frac{k^2/l + l}{g}} \quad \text{---(8)}$$

This is the time period of compound pendulum i,e same as that of simple pendulum of length $L = \frac{k^2}{l} + l$. This is called length of equivalent simple pendulum.
since k^2 is always positive, the length l is always greater the length of compound pendulum l. i,e the center of oscillation is always lies beyond the center of gravity.

* Interchangeability of center of suspension and oscillation -
we have, time period of compound pendulum,



$$T = 2\pi \sqrt{\frac{k^2/l + l}{g}} \quad \cdots (1)$$

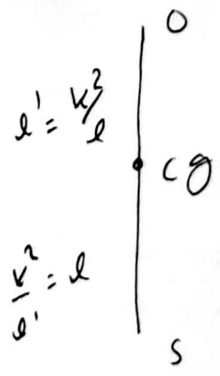
Let $\frac{k^2}{l} = l' \cdots (2)$

$\Rightarrow k^2 = l l' \cdots (3)$

From (1) and (2)

$$T = 2\pi \sqrt{\frac{l' + l}{g}} \quad \cdots (2)$$

Now, the center of oscillation is made center of suspension by inverting the pendulum,
Then, time period will be,



$$T' = 2\pi \sqrt{\frac{\frac{k^2}{l'} + l'}{g}} \quad \cdots (3)$$

From (1) and (3)

$$T' = 2\pi \sqrt{\frac{l + l'}{g}} \quad \cdots (4)$$

From (2) and (4)

$$T = T'$$

i,e center of suspension and oscillation can be interchanged.

* Maximum and minimum time period:

we have,

$$T = 2\pi \sqrt{\frac{\frac{k^2}{l} + l}{g}} \quad \cdots (1)$$

when $l = 0$, $T = \infty$ (maximum value)

$\therefore T_{max} = \infty$

Now, squaring eqn(1)

$$T^2 = \frac{4\pi^2}{g}\left[\frac{k^2}{l} + l\right]$$

Now, differentiating w. r. to l we get

$$2T\frac{dT}{dl} = \frac{4\pi^2}{g}\left[-\frac{k^2}{l^2} + 1\right]$$

For minima $\frac{dT}{dl} = 0$

$$\therefore \quad 0 = \frac{4\pi^2}{g}\left[-\frac{k^2}{l^2} + 1\right]$$

$$\Rightarrow -\frac{k^2}{l^2} + 1 = 0$$

$$\Rightarrow \frac{k^2}{l^2} = 1$$

$$\Rightarrow k^2 = l^2$$

$$\Rightarrow k = l \quad \text{....}(*)$$

$\therefore$ time period is minimum when $k = l$.

$\therefore$ minimum. time period,

$$T_{min} = 2\pi\sqrt{\left(\frac{l^2}{l} + l\right)/g}$$

$$\Rightarrow T_{min} = 2\pi\sqrt{\frac{2l}{g}} \quad \text{---}(**)$$

* Determination of value of 'g':-

we have, time period of compound pendulum,

$$T = 2\pi\sqrt{\frac{\frac{k^2}{l} + l}{g}}$$

squaring both sides we get

$$T^2 = \frac{4\pi^2}{g}\left[\frac{k^2}{l} + l\right]$$

$$\Rightarrow T^2 = \frac{4\pi^2 k^2}{gl} + \frac{4\pi^2 l}{g}$$

multiplying by l on both sides

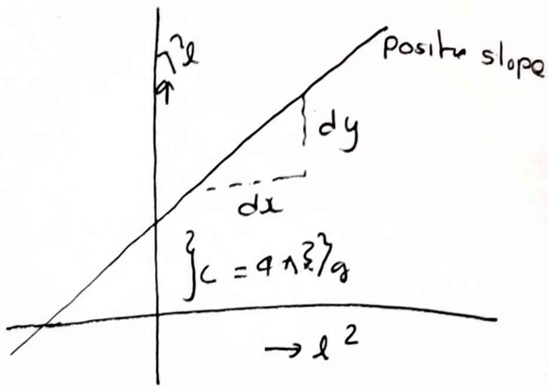
$$T^2 l = \frac{4\pi^2 l^2}{g} + \frac{4\pi^2 k^2}{gl}$$

comparing this eqn with $y = mx + c$, we get

$y = T^2 l$, $m = \frac{4\pi^2}{g}$, $x = l^2$ and $c = \frac{4\pi^2 k^2}{g}$

$\therefore$ slope of straight line, $m = \frac{4\pi^2}{g}$ ....(*)

Also, slope $= dy/dx$ ...(**)



From (*) and (**)

$$\frac{4\pi^2}{g} = \frac{dy}{dx}$$

$$\Rightarrow g = \frac{4\pi^2}{(dy/dx)}$$

This gives the accln due to gravity 'g'

* Energy conservation in simple harmonic motion - simple harmonic motion is defined by the ~~rotation~~, equation,

$$F = -kx \quad ---(1)$$

The work done by force during the displacement from x to $x+dx$ is

$$dw = F\,dx \quad ---(2)$$

From (1) and (2)

$$dw = -kx\,dx$$

The workdone to displace the particle from $x = 0$ to x is

$$\int_0^w dw = \int_0^x -kx\,dx$$

$$\Rightarrow w = -\frac{kx^2}{2}$$

This much amount of work is stored in the form of potential energy

$\therefore$ P.E. $= \frac{1}{2}kx^2$

P.E. $= \frac{1}{2}m\omega^2 A^2 \sin^2(\omega t + \delta)$ $[\because k = m\omega^2, x = A\sin(\omega t + \delta)]$

Also, kinetic energy, K.E. $= \frac{1}{2}mv^2$

K.E. $= \frac{1}{2}m\omega^2 A^2 \cos^2(\omega t + \delta)$

$\therefore$ total energy, E = K.E + P.E.

$$= \frac{1}{2}m\omega^2 A^2 \cos^2(\omega t + \delta) + \frac{1}{2}m\omega^2 A^2 \sin^2(\omega t + \delta)$$

$$E = \frac{1}{2}m\omega^2 A^2$$

since the total energy (mechanical) at time t is independent of

Wave motion-

Prepared By: CPS

In order to transfer energy from one point to another, there are different methods involved. In one, there is an actual transfer of the matter carrying the energy as in the case of throwing a stone with some velocity. And in the other, a disturbance is set up into the medium by the body at one point and this disturbance carries the energy forward without the actual transfer of the matter from one point to other. For example when a stone instead being thrown with a velocity is made to strike the surface of water in pond, the energy of stone is carried by the waves setup in the water which moves stedily along the surface carrying the energy from point to point.

wave is the disturbance that travels onwards through the medium due to the repeated periodic motion of its particles about their mean position. Thus wave transfer energy from one place to another place without bulk motion of their intervening medium. i,e wave tranfer energy and it is the mode of transfer of energy.

* Mechanical wave-

Mechanical waves or elastic wave are governed by Newton's laws of and required a material medium for their propogation. sound waves, seismic waves, water waves are example of mechanical wave.

* Electromagnetic wave -

The waves which do not require material medium to travel onwards are called electromagnetic waves. visible light, radio waves, micro-waves, x-rays -rays belong to this category. Electromagnetic waves consists of oscillating lectric and magnetic fields and travel with the same speed 'c' in free space

* Matter wave -

Atomic particles exhibit wave properties under cetain conditions. aws of quantum mechanics govern such matter waves.

: Gravitational wave -

It is suggested that cosmic bodies such as stars, galaxies produce ravitational waves and interact with each other through these waves. The ravitational waves are belived to propagate with the velocity of light.

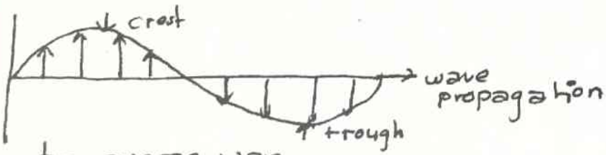
Two distinct classes of mechanical wave:

) Transverse wave -

The wave motion in which particles of the medium oscillates p and down about their mean position perpendiculer to the direction of ropagation of wave is called transverse wave. Thus it is in the form of est and trough. Ripples of water surfaces are transverse wave.

Longitudinal wave -

The wave motion in which particles of the medium vibrate in a rection parallel to the direction of propagation of wave is called longitu- nal wave motion. It is in the form of compression and rarefaction. For ample sound waves.



ig. transverse wave

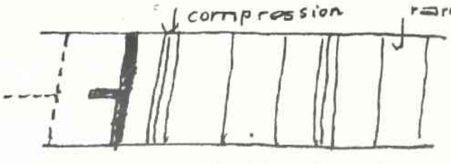


fig. Longitudinal wave.

* Terms associated with wave -

(1) Crest - The maximum displacement of particles of a medium above the equilibrium position is called crest.

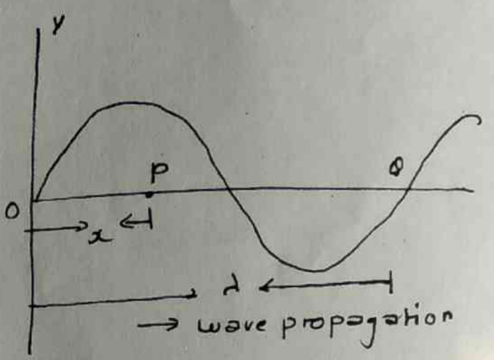
(2) Trough - The maximum displacement of the particles of the medium below the equillibrium position is called trough.

(3) Amplitude: The maximum displacement of the particles of medium from its equilibrium position when mechanical wave passes through the medium is called amplitude.

(4) wavelength - The distance travelled by the wave during the time at which any particle, of the medium completes one vibration about its mean position is called wavelength. It is denoted by λ. i,e distance between two nearest trough or crest.

* progressive wave or travelling wave -

The wave which travels onwards through the medium in a given direction without attenuation i,e with constant amplitude is called progressive wave. It may be transverse or longitudinal.



As the oscillations are communicated from point to point, the points are in different state of oscillations at different times. Therefore displacement of particle in the medium is function of space-coordinate and time.

We denote the displacement by

$$y = f(x,t) \quad \ldots (1)$$

It is called wave function.

Let a wave travels in positive x- direction. consider a particle at 'o' executes simple harmonic motion. Then equation of particle displacement will be

$$y = f(t) \quad \ldots (2)$$

Interms of trigonometric function,

$$y = A \sin \omega t \quad \ldots (3)$$

Since the successive particles to the right of 'o' receive and repeats its moments after definite interval of time, the phase lags goes on increasing as we proceed away from 'o' towards right. Let us consider a particle at point 'p' at a distance x from origin.

Let ϕ be the phase lag then at point p,

$$y = A \sin(\omega t - \phi) \quad \ldots (4)$$

In one complete oscillation total phase difference $= 2\pi$

$\therefore$ when path difference is λ, then phase difference $= 2\pi$

" " " is 1 then, " $= 2\pi/\lambda$

Therefore, for x displacement, phase difference $= \frac{2\pi}{\lambda} x$

$$\therefore \phi = \frac{2\pi}{\lambda} x \quad \ldots (5)$$

om (4) and (5)

$$y = A\sin(\omega t - \frac{2\pi}{\lambda}x)$$

$$\Rightarrow y = A\sin(\omega t - kx) \cdots (6) \qquad [\because \frac{2\pi}{\lambda} = k]$$

This is a progressive wave equation. If the particle is in negative x-direction then,

$$y = A\sin(\omega t + kx) \cdots (7)$$

* phase velocity (wave velocity) and particle velocity -

The distance travelled by wave in unit time is called phase velocity. It is denoted by u and given as

$$u = \frac{dx}{dt} \cdots (1)$$

As we know that the wave travelling in positive x-direction is represented as $y = A\sin(\omega t - kx) \cdots (2)$

where 'y' is the displacement of particle along y-direction in time 't'.

For a given wave,

$$\omega t - kx = \text{constant}$$

Differentiating w.r.to t we get

$$\Rightarrow \omega\frac{dt}{dt} - k\frac{dx}{dt} = 0$$

$$\Rightarrow \omega - ku = 0$$

$$\Rightarrow \omega = ku$$

$$\Rightarrow u = \frac{\omega}{k}$$

$$\Rightarrow u = \frac{2\pi f}{2\pi/\lambda}$$

$$\Rightarrow u = f\lambda \cdots (3)$$

i.e wavevelocity = frequency x wavelength.

The rate of change of displacement of a particle with time is called particle velocity. It is denoted by v and given by

$$v = \frac{dy}{dt} \cdots (4)$$

Differentiating eqn (2) w.r.to time,

$$\frac{dy}{dt} = A\omega\cos(\omega t - kx)$$

$$\Rightarrow v = A\omega\cos(\omega t - kx) \cdots (5)$$

Again, differentiating eqn (2) w.r.to x

$$\frac{dy}{dx} = -Ak\cos(\omega t - kx)$$

$$\Rightarrow A\cos(\omega t - kx) = -\frac{dy/dx}{k} \cdots (6)$$

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From (5) and (6)

$$V = \frac{-\omega (dy/dx)}{K}$$

$$\Rightarrow V = -u\frac{dy}{dx}$$

$$\Rightarrow V = u\left(-\frac{dy}{dx}\right)$$

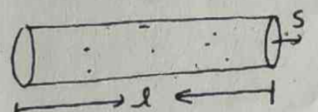
⇒ particle velocity = wave velocity times the slope (negative) of displacement.

* Intensity of wave –

The total energy flowing per sec per unit area is called intensity.

i,e intensity $= \frac{\text{total energy}}{\text{Area} \times \text{time}}$ ----(1)

As we have, the total energy of a particle executing periodic motion $= \frac{1}{2}m\omega^2A^2$ ---(2)



Now, consider a tube of cross-section area 's' and length 'l'.

Then volume of the tube = Sl

Let 'n' be the number of particles per unit volume. Then total numbers of particles inside the tube = nSl

∴ total energy of particles $= \frac{1}{2}m\omega^2A^2nSl$ --(3)

Then subsituting (3) in (1)

$$\Rightarrow I = \frac{\frac{1}{2}m\omega^2A^2n\cancel{S}l}{\cancel{S}t}$$

$$\Rightarrow I = \frac{1}{2}mn\frac{l}{t}A^2\omega^2$$

$$\Rightarrow I = \frac{1}{2}\rho u A^2\omega^2$$

$$\Rightarrow I = \frac{1}{2}\rho u A^2(2\pi f)^2$$

$$\Rightarrow I = \frac{1}{2}\rho u A^2 4\pi^2 f^2$$

$$\Rightarrow I = 2\pi^2 f^2 A^2 \rho u \quad \text{----(4)}$$

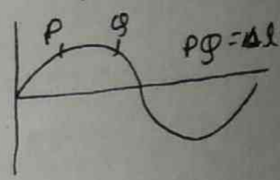
where, $mn = \rho$, density of the medium.

$l/t = u$, wave velocity.

$I \propto A^2$

the intensity of wave is directly proportional to the square of amplitude.

velocity of transverse wave along a stretched string -



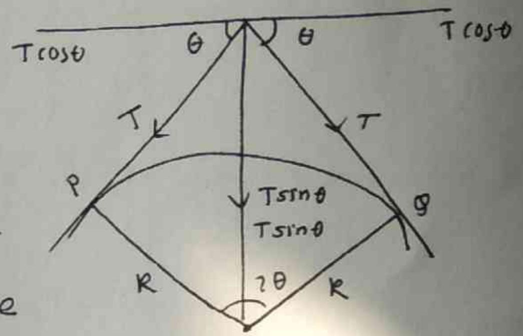
The wave velocity along a stretched string can be expressed interms of tension along the string and mass per unit length of the string.

when a jerk is given to the string fixed at one end and a transverse wave can be produced. consider an element length Δl of arc PQ of the string. Let 'T' be the tension along the string and μ be the mass per unit length of string. Then,

$$\mu = \frac{mass}{length}$$

$$\Rightarrow \mu = \frac{m}{\Delta l}$$

$$\Rightarrow m = \mu \Delta l \text{ ----(1)}$$



The element length Δl forms the arc PQ of radius R. The tension along the string can be resolved into two perpendicular components $T\cos\theta$ and $T\sin\theta$.
The horizontal component $T\cos\theta$ cancel eachother and the radial component $T\sin\theta$ provides the net tension which is along the center of arc PQ hence provides centripetal force.
∴ The resultant tension along radial component

$$= T\sin\theta + T\sin\theta$$

$$F = 2T\sin\theta \text{ ---(2)}$$

As the force provides centripetal force,

Also, $F = \frac{mu^2}{R}$ --(3)

From equation (1) and (3)

$$F = \frac{\mu \Delta l u^2}{R} \text{ --(4)}$$

From (2) and (4)

$$\frac{\mu \Delta l u^2}{R} = 2T\sin\theta$$

For small displacement, $\sin\theta \approx \theta$

$$\therefore \frac{\mu \Delta l u^2}{R} = 2\theta T \text{ --(5)}$$

From fig, angle made at center $= \frac{Arc}{radius}$

$$\Rightarrow 2\theta = \frac{PQ}{R}$$

$$\Rightarrow 2\theta = \frac{\Delta l}{R} \text{ -(6)}$$

From (5) and (6) $\frac{\mu \Delta l u^2}{R} = \frac{\Delta l T}{R}$

$\Rightarrow \mu u^2 = T$

$\Rightarrow u^2 = T/\mu$

$\Rightarrow u = \sqrt{T/\mu}$ ---(x)

This gives the velocity of transverse wave along the stretched string.

* Energy transmission in stretched string -

The kinetic energy of an element in stretched string can be written as

$$k.E = \frac{1}{2} mv^2$$

$$\Rightarrow k.E = \frac{1}{2} \mu \Delta l\, A^2 \omega^2 \cos^2(\omega t - kx) \qquad \left[\because \mu = \frac{\text{mass}}{\text{length}},\ y = A\sin(\omega t - kx)\right]$$

The rate of transmission of k.E

$$= \frac{1}{2} \mu \frac{\Delta l}{t} A^2 \omega^2 \cos^2(\omega t - kx) \quad ---- (1)$$

Now, the potential energy of the element length Δl is

$$P.E = \frac{1}{2} k y^2$$

$$= \frac{1}{2} m\omega^2 A^2 \sin^2(\omega t - kx)$$

$$= \frac{1}{2} \mu \Delta l\, \omega^2 A^2 \sin^2(\omega t - kx)$$

The rate of transmission of P.E

$$= \frac{1}{2} \mu \frac{\Delta l}{t} A^2 \omega^2 \sin^2(\omega t - kx) \quad --- (2)$$

$\therefore$ total rate of transmission of energy (power) =

$$\frac{1}{2} \mu \frac{\Delta l}{t} A^2 \omega^2 \cos^2(\omega t - kx) + \frac{1}{2} \mu \frac{\Delta l}{t} A^2 \omega^2 \sin^2(\omega t - kx)$$

$$= \frac{1}{2} \mu \frac{\Delta l}{t} A^2 \omega^2 \left[\sin^2(\omega t - kx) + \cos^2(\omega t - kx)\right]$$

$$= \frac{1}{2} \mu u A^2 \omega^2$$

$$= \frac{1}{2} \mu u A^2 (2\pi f)^2 \qquad \left[\because \frac{\Delta l}{t} = u, \text{ wave velocity}\right]$$

$$= \frac{1}{2} \mu u A^2 4\pi^2 f^2$$

$$P = 2\pi^2 f^2 \mu u A^2$$

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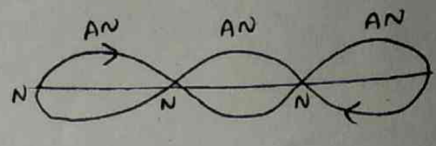
nonary (standing) wave -
nen two progressive waves equal in amplitude
nd wavelength travelling in opposite direction
iperimpose, a resultant wave is formed which
es not travel forward in medium and the



sultant wave formed is called stationary wave. In stationary wave there
e certain points like 'N' where particles vibration is minimum i,e particles
not vibrate (completely at rest). These positions are called nodes. where as
ere are certain points like 'AN' where particle vibrations are maximum
lled anti-nodes.

Let $y_1 = A\sin(\omega t - kx)$

and $y_2 = A\sin(\omega t + kx)$ be the two progressive waves travelling in opposite
rection. when these two waves overlap, the total displacement i,e resultant
splacement is

$$y = y_1 + y_2$$

$$\Rightarrow y = A\sin(\omega t - kx) + A\sin(\omega t + kx)$$

$$\Rightarrow y = A\left[2\sin\frac{(\omega t - kx + \omega t + kx)}{2} \cdot \cos\frac{(\omega t - kx - \omega t - kx)}{2}\right]$$

$$\Rightarrow y = 2A\sin\omega t\cos(-kx)$$

$$\Rightarrow y = 2A\cos kx\sin\omega t$$

$$\Rightarrow y = B\sin\omega t \text{ ---- } (*)$$

here $B = 2A\cos kx$ is the amplitude of resultant wave i,e eqn $(*)$ represents
e equation of stationary wave, of amplitude B.
he amplitude is maximum when,

$$\cos kx = \pm 1$$

$$\Rightarrow \cos kx = \cos n\pi \quad [n = 0, 1, 2, 3, \ldots$$

$$\Rightarrow kx = n\pi$$

$$\Rightarrow x = \frac{n\pi}{k}$$

$$\Rightarrow x = \frac{n\pi\lambda}{2\pi}$$

$$\Rightarrow x = \frac{n\lambda}{2}$$

$$\Rightarrow x = 0, \frac{\lambda}{2}, \lambda, \frac{3\lambda}{2}, \ldots$$

ese are the position of anti-nodes.
amplitude is minimum when,

$$\cos kx = 0$$

$$\Rightarrow \cos kx = \cos(2n+1)\frac{\pi}{2} \quad , [n = 0, 1, 2, 3, \ldots$$

$$\Rightarrow kx = (2n+1)\frac{\pi}{2}$$

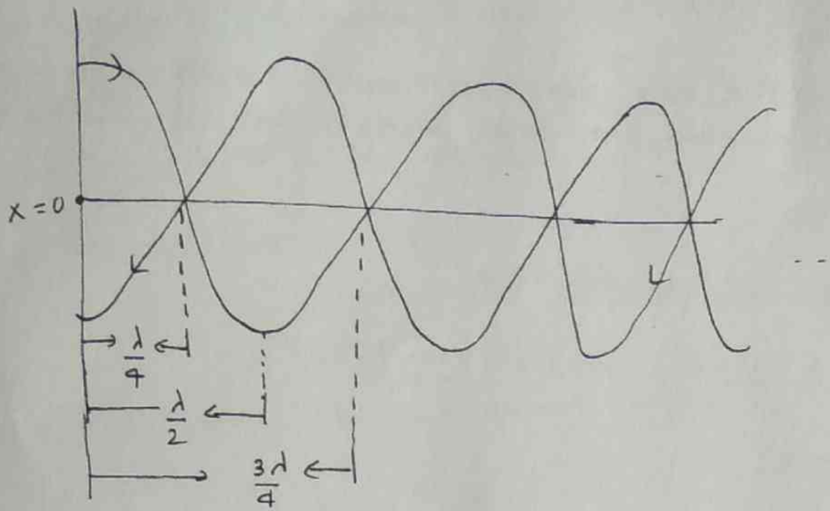
$$\Rightarrow x = \frac{(2n+1)\pi}{2k}$$

$\Rightarrow x = (2n+1)\frac{\pi}{2.2\pi}\lambda$

$\Rightarrow x = (2n+1)\frac{\lambda}{4}$

$\Rightarrow x = \frac{\lambda}{4}, \frac{3\lambda}{4}, \frac{5\lambda}{4}, \ldots$

hese are the position of nodes.
hus we see that anti-nodes and nodes are formed alternatively.

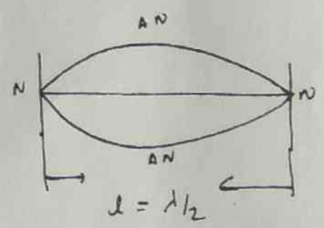


* Resonance of stationary wave -

In general standing waves are formed in a bounded medium. For instance, when a string is tied at both ends, standing wave is formed. but the standing wave is set up only for a certain discrete set of frequencies. we can say that the system resonates at these frequencies and called resonance of stationary wave

The minimum frequency at which resonance occur is called fundamental frequency or first harmonic.

First harmonic occurs at



$$l = \frac{\lambda}{2}$$

$$\Rightarrow \lambda = 2l \quad \text{---(1)}$$

we have, $u = f\lambda$

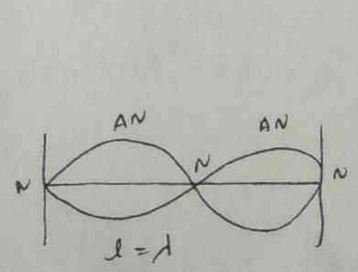
If f_0 be the fundamental frequency then,

$$u = f_0\lambda \quad \text{---(2)}$$

From (1) and (2)

$$u = f_0 2l$$

$$\Rightarrow f_0 = \frac{u}{2l} \quad \text{---(3)}$$



For 2nd harmonic, occurs at

$$l = \lambda \quad \text{---(4)}$$

f_1 be the frequency of 2nd harmonic then,

$$u = f_1 \lambda \quad ---(5)$$

from (4) and (5), $u = f_1 l$

$$\Rightarrow f_1 = \frac{u}{l}$$

$$\Rightarrow f_1 = \frac{2u}{2l}$$

$$\Rightarrow f_1 = 2f_0 \quad ---(6)$$ [This is called 1st overtone]

similarly, for third harmonics

$$f_2 = 3f_0 \quad --(7)$$ [2nd overtone]

In general, $f_n = (n+1) f_0$

* Difference betn progressive and standing wave-

* progressive -

1. The disturbance travels forward in the medium and is handed over from one particle to next after some time
2. The amplitude of oscillation is same at all positions in the medium.
3. No particles is permanently at rest.
4. Energy is transmitted from particle to particle across every section of the medium.
5. As the disturbance moves ~~from~~ every part of the medium suffers a change in density.
6. At every point there is variation in pressure.
7. Regular phase difference exists between successive particles.
8. The value of maximum velocity for all particles of the medium is same.

* standing-

1. The disturbance is at rest and does not move at all. so there is no transfer of disturbance to the neighbouring particles.
2. The amplitude of oscillation varies from zero at node to maximum at anti-node.
3. The particles at nodes are permanently at rest.
4. Energy is not transmitted from particle to particle i,e no transfer of energy across every section of the medium.
5. At anti-nodes, there is no change in density but at node there is maximum.
6. pressure variation is maximum at nodes and zero at anti-nodes.
7. All the particles ~~bein~~ in between two successive nodes are in phase.
8. The value of maximum velocity for different particles is different and velocity of the particles at the node is always zero.

Acoustics

prepared by cps

he word acoustics is derived from Greek word, meaning to 'hear.' Hence acoustics is defined as the science of sound. It conveys a ~~b~~double meaning and refers to the mental sensation perceived by ears and the cause responsible for that perception, namely, the physical phenomena external to the air, the wave motion which excites the auditory nerve.

Acoustic is a branch of physics dealing with production, propagation ~~and~~ perception and analysis of sound. It deals with design of and construction of different units of buildings to get proper acoustic conditions and also with the correction of the corresponding defects existing rooms.

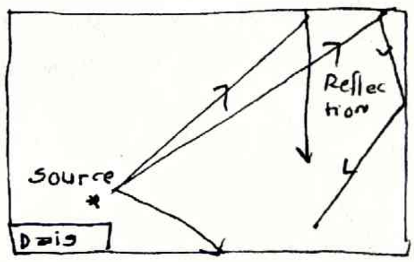
The science of acoustic of buildings has now achieved a unique place in design of modern buildings.

* Factors affecting good acoustic results -

(1) Reverberation time
(2) Loudness
(3) Focusing
(4) Echoes
(5) Echelon effect
(6) Resonance
(7) Noises.

* Reverberation -

when sound produced in a room or hall, It is noted that sound continues to be heared for sometime. sound produced in a hall or room undergoes multiple reflections from the walls, floor and ceiling before It becomes inaudible. The listner does not hear a single sharp sound but roll of sound.



Reverberation is the persistence and prolongation of sound in a room or hall due to the successive reflections from surfaces even when the source of sound has stop to emit sound. And time taken for sound to fall below the minimum audible range measured from the instant when the source of sound stopped sounding is called reverberation time.

The reverberation time in a hall should not be too large and also should not be too short. If the reverberation time is too short, the sound may not be sufficiently loud in all portion of the hall and the hall sounds dead.

If the reverberation time is too long, echoes will be present which results in speech being unitelligible. Therefore, value of reverberation time is maintained at an optimum value.

The satisfactory reverberation time are

* speech → 0.5 sec
* music → 1.0 to 0.2 sec
* Theatres → 1.1 to 0.15 sec

The reverberation time is controlled by
1. providing windows and ventillators.
2. covering the ceiling, part of the walls and even back of the chairs with absorbent materials like felt, fiber board, glass, wool etc.
3. using heavy curtains with folds.
4. covering the floor with carpets.
5. having a good sized audience.
6. Decorating the walls by pictures and maps.

* Absorption coefficient-

The ratio of sound intensity absorbed by a surface to the total energy incident on the surfare is known as absorbing power or absorbing coefficient of the surface. It is denoted by 'a'.

$$\therefore \text{Absorption coefficient (a)} = \frac{\text{sound energy absorbed}}{\text{total sound energy incident}}$$

An open window is an ideal of perfect sound absorbent. It is so because whole of the sound energy falling on an open window passes out and none is reflected.

* sabine's formula or law
(standard reverberation time)

This law states that the standard reverberation time is the time taken by the intensity of sound to fall one millionth (10^{-6}) of its original intensity after the original sound is cut-off.

Let $a_1, a_2, a_3 \ldots a_n$ be the absorbtion coefficients of various surfaces inside the room or hall and $s_1, s_2, s_3 \ldots s_n$ be there respective areas of surfaces. Then the average value of the absorption coefficient $\bar{a}$ is given by

$$\bar{a} = \frac{a_1 s_1 + a_2 s_2 + \cdots + a_n s_n}{s_1 + s_2 + \cdots + s_n}$$

$$\Rightarrow \bar{a} = \frac{\Sigma a s}{S}$$

$$\Rightarrow \Sigma a s = \bar{a} S \quad \text{----(1)} \qquad [\because S = \text{total surface area}]$$

By statistical method, Jager showed that the average distance traveled by sound between two successive reflection is $\frac{4V}{S}$.
Where V = volume of room or hall.
If v be the velocity of sound in air, then time between two successive

$$\text{reflection} = \frac{\text{distance travelled}}{\text{velocity}}$$

$$= \frac{4V/S}{v} = \frac{4V}{Sv} \text{ sec.}$$

In $\frac{4V}{S\upsilon}$ second the number of reflection = 1

In 1 second the number of reflection = $\frac{1}{4V/S\upsilon} = \frac{S\upsilon}{4V}$

The number of reflections in 't' sec $= \frac{S\upsilon t}{4V}$

hat means the average no. of reflections in time t, $= \frac{S\upsilon t}{4V}$

At one reflection fraction of sound absorbed $= \bar{a}$

Fraction of sound reflected $= (1-\bar{a})$

After two reflections, fraction of sound reflected $=(1-\bar{a})(1-\bar{a})$

In $\frac{S\upsilon t}{4V}$ reflections fraction of sound reflected

$$= (1-\bar{a})^{S\upsilon t/4V} \quad \text{--(2)}$$

f I_0 be the initial intensity of sound and I_t be the intensity of sound after time 't' then,

Fraction of sound reflected $= \frac{I_t}{I_0}$ --(3)

rom (2) and (3)

$$\frac{I_t}{I_0} = (1-\bar{a})^{S\upsilon t/4V} \quad \text{--(4)}$$

But according to the definition of reverberation time,

$$\frac{I_t}{I_0} = 10^{-6} \quad \text{--(5)} \quad \text{and } t = T$$

rom (4) and (5)

$$(1-\bar{a})^{S\upsilon T/4V} = 10^{-6}$$

aking log on both sides

$$\frac{S\upsilon T}{4V}\{\log(1-\bar{a}) = (-6)\log_e 10^1$$

$$\Rightarrow \frac{S\upsilon T}{4V}\left[-\bar{a} - \frac{\bar{a}^2}{2} - \frac{\bar{a}^3}{2}\dots\right] = (-6)\log_{10}10^1 \times 2.3026$$

$$\Rightarrow \frac{S\upsilon T}{4V}(-\bar{a}) = (-6)\times 2.3026 \times 1$$

[neglecting higher order terms]
And $\log_{10}10^1 = 1$

$$\Rightarrow T = \frac{(-6)\times 2.3026 \times 4V}{S\upsilon(-\bar{a})}$$

$$T = \frac{6\times 2.3026 \times 4V}{\upsilon(\bar{a}S)} \quad \text{--(7)}$$

om eqn (1) and (7)

$$\Rightarrow T = \frac{6\times 2.3026 \times 4V}{330\ \Sigma aS}$$

[$\bar{a}S = \Sigma aS$ and $\upsilon = 330\,m/s$]

$$\Rightarrow T = \frac{0.167V}{\Sigma aS} \quad \text{--(x)}$$

he reverberation time depends upon the valume of room and total absorption or absorbing power of the hall.

* ultrasound -

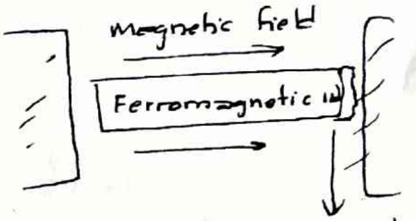
Sound waves with frequencies between 20Hz to 20kHz heard by human ear is called audible range. The sound wave having frequency above the audible range i.e above 20KHz are called ultrasonic waves.

The wavelengths of ultrasonic waves are very small as compared to audible sound.

* ultrasonic production -

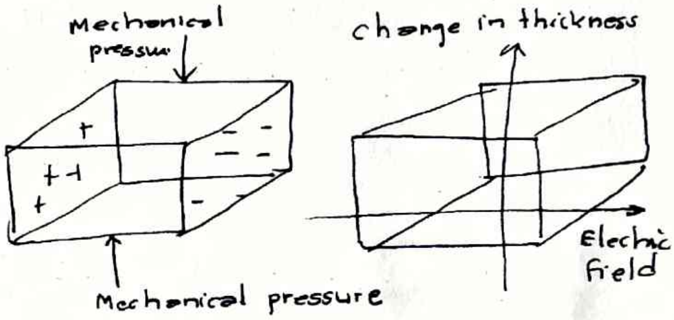
(1) Magnetostriction method -

When a ferromagnetic rod like iron or nickel is placed in a magnetic field parallel to its length, (strong magnetic field) the rod experiences a small change in its length. This is called magnetostriction effect. The change in length increase or decrease depends upon the strength of magnetic field, the nature of the materials and independent of magnetic field applied. This effect is very small and can be detected by a sensitive device.



If a ferromagnetic rod is placed in an alternating magnetic fields, the rod expands and contracts in length alternately. When the frequency of the alternating field is adjusted to the natural frequency of vibration of the rod, resonance will occur. The rod vibrates longitudinally with large amplitude and the rod generates ultrasonic waves from its ends.

(2) piezo-electric method:



If mechanical pressure is applied to one pair of opposite faces of certain crystal like quartz, equal and opposite electrical charges will appear across its other faces. This is called piezo-electric effect.

The converse of piezo-electric effect is also true. If an electric field is applied to one pair of faces, the corresponding changes in the dimensions of other pair of faces of the crystal are produced.

This is known as inverse piezoelectric effect or electrostriction. If an alternating voltage is applied to the opposite faces of quartz crystal pressure is developed along the other opposite faces of crystal. The quartz crystal continuously contracts and expands for the applied alternating field. Then the crystal is set into mechanical vibrations and hence produced ultrasonic waves.

It is more efficient than magnetostriction oscillator. Almost all the modern ultrasonic generators are of this type. Moreover, ultrasonic frequencies are high as 5×10^8 or 500 MHz can be obtained with the arrangement.

The output of the oscillator is very high and it is not effected by temperature and humidity. but the demerits is that the cost of piezoelec quartz is very high and cutting and shaping of quartz crystal are very complex.

* properties of ultrasonic waves:

1. They have high energy content.
2. Just like ordinary sound wave, ultrasonic waves get reflected, refracted and absorbed.
3. They can be transmitted over long distances with no appreciable loss of energy.
4. If an arrangement is made to form stationary waves of ultrasonic in liquid, it serves as a diffraction grating. It is called acoustic grating
5. They produce intense heating effect when passed through a substance.

* Applications-

(1) ultrasonic waves are used to detect the presence of flaws or defects in the form of crack, blowholes, poprosity etc in the internal structure of a material.

2) ultrasonics are used for making holes in very hard materials like glass, diamond etc.

3. The properties of some metals changes on heating and therefore, can not be welded by electric or gas welding. In such cases, the sheets are welded together at room temperature by using ultrasonics.

4. ultrasonic waves are used for cutting and machining.

5. It is used as a direction signalling.

6. It is used to measure the depth of the sea.

7. small organism and bacteria are either killed or maimed when ultrasonic waves fall on them.

8. strong ultrasonic waves are used for cleaning and washing cloths

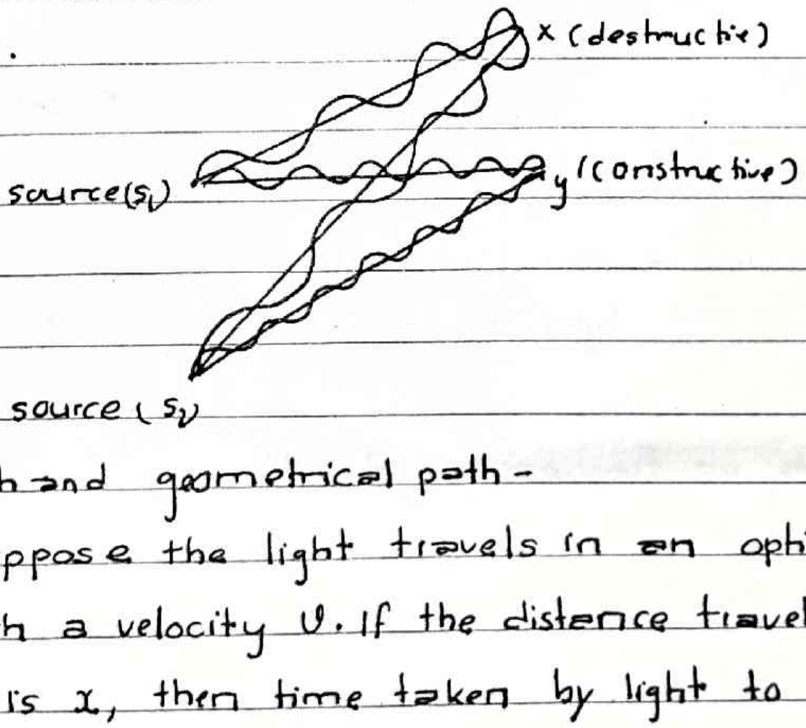
9. ultrasonic waves are used to get relief from neurological pain.

physical optics

Date . .

* Interference -

The source of light send out its energy uniformly in all direction but If there are two sources of light, the energy distribution is non-uniform in all direction. The phenomenon of non-uniform distribution of light energy due to the superposition of two light waves is called interference of light wave. If the crest of one wave falls on crest of the other or trough of one falls on trough of other the resultant amplitude becomes maximum i,e intensity of light is maximum at these points. At certain point crest of one falls on trough of other and vice versa, the corresponding point will be of minimum intensity. The former is called constructive interference and later one is called destructive interference.



* optical path and geometrical path -

suppose the light travels in an optically denser medium with a velocity v. If the distence travelled by light in the medium is x, then time taken by light to travel this distane is given by $t = \frac{x}{v}$ --(1)

If the light travels a distance L in the free space during the same time as it travels in medium then

$$L = ct \quad ---(2)$$

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From (1) and (2) $L = \frac{Cx}{v}$ ...(3)

If the refractive index of the medium is μ then,

$$\mu = C/v \quad ...(4)$$

From (3) and (4) $L = \mu x$ --(*)

$\Rightarrow$ optical path = refractive index x geometrical path.

optical path is equal to the product of refractive index of a medium and geometrical path.

Geometrical path is the distance travelled by light wave in medium and optical path is distance travelled in vacuum ...ing the same time.

sustained interference-

The interference patterns on the screen are said to be ...stained if they remain as permanent pattern. The conditions ... sustain interference are as follows-

The two sources of light must be coherent. The two ...urces are said to be coherent if they emit waves continuously ... same wavelength or frequency and always have a constant ...ase difference.

The amplitudes of waves should be equal.

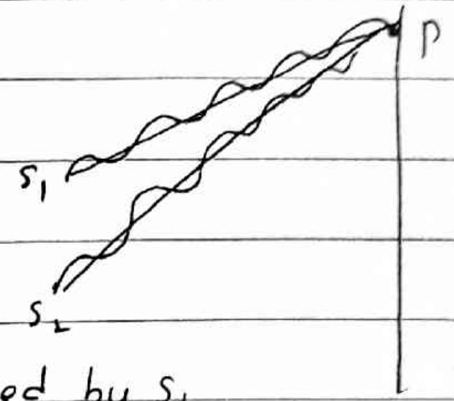
The sources should be monochromatic.

They must be very close to each other.

They should be point sources or very narrow sources.

The screen should be far from sources.

* Condition for maxima and minima -



Let us consider S_1 and S_2 be two ~~w~~ coherent sources of light which produce interference fringes. Let 'p' be the point on the screen.

suppose $y_1 = a \sin \omega t$ be the wave produced by S_1

and $y_2 = a \sin(\omega t + \phi)$ be the wave produced by S_2. where ϕ is the phase difference betn waves from S_1 and S_2.

Then the resultant displacement of wave is given by

$$y = y_1 + y_2$$

$$\Rightarrow y = a \sin \omega t + a \sin(\omega t + \phi)$$

$$\Rightarrow y = a[\sin \omega t + \sin \omega t \cos \phi + \cos \omega t \sin \phi]$$

$$\Rightarrow y = a[\sin \omega t(1 + \cos \phi) + \cos \omega t \sin \phi] \quad --(1)$$

Let $a(1 + \cos \phi) = A \cos \theta$ --(2)

and $a \sin \phi = A \sin \theta$ --(3)

subsbituting (2) and (3) in (1)

$$\Rightarrow y = A[\sin \omega t \cos \theta + \cos \omega t \sin \theta]$$

$$\Rightarrow y = A \sin(\omega t + \theta) \quad ---(4)$$

This is the resultant wave equation with amplitude A.

Now, squaring and adding eqn (2) and (3)

$$\Rightarrow A^2 \cos^2 \theta + A^2 \sin^2 \theta = a^2(1 + \cos \phi)^2 + a^2 \sin^2 \phi$$

$$\Rightarrow A^2[\sin^2 \theta + \cos^2 \theta] = a^2[1 + 2\cos \phi + \cos^2 \phi + \sin^2 \phi]$$

$$\Rightarrow A^2 = a^2[2 + 2\cos \phi]$$

$$\Rightarrow A^2 = 2a^2(1 + \cos \phi)$$

$$\Rightarrow A^2 = 4a^2 \cos^2 \frac{\phi}{2} \quad ----(5)$$

since the intensity of wave is directly proportional to the quare of the amplitude, we can write,

$$\text{Intensity } (I) \propto A^2$$

$$\therefore I \propto 4a^2\cos^2\phi/_2 \quad ---(*)$$

- maximum intensity, $\cos^2\frac{\phi}{2} = 1$

$$\Rightarrow \cos\phi/_2 = \pm 1$$

$$\Rightarrow \cos\phi/_2 = \cos n\pi, \quad n = 0, 1, 2, 3, \ldots$$

$$\Rightarrow \frac{\phi}{2} = n\pi$$

$$\Rightarrow \phi = 2n\pi \quad ----(6) \quad [\text{phase difference}]$$

have, relation between path difference and phase difference,

$$\text{path diff.} = \frac{\lambda}{2\pi} \times \text{phase diff.}$$

$$= \frac{\lambda}{2\pi} \times 2n\pi$$

$$\text{path diff.} = n\lambda \quad ----(7)$$

s is the condition for maximum intensity. i.e bright fringe.

For minimum intensity, $\cos^2\phi/_2 = 0$

$$\Rightarrow \cos\frac{\phi}{2} = 0$$

$$\Rightarrow \cos\phi/_2 = \cos(2n+1)\pi/_2, \quad n = 0, 1, 2, 3, -$$

$$\Rightarrow \phi = (2n+1)\pi \quad --(8)$$

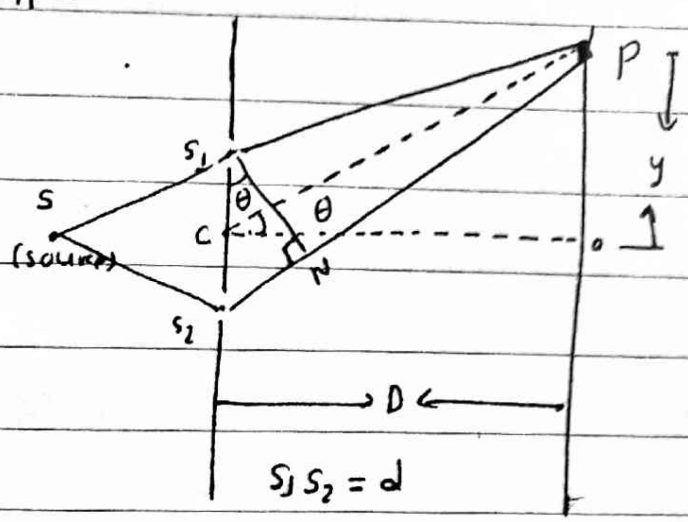
$$\text{path diff} = \frac{\lambda}{2\pi} \times (2n+1)\pi$$

$$\Rightarrow \text{path diff} = (2n+1)\frac{\lambda}{2} \quad --(9)$$

his is the condition for minimum intensity

i.e dark fringe.

* Young's Double slit experiment -



consider a monochromatic source of light 's'. Let S_1 and S_2 be the two slits which are equidistance from source s. S_1 and S_2 acts as two coherent sources of light. Light waves from S_1 and S_2 overlap each other so as to produce alternative dark and bright fringes.

From fig. the path difference between the waves from S_1 and S_2 reaching at point 'p' on the screen is S_2N_1 as drawn normal for S_1 to S_2P.

since S_1 and S_2 are close to each other and so as to center C; S_1N meets cp at 90°. Then $\angle CS_1N = \theta$ If $\angle PCO = \theta$

$\therefore$ considering triangle $S_1S_2N_1$

$$\sin\theta = S_2N / S_1S_2$$

$$\Rightarrow \sin\theta = S_2N/d$$

$$\Rightarrow S_2N = d\sin\theta \quad \text{---(1)}$$

where d = distance bet^n slits

Again, from Δ PCO, $\tan\theta = \dfrac{y}{D}$ ---(2)

where y = position of fringe from centre of screen and
D = distance of screen from sits.

The point 'p' on the screen will correspond to bright fringe if

$$\text{path diff} = n\lambda \quad \text{---(3)}$$

From (1) and (3)

$$d\sin\theta = n\lambda \quad \cdots (4)$$

If θ is small then $\sin\theta \approx \theta$ and $\tan\theta \approx \theta$

from (2) $\theta = y/D$ ---(5)

d from (4) $d\theta = n\lambda$ --(6)

m(5) and (6) $\frac{dy}{D} = n\lambda$

$$\Rightarrow y = \frac{n\lambda D}{d} \quad \text{--(7)}$$

when $n=0$, $y_0 = 0$

when $n=1$, $y_1 = \frac{\lambda D}{d}$

when $n=2$, $y_2 = \frac{2\lambda D}{d}$

distance betn the two consecutive bright fringes will be,

$y_1 - y_0$ or $y_2 - y_1$, ...

ry time we get the diff $= \frac{\lambda D}{d}$

is called fringe width denoted by $\beta = \frac{\lambda D}{d}$ .. (8)

imilarly, the point p will correspond to dark fringe If

path diff $= (2n+1)\lambda/2$ --(9)

From (1) and (9)

$$d\sin\theta = (2n+1)\lambda/2$$

$$\Rightarrow d\theta = (2n+1)\lambda/2 \qquad [\because \theta = y/D, \text{ from (5)}]$$

$$\Rightarrow \frac{dy}{D} = (2n+1)\frac{\lambda}{2}$$

$$\Rightarrow y = \frac{(2n+1)\lambda D}{2d} \quad \text{---(10)}, \quad n = 0, 1, 2, 3, ..$$

$\therefore \quad y = \frac{\lambda D}{2d}, \frac{3\lambda D}{d}, \frac{5\lambda D}{d}, \ldots$

Then distance betⁿ two consecutive dark fringes will again, $\lambda D/d$

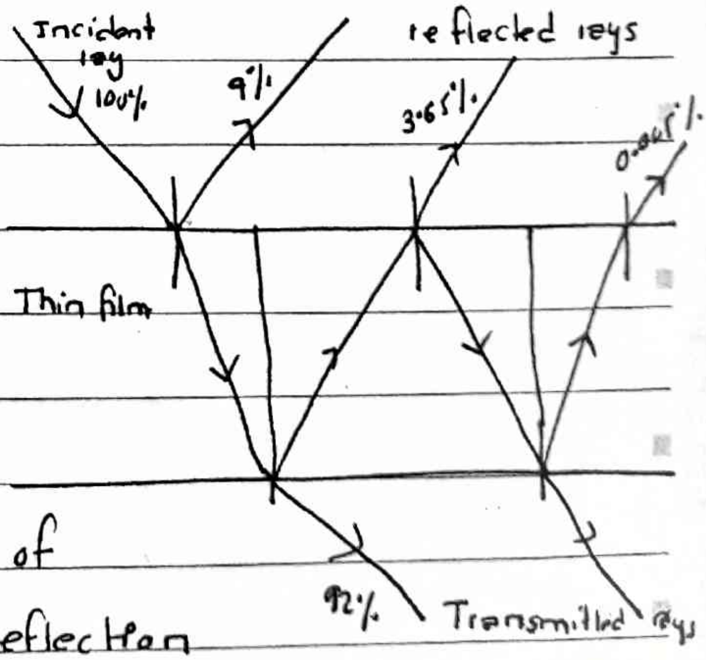
$\therefore \quad \beta = \lambda D/d \quad \text{--}(**)$

we can conclude that bright and dark fringes are equally spaced.

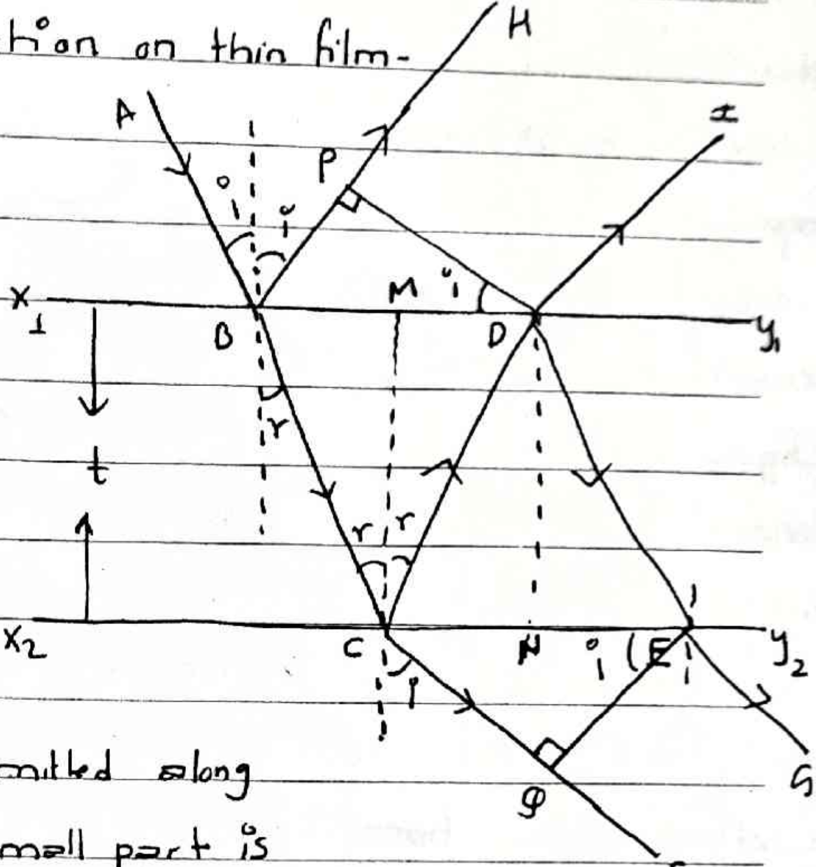
* Thin film -

An optical medium is called thin film when the thickness is about 1 order of wavelength of light is visible spectrum. Thus, a film of thickness in the range 0.5 μm to 10 μm, may be considered as a thin film. It may be the thin sheet of transparent medium such as glass, mica or air film enclosed between two transparent plates or a soap bubble.

If the light incident on thin film a small part of its gets reflected from the top surface and major part is transmitted through film. Again a small part of the transmitted component is reflected back into the film by the bottom surface and rest of it emerges out of the film. Therefore only the first reflection at top surface and first reflection on the bottom surface will be of appreciable strength.



Interference due to reflection on thin film-



suppose a ray of light
s incident on the scteer
face $X_1 Y_1$ of thin film of
ckness t and refractive
lex μ.
part of ray incident at
at B such that a part
light is reflected along
and major part is transmitted along
similarly at point 'C' small part is
ected along CD and major part is transmitted along CF.
lso at 'D' major part is transmitted along DI and small
rtion reflected back along DE. And at point E, major part
ansmitted along EG and remaining part reflected back to
m. and so on...

Only the rays reflected along BH and CD are of appreciable
rength.

From fig. draw DP normal on BH.

'i' be the angle of incidence and 'r' be the angle of refraction,
hen, from Δ BPD,

en, path diff. betn reflected rays from point B and C will be

path diff = BC + CD − BP

optical path diff = μ(BC + CD) − BP --(1)

om, Δ BCM

$\cos r = \frac{MC}{BC} \Rightarrow BC = \frac{t}{\cos r}$ ---(2) $[MC = t]$

Date . .

As ΔBCM and ΔCMD are similar, then, also,

$$CD = \frac{t}{\cos r} \quad --(3)$$

Again, from ΔBPD, $\sin i = BP/BD$

$$\Rightarrow BP = BD \sin i$$

$$\Rightarrow BP = 2BM \sin i \quad ---(4) \quad [BM = MD]$$

Also, from ΔBMC, $\tan r = \frac{BM}{MC}$

$$\Rightarrow BM = MC \tan r$$

$$\Rightarrow BM = t \tan r \quad --(5)$$

From (4) and (5) $BP = 2t \tan r \sin i$

$$BP = 2t \tan r \mu \sin r \qquad [\because \mu = \sin i / \sin r]$$

$$\Rightarrow BP = 2t\mu \frac{\sin^2 r}{\cos r} \quad --(6)$$

From (1) and (2), (3), (6) we got,

optical path diff. $= \mu \left[\frac{t}{\cos r} + \frac{t}{\cos r} \right] - 2\mu t \frac{\sin^2 r}{\cos r}$

$$= \frac{2\mu t}{\cos r} - \frac{2\mu t \sin^2 r}{\cos r}$$

$$= \frac{2\mu t}{\cos r} \left[1 - \sin^2 r \right]$$

$$= \frac{2\mu t \cos^2 r}{\cos r}$$

path diff. $= 2\mu t \cos r \quad ---(7)$

But at point 'B' the ray is reflected from the surface of denser medium so that there is change in phase by π or path diff is $\lambda/2$ i.e. wave losses half of its wavelength on reflection.

rom the boundary of rarer to denser medium.

$\therefore$ Additional path diff $= \lambda/2$.

total path diff $= 2\mu t \cos r + \frac{\lambda}{2}$ --- (8)

hen the thickness of the film is negligible, $t=0$ then

$$\text{path diff} = \frac{\lambda}{2}$$

the position of bright fringe,

$$2\mu t \cos r + \frac{\lambda}{2} = n\lambda$$

$$\Rightarrow 2\mu t \cos r = (2n-1)\lambda/2 \quad \cdots (9)$$

the position of dark fringe

$$2\mu t \cos r + \frac{\lambda}{2} = (2n+1)\frac{\lambda}{2}$$

$$\Rightarrow 2\mu t \cos r = n\lambda \quad \cdots (10)$$

If we draw normal EQ on CF and consider triangle Q we can get the condition of interference due to transmitted ys in thin film. Between rays GE and CF,

the path diff $= CD + DE - CQ$

optical path diff $= \mu(CD + DE) - CQ$

$= 2\mu t \cos r$

here is no additional path difference as at point D, the ys reflected from the surface of rarer medium i.e there is o phase change.

$\therefore$ total path diff $= 2\mu t \cos r$ --- (11)

transmitted rays overlap to give bright fringe if $2\mu t \cos r = n\lambda$ d give dark fringe if $2\mu t \cos r = (2n+1)\lambda/2$

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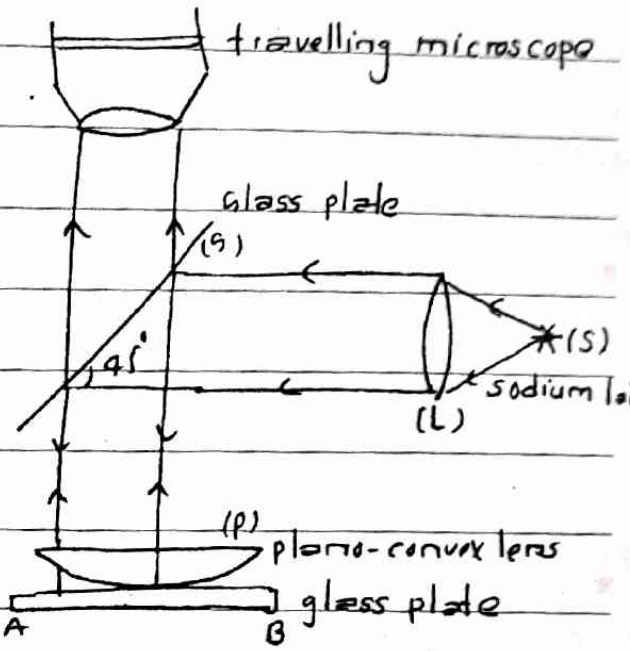
* Newton's Ring -

Newton's Rings are the example of fringes of equal thickness.

The experimental arrangement for observing Newton's Ring is shown in fig.

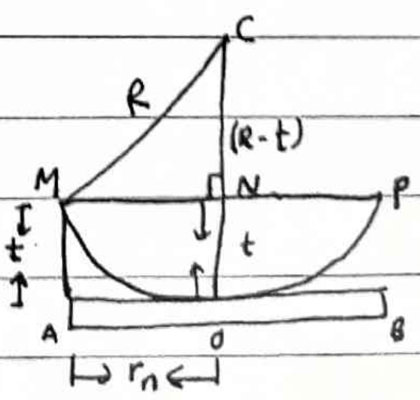


Monochromatic light from the extended source S is spread parallel by lens 'L'. These rays are partially reflected by the glass plate G inclined 45° to horizontal. The reflected rays from G incident normally to the plane surface of planoconvex lens P kept on glass plate AB. The lens 'P' and glass plate AB enclases air film of variable thickness.

The rays reflected from upper surface of the film and lower surface of the film overlap so as to produce interference fringes. The fringes are formed between the lens 'P' and plate 'AB' as circular fringes are called Newton's Ring.

As the thickness of air film at the point contact betn lens 'P' and plate 'AB' is zero and gradually increases as we move outward the locus of points where the air film has equal thickness fall on a circle whose centre is the point of contact. Thus the fringes are in circular form.

Let 'R' be the radius of curvature of plano-convex lens. consider the air film of thickness 't' such that r_n be the radius of n^{th} ring whose thickness from center is 't'.



or the thin film, the path difference for reflected rays

$= 2\mu t \cos r + \lambda/2$ --- (1)

- air film, $\mu = 1$

- normal incidence, $r = 0$

q^n (1) becomes, path diff $= 2t + \lambda/2$ --- (2)

or bright fringes, path diff $= n\lambda$ --- (3)

om (2) and (3)

$$2t + \frac{\lambda}{2} = n\lambda$$

$$\Rightarrow 2t = n\lambda - \lambda/2$$

$$\Rightarrow 2t = (2n-1)\frac{\lambda}{2} \quad \text{---(4)}$$

om fig.

n Δ MCN, $(MC)^2 = (CM)^2 + (MN)^2$

$$\Rightarrow R^2 = (R-t)^2 + r_n^2$$

$$\Rightarrow R^2 = R^2 - 2Rt + t^2 + r_n^2$$

$$\Rightarrow r_n^2 = 2Rt - t^2 \quad \text{---(5)}$$

nce, lens is of large aperature, $R >> t$

From eq^n (5) t^2 can be neglected.

eq^n (5) becomos,

$$r_n^2 = 2Rt \quad \text{---(6)}$$

$$\Rightarrow 2t = \frac{r_n^2}{R} \quad \text{....(7)}$$

m (4) and (7) $\frac{r_n^2}{R} = (2n-1)\frac{\lambda}{2}$

$$\Rightarrow r_n^2 = (2n-1)\lambda R/2$$

$$\Rightarrow D_n^2 = 2(2n-1)\lambda R \quad \text{---(8)} \qquad [\because r_n = D_n/2]$$

milarly

For m^{th} bright ring, $D_m^2 = 2(2m-1)\lambda R$ --(9)

subtracting eqn (8) from (9) we get.

$$D_m^2 - D_n^2 = 2(2m-1)\lambda R - 2(2n-1)\lambda R$$

$$= 2\lambda R[2m-1-2n+1]$$

$$= 2\lambda R[2m-2n]$$

$$D_m^2 - D_n^2 = 4(m-n)\lambda R$$

$$\Rightarrow \lambda = \frac{D_m^2 - D_n^2}{4(m-n)R} \quad ----(10)$$

This is the wavelength of monochromatic light. Measuring the diameter of n^{th} and m^{th} ring we can determine the wavelengths. similar condition can be obtained by considering the condition of dark fring.

For dark fringe, path diff $= (2n+1)\lambda/_2$ ---(11)

From (2) and (11)

$$2t + \frac{\lambda}{2} = (2n+1)\frac{\lambda}{2}$$

$$\Rightarrow 2t = n\lambda \quad ---(12)$$

Now from (7) and (12)

$$\frac{r_n^2}{R} = n\lambda$$

$$\Rightarrow r_n^2 = n\lambda R$$

$$\Rightarrow D_n^2 = 4n\lambda R \quad ---(13)$$

similaly for m^{th} ring, $D_m^2 = 4m\lambda R$ ---(14)

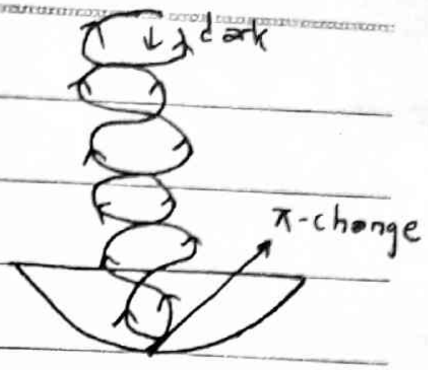
From (13) and (14) $D_m^2 - D_n^2 = 4(m-n)\lambda R$

$$\Rightarrow \lambda = \frac{D_m^2 - D_n^2}{4(m-n)R} \quad ---(15)$$

As path diff $= 2t + \lambda/_2$

t the point of contact, $t=0$

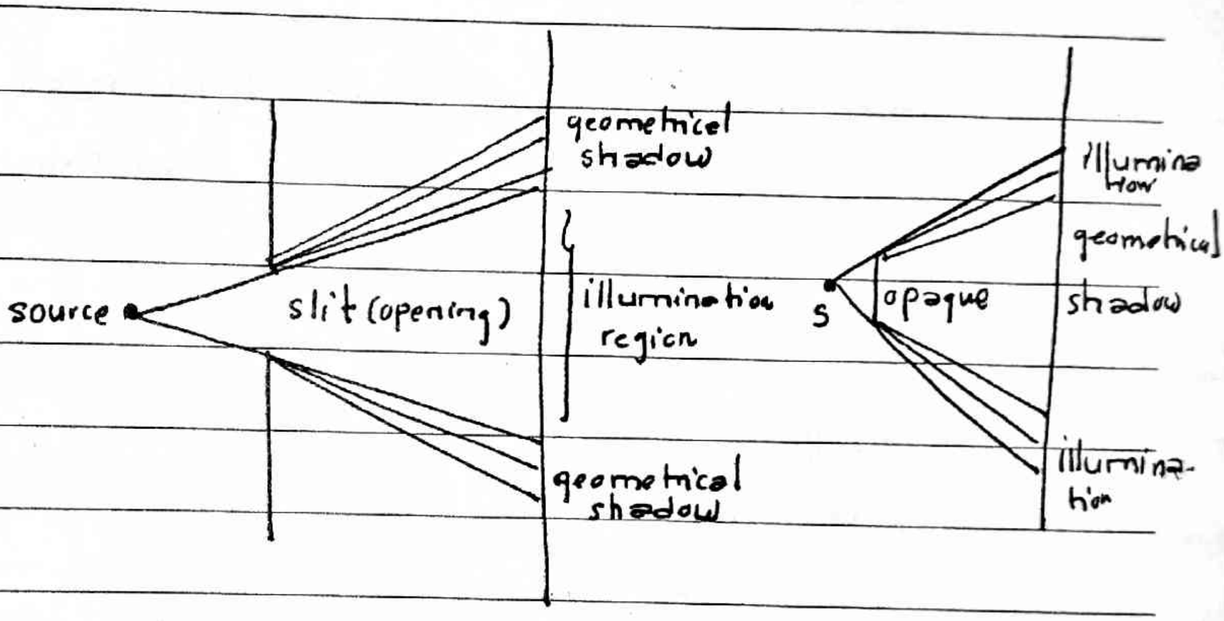
$\therefore$ total path diff $= \lambda/2$



ich is the condition of dark fringes.

at means at the point of contact the ase ~~di~~ changes by π or path change by $\lambda/2$ or the wave lost lf wave on reflection. Therefore, the center of the Newton's ring always dark.

Diffraction-



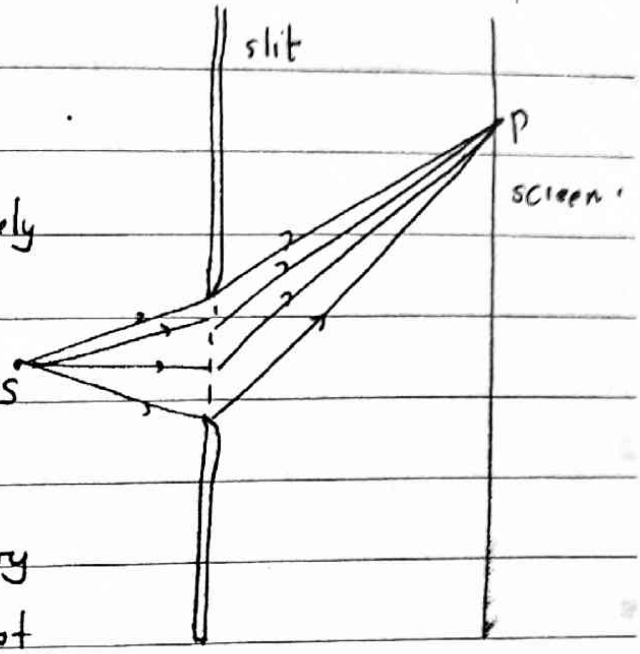
The phenomenon of bending of light from the sharp edge of e obstacle and spreading around the geometrical shadow region called diffraction.

When the opening is large compared to the wavelength of light e wave do not bend round the edges. When the opening is nell the bending round the edges is noticeable. When the ening is very small, the wave spread over the entire surface hind the opening. Therefore, diffraction is obserable only when ze of the obstacle is compareable to wavelength of light.

The diffracted rays from the obsdacle overlap each other produce diffracted fringes.

* Fresnel diffraction-

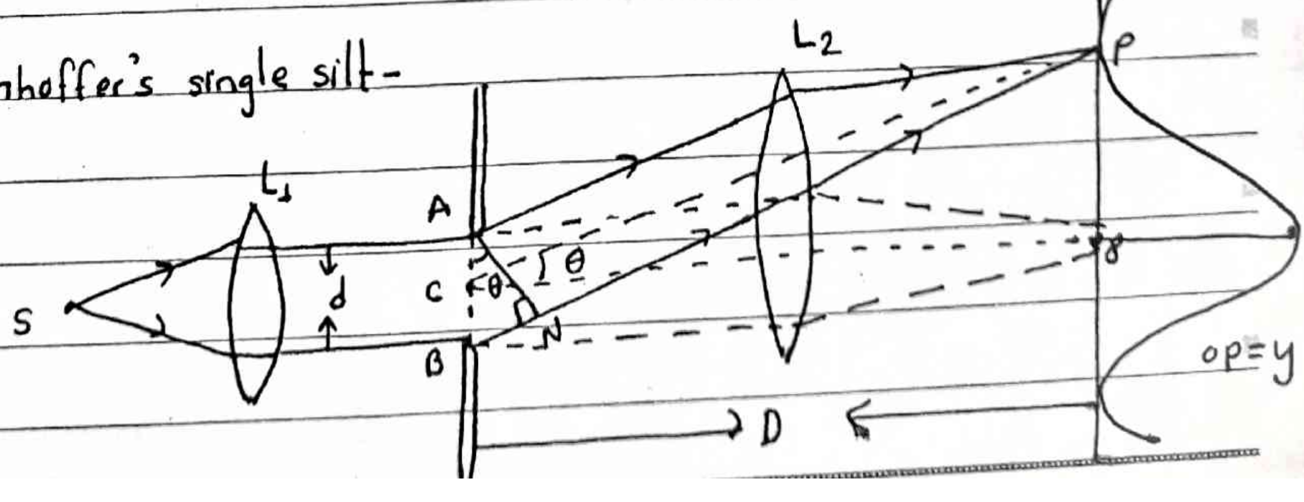
In fresnel type of diffraction the source and slit screen are effectively at finite distance from obstacle. It does not require any lenses. The incident wavefront is not planar. Therefore, the phase of the secondary wavelets are not in same phase at all the points in the plane of obstacle. The resultant amplitude at any point on the screen is obtained by mutual interference of secondary wavelets from different elements of unblocked portions of wavefront.



* Fraunhoffer's diffraction-

In this type of diffraction the source and screen are effectively infinite distance from the obstacle. The two converging lens are used. The first one is used to make rays parallel and 2nd one is used to converse the diffracted rays on the screen. The wavelets emerges after unblocked portion of the obstacle are of same phase, which overlap to give diffracted fringes.

* Fraunhoffer's single silt-



A ray of parallel beem of light (monochromatic) is incident 1 a slit AB of width 'd'. The diffracted rays are focused on a creen by a convex lens, L_2 where we get diffraction fringes.

The light wave travelling in the same direction as the incident it wave, focus at point 'O' i,e center of the screen. For the nter of screen all the rays meet at same phase i,e there is path difference. Therefore, central fringe is bright called incipal maxima.

The path difference between the rays from A and B aching at point P = BN

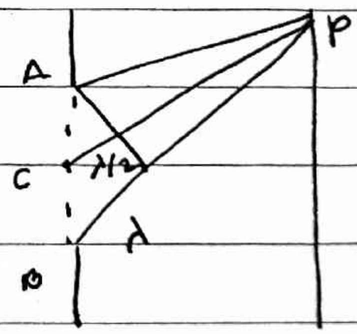
2, from Δ ABN, $\sin\theta = \frac{BN}{AB}$

$$\Rightarrow BN = AB\sin\theta$$

$$\Rightarrow BN = d\sin\theta \quad --(1)$$

path difference is equal to the wavelength of light then point will be the position of minimum intensity

i.e If $d\sin\theta = \lambda$



n this case the slit is divided into wo equal parts i,e the whole wave front divided into AC and BC and if the ath difference between the wave from A and B is λ then th difference between waves from A and C will be $\lambda/2$ and th difference between waves from B and C will be $\lambda/2$. As the nt in uper half of AC the corresponding point in lower half BC, they interfere destructively and corresponding point will be dark.

similarly If the path diff $= 2\lambda$, then we can divide whole avefront into 4 equal parts so as the wave reaching at screen will be minimum intensity.

In general for dark fringe, $d \sin\theta_n = n\lambda$ ---- (2) $[n = 1, 2, 3, \cdots$

If we divide the whole wave-front into, 3, 5, 7, ... two of the wave cancel eachother and remaining one reinforces the bright fringe and corresponding point will be bright.

In general for bright fringe,

$$d \sin\theta_n = (2n+1)\frac{\lambda}{2} \quad \cdots (4) \quad n = 1, 2, 3, \cdots$$

Let y_n be the position of n^{th} minima and 'D' be the distance between slit and screen then,

From Δ CPO, $\tan\theta_n = y_n/D$ ---(5)

For small θ, $\tan\theta_n \approx \sin\theta_n \approx \theta_n$

$\therefore$ From (3) and (5)

$$d\theta_n = n\lambda \quad \cdots (6) \qquad \text{[For minima]}$$

and $\theta_n = y_n/D$ --(7)

From (6) and (7)

$$\frac{d\, y_n}{D} = n\lambda$$

$$\Rightarrow y_n = \frac{n\lambda D}{d} \quad \text{--(8)}$$

the difference betn two consecutive minima,

$$y_{n+1} - y_n = \beta$$

$$\Rightarrow \frac{(n+1)\lambda D}{d} - \frac{n\lambda D}{d} = \beta$$

$$\Rightarrow \frac{\lambda D}{d}[n+1-n] = \beta$$

$$\Rightarrow \beta = \frac{\lambda D}{d} \quad \cdots (9)$$

similarly for maxima,

...m eq^n (4) and (5) $\frac{d\,y_n}{D} = (2n+1)\frac{\lambda}{2}$

$$\Rightarrow y_n = (2n+1)\frac{\lambda D}{2d} \quad --(10)$$

...e difference bet^n two consecutive maxima,

$$y_{n+1} - y_n = \beta$$

$$\Rightarrow [2(n+1)+1]\frac{\lambda D}{2d} - (2n+1)\frac{\lambda D}{2d} = \beta$$

$$\Rightarrow \frac{\lambda D}{2d}\left[2n+2+1-2n-1\right] = \beta$$

$$\Rightarrow \beta = \frac{\lambda D}{d} \quad ...(11)$$

... width of central maxima is the distance between the first ...ima on either side of central maxima.

... α be the distance of first minima from center of screen then ...idth of central maxima $= 2\alpha$

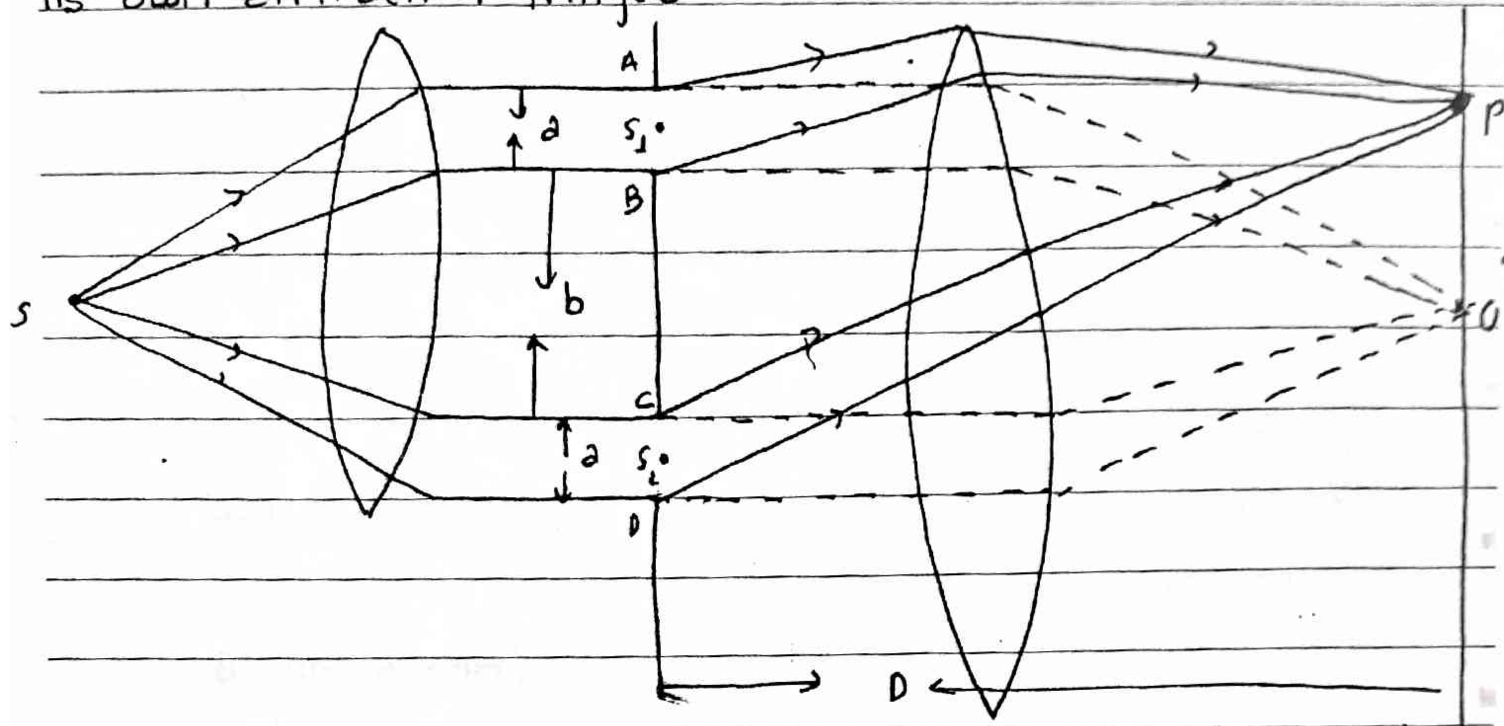
$$\text{but } \alpha = \lambda D/d$$

$$\therefore \text{central width} = 2\lambda D/d$$

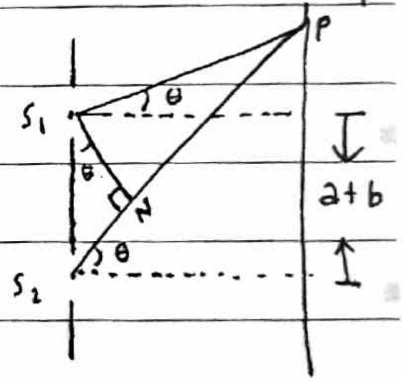
Fraunhoffer's diffraction through Double slit -

suppose AB and CD are two identical slits each of ...idth 'a' separated, by opaque of width 'b'. when the ...arallel beam of light incident on the slits, these rays are ...cused at the center of screen by a convex lens where ...ntral maxima is obtained. These two slits together produces ...ight and dark interference fringes. Also each slits produces

Its own diffraction fringes.



* Interference maxima and minima -



The mid point of each slit behave as a source. Therefore S_1 and S_2 are two coherent sources and produces interference fringes. The secondary wavelets emerging from S_1 and S_2 at angle θ have path difference $= S_2N$

From fig, ΔS_1S_2N

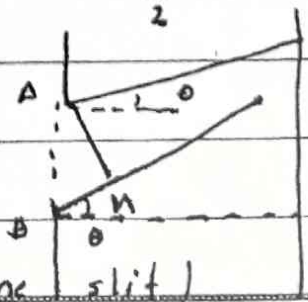
$$\sin\theta = \frac{S_2N}{S_1S_2}$$

$$\Rightarrow S_2N = S_1S_2 \sin\theta$$

$$\Rightarrow S_2N = (a+b)\sin\theta$$

∴ For interference maxima, $(a+b)\sin\theta = n\lambda$ $[n = 1, 2, 3, \ldots]$

For interference minima, $(a+b)\sin\theta = (2n+1)\frac{\lambda}{2}$



* Diffraction maxima and minima -

The path difference between the rays diffracted through angle θ (considering any one slit) om the upper and lower surface of slit = BM.

om fig In Δ BAM, $\sin\theta = BM/AB$

$$\Rightarrow BM = AB\sin\theta$$

$$\Rightarrow BM = a\sin\theta \quad \text{---(x)}$$

· diffraction minima, $a\sin\theta = n\lambda$ [$n = 1, 2, 3, \ldots$

· diffraction maxima, $a\sin\theta = (2n+1)\lambda/2$

Thus we can conclude that the interference depends upon th slits and opaque width but diffraction depends upon only width. The resultant pattern on the screen is the combination interference as well as diffraction fringes. The diffraction inges intensity is 4 times that of single slit diffraction fringes.

Diffraction grating -

Diffraction grating is an arrangement of ge number of equidistance parallel slits separated identical opaques. It is made by rulling 15000 to 200 lines on one inch glass plate by diamond point. ch rulled line behaves like opaque and glass ween two such line behaves as a slit.

If the rullings are made on transparent sheet of ss then it is called transmission grating and if rullings e made on silvered glass then it is called reflection grating. reflection grating the rulling line behaves as slit.

he width of slit and opaque is called grating element. 'a' be the slit width and 'b' be the opaque width then

grating element = a+b

f N be the number of rulling lines in 1 inch glass plate then

$$N(a+b) = 1 \text{ inch.}$$

$$\Rightarrow a+b = \frac{1}{N} \text{ inch}$$

$$\Rightarrow a+b = \frac{2.54}{N} \text{ cm.}$$

If the path difference betn the rays emerges from two consecutive slit is $n\lambda$ then corresponding point will be bright and if the path difference is $(2n+1)\lambda/2$ then corresponding point will be dark.

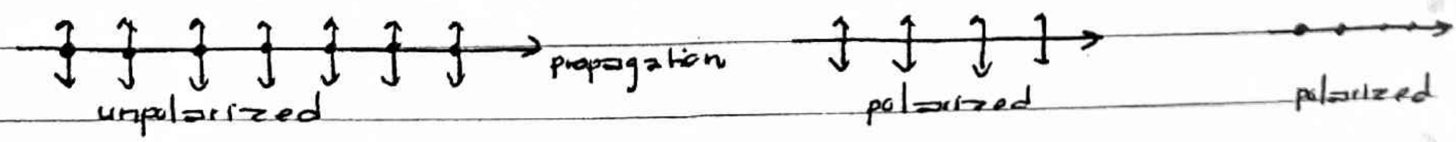
$\therefore$ For maxima, path diff $\Rightarrow (a+b)\sin\theta = n\lambda$

For minima, $(a+b)\sin\theta = (2n+1)\lambda/2$

* polarization.

The light rays in which vibrations are symmetrical and perpendicular to the direction of light wave is called unpolarized light wave. The vibration in horizontal direction is denoted by arrows and vibrations in vertical plane are denoted by dots.

The light wave in which vibrations are confined only in one plane perpendicular to wave propagation is called polarized light and phenomenon is called polarization. Therefore the polarized light wave contains either dot vibrations or arrow vibrations.



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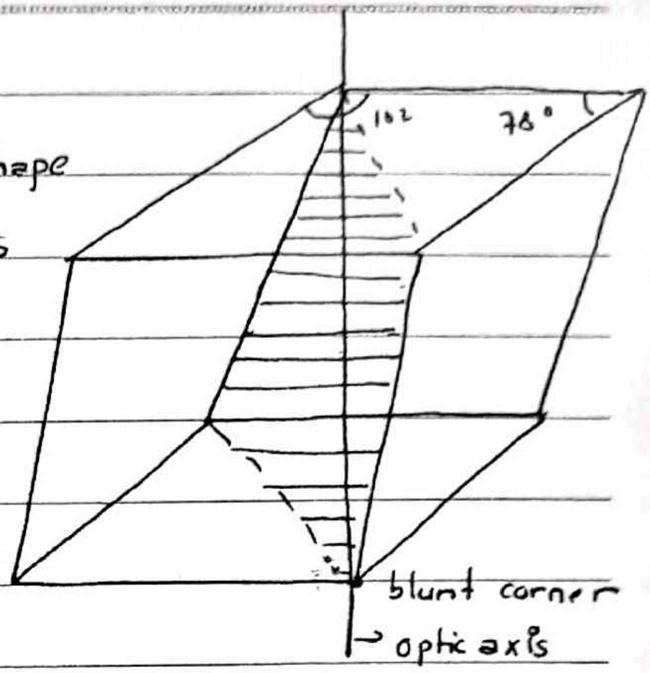
* Calcite crystal-

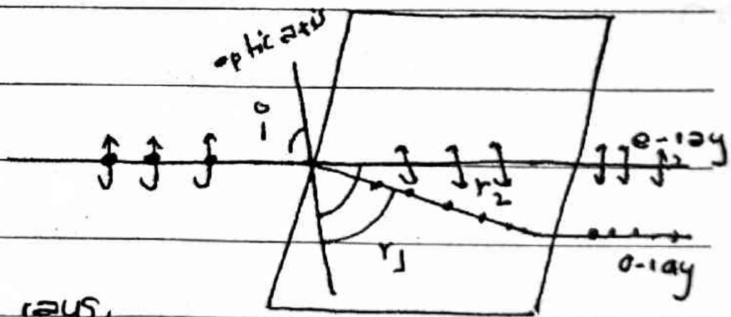
It is the crystal of rhombohedral shape bounded by six faces each of which is parallelogram with angle 102° and 78°. Therefore there are two corners at which all three faces meet at obtuse angle. These corners are called blunt corners. The line joining the blunt corners is called optic axis and any line parallel to this line behave optic axis.



The plane which contains optic axis and perpendicular to the opposite pair of faces is called principle plane. In the fig. shadded area represents a principle plane. There are three such planes.

· Double refraction-

when a ray of monochromatic light incident on one face of calcite crystal, It splits into two rays, one containing arrow vibration called extra-ordinary ray which does not follow the snell's law strictly. The velocity of e-ray is different at different surface of the crystal while other ray containing dot vibration called ordinary ray. The velocity of ordinary ray throughout the crystal is same but ~~+~~ along the optic axis both rays travel with same speed and we can not separate o-ray and e-ray.



The vibrations of e-ray are parallel to optic axis and vibrations of o-rays are perpendicular to the axis.

From fig, for ordinary ray, $\mu_o = \sin i / \sin r_1$

for extra-ordinary ray $\mu_e = \sin i / \sin r_2$

but $r_2 > r_1$

$\therefore \mu_e < \mu_o$

$\Rightarrow \frac{c}{v_e} < \frac{c}{v_o}$ [v_e = velocity of e-ray

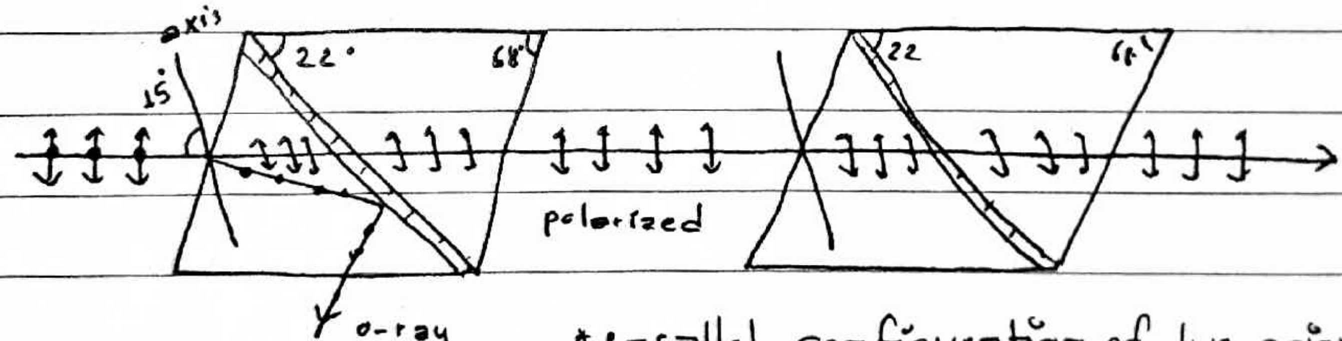
v_o = velocity of o-ray

$\Rightarrow v_o < v_e$ c = speed of light in vaccum]

i.e velocity of extra-ordinary ray is grater than velocity of ordinary ray. Such a crystal is called negative double refracting crystal. If $v_o > v_e$ then such a crystal is called positive double refracting crystal. Calcite is negative crystal and quartz is positive crystal.

The phenomenon in which an unpolarized splits into two rays is called double refractions.

* Nicol prism:



*parallel configuration of two prisms

A Nicol prism is made from calcite crystal whose length is three times its width. The calcite crystal is grounded with until the acute angle of principle plane reaches 68° instead of 71°. The piece is the cut into two parts perpendicular to principle axis and the two new end surfaces again combined together with cement called canada balsam.

The combination is then called Nicol prism. The canada Balsam behaves like rarer medium for ordinary ray and denser medium for extra-ordinary ray. The refractive index of canada Balsam lies between the refractive index of o-ray and e-ray.

$\mu_o = 1.66$, $\mu_b = 1.55$ and $\mu_e = 1.48$

when unpolarized light is incident on Nicol prism at an angle about 15° the ray after entering the crystel splits into two rays i.e ordinary ray and extra-ordinary ray. The refractive index of canada Balsam is such that the e-ray is transmitted while o-ray gets total internal reflection. Therefore the transmitted light is polarized with vibration parallel to optic axis. Therefore Nicol prism acts as polarizer.

If another Nicol prism is kept parallel to the 1st Nicol prism then the transmitted vibration is also transmitted through 2nd one if the optic axis of both prisms are parallel otherwise not. Thus Nicol prism acts as both polarizer and analyzer.

* optical activity-

If two Nicol prisms are held in crossed configuration and if the field of view of polarized light is observed, it will be completely appeared dark. Now if the quartz or calcite crystal cut its face perpendicular to its optic axis is inserted between the prisms such that the light normally incident on the crystal the field of view appears bright indicating that light is not cutoff by the analyzer. In order to cutoff the transmitted light the analyzer has to be rotate through certain angle.

This shows that the plane of polarization is rotated through certain angle when light passes through the crystal.

The ability to rotate the plane of polarization of plane polarized light by certain substance is called optical activity and substance is called optical active substance. For example sugar solution, sodium chloride etc.

* specific rotation-

The specific rotation for a given wavelength of light at given temperature is defined as the rotation produced by one decimeter long column of the solution containing 1 gm of optically active substance per cc of solution.

If 'S' be the specific rotation, θ be the rotation produced and 'c' be the concentration.

Then, $$S = \frac{\theta}{lc}$$

$$\Rightarrow \text{specific rotation} = \frac{\text{rotation produced in degree}}{\text{length in cm} \times \text{concentration gm/cc}}$$

$$\Rightarrow S = \frac{10\theta}{lc} \quad \text{....}(*)$$

[Here l in decimeter]

* Quarter wave plate-

A quarter wave plate is the double refracting crystal having optic axis parallel to its refracting face and thickness is adjusted in such a way that it introduced a path difference $\lambda/4$ or phase difference $\pi/2$ between o-ray and e-ray, propagating through it.

If μ_o be the refractive index of o-ray, μ_e be the refractive index of e-ray and t be the thickness of plate then,

optical path difference between o-ray and e-ray is

For o-ray $= \mu_o t$

For e-ray $= \mu_e t$

total path diff $= \mu_o t - \mu_e t$

For quarter wave plate, path diff $= \lambda/4$

$$\therefore \mu_o t - \mu_e t = \lambda/4$$

$$\Rightarrow (\mu_o - \mu_e)t = \frac{\lambda}{4} \quad \cdots (*) \quad [\text{For negative crystal}]$$

$$\text{or} \quad (\mu_e - \mu_o)t = \frac{\lambda}{4} \quad \cdots (**) \quad [\text{for positive crystal}]$$

* half wave plate -

A half wave plate is the double refractive crystal having optic axis parallel to its refracting face and thickness is adjusted in such a way that it introduced the path difference $\lambda/2$ or phase difference π between e-ray and o-ray, emerging from the plate.

$$\text{i.e} \quad (\mu_e - \mu_o)t = \frac{\lambda}{2} \quad \cdots (*)$$

$$\text{or} \quad (\mu_o - \mu_e)t = \frac{\lambda}{2} \quad \cdots (**)$$

respectively for positive and negative crystal.

chapter-5 2 ~~23~~ copy

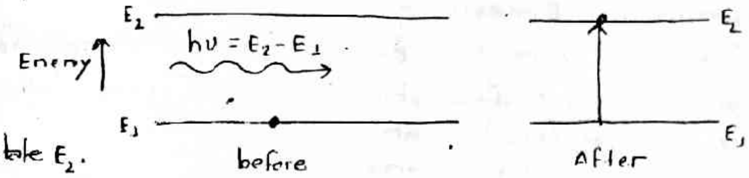
copy Laser and Fibre optics

* The word laser stands for Light amplification by stimulated emission of radiation. It becomes a valuable tool in a variety of field starting with medicine to communications. Laser is a light source but it is very much different from many of traditional light sources. Laser is not used a for illumination purposes as we use the other light sources. Lasers produces a highly directional and high intensity beam with narrow frequency range than that available from the common type of light sources. They are more widely used as a high power electromagnetic beam rather than a light beam.

* The processes of laser beam -

① Absorption -

An atom at lower energy level E_1 may absorb the incident photon and jumps to the excited state E_2. This transition is called induced absorption. corresponding to each absorption transition one photon disappears and one atom adds to the population at excited energy level.



This process may be represented as

$$A + h\upsilon \longrightarrow A^*$$

A = ground state atom

A^* = Excited atom.

The number of atoms per unit volume that makes upward transition from lower state to upper state per sec is called absorption transition rate. It is represented by

$$R_{abs} = -\frac{dN_1}{dt}$$

- sign indicates that population of level E_1 is decreasing.

In terms of increase population in level E_2, the transition rate will be

$$R_{abs} = \frac{dN_2}{dt}$$

N_1 = number of atoms per unit volume in state E_1

N_2 = number of atoms per unit volume in state E_2

This transition rate is directly proportional to the number of photons per unit volume in incident beam and population of lower level.

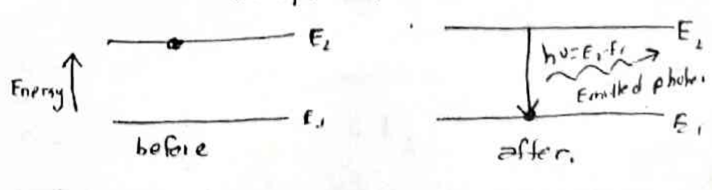
i.e $R_{abs} \propto \rho(\nu) N_1$

$\Rightarrow R_{abs} = B_{12}\, \rho(\nu)\, N_1$ - - - (*)

where B_{12} is called Eistein coefficient of Induced absorption.

② spontaneous emission -

An excited atom can stay at the excited level for an average lifetime if it is not stimulated by



any other agent during its short life time, the excited atom undergoes a transition to lower energy level by its own. During this transition, it gives up excess of energy in the form of photon. This process in which an excited atom emits a photon all by itself and without any external impetus is known as spontaneous emission.

The process is represented by

$$A^* \longrightarrow A + h\nu$$

The rate of spontaneous transition, $R_{sp} = -\frac{dN_2}{dt}$

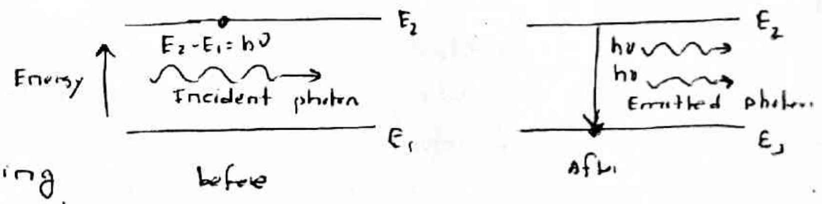
The number of photons generated will be proportional to the population of excited level only.

$$\therefore \; R_{sp} = A_{21} N_2 \quad --(**)$$

A_{21} = Einstein coefficient for spontaneous emission and is function of frequency and properties of the material, or it is the probability of spontaneous transition from level $2 \rightarrow 1$.

(3) Stimulated Emission -

An atom in the excited state need not to wait for spontaneous emission to occur. If a photon with appropriate energy ($E_2 - E_1 = h\nu$) interacts with the excited atom, it can trigger the atom to undergo transition to lower level and to emit another photon. The process of emission of photons by an excited atom through a forced transition occuring under the influence of an external agent is called stimulated emission. The process may be represented as



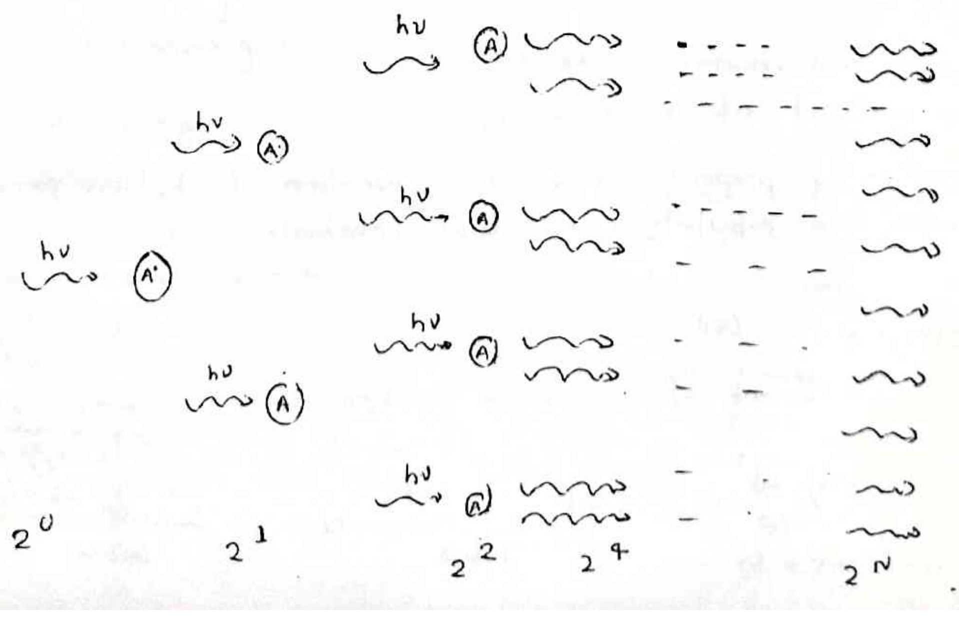
$$A^* + h\nu \longrightarrow A + 2h\nu$$

The rate of stimulated emission of photons is

$$R_{st} = B_{21}\, \rho(\nu)\, N_2 \quad ---(***)$$

The process of stimulated emission is controllable from outside. The photon emitted propagates in the same direction as that of stimulating photon. The light produced is directional, coherent and monochromatic.

Light amplification -



* population -

The atoms of each chemical element have their own characteristic system of energy levels. The energy difference betn the successive energy levels of an atom is of the order of 1 ev to 5 ev. The energy levels are common to all the atoms in a system which is composed of identical atoms. we can therefore say that a certain number of atoms occupy a certain energy state. The number of atoms per unit volume that occupy a given energy state is called population of that energy state. The population N of an energy level E depends on the temp. Thus, $N = e^{-E/KT}$ ... [Boltzmann's equation]

In a material, atoms are distributed differently in different energy states. The atoms normally tend to be their lowest possible energy level which need not be the ground state. At temperatures above 0 K, the atoms always have some thermal energy and therefore, they are distributed among the available energy levels according to their energy.

* population inversion -

In general, the number of atoms in ground state (lower state) N_1, is higher than the atoms in excited state (N_2). If $N_2 > N_1$ then we can say that the population is inversed.

under the population inversion condition the stimulated emission can produce a cascade of light. The first few randomly emitted photons (spontaneous) trigger stimulated emission of more photons and those stimulated photons induce still more stimulated emission and so on.

* pumping -

In order to realize and maintain the state of population inversion, it is necessary that atoms must be continuously promoted from lower state to exited state. Energy is to be supplied somehow to the laser medium to raise atoms from the lower to excited lowel and for maintaining population at the excited level at a value grater then that of lower energy. The process by which atoms are raised from the lower to excited level is called pumping.

* pumping methods -
(a) optical pumping
(b) Electrical pumping
(c) X-ray pumping
(d) chemical pumping

* optical pumping -

optical pumping is the use of photons to excite the atoms. A light source such as a flash discharge tube is used to illuminate the laser medium and the photons of appropriate frequency excite the atoms to an uppermost level. From there, they drop to metastable upper laser level and create the state of population inversion. optical pump sources are flash discharge tubes continously operating lamps, spark gaps or an auxiliary laser is sometimes used as the pump source. The pump source must have frequency higher than emitted photon. And the pumping level of the atom must not be a narrow level.

* Active medium -

Atoms in general are characterised by a large number of energy levels. However, all types of atoms are not suitable for laser operation. Even in a medium consisting of different species of atoms, only a small fraction of atoms of particular species are suitable for stimulated emission and laser action. Those atoms which causes light amplification are called active centers. The rest of the medium acts as host and supports active centres. The medium hosting the active centers is called as active medium.

An active medium is thus a medium which when excited, reaches the state of population inversion and eventually causes light amplification. The active medium may be solid, liquid or gas.

* Metastable states -

The metastable states are those state at which excited atoms lived upper level for an appreciable time. Atoms stay in metastable states for about 10^{-6} to 10^{-3} sec. This is 10^3 to 10^4 times longer than the time of stay of atom at excited levels. Therefore, It is possible for a large number of atoms to accumulate at a metastable state. The metastable state population can exceed the population of lower state and lead to the state of population inversion. If metastable states do not exist, there could be no population inversion, no stimulated emission and hence no laser operation.

Thus foundation of laser operation is the existence of metastable state.

* Pumping scheme:

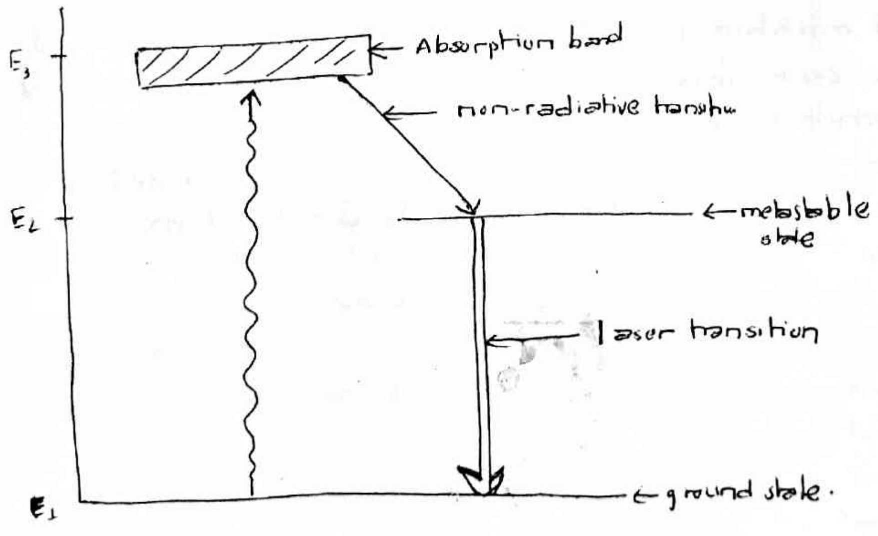


Fig: Three level scheme.

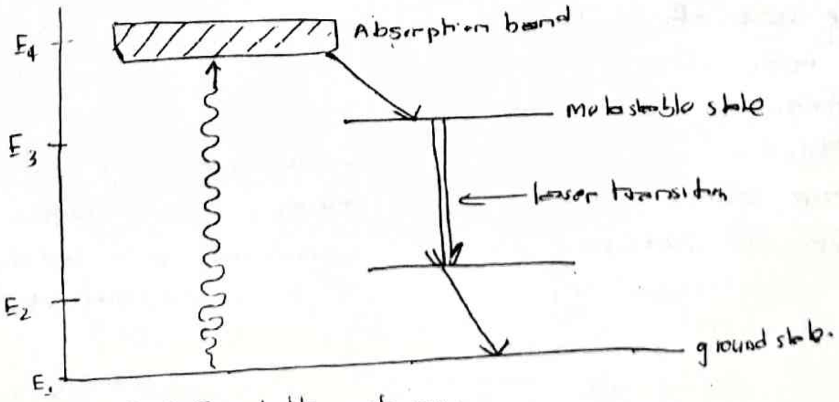
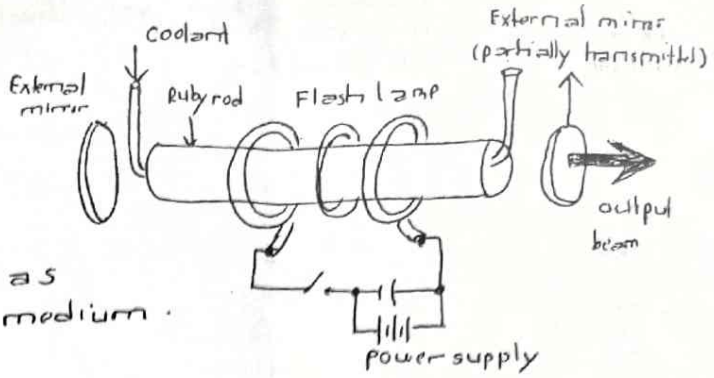


Fig: Four lable scheme.

* Ruby Laser -

The Ruby laser consists of a synthetic ruby crystal Al_2O_3, doped with chromium ions at a concentration of about 0.05 % by weight. The chromium ions constitute the active centres as they have a set of three energy levels suitable for realzing lasing action where as aluminium and oxygen atoms ae active medium.

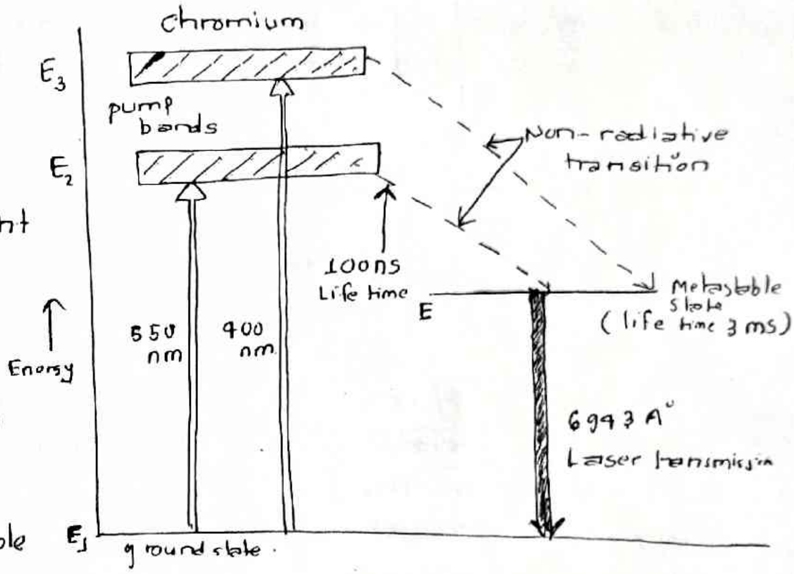


construction -

The ruby rod is in the form of cylinder with 4 cm long and 0.5 cm in diameter. The rod is fitted with helical flash tube. The tube is filled with xenon gas. At the end of the tube two external mirrors are fixed, one is fully reflecting and other is partially transmitting type. The system is cooled with the help of a coolant circulating around the ruby rod.

working -

It is a three level laser system. The energy level of chromium ions are as shown in fig. when the ruby rod is irradiate with an intense brust of white light from the xenon lamp the ground state cr^{+3} ions absorbs light in two pump bands one centered near 5500 A° and other at 4000 A°. They have average life time of 10^{-9} sec. Therefore, the excited cr^{+3} ions rapidly lose some of their energy and undergo non-radiative transition to level E. The level E becomes metastable states having lifetime of 3×10^{-3} sec.



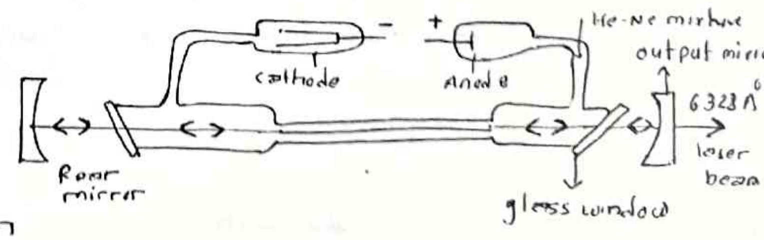
The transition from E to E_1 is radiative and under normal population condition produces spontaneous red fluorescence typical of ruby with a peak near 6943 A°.

under the intense excitation, population inversion occurs in E with respect to ground state E_1. Then one of the spontaneously emitted photons travelling parallel to the axis of ruby rod would initiate stimulated emissions. The photons make many passes through the medium building up the stimulated emissions in large way. The photons travelling in any other direction would be lost after few reflections.

As the one flash is used then another flash is followed. Therefore Ruby laser is a pulse laser and is not contineous.

* Helium - Neon laser -

Helium Neon laser is an atomic laser which employs a four-level pumping scheme. The active medium is the mixture of 10 parts of Helium and 1 part of Neon. Neon atoms are active centre and have energy level



suitable for laser transition.

construction- It consists of glass discharge tube about 30 cm long and 1.5 cm in diameter. The tube is filled with Helium and Neon gas in the ratio 10:1. Electrodes are provided in the tube to produce a discharge in a gas. They are connected to high voltage (10 kV) power supply. The tube is sealed with glass windows oriented at Brewster angle to the axis of tube. And mirrors are arranged externally.

Working-

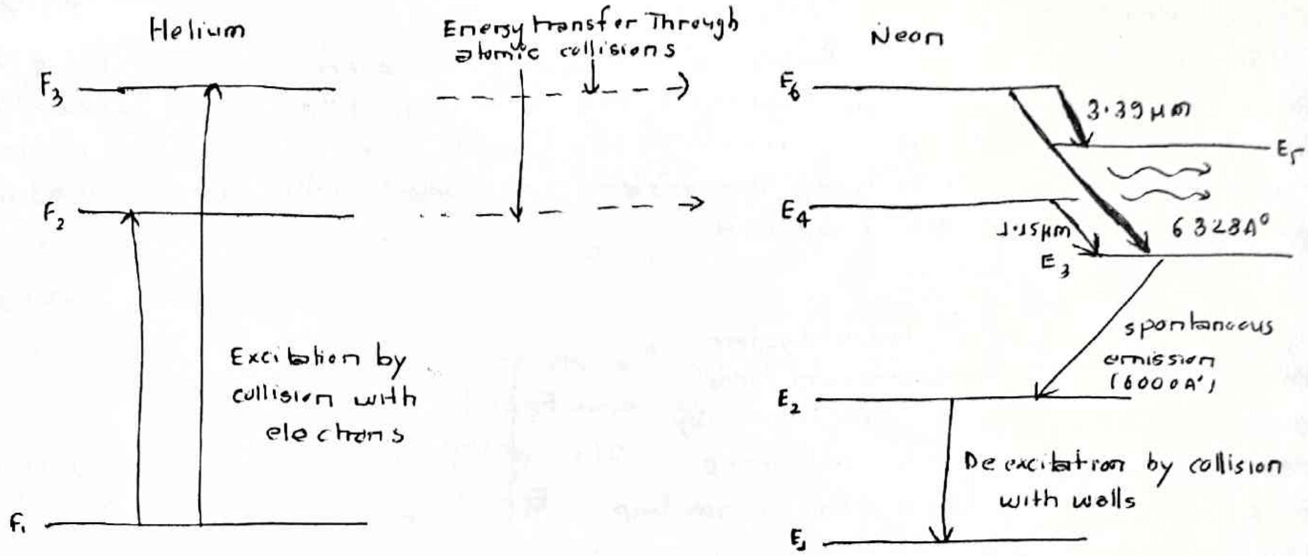
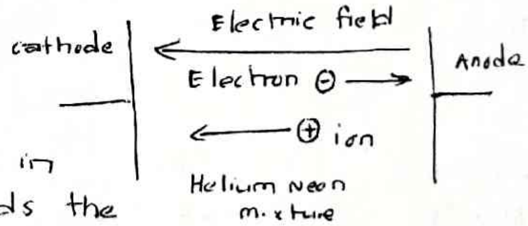


Fig: Energy levels of Helium and Neon atoms and transition betn the levels.



when the power is switched on with high voltage about 10 kV across the gas, It is sufficient to ionize the gas. The electrons and ions produced in the process of discharge are accelerated towards the anode and cathode resp. As shown in fig.

As electrons have smaller mass, they have higher velocity and transfer their kinetic energy to atoms of Helium and exites them to the level F_2 and F_3 as in fig at energy 19.81 eV and 20.16 eV resp. The excited helium atoms can return to the normal state by transfering their energy to neon atoms through collision. It is called resonance transfer of energy.

The energy level E_4 and E_6 of Neon concide with energy level F_2 and F_3 of Helium. Therefore Neon atoms excited to level E_4 and E_6. This is the pumping mechanism of He-Ne laser.

The levels E_6 and E_4 of Neon atoms are metastable states. Therefore, as the collisions go on, neon atoms accumulate in E_4 and E_6. At ordinary temperature E_5 and E_3 levels are sparsely populated thus population inversion is acheived betn E_6 to E_5, E_6 to E_3 and E_4 to E_3.

(i) $E_6 \rightarrow E_3$ corresponds to the laser transition of red colour at 6328 A°

(ii) $E_4 \rightarrow E_3$ corresponds to IR beam at wavelength 11500 A°

(iii) $E_6 \rightarrow E_5$ corresponds to IR region at 33900 A°

The one of the spontaneously emitted photon (E_3 to E_2) may trigger laser action. Finally the Neon atom is de excited by collision with walls and fresh cycle is started.

process

$He + e \longrightarrow He^* + e_1$ (same electron with laser action)

$He^* + Ne \longrightarrow Ne^* + He$

* uses of laser –

① Laser is used in material processing such as cutting, drilling, welding etc.

② Laser is used in nuclear energy.

③ It is used in micromaching. In this process a laser beam is focused which gives extremly high energy density on a very small area which causes vaporization of material.

④ It is also used in communication. since laser beam has enormous bandwdth and it permits 10 million telephone conversation or 8000 TV programmes simultanealy.

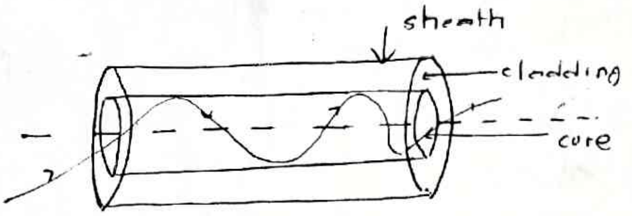
⑤ It is also used by military for war purposes.

⑥ It is also used in Radars.

⑦ Lasers are presently used for a variety of application in the medical field. Lasers are especially successful in the areas of opthalmology, Neurosurgery, Gastroenterology, dermatology, Gynecology etc.

* optical fibre-

An optical fibre is made of glass or clear plastic design to transmit light signal along its length. It works on the principla of total internal reflection.



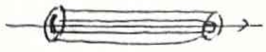
A practical optical fibre has three co-axial region. The central layer is the light guiding region known as core. The middle layer is known as cladding whose function is to make the light to be confined to the core. The outer most region is called sheath whose function is to protect the cladding and core from harmful influence of moisture. The diameter is about 150 μm.

* Types of optical fibre

- ophical fibre
  - monomode
  - multi-mode
    - step index
    - gradded index

* mono-mode optical fibre –

The mono-mode optical fibre is one in which core is thin about 9 μm in diameter and cladding is very thick. The mode is the path followed by the light signal, therefore in mono-mode fibre there is only one path for the transmission and it is along the axis of fibre.

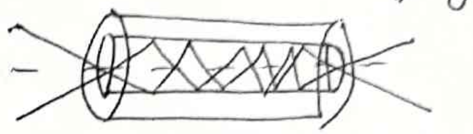


* Multi-mode -

The multimode fibre is one in which core is thick about 100 μm in diameter. It is further divided into two types -

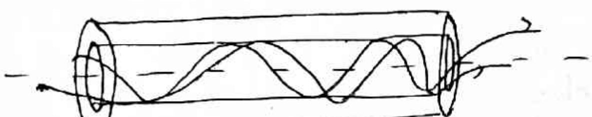
(a) step index -

In The step index fibre the refractive index of core is about 1.52 and the refractive index of cladding is about 1.48 and both refractive index are constant. Therefore there is a certain change in refractive index as we go towards cladding surface. Thus the path followed by light signal is zig-zag. The transmission of signal is not ~~com~~ smooth.

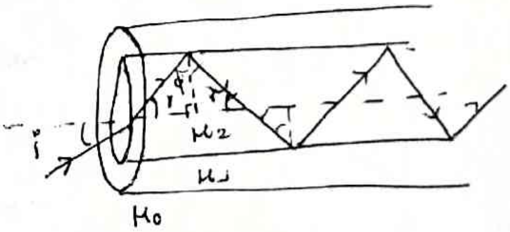


(b) Gradded index -

In case of gradded index fibre the refractive index is gradually decreasing from core to cladding surface. Therefore refractive index is maximum at core and minimum at cladding surface. Therefore It causes the periodic focusing of light propagating through the fibre, thus manufacturing is more complex.



* working of optical fibre
(Angle of acceptance)



optical fibre works on the principle of total internal reflection at the cladding surface. The signal is transmitted through the core and finally emerges out through another end due to multiple reflection. The maximum angle of incidence at which total internal reflection takes place is called angle of acceptance. The angle of incidence at cladding surface must be equal to or greater than critical angle.

Let μ_0 be the refractive index of the medium from which light enters and μ_1 and μ_2 be the refractive index of cladding and core resp. such that $\mu_2 > \mu_1$.

Let i be the angle at which light enters the fibre and 'r' be the angle of refraction. Let φ be the angle of incidence at cladding surface.

From snell's law, to the launching face,

$$ {}_0\mu_2 = \frac{\sin i}{\sin r} \quad ---(1)$$

but $r = 90 - \phi$

$$\Rightarrow \sin r = \sin(90-\phi)$$

$$\Rightarrow \sin r = \cos\phi \quad --(2)$$

From (1) and (2) $\quad {}_0\mu_2 = \dfrac{\sin i}{\cos\phi}$

$$\Rightarrow \frac{\mu_2}{\mu_0} = \frac{\sin i}{\cos\phi}$$

$$\Rightarrow \mu_2 = \frac{\sin i}{\cos\phi} \quad [\mu_0 = 1]$$

If $\phi = \phi_c$ (critical angle) then,

$$\mu_2 = \frac{\sin i}{\cos\phi_c}$$

$$\Rightarrow \cos\phi_c = \frac{\sin i}{\mu_2} \quad ---(3)$$

Again, from core to cladding, $\mu_1 = \frac{\sin\phi_c}{\sin 90}$ [when $\phi = \phi_c$]

$$\Rightarrow \frac{\mu_1}{\mu_2} = \sin\phi_c \quad ---(4) \quad [\sin 90^\circ = 1]$$

squaring and adding (3) and (4)

$$\sin^2\phi_c + \cos^2\phi_c = \frac{\mu_1^2}{\mu_2^2} + \frac{\sin^2 i}{\mu_2^2}$$

$$\Rightarrow 1 = \frac{\mu_1^2 + \sin^2 i}{\mu_2^2}$$

$$\Rightarrow \sin^2 i = \mu_2^2 - \mu_1^2$$

$$\Rightarrow \sin i = \sqrt{\mu_2^2 - \mu_1^2}$$

$$\Rightarrow i = \sin^{-1}\sqrt{\mu_2^2 - \mu_1^2} \quad ----(5)$$

This is the angle of acceptance at air side or angle of acceptance at which the light signal is transmitted from one end to another.

* uses of optical fibre -

① optical fibre is used mainly for the transmission of signal to a large distance in the form of cable. It is used in telecommunication, about 500000 talks through one fibre at a time.

② It is used for Television broadcasting.

③ It is useful in computer networking.

④ The fibre optic endoscope is used to inspect internal organ for diagonstic purposes.

⑤ It is used as direction signalling radars for war weapon

* Advantages -

① optical fibres are made from silica (SiO_2) which is one of the most abundant materials of the earth. The overall cost of fiber optic communication is lower than that of an equivalent cable communication system.

② They are smaller in size, lighter in weight and flexible yet strong.

③ In optical fibres information is carried by photons. photons are electrically neutral and can not be distrubed by high voltage fields, lightining etc.

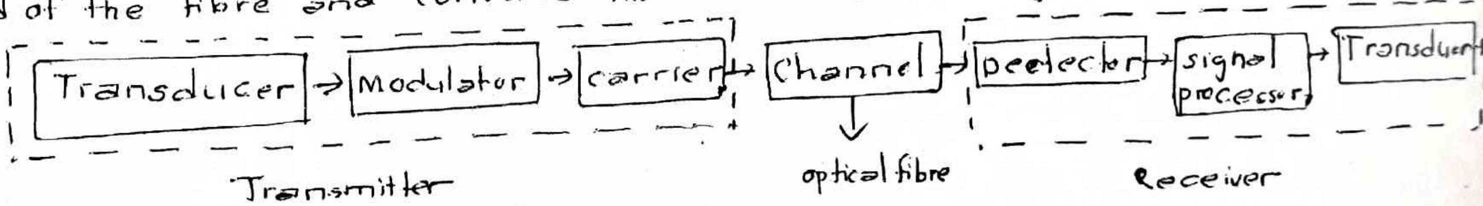
(4) As the optical fibre is made from insulator, It is not hazardous and no chance of sparking.

(5) The light waves propagating along the optical fibre are completly trapped within the fibre and can not leak out. The possibility of cross talk is minimized when optical fibre is used. Therefore transmission is more secure and private.

(6) The transmission loss per unit length of an optical fibre is about 4 dB/km. Therefore, longer cable runs betn repeaters are feasible. If copper cables are used, the repeaters are to be spaced at interval of 2 km. In case of optical fibres, the interval can be as large as 100 km and above.

* Fibre optic communication system -

A fibre optic communication system is very much similar to a traditional communications system and has three major components. A transmitter converts electrical signal to light signals, an optical fibre transmits the signals and a receiver captures the signals at the other end of the fibre and converts them to electrical signals.



Electrostatics
(chapter-6)

Date . .

* Electric field:

The electric field of the charge is defined as the region around the charge in which its influence can be experienced.

* Electric field intensity:

The force experienced by a unit positive charge placed at a point in an electric field of an another charge is called electric field intensity.

Let 'q' be the charge surrounded by its own electric field and q_0 be a test charge placed at a distance 'r' from charge 'q'. Then according to coulombs law, force between the charges q and q_0 is

$$\vec{F} = \frac{1}{4\pi\epsilon_0}\frac{qq_0}{r^2} \quad ----(1)$$

If $\vec{E}$ be the electric field strength then,

$$\vec{E} = \frac{\vec{F}}{q_0} \quad ----(2)$$

$$\Rightarrow \vec{E} = \frac{1}{4\pi\epsilon_0}\frac{q}{r^2} \quad ----(3) \quad [\text{From (1) and (2)}]$$

where $1/4\pi\epsilon_0$ is constant

and ϵ_0 = permitivity in free space

Numerically $1/4\pi\epsilon_0 = 9\times10^9\ Nm^2C^{-2}$

or $\epsilon_0 = 8.85\times10^{-12}\ N^{-1}m^{-2}C^2$

The electric field intensity is also defined as the number of electric lines of force passing normally through unit area.

i.e $\vec{E} = \frac{\text{electric lines of force (normally crossing)}}{\text{Area}}$

Date

$\Rightarrow \vec{E} = \frac{\text{flux}(\phi)}{\text{Area}(A)}$

$\Rightarrow \phi = \vec{E}A$ ---- (1)

* Gauss law of electrostatics:

It states that the total number of electric lines of forces crossing normally through certain area i,e electric flux is equal to $1/\epsilon_0$ times toal charge enclosed by that surface.

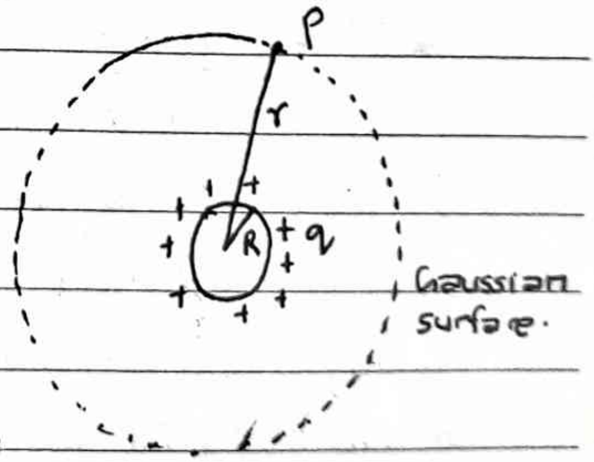
i,e electric flux $(\phi) = \frac{1}{\epsilon_0} \times$ charge enclosed (q)

$$\Rightarrow \phi = \frac{1}{\epsilon_0} \times q \quad \text{....} (*)$$

The surface enclosed is celled Gaussian surface.

* Application of Gauss law

① Electric field intensity due to charged sphere:



Let us consider a sphere of radius R having total charge 'q'. Let the point 'p' lies at a distance 'r' from the center of sphere where we have to find the intensity $\vec{E}$.

Now draw a Gaussian surface through point 'p' in the form of sphere of radius 'r'.

case I

when point 'p' lies outside the sphere $r > R$,

Now, from Gauss law, total flux $(\phi) = \frac{1}{\epsilon_0} \times$ charge enclosed (q)

$\Rightarrow \phi = q/\epsilon_0$

$\Rightarrow \vec{E}A = \frac{q}{\epsilon_0}$

$\Rightarrow \vec{E} = q/A\epsilon_0$ ----- (1)

where A = Area of Gaussian surface

$A = 4\pi r^2$ --- (2)

∴ From (1) and (2) $\vec{E} = \frac{1}{4\pi\epsilon_0}\frac{q}{r^2}$ -- (3)

Case II

when point 'p' lies at the surface of the sphere.

In this case $r = R$

∴ Field intensity at surface, $\vec{E} = \frac{1}{4\pi\epsilon_0}\frac{q}{R^2}$ ---- (4)

when point 'p' lies inside the sphere (III case):

In this case, $r < R$, and no charge enclosed by the Gaussian surface.

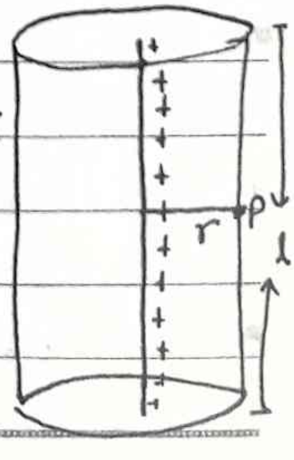
$\therefore \phi = \frac{1}{\epsilon_0} \times 0$

$\Rightarrow \vec{E}A = 0$

$\Rightarrow A \neq 0$

$\therefore \vec{E} = 0$ .... (5)

(2) Electric field intensity due to linear charged conductor:



Let us consider an infinitely long conductor with linear charge density λ.

i.e $\lambda = \frac{\text{charge}(q)}{\text{length}(l)}$

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Let 'p' be the point at a perpendicular distance 'r' from the conductor. To determine the electric field intensity draw the Gaussian surface in the form of cylinder of length l and radius 'r'.

From the Gauss law,

$$\text{electric flux } (\phi) = \frac{1}{\epsilon_0} \times \text{charge enclosed } (q)$$

$$\Rightarrow \phi = 1/\epsilon_0 \, q$$

$$\Rightarrow \vec{E} A = \frac{1}{\epsilon_0} \lambda l \quad \text{---}(*)$$

$$\Rightarrow \vec{E} = \frac{\lambda l}{A \epsilon_0}$$

but $A = 2\pi r l$ [Area of Gaussian surface]

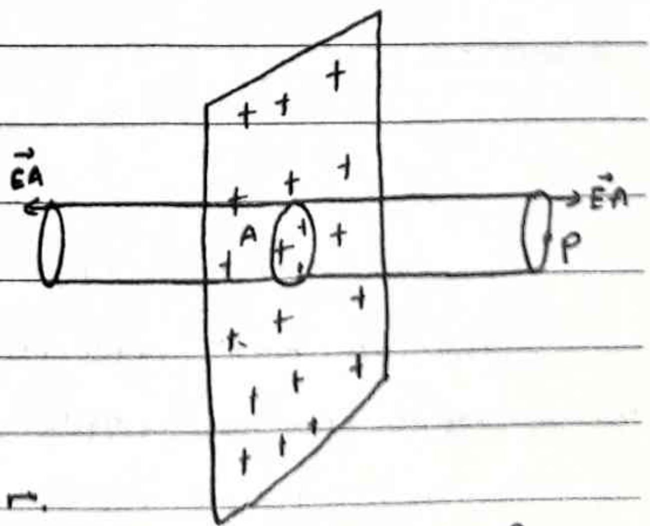
$$\therefore \vec{E} = \frac{\lambda l}{2\pi r l \epsilon_0}$$

$$\Rightarrow \vec{E} = \frac{\lambda}{2\pi \epsilon_0 r} \quad \text{---}(**)$$

(3) Electric field intensity due to infinite plane charged conductor:

consider a plane charged conductor having surface charge density σ. i.e

$$\sigma = \frac{\text{charge } (q)}{\text{Area } (A)} \quad \text{---}(*)$$



Let us calculate the field intensity at a point 'p' close to the conductor.

A convenient Gaussian surface is a closed cylinder of cross-section area 'A' to pierce the plane. since the flux from the charged plane conductor is normal to the plane, no flux cross through the curved surface of the cylinder. From symmetry the field has the same magnitude at the ends caps.

The flux through each end cap $= \vec{E}A$

∴ From Gauss law,

Total flux (ϕ) $= \frac{1}{\epsilon_0}$ times total charge enclosed (q)

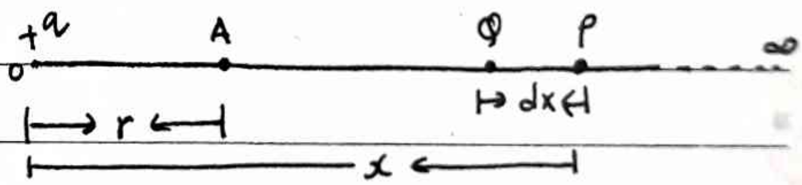
$$\Rightarrow \vec{E}A + \vec{E}A = \frac{1}{\epsilon_0}\sigma A$$

$$\Rightarrow 2\vec{E}A = \frac{\sigma A}{\epsilon_0}$$

$$\Rightarrow \vec{E} = \frac{\sigma}{2\epsilon_0} \quad \text{---}(**)$$

* Electric potential at a point:

The potential at a point in an electric field is defined as the amount of work done in moving a unit positive charge from infinity to that point against electric forces.



consider a point 'A' in an electric field at a distance 'r' from an isolated point charge $+q$ placed at 'O'.

Let $W_{\infty A}$ be the amount of workdone in moving unit positive charge from ∞ to point 'A' then potential at point 'A' will be

$$V_A = W_{\infty A} \quad \text{---}(1)$$

consider at any instant of time the positive charge is at point 'p' at a distance 'X' from 'O' then force experienced by unit charge is

$$\vec{E} = \frac{1}{4\pi\epsilon_0}\frac{q}{x^2} \quad --(2)$$

If a test charge moves an infinitesimal displacement $pq = dx$, then small amount of workdone from p to q is

$$dW = \vec{E}(-dx) \quad --(3)$$

From (2) and (3) $\quad dW = -\frac{1}{4\pi\epsilon_0}\frac{q}{x^2}\,dx \quad --(4)$

The negative sign indicates that the work is done againsts the electrostatic force.

$\therefore$ Amount of workdone from ∞ to A

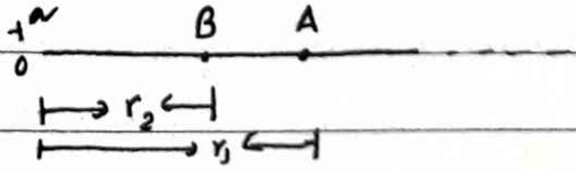
$$\int_{\infty}^{A} dW = -\frac{q}{4\pi\epsilon_0}\int_{\infty}^{r}\frac{1}{x^2}\,dx$$

$$\Rightarrow W_{\infty A} = -\frac{q}{4\pi\epsilon_0}\left[\frac{-1}{x}\right]_{\infty}^{r}$$

$$\Rightarrow V_A = \frac{q}{4\pi\epsilon_0}\frac{1}{r}$$

$\therefore$ potential at 'A', $V_A = \frac{1}{4\pi\epsilon_0}\frac{q}{r} \quad ----(5)$

If charge moved from distance r_1 to r_2 then there will be potential difference between them. If V_A be the potential at A and V_B be the potential at B then,



$$V_B - V_A = \frac{q}{4\pi\epsilon_0}\left[\frac{1}{r_2} - \frac{1}{r_1}\right]$$

$\Rightarrow$ potential difference $= \dfrac{\text{change in work done}}{\text{charge}}$

⇒ change in workdone = potential x charge

⇒ The much amount of workdone is converted into potential energy. Therefore,

Potential energy = potential x charge.

* Relation betn electric field and potential:

suppose the electric field at a point $\vec{r}$ due to charge distribution is $\vec{E}$ and electric potential at the same point is 'V'. suppose a point charge 'q' is displaced slightly from the point $\vec{r}$ to $\vec{r} + d\vec{r}$. Then force on the charge is

$$\vec{F} = q\vec{E} \quad \text{---- (1)}$$

Then workdone by this force during the displacement $d\vec{r}$ is

$$dw = \vec{F} \cdot d\vec{r}$$

$$\Rightarrow dw = q\vec{E} \cdot d\vec{r}$$

The change in workdone, $dw = -q\vec{E} \cdot d\vec{r}$ -- (2)

we have, Change in potential = $\frac{\text{change in workdone}}{\text{charge}}$

$$\Rightarrow dv = \frac{-q\vec{E} \cdot d\vec{r}}{q}$$

$$\Rightarrow dv = -\vec{E} \cdot d\vec{r}$$

$$\Rightarrow \vec{E} = -\frac{dv}{dr} \quad \text{---- (3)}$$

This is the relation between $\vec{E}$ and V.

It concludes that electric field strength is the negative gradient of potential.

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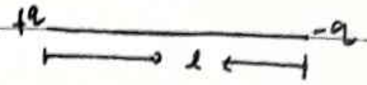
* Electric dipole:

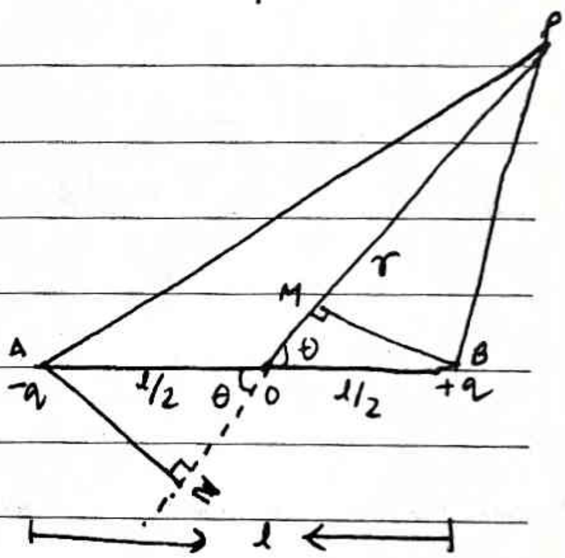


Two equal and opposite charges separated by a finite distance constitutes an electric dipole. The product of length of separation and magnitude of charge gives the dipole moment. It is denoted by $\vec{p}$ and

$$\vec{p} = \text{charge } (q) \times \text{dipole length } (l)$$

$$\Rightarrow \vec{p} = q\vec{l}$$

* Electric potential and field due to electric dipole:



Suppose AB is an electric dipole of length l consitutes by two equal and opposite charges $-q$ and $+q$ placed at A and B resp.

We have to find the electric potential and field at point 'p' at a distance 'r' from the center of dipole 'o'.

Let $\angle BOP = \theta$, then $\angle AON = \theta$

since $AB = l$, then $AO = BO = l/2$

Now, Draw the normal BM on op ($BM \perp op$) and normal AN on producing op. ($AN \perp$ to producing op)

Now, from Δ OBM,

$$\cos\theta = \frac{OM}{OB}$$

$$\Rightarrow OM = OB\cos\theta$$

$$\Rightarrow OM = \frac{l}{2}\cos\theta \quad \text{--- (1)}$$

Date

similarly from Δ OAN, $ON = \frac{l}{2}\cos\theta$ ---- (2)

Now, $PM \approx PB$

$\therefore PB = OP - OM$ $\quad [\because PM = OP - OM]$

$\Rightarrow PB = r - \frac{l}{2}\cos\theta$ ---- (3)

Again, $PN = ON + OP$

$PN = r + \frac{l}{2}\cos\theta$

but $PN \approx AP$

$\therefore AP = r + \frac{l}{2}\cos\theta$ ---- (4)

Then potential at P, due to $-q$ charge is

$$V_1 = -\frac{1}{4\pi\epsilon_0}\frac{q}{AP} \quad \text{---- (5)}$$

and potential at 'p' due to $+q$ charge at B is

$$V_2 = \frac{1}{4\pi\epsilon_0}\frac{q}{BP} \quad \text{---- (6)}$$

Then total potential at 'p' due to dipole,

$$V = V_1 + V_2$$

$$V = \frac{q}{4\pi\epsilon_0}\left[-\frac{1}{AP} + \frac{1}{BP}\right] \quad \text{---- (7)}$$

Now, substituting AP and BP from (3) and (4) in (7) we get

$$V = \frac{q}{4\pi\epsilon_0}\left[-\frac{1}{r + l/_2\cos\theta} + \frac{1}{r - l/_2\cos\theta}\right]$$

$$\Rightarrow V = \frac{q}{4\pi\epsilon_0}\left[\frac{-r + l/_2\cos\theta + r + l/_2\cos\theta}{r^2 - l^2/_4\cos^2\theta}\right]$$

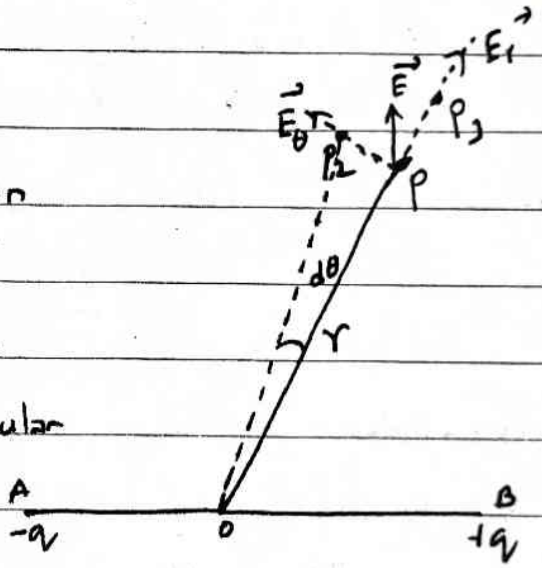
Date . .

$$\Rightarrow V = \frac{q}{4\pi\epsilon_0} \frac{l\cos\theta}{r^2 - l^2/4 \cos^2\theta}$$

For small dipole, $r^2 >> \frac{l^2}{4} \cos^2\theta$

$$\therefore V = \frac{ql\cos\theta}{4\pi\epsilon_0 r^2}$$

$$\Rightarrow V = \frac{p\cos\theta}{4\pi\epsilon_0 r^2} \quad ----(8) \qquad [\because ql = p]$$



The electric field at point 'p' can be obtained by resolving the field into two perpendicular components $\vec{E_r}$ along the op and $\vec{E_\theta}$ perpendicular to it.

In going from P to P_1 the angle does not change and distance changes by small amount 'dr'

$$\therefore PP_1 = dr \quad ---(9)$$

But in going from P to P_2 the angle changes by small amount 'dθ' while distance remains almost same from centre of dipole.

$$\therefore PP_2 = r\,d\theta \quad ----(10)$$

Now, electric field at p in PP_1 direction,

$$\vec{E_r} = -\frac{dV}{dP_1}$$

$$\Rightarrow \vec{E_r} = -\frac{dV}{dr}$$

$$= -\frac{d}{dr}\left[\frac{p\cos\theta}{4\pi\epsilon_0 r^2}\right]$$

$$\Rightarrow \vec{E_r} = -\frac{p\cos\theta}{4\pi\epsilon_0}\left[-\frac{2}{r^3}\right].$$

$$\Rightarrow \vec{E_r} = \frac{2p\cos\theta}{4\pi\epsilon_0 r^3} \quad ----(11)$$

Again, electric field at p in pP_2 direction,

$$\vec{E_\theta} = -\frac{dv}{pP_2}$$

$$\Rightarrow \vec{E_\theta} = -\frac{d}{rd\theta}\left[\frac{p\cos\theta}{4\pi\epsilon_0 r^2}\right]$$

$$= -\frac{p}{4\pi\epsilon_0 r^3}\left[-\sin\theta\right]$$

$$\therefore \vec{E_\theta} = \frac{p\sin\theta}{4\pi\epsilon_0 r^3} \quad ----(12)$$

$\therefore$ The resultant field at p.

$$E = \sqrt{E_r^2 + E_\theta^2}$$

$$= \sqrt{\left(\frac{2p\cos\theta}{4\pi\epsilon_0 r^3}\right)^2 + \left(\frac{p\sin\theta}{4\pi\epsilon_0 r^3}\right)^2}$$

$$= \frac{p}{4\pi\epsilon_0 r^3}\sqrt{4\cos^2\theta + \sin^2\theta}$$

$$= \frac{p}{4\pi\epsilon_0 r^3}\sqrt{4\cos^2\theta + 1 - \cos^2\theta}$$

$$E = \frac{p}{4\pi\epsilon_0 r^3}\sqrt{3\cos^2\theta + 1}$$

when $\theta = 0$, $V = \dfrac{p}{4\pi\epsilon_0 r^2}$

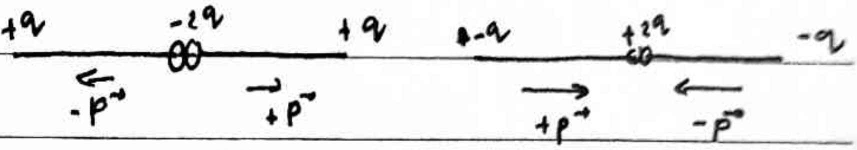
and $E = \dfrac{2p}{4\pi\epsilon_0 r^3}$

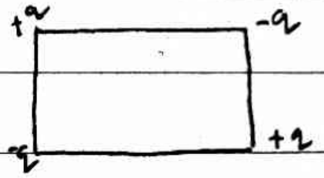
$\Big\}$ along the axis (position of end on) if $\theta = 90^\circ$ then,

$$\left.\begin{aligned} V &= 0 \\ \text{and } E &= \frac{p}{4\pi\epsilon_0 r^3} \end{aligned}\right\} \begin{aligned} &\text{(perpendicular bisector)} \\ &\text{(Broad side on position)} \end{aligned}$$

* Quadrupole:

An electric quardrupole is two equal and opposite dipoles that doesnot coincide in space so that their resultant effects at distant points do not quite cancel. The various configurations of quadrupole system is shown in fig.



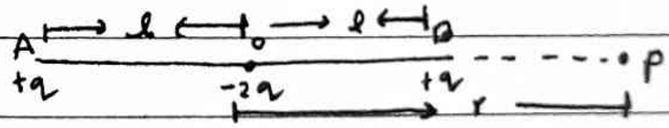


The quadrupole moment, $Q = 2ql^2$ ---(*)

where l is the distance between two charges in each dipole.

when point 'p' lies along the quadruple:

Let OA and OB be two dipoles each of length 'l'.



It may be noted that though the total charge on the system as a whole is zero, the potential and intensity is not zero.

Let us consider a point 'p' at a distance 'r' from the centre of quadrupole along the axis.

Let V_1, V_2 and V_3 be the potential at 'p' due to charges at A, O, and B resp. Then

$$V = V_1 + V_2 + V_3 \quad \text{---(1)}$$

Date . .

where, $V_1 = \frac{q}{4\pi\epsilon_0}\frac{1}{r+l}$ ---(1)

$V_2 = \frac{1}{4\pi\epsilon_0}\frac{(-2q)}{r}$ ---(3)

and $V_3 = \frac{1}{4\pi\epsilon_0}\frac{q}{r-l}$ ---(4)

$\therefore$ From (1), (2), (3) and (4)

$$V = \frac{q}{4\pi\epsilon_0}\left[\frac{1}{r+l} - \frac{2}{r} + \frac{1}{r-l}\right]$$

$$= \frac{q}{4\pi\epsilon_0}\left[\frac{r^2 - rl - 2r^2 + 2l^2 + r^2 + rl}{r(r^2 - l^2)}\right]$$

$$= \frac{q}{4\pi\epsilon_0}\frac{2l^2}{r^3(1 - l^2/r^2)}$$

$$V = \frac{Q}{4\pi\epsilon_0 r^3} \quad \text{-----(5)}$$ $\left[\frac{l^2}{r^2} \text{ is neglected}\right]$

And field,

$$E = -\frac{dV}{dr}$$

$$\Rightarrow E = -\frac{d}{dr}\left[\frac{Q}{4\pi\epsilon_0 r^3}\right]$$

$$\Rightarrow E = \frac{Q}{4\pi\epsilon_0}\frac{3}{r^4}$$

$$\Rightarrow E = \frac{3Q}{4\pi\epsilon_0 r^3} \quad \text{-----(6)}$$

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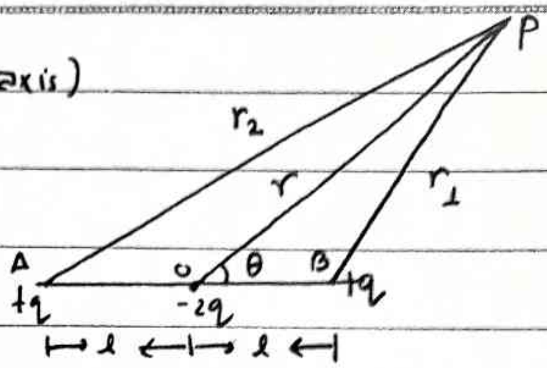
Case II: (when point p doesnot lies along axis)



The electric potential at P is

$$V = \frac{1}{4\pi\epsilon_0}\left[\frac{q}{r_1} - \frac{2q}{r} + \frac{q}{r_2}\right].$$

$$\Rightarrow V = \frac{q}{4\pi\epsilon_0}\left[\frac{1}{r_1} - \frac{2}{r} + \frac{1}{r_2}\right] \quad ----(7)$$

Let $\angle POB = \theta$ then

From Δ OPB, $r_1^2 = r^2 + l^2 - 2rl\cos\theta$

$$\Rightarrow r_1^2 = r^2\left[1 + \frac{l^2}{r^2} - \frac{2l\cos\theta}{r}\right].$$

$$\Rightarrow r_1 = r\left[1 + \frac{l^2}{r^2} - \frac{2l\cos\theta}{r}\right]^{1/2}$$

$$\Rightarrow \frac{1}{r_1} = \frac{1}{r}\left[1 - \frac{2l\cos\theta}{r} + \frac{l^2}{r^2}\right]^{-1/2}$$

$$\Rightarrow \frac{1}{r_1} = \frac{1}{r}\left[1 - \frac{1}{2}\left(-\frac{2l\cos\theta}{r} + \frac{l^2}{r^2}\right) + \frac{3}{8}\left(-\frac{2l\cos\theta}{r} + \frac{l^2}{r^2}\right)^2 + \cdots\cdots\right.$$

$$\Rightarrow \frac{1}{r_1} = \frac{1}{r}\left[1 + \frac{l\cos\theta}{r} - \frac{l^2}{2r^2} + \frac{3}{8}\times\frac{4l^2\cos^2\theta}{r^2} - \frac{3}{8}\times 4\frac{l\cos\theta}{r}\frac{l^2}{r^2} + \frac{l^4}{r^4} + \cdots\right.$$

$$\Rightarrow \frac{1}{r_1} = \frac{1}{r} + \frac{l\cos\theta}{r^2} - \frac{l^2}{2r^3} + \frac{3l^2\cos^2\theta}{2r^3} - \frac{3}{2}\frac{l^3\cos\theta}{r^4} + \frac{l^4}{r^5} + \cdots\cdots$$

Retaining the terms upto r^3 and neglecting higher order terms

$$\Rightarrow \frac{1}{r_1} = \frac{1}{r} + \frac{l\cos\theta}{r^2} + \frac{l^2}{2r^3}(3\cos^2\theta - 1) \quad -----(8)$$

similarly,

from Δ OPA, $r_2^2 = r^2 + l^2 + 2rl\cos\theta \quad --(9)$

$$\text{and} \quad \frac{1}{r_2} = \frac{1}{r} - \frac{l\cos\theta}{r^2} + \frac{l^2}{2r^3}(3\cos^2\theta - 1) \quad ---(9)$$

From (7), (8) and (9)

$$V = \frac{q}{4\pi\epsilon_0}\left[\frac{1}{r} + \frac{l\cos\theta}{r^2} + \frac{l^2}{2r^3}(3\cos^2\theta - 1) - \frac{2}{r} + \frac{1}{r} - \frac{l\cos\theta}{r^2} + \frac{l^2}{2r^3}(3\cos^2\theta - 1)\right]$$

$$\Rightarrow V = \frac{q}{4\pi\epsilon_0}\left[\frac{2l^2}{2r^3}(3\cos^2\theta - 1)\right]$$

$$\Rightarrow V = \frac{2ql^2}{4\pi\epsilon_0 r^3}\frac{(3\cos^2\theta - 1)}{2}$$

$$\Rightarrow V = \frac{Q}{4\pi\epsilon_0 r^3}\frac{(3\cos^2\theta - 1)}{2} \quad \text{....(10)}$$

* capacitance:

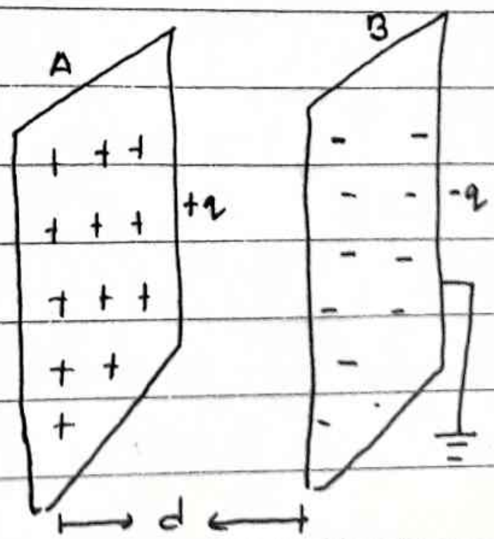
when a conductor given some charge, It is raised to some potential. If more and more charge is given, Its potential increases accordingly. If 'q' be the charge given and 'V' be the potential then

$$q \propto V$$

$$\Rightarrow q = CV \quad \text{...(1)}$$

'C' is constant called capacitance of the conductor.

* parallel plate capacitor:



The parallel plate capacitor consists of two conducting plates placed parallel to each other. Let 'A' be the area of each plate and 'd' be the distance between them. Let $\vec{E_0}$ be the field intensity.

If charge $+q$ is given to plate 'A' the charge $-q$ is induced on the left of plate B and $+q$

on the right side of plate B. when plate B is earthed the +q charges of plate B is flows to earth. Hence plate 'A' is completely positive charged and plate B is completely negative charged. Hence electric field ($\vec{E_0}$) is developed between the plates.

If 'σ' be the surface charge density then total charge enclosed,

$q = \sigma A$ ---(1)

According to Gauss law

$$\text{flux } (\phi) = \frac{1}{\epsilon_0} \times \text{charge enclosed } (q)$$

$$\Rightarrow \vec{E_0} A = 1/\epsilon_0 \times \sigma A$$

$$\Rightarrow \vec{E_0} = \frac{1}{\epsilon_0} \sigma \text{ ----(2)}$$

Also, the potential difference betn plates

$$V = \vec{E_0} d \text{ ---(3)}$$

From (2) and (3)

$$V = \frac{\sigma}{\epsilon_0} d \text{ ---(4)}$$

If 'c' be the capacitance of parallel plates then,

$$C = \frac{q}{V} \text{ ----(5)}$$

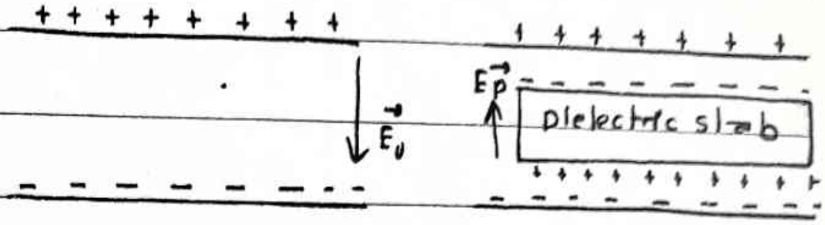
From (4) and (5) $$C = \frac{q \epsilon_0}{\sigma d}$$

$$\Rightarrow C = \frac{q}{\frac{q}{A} d} \epsilon_0$$

$$\Rightarrow C = \frac{\epsilon_0 A}{d} \text{ ---(*)}$$

* Dielectric constant:

consider a charged parallel plate capacitor as shown in fig. Let $\vec{E_0}$ be the electric field strength.



Now introduced a dielectric slab of non-polar molecules between the plates then due to induction, at the top of the slab negative charges -q and at the bottom surface positive charges +q appears. Due to these induced charges the electric field $\vec{E_p}$ is setup inside the slab which is opposite to $\vec{E_0}$.

Therefore resultant field, $\vec{E} = \vec{E_0} - \vec{E_p}$ .... (*)

The dielectric constant is defined as the ratio of electric field in absence of dielectric to the field in presence of dielectric and denoted by k.

Therefore, $k = \dfrac{\vec{E_0}}{\vec{E_0} - E_p}$

$\Rightarrow \quad k = \dfrac{\vec{E_0}}{\vec{E}}$ .... (**)

i.e $k > 1$

The dielectric constant is also defined interms of capacitance. If V be the potential in absence of slab and V' be the potential in presence of slab and 'd' be the distance betn plates then,

$V = \vec{E_0}\, d$ ....(***)

and $V' = \vec{E}\, d$ ...(****)

From (***) and (****) $\dfrac{V}{V'} = \dfrac{\vec{E_0}}{\vec{E}}$ .... (#)

From (**) and (#) $\Rightarrow V/V' = k$ .... (##)

If 'c' be the capacitance in absence of dielectric and 'c'' be the capacitance in presence of slab then,

$$V = q/C \quad \ldots\ldots (\cdot)$$

and $$V' = q/C' \quad \ldots\ldots (\cdot\cdot)$$

substituting $(\cdot)$ and $(\cdot\cdot)$ in (# #) we get

$$k = \frac{q/C}{q/C'}$$

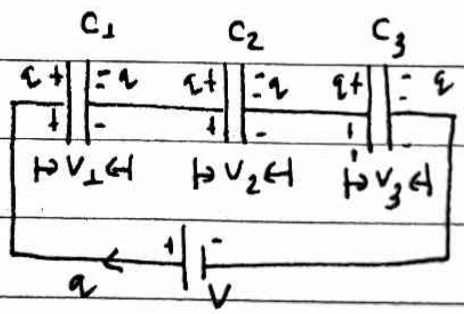
$$\Rightarrow k = \frac{C'}{C}$$

$$\Rightarrow C' = kC \quad \ldots\ldots (\cdot\cdot\cdot)$$

As $k > 1 \Rightarrow C' > C$

Therefore the significance of introducing the slab is to increase the capacitance of the capacitor.

* capacitor in series:



Let C_1, C_2 and C_3 be the capacitance of three capacitors connected in series with source of voltage 'v'.

If q be the charge supplied by the source then each capacitor has same amount of charge (q).

If V_1, V_2 and V_3 be the potential across the three capacitor resp

Then, $V_1 = \frac{q}{C_1}$, $V_2 = \frac{q}{C_2}$ and $V_3 = \frac{q}{C_3}$

but, $V = V_1 + V_2 + V_3$

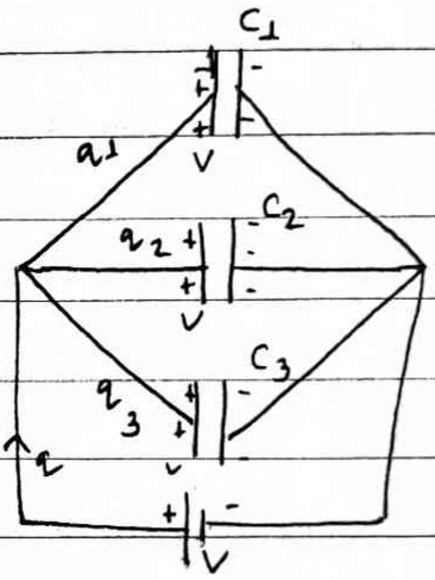
$$\Rightarrow V = \frac{q}{C_1} + \frac{q}{C_2} + \frac{q}{C_3}$$

$$\Rightarrow \frac{V}{q} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}.$$

If C be the total capacitance of series combination then,

$$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$$

* capacitors in parallel:



Let C_1, C_2 and C_3 be the capacitance of three capacitors connected in parallel with source of voltage 'V'.

As each capacitor is directly connected to the source each capacitor gains same voltage as of source (V) but charges on capacitor is different.

If q_1, q_2 and q_3 be the charges on the three capacitors and 'q' be the charge supplied by source

Then, $q = q_1 + q_2 + q_3$

but,

$$q_1 = VC_1, \quad q_2 = VC_2 \quad \text{and} \quad q_3 = VC_3$$

$$\therefore q = VC_1 + VC_2 + VC_3$$

$$\Rightarrow \frac{q}{V} = C_1 + C_2 + C_3$$

If C be the capacitance of parallel combination then

$$q/V = C$$

$$\therefore C = C_1 + C_2 + C_3$$

* Energy stored in a capacitor:

Let us consider a capacitor having capacitance 'C'. Let V be the potential of the capacitor being connected to a battery. If 'q' be the charge on plate of capacitor then,

$$q = VC \quad \ldots (1)$$

suppose a battery supplies a charge dq to the capacitor at constant potential V. Then according to the definition of potential difference the small amount of workdone

$$dW = V\,dq$$

$$\Rightarrow dW = \frac{q}{C}\,dq \qquad [\text{From } (1)]$$

Total workdone to add charge 'q' on the capacitor is

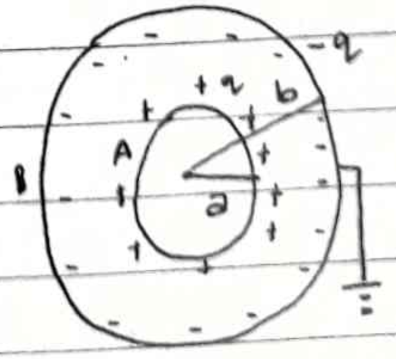
$$\int dW = \int \frac{q}{C}\,dq$$

$$\Rightarrow W = \frac{1}{2}\frac{q^2}{C} \quad \ldots (*)$$

since the workdone is stored inside the capacitor in the form of electric potential energy

$$\therefore \text{Energy stored} = \frac{1}{2}\frac{q^2}{C} \quad \ldots (*)(*)$$

* spherical capacitor:



spherical capacitor consists of two spherical shell (co-centric) of radius 'a' and 'b' resp. as shown in fig. when a positive charge +q is given to the inner spherical shell, it induces negative charge

Date . .

$-q$ on inner surface of outer shell and positive charge $+q$ on outer surface. If outer spherical shell is earthed then inner shell is completely positive and outer shell is completely negative.

The potential at any point on the surface of inner spherical shell, $V_A = \frac{1}{4\pi\epsilon_0}\frac{q}{a}$ ---(1)

similarly, the potential at any point on the surface of outer shell, $V_B = \frac{1}{4\pi\epsilon_0}\frac{(-q)}{b}$ ---(2)

Then total potential of capacitor,

$$V = V_A + V_B$$

$$\Rightarrow V = \frac{1}{4\pi\epsilon_0} q \left[\frac{1}{a} - \frac{1}{b}\right]$$

$$\Rightarrow V = \frac{q}{4\pi\epsilon_0}\left(\frac{b-a}{ab}\right)$$

$$\Rightarrow \frac{q}{V} = 4\pi\epsilon_0\left(\frac{ab}{b-a}\right)$$

If 'C' be the capacitance of the spherical shells then, $q/V = C$

$$\therefore \quad C = 4\pi\epsilon_0\left(\frac{ab}{b-a}\right) \quad ---(3)$$

If C' be the capacitance of inner spherical shell then,

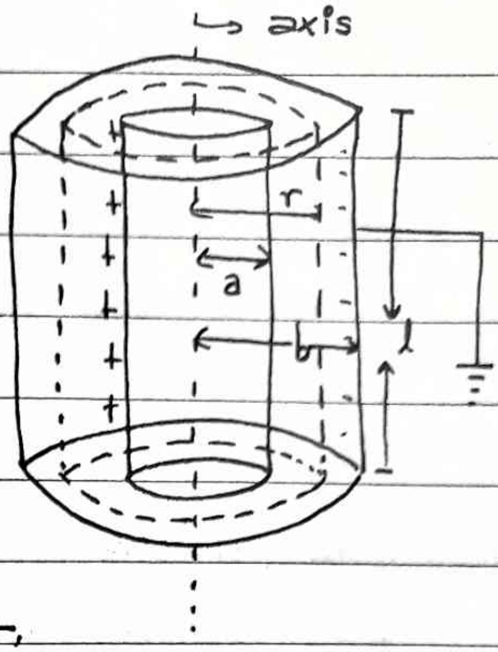
From eq^n (1) $\frac{q}{V_A} = 4\pi\epsilon_0 a$

$$\Rightarrow C' = 4\pi\epsilon_0 a \quad ---(4)$$

comparing (3) and (4), $C > C'$

This shows that two spherical ~~capacitors~~ shells leads to the increase in capacitance.

* cylindrical capacitor-



It consists of two co-axial cylinders of same length. The inner cylinder of radius 'a' is given some positive charge and outer cylinder of radius 'b' is connected to earth.

Let 'l' be the length of the cylinder, and $\vec{E}$ be the electric field strength at a distance 'r' from the axis. Then draw a Gaussian surface in the form of cylinder of radius 'r'.

Then according to Gauss law,

$$\phi = \frac{1}{\epsilon_0} \times \text{charge enclosed } (q)$$

$$\Rightarrow \phi = q/\epsilon_0$$

$$\Rightarrow \vec{E}A = q/\epsilon_0$$

$$\Rightarrow \vec{E} = \frac{q}{A\epsilon_0} \quad \text{----} (1)$$

Now the potential difference betn the cylinders

$$dV = -\vec{E}\cdot d\vec{r}$$

Integrating $\int dV = \int -E\,dr$

$$\Rightarrow V = \int -\frac{q}{A\epsilon_0}\,dr \quad \text{[From (1)]}$$

$$\Rightarrow V = \int_b^a -\frac{q}{2\pi r l \epsilon_0}\,dr \quad \left[A = \text{Area of Gaussian surface}\right]$$

$$\Rightarrow V = \frac{-q}{2\pi l \epsilon_0} \int_b^a \frac{1}{r}\, dr$$

$$\Rightarrow V = \frac{-q}{2\pi \epsilon_0 l} \left[\log_e r \right]_b^a$$

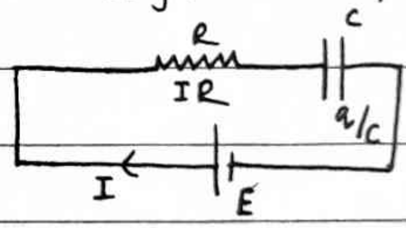
$$\Rightarrow V = \frac{-q}{2\pi \epsilon_0 l} \left[\log_e a - \log_e b \right]$$

$$\Rightarrow V = \frac{q}{2\pi \epsilon_0 l} \left[\log_e \left(\frac{b}{a}\right) \right]$$

$$\Rightarrow \frac{q}{V} = \frac{2\pi \epsilon_0 l}{\log_e (b/a)}$$

$$\Rightarrow C = \frac{2\pi \epsilon_0 l}{\log_e (b/a)} \quad \text{-----}(*)$$

* charging and discharging of capacitor through resistor:
(R-C circuit)



The resistor 'R' is connected to the capacitor 'C' and to the source of emf 'E' in series as shown in fig.

suppose the charge on the capacitor and current in the circuit are 'q' and 'I' resp. at time 't'.

Then potential drop on the capacitor $= q/C$ ---(1)

and on the resistor $= IR$ --(2)

Also the charge deposited on the positive plate of capacitor in time dt is, $dq = I\,dt$ ---(3)

$$\Rightarrow I = \frac{dq}{dt} \quad \text{---}(4)$$

But from the circuit above, $E = IR + \frac{q}{C}$ ----(5)

$\Rightarrow E - q/C = IR$

$\Rightarrow EC - q = RC\,I$

$\Rightarrow EC - q = RC \dfrac{dq}{dt}$

$\Rightarrow \dfrac{dq}{EC-q} = \dfrac{1}{RC}\, dt$ ----(6)

Let $EC - q = x$ --(7)

then differentiating (7) w.r.to x

$-dq = dx$ --(8)

substituting (7) and (8) in (6)

$\Rightarrow \dfrac{-dx}{x} = \dfrac{1}{RC}\, dt$

Integrating on both sides $\displaystyle\int \frac{dx}{x} = \int \frac{-1}{RC}\, dt$

$\Rightarrow \log_e x = -\dfrac{1}{RC}\, t + k$ (constant) ---(9)

From (7) and (9) $\log_e (EC-q) = \dfrac{-t}{RC} + k$ --(10)

when $t=0,\ q=0$

$\therefore$ From (10) $\log_e (EC) = k$ --(11)

substituting (11) in (10)

$\log_e (EC-q) = \dfrac{-t}{RC} + \log_e (EC)$

$\Rightarrow \log(EC-q) - \log(EC) = -t/RC$

$\Rightarrow \log\left(\dfrac{EC-q}{EC}\right) = -\dfrac{t}{RC}$

$\Rightarrow 1 - q/EC = e^{-t/RC}$

Date . . .

$$\Rightarrow q = EC(1 - e^{-t/RC}) \quad \text{---- (12)}$$

If $t=0$, $q=0$

If $t=\infty$, $q = EC$ i.e maximum charge on capacitor.

$\therefore$ max. charge, $q_0 = EC$ -- (13)

$\therefore$ From (12) and (13)

$$q = q_0(1 - e^{-t/RC}) \quad \text{--- (14)}$$

This gives the relation of charging of capacitor.

The constant 'RC' has dimension of time and is called time constant of circuit. In one time constant, $t/RC = 1$

$$\therefore \quad q = q_0[1 - e^{-1}]$$

$$\Rightarrow q = 63\% \text{ of } q_0 \quad \text{--- (15)}$$

i.e At one time constant the charge on capacitor is 63% of maximum charge.

Discharging:

The resistor and capacitor are disconnected with the source after charging the capacitor fully. i.e If the plates of a charged capacitor are connected through a conducting wire, the capacitor gets discharged. Again there is a flow of charge through the wire and hence there is a current.

suppose the capacitor of capacitance 'C' has a charge q_0, at time $t=0$, the plates are connected to the resister R.

Let the charge on the capacitor be 'q' and current be I_0 at time 't' then, from circuit,

$$\frac{q}{C} + IR = 0$$

$$\Rightarrow q/C = -IR$$

$$\Rightarrow \frac{q}{C} = -R\frac{dq}{dt}$$

Date

$$\frac{dq}{q} = -\frac{1}{RC}\,dt$$

Integrating $\int \frac{dq}{q} = \int -\frac{1}{RC}\,dt$

$$\Rightarrow \log_e q = -\frac{t}{RC} + k \text{ (constant)} \quad ----(16)$$

when $t=0$, $q = q_0$

$\therefore \log_e(q_0) = k \quad ---(17)$

substituting (17) in (16)

$$\log_e q = -\frac{t}{RC} + \log_e q_0$$

$$\Rightarrow \log_e(q/q_0) = -t/RC$$

$$\Rightarrow q = q_0 e^{-t/RC} \quad ---- (18)$$

In principle, discharging is completed only at $t=\infty$

The constant RC is time constant.

At one time constant $t/RC = 1$

then $q = q_0 e^{-1}$

$$\Rightarrow q = \frac{q_0}{e}$$

$$\Rightarrow q = \frac{q_0}{2.178}$$

$$\Rightarrow q = 0.37\, q_0$$

$$\Rightarrow q = 37\% \text{ of } q_0$$

i.e 37% of discharging is completed in one time constant

Date

* Mechanism of metalic conduction-

In a metal valence electrons are alomost free and move randomly in all direction. The moving electrons collides with the atom and changes its path contineously. The distance betn two successive collision is called free path and the time betn two collision is called free time.

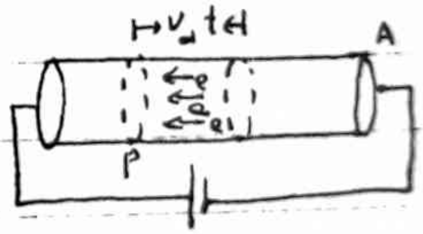
Let $s_1, s_2, \cdots$ be the free path of electron then mean free path, $\lambda = \frac{s_1 + s_2 + \cdots}{n}$

If $t_1, t_2 \cdots$ be the free time then mean free time,

$$\tau = \frac{t_1 + t_2 + \cdots}{n}$$

Then average velocity of electron, $\bar{v} = \frac{\lambda}{\tau}$ ----(*)

* current density of a conductor:



consider a conductor of area of cross-section 'A' containing 'n' electrons per unit volume. when an electric field is applied the free electrons move from right to left. If 'q' be the charge crossing the plate 'p' in time 't' then current fllowing through the conductor is $I = \frac{q}{t}$ ....(1)

In time 't' all the electrons cross the plate 'p' which line up to a distance $v_d t$ from it.

where v_d = drift velocity of electron.

The volume of a conductor, $= Av_d t$ ---(2)

∴ The total number of electrons through the plane 'P' in time $t = nAv_d t$ ....(3)

If 'e' be the charge of electron then total charge

$$q = enAv_d t \quad ....(4)$$

From (1) and (4)

$$I = \frac{enAv_d t}{t}$$

$$\Rightarrow I = nev_d A \quad ....(5)$$

$$\Rightarrow \frac{I}{A} = nev_d$$

$$\Rightarrow \vec{J} = ne\vec{v_d} \quad ...(6)$$

where $\vec{J}$ is called current density. i.e. current flowing per unit area.

* ohm's law:

It states that the current flowing through the conductor is directly proportional to the potential difference across its two ends. If I be the current and v be the potential then

$$V \propto I$$

$$\Rightarrow V = RI$$

$$\Rightarrow R = V/I \quad ..(*)$$

R is called resistance of the conductor.

* Resistivity and conductivity:

Resistance is the abstruction offered to flow of current by a conductor and depends upon the properties of the object where as resistivity is the obstruction offered to flow the current by unit area of the conductor and depends upon the properties of metal and does not depend upon shape and size of a material. It is a constant quantity where as resistance is a variable.

Resistivity of the conductor is defined as the ratio of electric field to the current density. It is denoted by ρ

$$\therefore \text{Resistivity } (\rho) = \frac{\text{Electric field } (E)}{\text{current density } (J)}$$

$$\Rightarrow \rho = \vec{E}/\vec{J} \quad \cdots (1)$$

If V be the potential difference applied across the length 'l' then,

$$\vec{E} = \frac{V}{l} \quad \cdots (2)$$

If I be the current flowing per unit area 'A' then

$$\vec{J} = \frac{I}{A} \quad \cdots (3)$$

substituting (2) and (3) in (1)

$$\rho = \frac{V/l}{I/A}$$

$$\Rightarrow V = \frac{I\rho l}{A} \quad \cdots (4)$$

comparing eq^n (4) with $V = IR$ we get,

$$R = \frac{\rho l}{A}$$

$$\Rightarrow \rho = RA/l \quad \cdots (5)$$

If $l = 1m$ and $A = 1m^2$ then, $\rho = R$ i.e. resistivity is the resistance of the conductor of unit length of unit area. Its unit is ohm-m.

The reciprocal of resistivity is called conductivity. It is given by $\sigma = 1/\rho$ ....(6)

* Relation between current density and conductivity:

we have from ohm's law, $I = V/R$ ...(1)

but $V = El$ ...(2)

$\therefore$ From (1) and (2) $I = \frac{El}{R}$ ...(3)

Again, resistivity, $\rho = \frac{RA}{l}$

$$\Rightarrow R = \rho l/A \quad ---(4)$$

From (3) and (4) $I = \frac{El}{\rho l} A$

$$\Rightarrow \frac{I}{A} = \frac{E}{\rho}$$

$$\Rightarrow \vec{J} = \sigma \vec{E} \quad ...(5)$$

* Resistivity interms of mean free path

(Atomic view of resistivity)

Let a metal wire is subjected to an electric field $\vec{E}$. The free electron experiences a force,

$$\vec{F} = e\vec{E} \quad --(1)$$

The acceleration of electron, $\vec{a} = \frac{\vec{F}}{m}$ ...(2)

From (1) and (2) $\vec{a} = \frac{e\vec{E}}{m}$ ...(3)

where m = mass of electron

e = charge of electron.

Let τ be the mean free time and $\vec{v_d}$ be the drift velocity of electron in time τ then

$$\vec{v_d} = \vec{u} + \vec{a}\tau$$

Initial velocity, $\vec{u} = 0$

$$\therefore v_d = a\tau \quad ---(4)$$

From (3) and (4)

$$\vec{v_d} = \frac{e\vec{E}}{m}\tau \quad ---(5)$$

Again we have resistivity, $\rho = \vec{E}/\vec{J}$

$$\Rightarrow \rho = \frac{\vec{E}}{ne\vec{v_d}} \quad ---(6)$$

From (5) and (6) $\rho = \dfrac{\vec{E}}{ne\, e\vec{E}/m\, \tau}$

$$\Rightarrow \rho = \frac{m}{ne^2\tau} \quad ---(7)$$

Also, average velocity of electron $\bar{v} = \dfrac{\lambda}{\tau} \quad ...(8)$

substituting τ from (8) in ρ

$$\Rightarrow \rho = \frac{m\bar{v}}{ne^2\lambda} \quad ---(9)$$

This is the required resistivity interms of mean free path (λ)

* Mobility:

Drift velocity per unit electric field is called mobility of the electrons. It is denoted by μ.

we have the relation between electrical conductivity and current density with electric field as

$$\vec{J} = \sigma \vec{E} \quad \text{---(1)}$$

Also If 'n' be the number of electrons per unit volume and 'e' be the charge of electron with drift velocity $\vec{v_d}$ then

$$\vec{J} = ne\vec{v_d} \quad \text{...(2)}$$

From (1) and (2)

$$\sigma \vec{E} = ne\vec{v_d}$$

$$\Rightarrow \frac{\sigma}{ne} = \frac{\vec{v_d}}{\vec{E}}$$

$$\Rightarrow \sigma = ne\frac{\vec{v_d}}{\vec{E}}$$

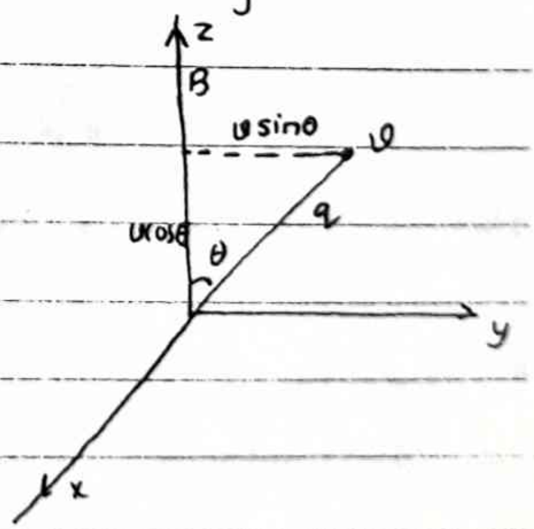
$$\Rightarrow \sigma = ne\mu \quad \text{----(3)}$$

* Magnetic effect of current:

when electric current passes through the conductor magnetic field is produced. For circular current magnetic lines of forces are linear and for linear current magnetic lines of forces are circular.

\# motion of charge particle in a uniform magnetic field:

consider a charge +q moving with velocity v inside the uniform magnetic field strength 'B'. suppose the magnetic field is along z-axis and charge particle moves along yz plane making angle 'θ' with the direction of magnetic field 'B'.



It is found that charge experience force F_m perpendicular to the plane of v and B such that

$$\vec{F_m} \propto q, \quad ---(1)$$

$$\vec{F_m} \propto \vec{B} \quad ---(2)$$

and also proportional to the component of velocity of charge in a direction perpendicular to the direction of B.

i,e $F_m \propto v \sin\theta \quad --(3)$

From (1), (2) and (3)

$$F_m \propto \vec{B} q v \sin\theta$$

$$\Rightarrow \vec{F_m} = k \vec{B} q v \sin\theta$$

For S.I unit $k=1$.

$$\therefore \vec{F_m} = \vec{B} q v \sin\theta \quad ---(*)$$

$$\Rightarrow \vec{F_m} = q \vec{V} \times \vec{B} \quad ----(4)$$

i,e $\vec{F_m}$ is perpendicular to both plane containing $\vec{v}$ and $\vec{B}$. Also $\vec{F_m}$ is always perpendicular to $\vec{V}$, therefore path of charge is circular in magnetic field

Therefore F_m provides the centripetal force,

$$\therefore F_m = \frac{mv^2}{r} \quad --(5)$$

From (*) and (5)

$$Bqv\sin\theta = \frac{mv^2}{r}$$

If $\theta = 90°$ then, $\quad Bqv = \frac{mv^2}{r}$

$$\Rightarrow Bev = mv^2/r$$

$$\Rightarrow \frac{Be}{m} = \frac{v}{r}$$

$$\Rightarrow \frac{Be}{m} = \omega$$

$\Rightarrow \quad \omega = Be/m$

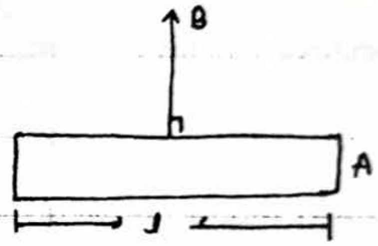
$\Rightarrow \quad 2\pi f = \dfrac{Be}{m}$

$\Rightarrow \quad f = \dfrac{Be}{2\pi m} \quad ---- (6)$

This is called cyclotron frequency.

× Magnetic force on a conductor:

consider a conductor of length 'l' and cross-section area 'A' carrying current I which is kept at right angle to the magnetic field $\vec{B}$.



If $\vec{J}$ be the current density, 'n' be the number of electrons per unit volume and 'V_d' be the drift velocity and 'e' be the charge on an electron then,

$$\vec{J} = ne\vec{v_d} \quad -- (1)$$

$$\Rightarrow \frac{I}{A} = ne\vec{v_d}$$

$$\Rightarrow I = nev_d A \quad ---(2)$$

we know that force on each electron $= Bev_d$ ---(3)

And the total number of electrons on conductor $= nAl$ ··(4)

∴ The force on a conductor will be

$$\vec{f_m} = Bev_d(nAl)$$

$$\Rightarrow \vec{f_m} = (nev_d A)Bl \quad --(5)$$

From (2) and (5)

$$\vec{F_m} = BIl \quad --(6)$$

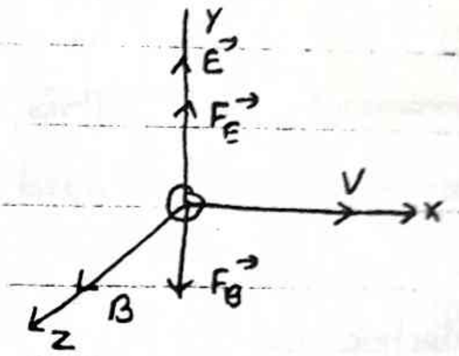
he fomagnetic force on a conductor.

* Lorentz force:

If both an electric and a magnetic field E and B acts on a charge particle, the total force on it can be expressed as

$$\vec{F} = q\vec{E} + q\,\vec{v}\times\vec{B} \quad \cdots (1)$$

This force is called lorentz force. The electric part of this force acts only on any charge particle but the magnetic part acts only on moving particles.



one common application of lorentz force occurs when a beam of charged particles passess through the region in which E and B fields are perpendicular to eachother and also perpendicular to the velocity of particles.

If B, V and E are oriented as shown in fig, then electric force $\vec{F_E} = q\vec{E}$ is in opposite direction to the magnetic force $\vec{F_B} = q\,\vec{v}\times\vec{B}$.

we can adjust the magnetic and electric field until the magnitude of forces are equal in which lorentz force is zero.

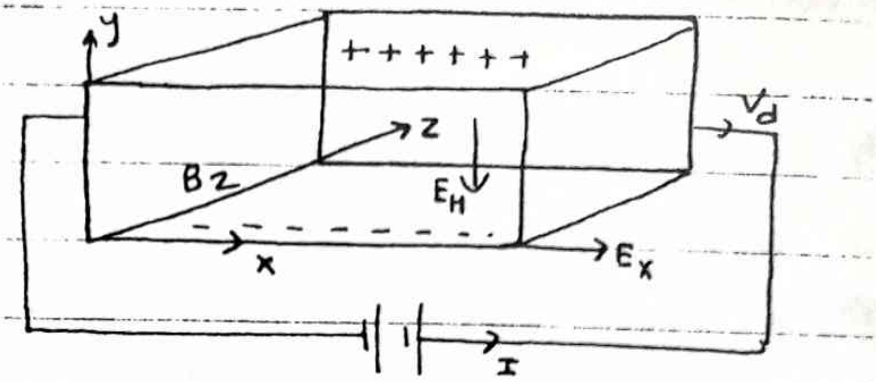
i.e $F_E = F_B$

$\Rightarrow qE = qVB$

$\Rightarrow V = \dfrac{E}{B} \quad \cdots\cdots (2)$

$\therefore$ The crossed E and B fields acts as velocity selector. only the particle with speed $V = E/B$ passess through the region unaffected by the magnetic force where as particles with other speeds are deflected. The value of speed is independent of the charge and mass of the particles.

* Hall effect:



when a magnetic field is applied perpendicular to a current carrying conductor, a voltage is developed across the specimen in a direction perpendicular to both current and magnetic field. This phenomena is called Hall effect. The voltage so developed is called Hall voltage.

consider a slab of materal subjected to an external electric field E_x along x-axis and magnetic field B_z along z-axis. As a result of electric field, a current density J_x will flow in the direction of E_x. Let the current be carried by electron of charge 'e'. The external magnetic field B_z will exert a transverse magnetic deflecting force on the electron which causes the electron to drift downward to the lower edge of the specimen. consequently the upper surface collects the excess of positive charge creats the transverse electric field E_H known as Hall field, which opposes the transverse drifting of electrons.

ultimately an equilibrium is reached in which force due to accumulation of electrons becomes equal to the magnetic force and so the flow of electrons stops. Thus net force on electron is zero. i,e lorentz force is zero.

i,e $\quad qE_H = qV_d B_z$

$\Rightarrow \quad E_H = V_d B_z \quad ---(1)$

Let 'n' be the free electron density, then current density is

$$J_x = -neV_d \quad --(2)$$

$$\Rightarrow \quad v_d = -\frac{J_x}{ne} \quad ---(3)$$

The negative sign indicates that the current is due to electron.

From (1) and (3) $E_H = -\frac{J_x}{ne} B_z$

$$\Rightarrow E_H = R_H J_x B_z \quad ---(4)$$

where $-1/ne = R_H$ is called Hall coefficient.

Also, we have, mobility $(\mu) = \frac{v_d}{E_x}$

$$\Rightarrow v_d = \mu E_x \quad ---(5)$$

From (1) and (5)

$$E_H = \mu E_x B_z \quad ---(6)$$

From (4) and (6)

$$R_H J_x B_z = \mu E_x B_z$$

$$\Rightarrow R_H = \mu \frac{E_x}{J_x}$$

$$\Rightarrow R_H = \mu/\sigma$$

$$\Rightarrow R_H = \rho\mu \quad ---(7)$$

If 'd' be the width and 't' be the thickness of material then

Hall voltage, $V_H = E_H d \quad ---(8)$

From (4) and (8)

$$V_H = R_H J_x B_z d$$

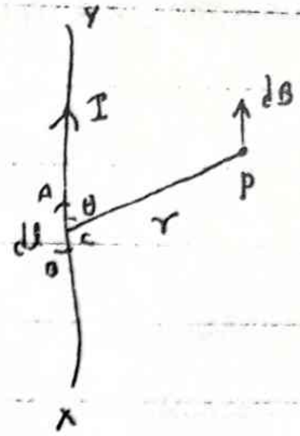
$$\Rightarrow V_H = R_H \frac{I}{A} B_z d$$

$$\Rightarrow V_H = R_H \frac{I}{dt} B_z d$$

$$\Rightarrow V_H = \frac{R_H B_z I}{t} \quad ---(9)$$

* Biot's and savarts law (Laplace law):

Biot's and savart's law is the law used to find the magnitude of magnetic field strength due to the electric current.



consider a conductor xy carrying current I as shown in fig. then arround xy the magnetic field is produced. Let 'p' be the point at a distance 'r' from the element length AB. Let θ' be the angle between the element and line joining point 'p' and to its centre 'C'.

According to Biot's and savart's law the strength of magnetic field 'dB' produced at 'p' due to current through element dl of AB is directly proportional to current, element length, sine angle betn dl and r. and inversely proportional to square of distance between element length to the point 'p'.

$\therefore \quad dB \propto I$

$dB \propto dl$

$dB \propto \sin\theta$

$dB \propto 1/r^2$

$\Rightarrow dB \propto \dfrac{I\, dl \sin\theta}{r^2}$

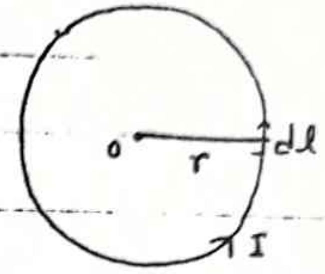
$\Rightarrow dB = \dfrac{\mu_0}{4\pi} \dfrac{I\, dl \sin\theta}{r^2}$

$\Rightarrow B = \int \dfrac{\mu_0}{4\pi} \dfrac{I\, dl \sin\theta}{r^2}$ -- (*)

where, $\mu = 4\pi \times 10^{-7} \mu m^{-1}$ called permeability of air or free space.

* Application of Biot's and savart's law -

(1)# consider a circular coil carrying current I. let r be the radius of the coil consider the element length dl of coil, then according to Biot's and savart's law, field at the centre of coil due to length dl is



$$dB = \frac{\mu_0}{4\pi} \frac{I\, dl \sin\theta}{r^2}$$

but $\theta = 90^0$

$$\therefore dB = \frac{\mu_0}{4\pi} \frac{I\, dl}{r^2}$$

Then total field due to whole coil,

$$\int dB = \int \frac{\mu_0}{4\pi} \frac{I\, dl}{r^2}$$

$$\Rightarrow B = \frac{\mu_0 I}{4\pi r^2} \int dl$$

$$\Rightarrow B = \frac{\mu_0 I}{4\pi r^2} 2\pi r \qquad \left[\because \int dl = \text{circumference of the coil}\right]$$

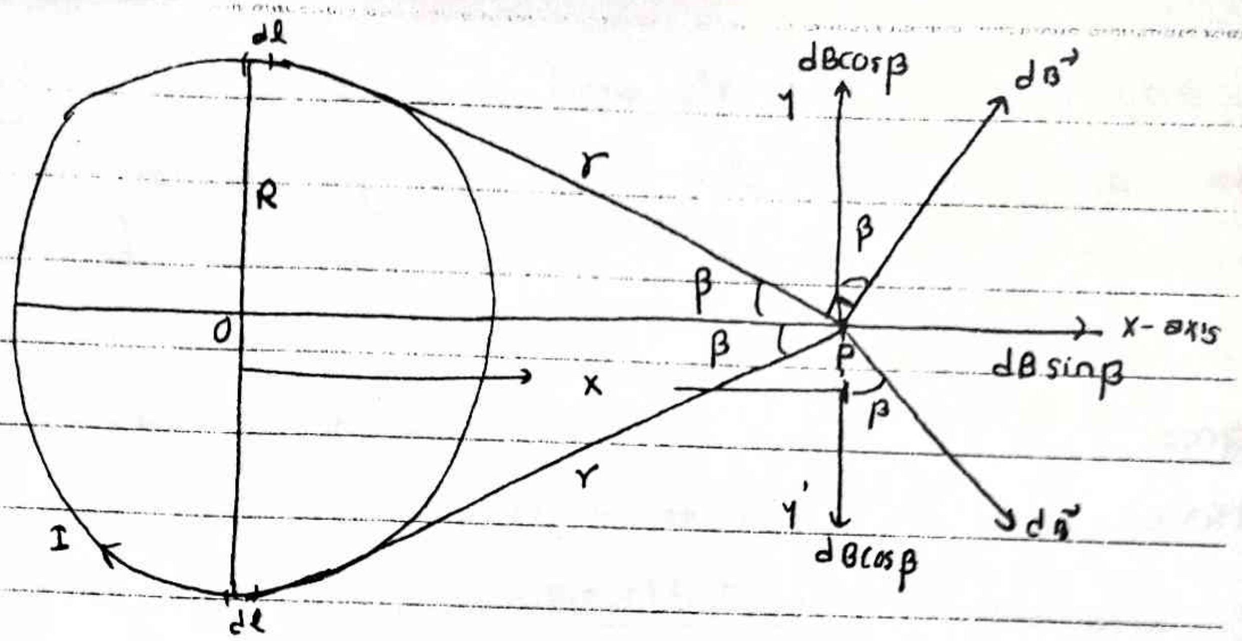
$$\Rightarrow B = \frac{\mu_0 I}{2r} \quad \text{----} (*)$$

This is the magnetic field strength at the centre of coil.

(2)# Magnetic field strength along the axis of coil:

consider a circular coil having radius 'R' carrying current 'I'. we have to find the field at a distance 'x' from the centre of coil, along the axis.

According to Biot's and savart's law, magnetic field



at point 'p' due to element length dl is (for upper element length]

$$dB = \frac{\mu_0}{4\pi}\frac{I\,dl\sin\theta}{r^2}$$

but $\theta = 90^\circ$, $\Rightarrow$ $dB = \frac{\mu_0}{4\pi}\frac{I\,dl}{r^2}$ ----(1)

This field is perpendicular to r.

The component of dB along two perpendicular axis as $dB\cos\beta$ along y-axis and $dB\sin\beta$ along x-axis. where β is the angle made by 'r' with axis.

If we consider a similar element diametrically opposite then the element would produced the same magnetic field dB at a point P. and would have the similar component $dB\cos\beta$ and $dB\sin\beta$. The components due to two current elements along yy' direction will cancel eachother as they are equal in magnitude and opposite in direction.

∴ magnetic field due to current element along px is $dB\sin\beta$.

∴ total field at P due to whole coil is

$$B = \int dB\sin\beta \quad ---(2)$$

From (1) and (2) $B = \int \frac{\mu_0}{4\pi} \frac{I\,dl}{r^2} \sin\beta$

$$= \frac{\mu_0 I \sin\beta}{4\pi r^2} \int dl$$

$$B = \frac{\mu_0 I}{4\pi r^2} \sin\beta \; 2\pi R \quad \text{--(3)} \quad \left[\because \int dl = \text{circumference of a coil}\right]$$

Also, from fig. $\sin\beta = R/r$ --(4)

From (3) and (4)

$$B = \frac{\mu_0 I}{4\pi r^2} \frac{R}{r} 2\pi R$$

$$\Rightarrow B = \frac{\mu_0 I R^2}{2r^3} \quad \text{---(5)}$$

Also, From fig. $r^2 = \sqrt{R^2 + x^2}$

$$\Rightarrow r^3 = (R^2 + x^2)^{3/2} \quad \text{--(6)}$$

From (5) and (6)

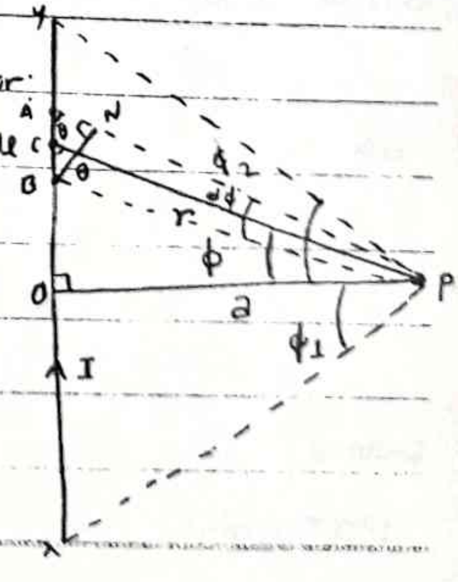
$$B = \frac{\mu_0 I R^2}{2[R^2 + x^2]^{3/2}}$$

If the coil contains N-turns then, $B = \frac{\mu_0 N I R^2}{2[R^2 + x^2]^{3/2}}$ ---(*)

(2)* magnetic field strength due to a striaght conductor:

consider a infinitely long streight conductor carrying current I. Let 'P' be the point at a perpendicular distance 'a' from the conductor where we have to find the strength of magnetic field.

consider an element length AB



of length dl whose center is 'C'.

Let $PC = r$, $\angle PCO = \theta$, $\angle APB = d\phi$, $\angle OPX = \phi_1$, $\angle OPY = \phi_2$

As the point C and A are very close to each other then,

$\angle BAN$ is also equal to θ

$\therefore \angle BAN = \theta$

Now Draw normal BN on AP then from Δ ABN

$$\sin\theta = \frac{BN}{AB}$$

$$\Rightarrow BN = AB\sin\theta$$

$$\Rightarrow BN = dl\sin\theta \quad ---(1)$$

Also, from Δ BPN,

$$\text{central angle, } (d\phi) = \frac{BN}{r}$$

[$d\phi$ can be taken as the angle substanded by BN of radius r at P]

$$\Rightarrow r\,d\phi = BN \quad --(2)$$

From (1) and (2)

$$dl\sin\theta = r\,d\phi \quad --(3)$$

According to Biot's and savart's law, field at P due to length dl is,

$$dB = \frac{\mu_0}{4\pi}\frac{I\,dl\sin\theta}{r^2} \quad --(4)$$

substituting (3) in (4) we get

$$dB = \frac{\mu_0}{4\pi}\frac{I}{r^2}r\,d\phi$$

$$\Rightarrow dB = \frac{\mu_0 I\,d\phi}{4\pi r} \quad --(5)$$

Also, from Δ CPO, $\cos\phi = a/r$

$$\Rightarrow r = \frac{a}{\cos\phi} \quad ---(6)$$

From (5) and (6)

$$dB = \frac{\mu_0 I}{4\pi a} \cos\phi \, d\phi$$

∴ The total field at 'p' due to whole conductor.

$$\int dB = \int_{-\phi_1}^{\phi_2} \frac{\mu_0 I}{4\pi a} \cos\phi \, d\phi$$

$$\Rightarrow B = \frac{\mu_0 I}{4\pi a} \left[\sin\phi \right]_{-\phi_1}^{\phi_2}$$

$$\Rightarrow B = \frac{\mu_0 I}{4\pi a} \left[\sin\phi_1 + \sin\phi_2 \right]$$

If the conductor is infinitely long then $\phi_1 = \phi_2 = \pi/2$

$$\therefore B = \frac{\mu_0 I}{4\pi a} [1+1]$$

$$\Rightarrow B = \frac{\mu_0 I}{2\pi a} \quad \text{----}(*)$$

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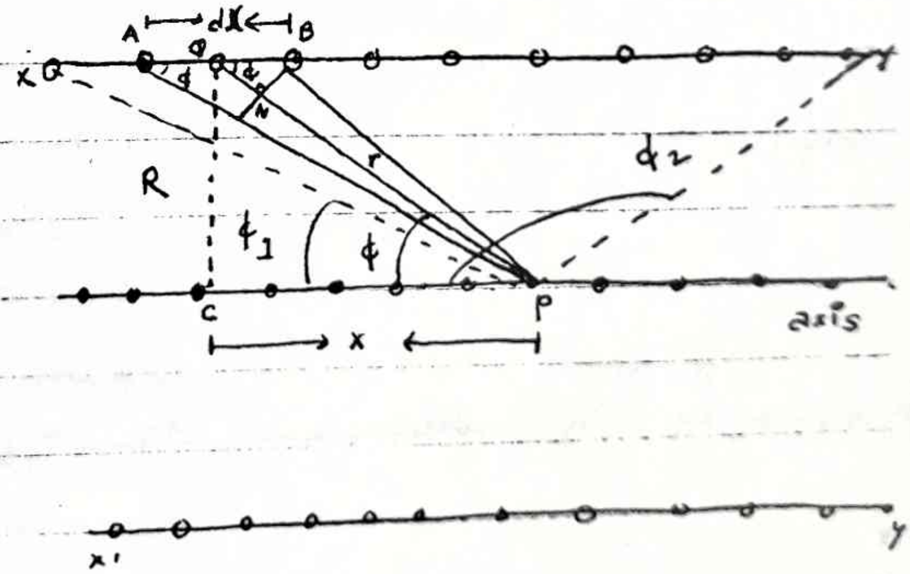
④ Magnetic field due to solenoid:



Let us consider a long solenoid of radius R carrying current I. Let the number of turns per unit length of the solenoid be 'n'. suppose we have to find the magnetic field strength B at any point 'p' on the axis of the solenoid at a distance 'r' from the centre of element length of AB of length dx and

The magnetic field B at point 'p' can be regarded as the resultant of field B due to number of coils such as AB of length dx, in which the solenoid may be imagined to be divided. Each of such coil has $n\,dx$ turns. If the point 'p' lies at a distance x from the center of the coil AB (perpendicular distance) then according to Biot's and savart's law magnetic field at a point on the axis of circular coil,

$$dB = \frac{\mu_0 I R^2}{2r^3} \times \text{number of turns}$$

$$\Rightarrow dB = \frac{\mu_0 I R^2 n\,dx}{2r^3} \quad \text{---(1)}$$

From fig.: $QP = r$, $\angle BQP = \phi = \angle QPC$, $QC \perp$ on axis.

$BN \perp AP$,

Join point p to two end points of solenoid.

Let $\angle XPC = \phi_1$, $\angle YPC = \phi_2$ and $\angle APB = d\phi$, $\angle BPA = \phi$

From Δ ABN,

$$\sin\phi = \frac{BN}{AB}$$

$$\Rightarrow BN = AB \sin\phi$$

$$\Rightarrow BN = dx \sin\phi \quad \text{---(2)}$$

Also, from fig.

$$d\phi = \frac{BN}{r}$$

$$\Rightarrow BN = r\,d\phi \quad \text{--(3)}$$

from (2) and (3) $\quad dx \sin\phi = r\,d\phi$

$$\Rightarrow \quad dx = \frac{r\,d\phi}{\sin\phi} \quad \text{--- (4)}$$

substituting (4) in (1)

$$dB = \frac{\mu_0 I R^2 n}{2r^3} \frac{r\,d\phi}{\sin\phi} \quad \text{-- (5)}$$

But $\sin\phi = \frac{R}{r}$ --- (6) [From Δ QPC]

From (5) and (6) $dB = \frac{\mu_0 n I R^2 r}{2r^3} \frac{d\phi}{R} r$

$$\Rightarrow dB = \frac{\mu_0 n I}{2} \frac{R}{r} d\phi$$

$$\Rightarrow dB = \frac{\mu_0 n I}{2} \sin\phi \, d\phi \quad \text{--- (7)}$$

Then total field due to solenoid.

$$\int dB = \int_{\phi_1}^{\phi_2} \frac{\mu_0 n I}{2} \sin\phi \, d\phi$$

$$\Rightarrow B = \frac{\mu_0 n I}{2} \left[-\cos\phi \right]_{\phi_1}^{\phi_2}$$

$$\Rightarrow B = \frac{\mu_0 n I}{2} \left[\cos\phi_1 - \cos\phi_2 \right]$$

For infinitely long solenoid, $\phi_1 = 0$, $\phi_2 = \pi$

$$\Rightarrow B = \frac{\mu_0 n I}{2} \left[1 - (-1) \right]$$

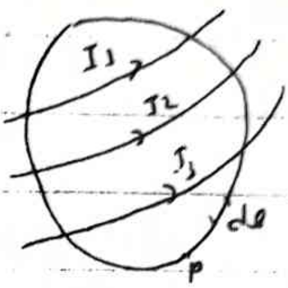
$$\Rightarrow B = \mu_0 n I \quad \text{--- (8)}$$

If point 'p' lies at one end of solenoid then $\phi_1 = 0$, $\phi_2 = \pi/2$

then $B = \frac{\mu_0 n I}{2} \left[1 - 0 \right] \Rightarrow B = \frac{\mu_0 n I}{2}$ --- (9)

* Ampere's law:

It states that the line integral of magnetic field round a closed path is equal to the μ_0 times the current enclosed by that path.



As shown in the fig the closed path 'p' encloses current $I_1, I_2, \ldots$ let B the magnetic field any point of path. Now consider an element length dl of the path at p. Then line integral of magnetic field $= \int \vec{B} \cdot \vec{dl}$

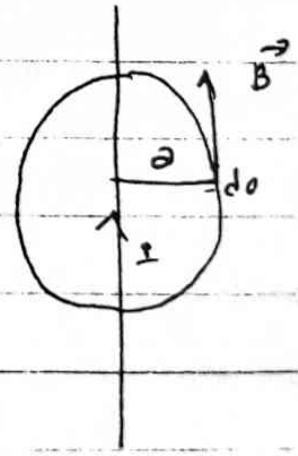
From Ampere's law,

$$\int \vec{B} \cdot \vec{dl} = \mu_0 (I_1 + I_2 + \ldots)$$

$$\Rightarrow \int \vec{B} \cdot \vec{dl} = \mu_0 I \quad -- (x)$$

Application of Ampere's law:

① Magnetic field due to a straight conductor.



Let us consider a straight conductor carrying current I. As we know that for linear current the magnetic lines of forces are circular and tangent to the lines of forces give the direction of magnetic field. To find the magnetic field at any point at a perpendicular distance 'a' from the conductor, draw a circle of radius 'a'.

Let 'B' be the magnetic field at a distance 'a' from the conductor, then according to Ampere's law

$$\int \vec{B} \cdot \vec{dl} = \mu_0 I$$

$$\Rightarrow \int B\,dl\cos\theta = \mu_0 I$$

since B has only tangential component, $\theta = 0$, i.e angle between B and dl.

$$\therefore \int B\,dl = \mu_0 I$$

$$\Rightarrow B\int dl = \mu_0 I \quad \text{---(1)}$$

But $\int dl = 2\pi a$, circumference of closed path.

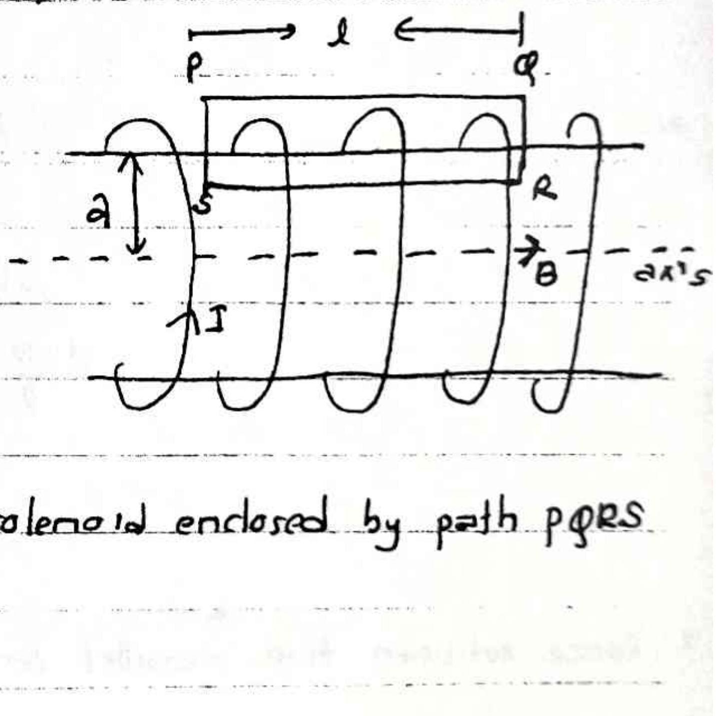
$\therefore$ From (1) $\quad B\,2\pi a = \mu_0 I$

$$\Rightarrow B = \frac{\mu_0 I}{2\pi a} \quad \text{---(2)}$$

(2) Magnetic field due to a solenoid:

The magnetic field inside the solenoid is parallel to the axis and magnetic field outside the solenoid is zero.

Let us consider a rectangular path PQRS as ~~s~~hown in fig.



Let N be the number of turns of solenoid enclosed by path PQRS and l be the length of the path.

i.e $PQ = RS = l$

Then total line integral of magnetic field of path PQRS is

$$\oint_{PQRS} \vec{B}\cdot d\vec{l} = \int_P^Q \vec{B}\cdot d\vec{l} + \int_Q^R \vec{B}\cdot d\vec{l} + \int_R^S \vec{B}\cdot d\vec{l} + \int_S^P \vec{B}\cdot d\vec{l}$$

But the line integral from P to Q, ~~the~~ lies outside the solenoid, $B = 0$

The line integral from Q to R, $\vec{B}$ and $d\vec{l}$ are perpendicular to each other, i,e $\theta = 90^\circ$

line integral from R to S, angle between $\vec{B}$ and dl is 0°

and line integral from S to P again $\theta = 90^\circ$

$$\therefore \oint_{PQRS} \vec{B}\cdot\vec{dl} = \int_P^Q \underset{0}{B}\, dl \cos 0^\circ + \int_Q^R B\, dl \underset{0}{\cos 90^\circ} + \int_R^S B\, dl \cos 0^\circ + \int_S^P B\, dl \cos 90^\circ$$

$$= 0 + 0 + \int_R^S B\, dl \cos 0^\circ + 0$$

$$\Rightarrow \int_{PQRS} \vec{B}\cdot\vec{dl} = \int_R^S B\, dl$$

$$\Rightarrow \int_{PQRS} \vec{B}\cdot\vec{dl} = B\int_R^S dl$$

$$\Rightarrow \int_{PQRS} \vec{B}\cdot\vec{dl} = Bl \quad ---- (1)$$

Now from ampere's law, $\int_{PQRS} \vec{B}\cdot\vec{dl} = \mu_0 N I \quad --(2)$ [For N turns]

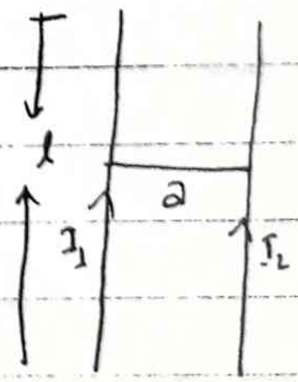
From (1) and (2) $\mu_0 N I = Bl$

$$\Rightarrow B = \frac{\mu_0 N I}{l}$$

$$\Rightarrow B = \mu_0 n I \quad --(3) \qquad [n = N/l]$$

* Force between two parallel conductors: (magnetic force)

consider two straight conductor carrying current I_1 and I_2 resp. Let 'a' be the perpendicular distance between the conductors. Then the wire carrying current I_1 produces magnetic field B_1 whose magnitude at the location of wire carrying current I_2 is



$$B_1 = \frac{\mu_0 I_1}{2\pi a} \quad ---(1)$$

Thus the wire which carries current I_2 can be considered to be in an external magnetic field B_1.

$\therefore$ The force experienced by conductor 2nd will be

$$F_{21} = B_1 I_2 l \quad --(2)$$

From (1) and (2)

$$F_{21} = \frac{\mu_0 I I_2 l}{2\pi a}$$

$$\Rightarrow \frac{F_{21}}{l} = \frac{\mu_0 I_1 I_2}{2\pi a} \quad ---(3)$$

similarly the wire carrying current I_2 produces magnetic field B_2 whose magnitude at the location of wire carrying current I_1 is $B_2 = \frac{\mu_0 I_2}{2\pi a} \quad ---(4)$

Thus the wire which carries current I_1 can be considered to be in an external magnetic field B_2

$\therefore$ Force experienced by conductor 1st will be

$$F_{12} = B_2 I_1 l \quad --(5)$$

From (4) and (5)

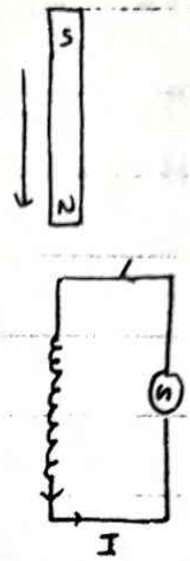
$$F_{12} = \frac{\mu_0 I_2 I_1 l}{2\pi a}$$

$$\Rightarrow \frac{F_{12}}{l} = \frac{\mu_0 I_1 I_2}{2\pi a} \quad ---(6)$$

From (3) and (6)

$$\vec{F_{21}} = \vec{F_{12}}$$

* Faradays law of electromagnetic induction



The phenomenon by which an electric current is produced in a closed coil due to the relative motion between it and magnetic field is called electromagnetic induction. The current so produced is called induced current and the emf under which the current flows is called induced emf.

The magnetic lines of forces per unit cross-section area represents the magnetic induction B, when the lines of forces are perpendicular to the area.

i,e $B = \frac{\text{magnetic lines of forces (perpendicular to area)}}{\text{Area}}$

$\Rightarrow B = \frac{\text{magnetic flux} (\phi_m)}{\text{Area} (A)}$

$\Rightarrow \phi_m = BA$

If the coil contains N turns then,

$$\phi_m = NBA \text{ ---- } (*)$$

whenever there is change in flux linked with the coil, induced emf is produced.

The induced emf last as the change in flux continuous.

The magnitude of induced emf is directly proportional to the rate of change of flux.

i.e $\quad E \propto d\phi_m/dt$

$\quad \Rightarrow E = -k\, d\phi_m/dt$

In S.I. unit k = 1. $\quad \therefore \quad E = -\frac{d\phi_m}{dt} \text{ --- } (**)$

- sign indicates that the induced emf oppose the change in flux.

* self induction:

when current is flowing in closed loop, it produces magnetic field and hence magnetic field has flux through the area bounded by the loop. If the current changes with time, the flux also changes and hence emf is induced and is called self induction.

∴ The flux linked with a coil (loop) is directly proportional to the current flowing through it.

i.e $\phi_m \propto I$

$\Rightarrow \phi_m = LI$ --- (1)

L is called coefficient of self induction.

If $I = 1$ A and then $\phi_m = L$,

the self inductance of coil is numerically equal to the flux linked with coil when unit current flowing through it.

Also, From Faraday's law,

$$E = -\frac{d\phi_m}{dt} \quad \text{---(2)}$$

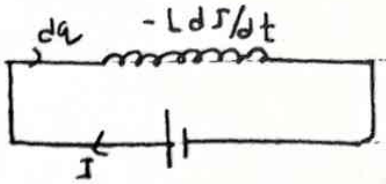
From (1) and (2) $\quad E = -L\frac{dI}{dt}$ --- (3)

when $\frac{dI}{dt} = 1\ \mathrm{A\,s^{-1}}$ then,

$$E = -L$$

The self inductance is also numerically equal to the back emf in the coil when the rate of change of current through the coil is 1 Amp sec^{-1}.

* workdone against back emf:



The current in the circuit due to charge dq in time dt as

$$I = \frac{dq}{dt}$$

$$\Rightarrow dq = I\,dt \quad \text{....(1)}$$

The workdone against back emf = emf x charge

$$\Rightarrow dW = E\,dq \quad \text{..(2)}$$

From (1) and (2) $\quad dW = E I\,dt \quad$ ..(3)

Also, back emf $\quad E = -L\dfrac{dI}{dt} \quad$ ...(4)

From (3) and (4) $\quad dW = L\dfrac{dI}{dt} I\,dt \quad$ ...(5) [Magnitude only]

Total workdone against back emf is

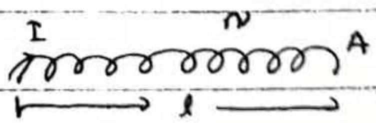
$$\int dW = \int L\frac{dI}{dt} I\,dt$$

$$\Rightarrow \int dW = L\int I\,dI$$

$$\Rightarrow W = \frac{1}{2}LI^2$$

The workdone is change into magnetic energy of a coil.

* self inductance of solenoid:



Let N be the number of turns l be the length and 'A' be the cross-section area of the coil, through which current I is flowing:

If 'n' be the number of turns per unit length then magnetic flux density along the axis of solenoid is

$$B = \mu_0 n I \quad ---(1)$$

Then flux through the solenoid,

$$\phi_m = NBA \quad ---(2)$$

From (1) and (2)

$$\phi_m = N\mu_0 n I A \quad \cdot\cdot(3)$$

Also, flux $(\phi_m) = IL \quad \cdot\cdot(4)$

From (3) and (4)

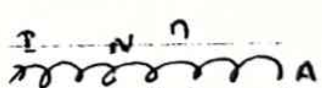
$$N\mu_0 n I A = IL$$

$$\Rightarrow L = \frac{\mu_0 N n I A}{I}$$

$$\Rightarrow L = \frac{\mu_0 N^2 A}{l} \quad ----(5) \qquad [n = N/l]$$

* Energy density of solenoid:

The magnetic energy per unit volume of the inductor (solenoid) is called energy density of the solenoid. It is denoted by U_m and given by



$$U_m = \frac{\text{~~Electr~~magnetic energy}}{\text{volume}}$$

$$U_m = \frac{\frac{1}{2} L I^2}{Al} \quad ---(1)$$

but $L = \mu_0 N^2 A / l \quad ---(2)$

From (1) and (2)

$$U_m = \frac{1}{2} \frac{\mu_0 N^2 A}{l} \frac{I^2}{Al}$$

$$\Rightarrow U_m = \frac{1}{2}\mu_0 \left(\frac{N}{l}\right)^2 I^2$$

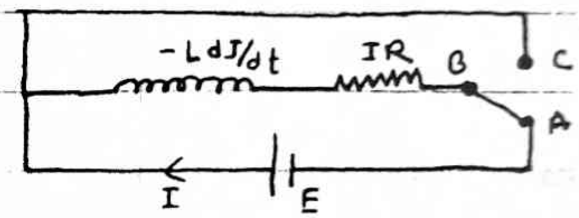
$$= \frac{1}{2\mu_0}\left(\mu_0 \frac{N}{l} I\right)^2$$

$$= \frac{1}{2\mu_0}\left(\mu_0 n I\right)^2$$

$$U_m = \frac{1}{2\mu_0} B^2 \quad ----(3)$$

* Growth and decay of current through L-R circuit:

The inductor of inductance 'L' and resistor of resistance 'R' is connected in series with the source of emf 'E' as shown in fig.



when the terminal A and B are connected, the current in the circuit starts growing. Let I be the current in the circuit at any time 't'

Then according to krichoff's 2nd law,

$$E + \left(-L\frac{dI}{dt}\right) = IR$$

$$\Rightarrow \frac{E}{L} - \frac{dI}{dt} = \frac{IR}{L}$$

$$\Rightarrow \frac{dI}{dt} = \frac{E}{L} - \frac{IR}{L}$$

$$\Rightarrow \frac{dI}{dt} = \frac{R}{L}\left[\frac{E}{R} - I\right]$$

$$\Rightarrow \quad \frac{dI}{\left[E/R - I \right]} = \frac{R}{L}\, dt \quad \text{----}(1)$$

Let $\quad E/R - I = x \quad \cdot\cdot(2)$

then differentiating eq^n (2) w.r.to x

$$- dI = dx \quad \text{--}(3)$$

substituting (2) and (3) in (1)

$$\frac{-dx}{x} = \frac{R}{L}\, dt$$

$$\Rightarrow \int \frac{dx}{x} = \int \frac{-R}{L}\, dt$$

Integrating $\Rightarrow \quad \log_e x = -\frac{R}{L}\, dt + k \text{ (constant)} \quad \text{---}(4)$

Now replace x by $E/R - I$ then eq^n (4) becomes

$$\log_e \left(\frac{E}{R} - I \right) = -\frac{R}{L}\, dt + k \quad \cdot\cdot(5)$$

when $t = 0$, $I = 0$ then (5) becomes

$$\log_e \left(\frac{E}{R} \right) = k \quad \text{--}(6)$$

From (5) and (6)

$$\log_e \left[\frac{E}{R} - I \right] = -\frac{R}{L}\, dt + \log_e \left(\frac{E}{R} \right)$$

$$\Rightarrow \log_e \left[\frac{E/R - I}{E/R} \right] = -\frac{R}{L}\, t$$

$$\Rightarrow 1 - \frac{I}{(E/R)} = e^{-R/L\, t}$$

$$\Rightarrow I = \frac{E}{R}\left[1 - e^{-\frac{R}{L} t} \right] \quad \text{---}(7)$$

when $t = 0$, $I = 0$

when $t = \infty$, $I = E/R$ i,e maximum current $= I_0$

$\therefore$ From eq^n (7)

$$I = I_0 \left[1 - e^{-\frac{R}{L}t} \right] \quad \cdots (8)$$

For one time constant, $\frac{R}{L} t = 1$

$$\therefore I = I_0 \left[1 - e^{-1} \right]$$

$$= I_0 \left[1 - \frac{1}{e} \right]$$

$$= I_0 \left[1 - \frac{1}{2.7} \right]$$

$$= 0.63 I_0$$

$$\Rightarrow I = 63\% \text{ of } I_0 \quad \cdots (9)$$

In one time constant the current in the circuit reaches 63% of maximum current.

Now, for decay:

when the current in the circuit reaches maximum then terminal A and B is disconnected and terminal A and C are connected, the current starts decaying.

The circuit eq^n becomes

$$-L \frac{dI}{dt} = IR$$

$$\Rightarrow \frac{dI}{I} = -\frac{R}{L} dt \quad \cdots (10)$$

Now, integrating $\int \frac{dI}{I} = \int -\frac{R}{L} dt$

Date

$$\Rightarrow \log_e I = -\frac{R}{L} t + k \text{ (constant)} \quad \cdots (11)$$

when $t=0$, $I = I_0$ then eq^n (11) becomes

$$\log_e I_0 = k \quad \cdots (12)$$

From (11) and (12)

$$\log_e I = -\frac{R}{L} t + \log_e I_0$$

$$\Rightarrow \log_e \left(\frac{I}{I_0}\right) = -\frac{R}{L} t$$

$$\Rightarrow \frac{I}{I_0} = e^{-\frac{R}{L} t}$$

$$\Rightarrow I = I_0 e^{-\frac{R}{L} t} \quad \cdots (13)$$

At ~~at~~ $t=0$, $I = I_0$

At $t=\infty$, $I = 0$

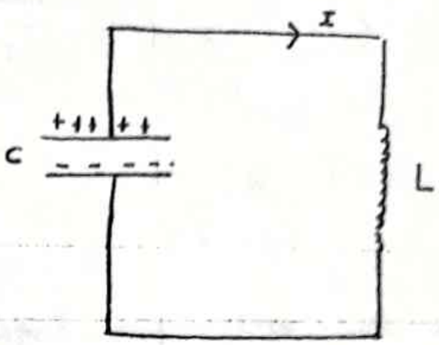
The eq^n (13) shows how the current in circuit decays.

Electromagnetism

Chapter-8

Date . .

L-C oscillation: (undamped)



when a charged capacitor is connected across the coil, the current set up in the circuit buid up magnetic field i,e the electrical energy is changed into magnetic energy. when capacitor is fully discharged Its whole electrical energy converted into magnetic energy.

After then, the magnetic energy starts to decey which changes the capacitor in opposite direction by sending an opposite current i,e magnetic energy is changed into electrical energy. In this way, an oscillation is set up in between L and C and is called L-C oscillation.

In the absence of resistence the total energy remains constant. The frequency of oscillation is called resonant frequency and the oscillation is undamped.

Total energy, U = Electrical energy + Magnetic energy

$$\Rightarrow U = \frac{1}{2}\frac{q^2}{C} + \frac{1}{2}LI^2$$

$$\Rightarrow \frac{dU}{dt} = \frac{1}{2C} 2q \frac{dq}{dt} + \frac{1}{2} L\, 2I \frac{dI}{dt}$$

$$\Rightarrow 0 = \frac{q}{C}\frac{dq}{dt} + LI\frac{dI}{dt} \quad \text{---(1)}$$

But, $I = \frac{dq}{dt}$ --(2)

and $\frac{dI}{dt} = \frac{d^2q}{dt^2}$ ..(3)

From (1), (2) and (3)

$$\Rightarrow \frac{q}{C} I + L I \frac{d^2q}{dt^2} = 0$$

$$\Rightarrow \frac{q}{C} + L \frac{d^2q}{dt^2} = 0$$

$$\Rightarrow \frac{d^2q}{dt^2} + \frac{1}{LC} q = 0 \quad ---(4)$$

This equation is analogous to the eq^n of simple harmonic motion.

i.e $\frac{d^2x}{dt^2} + \omega^2 x = 0 \quad --(5)$

comparing eq^n (4) and (5)

$$\omega^2 = \frac{1}{LC}$$

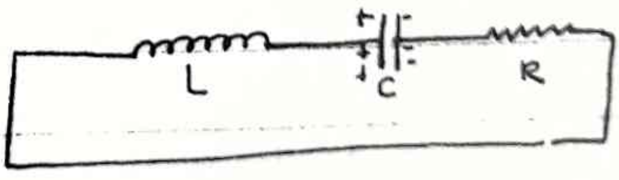
$$\Rightarrow \omega = \frac{1}{\sqrt{LC}}$$

$$\Rightarrow 2\pi f_0 = \frac{1}{\sqrt{LC}}$$

$$\Rightarrow f_0 = \frac{1}{2\pi\sqrt{LC}} \quad ---(6)$$

This is called resonant or undamped frequency.

* L-C oscillation (Damped)



when the charged capacitor is connected with the resistor and inductor as shown in fig. The energy is dissipated at the rate of I^2R and then the frequency of oscillation is continuously decreased or damped.

we have, total energy, U = Electrical energy + Magnetic energy

$$\Rightarrow U = \frac{1}{2}\frac{q^2}{C} + \frac{1}{2}LI^2$$

$$\Rightarrow \frac{dU}{dt} = \frac{1}{2}\frac{2q}{C}\frac{dq}{dt} + \frac{1}{2}L\,2I\frac{dI}{dt}$$

$$\Rightarrow -I^2R = \frac{q}{C}\frac{dq}{dt} + LI\frac{dI}{dt}$$

$$\Rightarrow -I^2R = \frac{q}{C}I + LI\frac{d^2q}{dt^2}$$

$$\Rightarrow -IR = \frac{q}{C} + L\frac{d^2q}{dt^2}$$

$$\Rightarrow -\frac{dq}{dt}R = \frac{q}{C} + L\frac{d^2q}{dt^2}$$

$$\Rightarrow -\frac{dq}{dt}\frac{R}{L} = \frac{q}{LC} + \frac{d^2q}{dt^2}$$

$$\Rightarrow \frac{d^2q}{dt^2} + \frac{R}{L}\frac{dq}{dt} + \frac{1}{LC}q = 0 \quad \text{---- } (*)$$

It is in the form of eqn of damped simple harmonic motion

i.e. $\frac{d^2x}{dt^2} + \frac{b}{m}\frac{dx}{dt} + \frac{k}{m}x = 0$ ---(**) $\left[\because \frac{d^2x}{dt^2} + r\frac{dx}{dt} + \omega^2 x = 0\right]$

comparing (*) and (**)

$$\frac{b}{m} = \frac{R}{L}$$

$$\frac{k}{m} = \frac{1}{LC}$$

then frequency of damped harmonic oscillation is given by

$$f = \frac{1}{2\pi}\sqrt{\left(\frac{k}{m}\right) - \left(\frac{b}{2m}\right)^2} \qquad \left[\because f = \frac{1}{2\pi}\sqrt{\omega^2 - \left(\frac{r}{2}\right)^2}\right]$$

$$\Rightarrow f = \frac{1}{2\pi}\sqrt{\frac{1}{LC} - \frac{R^2}{4L^2}} \quad \text{---(xxx)}$$

This is the damped frequency.

If $R = 0$ then, $f = f_0$

* Displacement current:

In the case of charging and discharging of the capacitor, the electric field betn the plates of capacitor changes continuously. Due to this change in electric field, the flux also changes which causes the current between the plates. This current due to the change in electric field is called displacement current and is denoted by I_d.

Therefore, the current in the circuit is always the sum of current due to a flow of charge and due to the change in electric field.

we have from Gauss law of electrostatics,

$$\text{flux}(\phi) = \frac{1}{\epsilon_0} \times \text{charge enclosed}(q)$$

$$\Rightarrow EA = \frac{q}{\epsilon_0}$$

$\therefore$ Field between the plates of capacitor,

$$E = \frac{q}{A\epsilon_0} \quad \text{----(1)}$$

$$\Rightarrow q = A\epsilon_0 E.$$

$$\Rightarrow dq/dt = A\epsilon_0\, dE/dt$$

$$\Rightarrow I_d = \epsilon_0 A \frac{dE}{dt} \quad ---(2)$$

$$\Rightarrow I_d = \epsilon_0 \frac{d}{dt}(EA)$$

$$\Rightarrow I_d = \epsilon_0 \frac{d\phi}{dt} \quad ----(3)$$

Eqn (2) can also be written as $\frac{I_d}{A} = \epsilon_0 \frac{dE}{dt}$

$$\Rightarrow J_d = \epsilon_0 \frac{dE}{dt} \quad ---(4)$$

This eqn represents the displacement current density.

Representation of integral:

① Line integral of any vector $\vec{A}$ is $\oint \vec{A}\cdot\vec{dl}$ ---(1)

② surface integral of a vector $\vec{A}$ is

$$\int_S \vec{A}\cdot\vec{da} \quad \text{or} \quad \iint \vec{A}\cdot\vec{da} \quad ---(2)$$

③ volume integral of a vector $\vec{A}$ is

$$\int_V \vec{A}\cdot\vec{dv} \quad \text{or} \quad \iiint \vec{A}\cdot\vec{dv} \quad ---(3)$$

The surface integral of any vector gives its flux.
For example if $\vec{A}$ vector is the electric field vector then, from eqn (2) replace $\vec{A}$ by $\vec{E}$ then

$$\int_S \vec{E}\cdot\vec{da} = \vec{E}\int_S \vec{da} = EA$$

$$= \phi \quad \text{(i.e electric flux)}$$

If $\vec{A}$ be the magnetic field vector ($\vec{B}$) then,

$$\int_s \vec{B}\cdot d\vec{a}$$

$$= B\int_s da$$

$$= BA$$

$$= \phi \text{ (magnetic flux).}$$

Gauss divergence theorem -

It states that the surface integral of a vector is equal to volume integral of divergence of that vector.

If $\vec{A}$ be the vector then the statement can be written as

$$\int_S \vec{A}\cdot d\vec{a} = \int_V (\text{div}\vec{A})\, dv$$

where $\text{div}\vec{A} = \vec{\nabla}\cdot\vec{A}$

and $\vec{\nabla}$ is the three dimensional operator.

$$\vec{\nabla} = \vec{i}\frac{\partial}{\partial x} + \vec{j}\frac{\partial}{\partial y} + \vec{k}\frac{\partial}{\partial z}$$

* stoke's theorem -

It states that the line integral of a vector is equal to the surface integral of curl of that vector.

If $\vec{A}$ be the vector then stoke's theorem can be written as

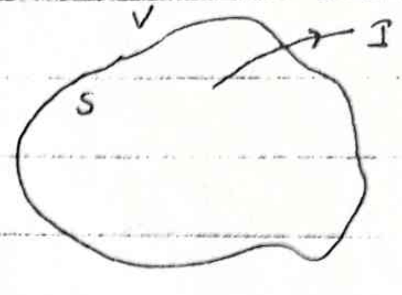
$$\int_l \vec{A}\cdot d\vec{l} = \int_S (\text{curl}\vec{A})\, da$$

where $\text{curl}\vec{A} = \vec{\nabla}\times\vec{A}$

$\vec{\nabla}$ is the three dimensional operator.

* Equation of continuity -

Let us consider a surface 's' enclosing volume 'v'. Let 'ds' be the small element of this surface. Further let $\vec{J}$ be the current density at a point on the surface element Then the current leaving the volume 'V' bounded by the surface 'ds' is given by



$$I = \int_S \vec{J} \cdot d\vec{s} \quad ---(1)$$

As some charge is leaving the volume, correspondingly the same amount of charge diminishes with in that volume. we can express this fact as

$$I = -\frac{dq}{dt} \quad ----(2)$$

If ρ be the volume charge density (i,e charge per unit volume) then,

$$q_\rho = \int_V \rho \, dv \quad ---(3)$$

From (2) and (3) $\quad I = -\frac{\partial}{\partial t}\int_V \rho \, dv \quad ---(4)$

eqn (4) becomes in form of partial derivative as ρ is the function of both position and time, and volume is fixed the time derivative operates only on the function ρ.

Now, from (1) and (4)

$$\int_S \vec{J} \cdot d\vec{s} = -\frac{\partial}{\partial t}\int_V \rho \, dv \quad --(5)$$

using Gauss divergence theorem,

$$\int_S \vec{J} \cdot d\vec{s} = \int_V (\operatorname{div} \vec{J}) \, dV \quad --(6)$$

From (5) and (6)

$$\int_V (\mathrm{div}\, \vec{J})\, dv = -\frac{\partial}{\partial t}\int_V \rho\, dv$$

$$\Rightarrow \mathrm{div}\, \vec{J} = -\frac{\partial \rho}{\partial t} \quad \text{--- } (*)$$

$$\Rightarrow \vec{\nabla}\cdot\vec{J} + \frac{\partial \rho}{\partial t} = 0$$

This is called equation of continuity.
From eq^n (*) we can conclude that charge cannot be flow away from the given volume without diminishing the amount of charge existing within the volume.

* Maxwell's electromagnetic wave equations:
(1) 1st equation:
From Gauss law of electrostatics,
we have, $\phi = \frac{1}{\epsilon_0} \times$ charged enclosed.

$$\Rightarrow \vec{E}A = \frac{1}{\epsilon_0} q$$

$$\Rightarrow \int_S \vec{E}\cdot d\vec{a} = \frac{1}{\epsilon_0}\int_V \rho\, dv \quad \text{--- } (1)$$

Now, using Gauss divergence theorem,

$$\int_S \vec{E}\cdot d\vec{a} = \int_V (\mathrm{div}\, \vec{E})\, dv \quad \text{--- } (2)$$

From (1) and (2)

$$\int_V (\mathrm{div}\, \vec{E})\, dv = \frac{1}{\epsilon_0}\int_V \rho\, dv$$

$\Rightarrow \operatorname{div} \vec{E} = \rho/\epsilon_0$

$\Rightarrow \vec{\nabla} \cdot \vec{E} = \frac{\rho}{\epsilon_0}$ ----(3) [Differential form]

(2) 2nd Equation:

The magnetic flux through a closed surface is given by the surface integral of magnetic field strength i.e

$$\text{magnetic flux} = \int_S \vec{B} \cdot d\vec{a} \quad ---(1)$$

but magnetic flux through a closed path is always zero. This means monopole of magnetic does not exists.

$$\therefore \int_S \vec{B} \cdot d\vec{a} = 0 \quad ----(2)$$

using Gauss divergence theorem, $\int_S \vec{B} \cdot d\vec{a} = \int_V (\operatorname{div} \vec{B}) dV$ (3)

From (2) and (3) $\int_V (\operatorname{div} \vec{B}) dV = 0$

$$\Rightarrow \operatorname{div} \vec{B} = 0$$

$$\Rightarrow \vec{\nabla} \cdot \vec{B} = 0 \quad ----(4) \quad \text{[Differential form]}$$

(3) ~~4th~~ 3rd equation -

~~From the faraday's law of electromagnetic induction, time integral of magnetic field round a closed~~ oriented

From the Faraday's law of electromagnetic induction, the induced emf is equal to the rate of change of magnetic flux.

$$\text{i.e} \quad e = -\frac{d\phi_m}{dt} \quad ---(1)$$

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where ϕ_m = magnetic flux

As the surface integral gives the flux,

$$\phi_m = \int_S \vec{B} \cdot d\vec{a} \quad \text{---}(1)$$

also, emf is the amount of workdone to move 1 coulomb charge round a closed path.

$$\therefore \quad e = \text{force} \times \text{displacement}$$

$$e = \int_l E \, dl \quad \text{---}(3)$$

ie line integral of electric field gives emf.

From (1), (2) and (3) [substituting (2) and (3) in (1)]

$$\int_l \vec{E} \, dl = -\frac{\partial}{\partial t} \int_S \vec{B} \cdot d\vec{a} \quad \text{---}(4)$$

⇒ using ~~Gauss divergence~~ stoke's theorem,

$$\int_l E \, dl = \int_S (\text{curl}\, \vec{E}) \, da \quad \text{---}(5)$$

From (4) and (5)

$$\int_S (\text{curl}\, \vec{E}) \, da = -\frac{\partial}{\partial t} \int_S \vec{B} \cdot d\vec{a}$$

$$\Rightarrow \quad \text{curl}\, \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

$$\Rightarrow \quad \vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} \quad \text{----}(6)$$

(4) 4th equation-

we have from Ampere's law, line integral of magnetic field round a closed path is equal to μ_0 times current enclosed:

$$\oint_{l} \vec{B} \cdot d\vec{l} = \mu_0 (I + I_d) \quad \text{----(1)}$$

But, current due to flow of charge and current due to change in field can be expressed as

$$I = \int_S \vec{J} \cdot d\vec{a} \quad \text{--(2)}$$

$$I_d = \int_S \vec{J_d} \cdot d\vec{a} \quad \text{---(3)}$$

substituting (2) and (3) in (1)

$$\oint_{l} \vec{B} \cdot d\vec{l} = \mu_0 \int_S (\vec{J} + \vec{J_d}) \cdot d\vec{a} \quad \text{--(4)}$$

Now, using stoke's law,

$$\oint_{l} \vec{B} \cdot d\vec{l} = \int_S (\text{curl}\,\vec{B})\, d\vec{a} \quad \text{---(5)}$$

From (4) and (5)

$$\int_S (\text{curl}\,\vec{B})\, da = \mu_0 \int_S (\vec{J} + \vec{J_d}) \cdot d\vec{a}$$

$$\Rightarrow \text{curl}\,\vec{B} = \mu_0 [\vec{J} + \vec{J_d}]$$

$$\Rightarrow \vec{\nabla} \times \vec{B} = \mu_0 \vec{J} + \mu_0 \vec{J_d} \quad \text{---(6)}$$

Also, $\vec{J_d} = \epsilon_0 \dfrac{d\vec{E}}{dt}$ --(7)

From (6) and (7)

$$\vec{\nabla} \wedge \vec{B} = \mu_0 \vec{J} + \mu_0 \epsilon_0 \frac{d\vec{E}}{dt} \quad \text{---(8)}$$

$\therefore$ Maxwell's eqns are

(1) $\vec{\nabla} \cdot \vec{E} = \rho / \epsilon_0$

(2) $\vec{\nabla} \cdot \vec{B} = 0$

(3) $\vec{\nabla} \times \vec{E} = -\partial \vec{B}/\partial t$

(4) $\vec{\nabla} \times \vec{B} = \mu_0 \vec{J} + \mu_0 \epsilon_0 \dfrac{d\vec{E}}{\partial t}$

In free space, there is no current density and charge density then the eq^{n}s becomes

(1) $\vec{\nabla} \cdot \vec{E} = 0$

(2) $\vec{\nabla} \cdot \vec{B} = 0$

(3) $\vec{\nabla} \times \vec{E} = -\partial \vec{B}/\partial t$

(4) $\vec{\nabla} \times \vec{B} = \mu_0 \epsilon_0 \dfrac{\partial \vec{E}}{\partial t}$

* speed of electromagnetic wave in free space:

we have from maxwell's third eq^n

$$\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} \quad --- (1)$$

Now taking curl on both sides we get

$$\vec{\nabla} \times (\vec{\nabla} \times \vec{E}) = -\frac{\partial}{\partial t} (\vec{\nabla} \times \vec{B}) \quad -- (2)$$

using vector triple product rule,

$$\vec{\nabla} \times (\vec{\nabla} \times \vec{E}) = (\vec{\nabla} \cdot \vec{E}) \vec{\nabla} - (\vec{\nabla} \cdot \vec{\nabla}) \vec{E} \quad -- (3)$$

and from Maxwell's Fourth equation,

$$\vec{\nabla} \times \vec{B} = \mu_0 \vec{J} + \mu_0 \epsilon_0 \frac{d\vec{E}}{\partial t} \quad -- (4)$$

substituting (3) and (4) in (2) we get

$$(\vec{\nabla} \cdot \vec{E}) \vec{\nabla} - (\vec{\nabla} \cdot \vec{\nabla}) \vec{E} = -\frac{\partial}{\partial t} \left[\mu_0 \vec{J} + \mu_0 \epsilon_0 \frac{\partial \vec{E}}{\partial t} \right]$$

but for free space $\vec{\nabla} \cdot \vec{E} = 0$, and $\vec{J} = 0$

$$-(\vec{\nabla}\cdot\vec{\nabla})E = -\frac{\partial}{\partial t}\left[\mu_0 \epsilon_0 \frac{\partial E}{\partial t}\right]$$

$$\Rightarrow \nabla^2 E = \mu_0 \epsilon_0 \frac{\partial^2 E}{\partial t^2} \quad ---(5)$$

This represents the electromagnetic wave equation of electric vectors similarly the wave eqn in magnetic vector form.

$$\nabla^2 B = \mu_0 \epsilon_0 \frac{\partial^2 B}{\partial t^2} \quad ---(6)$$

eqn (5) and (6) is analogous to the equation of wave equation,

$$\nabla^2 x = \frac{1}{c^2}\frac{\partial^2 x}{\partial t^2} \quad ---(7)$$

Comparing eqn (5) and (7) or (6) and (7)

$$\mu_0 \epsilon_0 = \frac{1}{c^2}$$

$$\Rightarrow c = \frac{1}{\sqrt{\mu_0 \epsilon_0}}$$

$$\Rightarrow c = \frac{1}{\sqrt{4\pi \times 10^{-7} \times 8.85 \times 10^{-12}}}$$

$$\Rightarrow c = 3 \times 10^8 \ m/s$$

Therefore electromagnetic wave travels with speed of light.

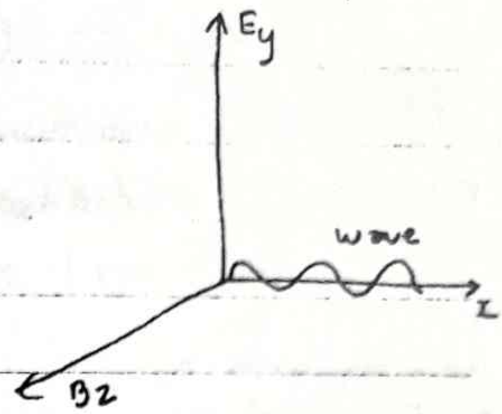
* To show $E_0/B_0 = C$

we have the electromagnetic wave equation in ~~the~~ electric vector is

$$\nabla^2 E = \mu_0 \epsilon_0 \frac{\partial^2 E}{\partial t^2} \quad ---(1)$$

and electromagnetic wave eqn in magnetic vector form is

$$\nabla^2 B = \mu_0 \epsilon_0 \frac{\partial^2 B}{\partial t^2} \quad \text{---(2)}$$



Let the wave travel in x- direction, electric field is along y direction and magnetic field is along z-direction. Then eqn (1) and (2) can be expressed in one dimensional case as

$$\frac{\partial^2 E_y}{\partial x^2} = \mu_0 \epsilon_0 \frac{\partial^2 E_y}{\partial t^2} \quad \text{---(3)}$$

and
$$\frac{\partial^2 B_z}{\partial x^2} = \mu_0 \epsilon_0 \frac{\partial^2 B_z}{\partial t^2} \quad \text{---(4)}$$

The solution of eqn (3) and (4) is

$$E_y = E_0 \sin(\omega t - kx) \quad \text{---(5)}$$

$$B_z = B_0 \sin(\omega t - kx) \quad \text{---(6)}$$

Now,

From Maxwell's third eqn.

$$\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

$$\Rightarrow \left(\vec{i}\frac{\partial}{\partial x} + \vec{j}\frac{\partial}{\partial y} + \vec{k}\frac{\partial}{\partial z}\right) \times (\vec{i} E_x + \vec{j} E_y + \vec{k} E_z) = -\frac{\partial}{\partial t}\left[\vec{i} B_x + \vec{j} B_y + \vec{k} B_z\right]$$

but $E_x = E_z = B_x = B_y = 0$

$$\therefore \left(\vec{i}\frac{\partial}{\partial x} + \vec{j}\frac{\partial}{\partial y} + \vec{k}\frac{\partial}{\partial z}\right) \times \vec{j} E_y = -\frac{\partial}{\partial t}\left[\vec{k} B_z\right]$$

$$\Rightarrow \vec{k}\frac{\partial E_y}{\partial x} + 0 - \vec{i}\frac{\partial E_y}{\partial z} = -\vec{k}\frac{\partial B_z}{\partial t}$$

Now, comparing the coeffici of $\vec{k}$

we got.

$$\frac{\partial E_y}{\partial x} = -\frac{\partial B_z}{\partial t} \quad \text{----} (7)$$

substituting (5) and (6) in (7)

$$\frac{\partial}{\partial x}\left[E_0 \sin(\omega t - kx)\right] = \frac{-\partial}{\partial t}\left[B_0 \sin(\omega t - kx)\right]$$

$$\Rightarrow E_0 \cos(\omega t - kx)(-k) = -B_0 \cos(\omega t - kx)(\omega)$$

$$\Rightarrow -E_0 k \cos(\omega t - kx) = -B_0 \omega \cos(\cancel{kx}\ \omega t - kx)$$

$$\Rightarrow E_0 k = B_0 \omega$$

$$\Rightarrow \frac{E_0}{B_0} = \frac{\omega}{k}$$

$$\Rightarrow \frac{E_0}{B_0} = \frac{2\pi f}{2\pi/\lambda}$$

$$\Rightarrow \frac{E_0}{B_0} = f\lambda$$

$$\Rightarrow \frac{E_0}{B_0} = c \quad \text{----} (8)$$

* poynting vector:

we can describe the energy transfer interms of the rate of energy flow per unit area or power per unit area, by a vector 's' is called poynting vector.

Let us calculate the energy 'du' passing during time 'dt' through a unit area perpendicular to the direction of propagation of the wave In a time dt, wave front moves a distance

$d = c\,dt$ ---- (1)

If 'u' be the electrical energy density then,

$$du = u\,c\,dt \quad \text{---} (2)$$

the value of $u = \frac{1}{2}\epsilon_0 E^2$ ---(3)

For electromagnetic wave electrical energy density is equal to magnetic energy density —

$$u = \frac{1}{2\mu_0} B^2 \text{ ---(4)}$$

From (3) and (4) $\frac{1}{2\mu_0} B^2 = \frac{1}{2}\epsilon_0 E^2$

$$\Rightarrow \frac{B}{\sqrt{\mu_0}} = \sqrt{\epsilon_0}\, E \text{ ---(5)} \Rightarrow B = \sqrt{\mu_0\epsilon_0}\, E \text{ ---(6)}$$

From eqn (2) and (3)

$$dU = \frac{1}{2}\epsilon_0 E^2 c\, dt \text{ ---(6)}$$

From (6) and (6) $dU = \frac{1}{2}\epsilon_0 \frac{B}{\sqrt{\mu_0\epsilon_0}} E\, c\, dt$ $\left[\because E = \frac{B}{\sqrt{\mu_0\epsilon_0}}\right]$

$$\Rightarrow dU = \frac{1}{2}\epsilon_0 c^2 E B\, dt \text{ ---(7)} \quad \left[\frac{1}{\sqrt{\mu_0\epsilon_0}} = c\right]$$

Or From (2) and (4)

$$dU = \frac{1}{2\mu_0} B^2 c\, dt \text{ ---(8)}$$

$$= \frac{1}{2\mu_0} B\, B\, c\, dt$$

$$= \frac{1}{2\mu_0}\sqrt{\mu_0\epsilon_0}\, E\, B\, c\, dt$$

$$dU = \frac{1}{2\mu_0} E B\, dt \text{ ---(8)} \quad \left[\because \sqrt{\mu_0\epsilon_0} = \frac{1}{c}\right]$$

From (7) and (8) we say that, the term EB represents the magnitude of energy flux density vector. This corresponding vector 'S' is then,

$$\vec{S} = \frac{1}{2\mu_0}(\vec{E} \times \vec{B}) \quad \text{---(9)} \quad [\text{From (8)}] \qquad \left(\frac{1}{A}\frac{dU}{dt} = S\right)$$

or, $$\vec{S} = \frac{\epsilon_0 c^2}{2}(\vec{E} \times \vec{B}) \quad \text{---(10)} \quad [\text{From (7)}]$$

'S' is the poyinting vector has dimension of ~~per~~ energy perunit time perunit area.

Intensity is the average power per unit area.

$$I = \frac{P_{avg}}{A}$$

$$\Rightarrow I = \frac{1}{2\mu_0 c} E_0^2$$

$$I \propto E_0^2$$

* Radiation pressure and momentum-

Electromagnetic waves transport energy. Therefore, they carry momentum and exhibit a force in the direction of propagation. The momentum is a property of the field alone and it is not associated with any moving mass. The momentum density, that is momentum dp per volume dV is given by

$$\frac{dp}{dV} = \frac{1}{2\mu_0 c^2} EB$$

$$\Rightarrow \frac{dp}{dV} = \frac{S}{c^2} \quad \text{--- (11)}$$

The volume dV occupied by an electronmagnetic wave that passes through an area 'A' in time dt is

$$\Rightarrow \quad dv = A c\, dt \quad \text{---(2)}$$

Therefore the momentum flow rate per unit area is

$$\frac{1}{A}\frac{dp}{dt} = \frac{S}{c} \quad \text{--(3)} \qquad [\because \text{substituting (2) in (1)}]$$

This is the momentum transfered per unit surface area per unit time. The average rate of momentum transfer per unit area is given by,

$$\frac{1}{A}\frac{dp}{dt} = \frac{S_{avg}}{c}$$

$$\frac{1}{A}\frac{dp}{dt} = \frac{I}{c} \quad \text{---(4)}$$

This momentum is responsible for the radiation pressure.

* Electrical Energy density and magnetic energy density.

we have, electrical energy density, $U_E = \frac{1}{2}\epsilon_0 E_0^2$ ---(1)

magnetic energy density, $U_B = \frac{1}{2\mu_0} B_0^2$ .. (2)

Also, from electromagnetism, $\frac{E_0}{B_0} = c$

$$\Rightarrow E_0 = B_0 c \quad \text{---(3)}$$

And, $\frac{1}{\sqrt{\mu_0 \epsilon_0}} = c$

$$\Rightarrow c^2 = 1/\mu_0\epsilon_0 \quad \text{....(4)}$$

From (1) and (3)

$$U_E = \frac{1}{2}\epsilon_0 B_0^2 c^2 \quad \text{---(5)}$$

From (4) and (5)

$$U_E = \frac{1}{2} \epsilon_0 B_0^2 \frac{1}{\mu_0 \epsilon_0}$$

$$\Rightarrow U_E = \frac{1}{2\mu_0} B^2$$

$$\Rightarrow U_E = U_B$$

i,e electrical energy density is equal to magnetic energy density.

photon and matter waves
chapter-9

Date . .

The light exhibits the phenomenon of interference, diffraction polarization, photoelectric effect, compton effect and emission and absorption. The phenomenon like interference, diffraction and polarization can only be explained on the basis of wave nature of light. This phenomenon shows that light passes wave nature.

On the other hand the phenomenon like photoelectric effect, compton effect, discrete emission and absorption can only be explained on the basis of the quantum theory of light, according to which light is propagated in a small packets or bundle of energy called photon or quanta. The energy of each quanta is given by

$$E = h\nu \quad --(1)$$

where h is a plank's constant, $h = 6.6 \times 10^{-34}$ J sec

and ν = frequency of photon.

The photon acts as particles. This phenomena indicates that light possess particle nature. From above discussion we say that light shows dual nature.

* De-Broglie wave-

As the electromagnetic wave possess dual nature i.e wave nature and particle nature, matters (electron, proton) also should possess dual nature. According to De-Broglie, a moving particle whatever its nature has wave properties associated with it. The wave associated with the particle is called De-Broglie wave or matter wave, and the wavelength associated with matter is called De-Broglie wavelength.

we know, the energy of photon

$$E = h\nu \quad ---(1)$$

If photon is considered as a particle of mass 'm' moving with velocity 'c' then according to mass energy relation.

$$E = mc^2 \text{ ---(2)}$$

From (1) and (2)

$$h\nu = mc^2$$

$$\Rightarrow \frac{hc}{\lambda} = mc^2$$

$$\Rightarrow \lambda = \frac{h}{mc} \text{ ---(3)}$$

where mc = momentum of photon.

eq^n (3) is the expression of De-Broglie wavelength.

For other particles eq^n (3) becomes:

$$\lambda = \frac{h}{mv} \text{ ---(4)}$$

* phase velocity (wave velocity):

A particle of mass 'm' having velocity 'v' has a wave associated with it whose wavelength is given by

$$\lambda = \frac{h}{mv} \text{ ---(1)}$$

let E be the total energy of the particle and ν be the frequency of the associated wave then.

$$E = h\nu \text{ ---(2)}$$

Also, energy associated with relativistic formula is

$$E = mc^2 \text{ ---(3)}$$

From (2) and (3)

$$h\nu = mc^2$$

$$\Rightarrow \nu = \frac{mc^2}{h} \quad ----(4)$$

Let V_p be the De-Broglie wave velocity (phase velocity) then,

v_p = frequency x wavelength

$$v_p = \nu\lambda \quad ---(5)$$

substituting ν and λ from (1) and (4) in (5) we get,

$$v_p = \frac{mc^2}{h} \frac{h}{mv}$$

$$v_p = \frac{c^2}{v} \quad --(6)$$

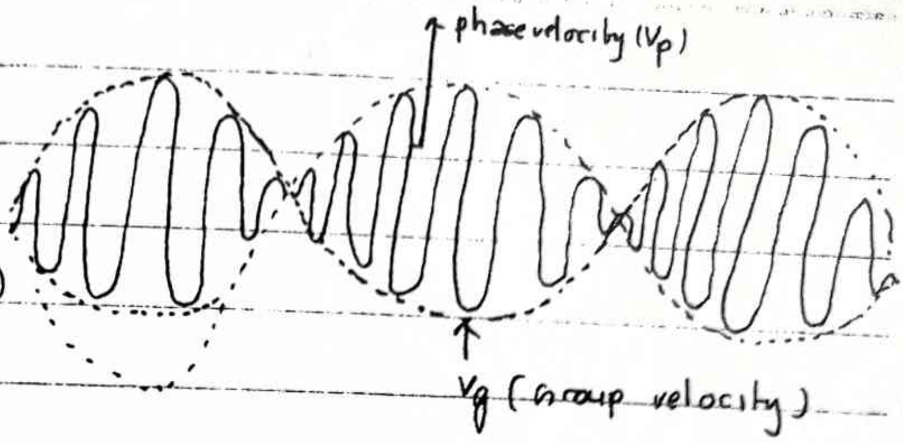
As particle velocity is always less than 'C' but eq^n (6) shows that $V_p > C$, which is not possible. However this problem is solved by De-Broglie by showing that the wave always travel in the form of group with velocity called group velocity.

* Group velocity -

In general real waves are of complex form. In practice waves are far from monochromatic and can be regarded as the result of superposition of waves of number of frequencies. The propagating velocity of a wave varies with frequency. The superposition of a very large number of harmonic waves differing small in frequency will produced a single wave packet.

The wave packet amplitude varies with position and time. such variation in amplitude is called modulation of the wave. The velocity of propagation of the modulation is known as group velocity. It is denoted by V_g and given as

$$V_g = \frac{d\omega}{dk} \quad --(1)$$



where $W = 2\pi\nu \quad \cdots(2)$

and $k = \frac{2\pi}{\lambda} \quad ---(3)$

Also, $\nu = \frac{mc^2}{h} \quad ---(4)$

The rest mass ~~energy~~ interms of moving mass can be written as (using relativistic law)

$$m = \frac{m_0}{\sqrt{1 - v^2/c^2}} \quad ---(5)$$

where m = moving mass

m_0 = rest mass

$\therefore$ From eq^n (2) and (4)

$$W = 2\pi \frac{mc^2}{h} \quad \cdots(6)$$

From (5), substitue m in eq^n (6)

$$\omega = \frac{2\pi c^2 m_0}{h\sqrt{1 - \frac{v^2}{c^2}}} \quad --(7)$$

Now differentiating eq^n (7) w.r.to v

$$\frac{d\omega}{dv} = \frac{2\pi c^2 m_0}{h} \frac{d}{dv}\left(1 - \frac{v^2}{c^2}\right)^{-1/2}$$

$$= \frac{2\pi c^2 m_0}{h} \left(\frac{-2v}{c^2}\right)\left(1 - \frac{v^2}{c^2}\right)^{-3/2}\left(-\frac{1}{2}\right)$$

$$\frac{d\omega}{dv} = \frac{2\pi m_0 v}{h\left(1 - v^2/c^2\right)^{3/2}} \quad ---(8)$$

we have, wavelength of De-Broglie wave

$$\lambda = \frac{h}{mv} \quad ---(9)$$

From (3) and (9)

$$k = \frac{2\pi mv}{h}$$

Interms of rest mass,

$$k = \frac{2\pi v}{h} \frac{m_0}{\sqrt{1-\frac{v^2}{c^2}}} \quad ---(10)$$

Now Differentiating with respect to v, we get

$$\frac{dk}{dv} = \frac{2\pi m_0}{h} \frac{d}{dv}\left(\frac{v}{[1-v^2/c^2]^{1/2}}\right)$$

$$= \frac{2\pi m_0}{h}\left[\frac{d}{dv}\left\{v\left(1-\frac{v^2}{c^2}\right)^{-1/2}\right\}\right]$$

$$= \frac{2\pi m_0}{h}\left[\left(1-\frac{v^2}{c^2}\right)^{-1/2} + v\left(\frac{-1}{2}\right)\left(1-\frac{v^2}{c^2}\right)^{-3/2}\left(-\frac{2v}{c^2}\right)\right]$$

$$= \frac{2\pi m_0}{h}\left[\left(1-\frac{v^2}{c^2}\right)^{-1/2} + \frac{v^2}{c^2}\left(1-\frac{v^2}{c^2}\right)^{-3/2}\right]$$

$$= \frac{2\pi m_0}{h}\left(1-\frac{v^2}{c^2}\right)^{-3/2}\left[1-\frac{v^2}{c^2}+\frac{v^2}{c^2}\right]$$

$$\frac{dk}{dv} = \frac{2\pi m_0}{h\,(1-v^2/c^2)^{3/2}} \quad ---(11)$$

From (8) and (11)

$$\frac{d\omega}{dk} = v$$

$$\Rightarrow v_g = v \quad ---(12)$$

* Relation between phase velocity and group velocity:

we have phase velocity,

$$v_p = \nu\lambda \quad ---(1)$$

Group velocity,

$$v_g = \frac{d\omega}{dk} \quad ---(2)$$

but, $\omega = 2\pi\nu$

$$\Rightarrow \omega = 2\pi\frac{v_p}{\lambda} \quad ---(3) \quad [\text{From } (1)]$$

Now, differentiating eqn (3) w.r.to λ

$$\frac{d\omega}{d\lambda} = 2\pi\left[-\frac{v_p}{\lambda^2} + \frac{1}{\lambda}\frac{dv_p}{d\lambda}\right]$$

$$\Rightarrow \frac{d\omega}{d\lambda} = -\frac{2\pi}{\lambda^2}\left[v_p - \lambda\frac{dv_p}{d\lambda}\right] \quad ----(4)$$

Also,

$$k = \frac{2\pi}{\lambda}$$

$\Rightarrow$ Differentiating w.r.to λ,

$$\frac{dk}{d\lambda} = -\frac{2\pi}{\lambda^2} \quad ---(5)$$

From (4) and (5)

$$\frac{d\omega/d\lambda}{dk/d\lambda} = \frac{-2\pi/\lambda^2\left[v_p - \lambda\frac{dv_p}{d\lambda}\right]}{-2\pi/\lambda^2}$$

$$\Rightarrow \frac{d\omega}{dk} = v_p - \lambda\frac{dv_p}{d\lambda}$$

$$\Rightarrow v_g = v_p - \lambda \frac{dv_p}{d\lambda} \quad ---(6)$$

Group velocity will be the same as the phase velocity If the entire constituent waves travel with same velocity. It means in a non-dispersive medium, $v_g = v_p$. However the waves of different wavelengths travel in a medium with different velocity. Therefore the group velocity is in general less than phase velocity.

* schrodinger wave equation - (Time dependent)

The quantity that characterises the De-Broglie wave is called wave function. It is denoted by ψ. It may be the complex function. Let us assume that ψ is specified in the x- direction by

$$\psi = A e^{-i(\omega t - kx)} \quad ----(1)$$

If ν be the frequency and λ be the wavelength then

$$\omega = 2\pi\nu \quad \text{and} \quad k = 2\pi/\lambda$$

then eq^n (1) becomes

$$\psi = A e^{-i[2\pi\nu t - 2\pi/_\lambda x]}$$

$$\psi = A e^{-2\pi i(\nu t - x/\lambda)} \quad --(2)$$

Let E be the total energy and p be the momentum of the particles then,

$$E = h\nu$$

$$\Rightarrow \nu = E/h \quad ---(3)$$

and

$$\lambda = h/mv$$

$$\lambda = \frac{h}{p} \quad ---(4)$$

substituting (3) and (4) in (2)

Date

$$\psi = A e^{-2\pi i\left[\frac{E}{h}t - \frac{x}{h}p\right]}$$

$$\Rightarrow \psi = A e^{-2\pi i/h\,[Et - px]} \quad \text{---- (5)}$$

since, the total energy E, of the particle is the sum of kinetic energy and potential energy.

$$E = \frac{1}{2}mv^2 + V$$

$$\Rightarrow E = \frac{1}{2m}(mv)^2 + V$$

$$\Rightarrow E = \frac{1}{2m}p^2 + V$$

Multiplying both sides by ψ

$$\Rightarrow E\psi = \frac{1}{2m}p^2\psi + V\psi \quad \text{---(6)}$$

to find $E\psi$ and $p^2\psi$ we use eq^n (5)

Now, differentiating eq^n (5) w.r.to x

$$\frac{\partial \psi}{\partial x} = A e^{-2\pi i/h\,[Et - px]}\left(-\frac{2\pi i}{h}\right)(-p)$$

Again, differentiating w.r.to x

$$\frac{\partial^2 \psi}{\partial x^2} = A e^{-2\pi i/h\,[Et - px]}\left(\frac{-2\pi i}{h}\right)^2(-p)^2$$

$$= \frac{i^2 4\pi^2}{h^2}p^2 A e^{-2\pi i/h\,(Et - px)}$$

$$\frac{\partial^2 \psi}{\partial x^2} = \frac{i^2 4\pi^2}{h^2}p^2\psi$$

$$\Rightarrow p^2\psi = \frac{h^2}{i^2 4\pi^2}\frac{\partial^2 \psi}{\partial x^2} \quad \text{---- (7)}$$

Now, differentiating (5) w.r.to 't'

$$\Rightarrow \frac{\partial \psi}{\partial t} = A e^{-2\pi i/h [Et - px]} \left(\frac{-2\pi i}{h}\right) E$$

$$\Rightarrow \frac{\partial \psi}{\partial t} = -\frac{2\pi i}{h} E \psi$$

$$\Rightarrow E\psi = \frac{-h}{2\pi i} \frac{\partial \psi}{\partial t} \quad \text{---} (8)$$

Now substituting (7) and (8) in (6)

$$\frac{-h}{2\pi i} \frac{\partial \psi}{\partial t} = \frac{1}{2m} \frac{h^2}{i^2 4\pi^2} \frac{\partial^2 \psi}{\partial x^2} + V\psi \qquad [\because -1 = i^2]$$

$$\Rightarrow \frac{ih}{2\pi} \frac{\partial \psi}{\partial t} = \frac{-h^2}{8\pi^2 m} \frac{\partial^2 \psi}{\partial x^2} + V\psi$$

$$\Rightarrow i\hbar \frac{\partial \psi}{\partial t} = -\frac{\hbar^2}{2m} \frac{\partial^2 \psi}{\partial x^2} + V\psi \quad \text{---} (9) \qquad \left[\because \hbar = \frac{h}{2\pi}\right]$$

This is the time dependent form of schrodinger wave equation.

* schrodinger wave equation (Time independent)

We have, the time dependent form of schrodinger wave eqn.

$$i\hbar \frac{\partial \psi}{\partial t} = -\frac{\hbar^2}{2m} \frac{\partial^2 \psi}{\partial x^2} + V\psi \quad \text{---} (1)$$

where, $\psi = A e^{-2\pi i/h [Et - px]}$ --- (2)

since the potential is the function of space ie vary with position only, the Schrodinger wave equation may be simplified by removing ~~ett~~ the reference to 't'.

From eqn (2)

$$\psi = A e^{-2\pi i/h \, Et} \cdot e^{2\pi i/h \, px}$$

$$\Rightarrow \psi = \psi_0 e^{-2\pi i E t/h} \quad --- (3)$$

where, $\psi_0 = A e^{2\pi i p x/h}$ --- (4)

This means eqn (3) is the product of a position dependent and time dependent function.

Now, differentiating eqn (3) w.r.to x

$$\Rightarrow \frac{\partial \psi}{\partial x} = e^{-2\pi i E t/h} \frac{\partial \psi_0}{\partial x}$$

Again, differentiating w.r.to x,

$$\frac{\partial^2 \psi}{\partial x^2} = e^{-2\pi i E t/h} \frac{\partial^2 \psi_0}{\partial x} \quad ---(5)$$

Now, differentiating eqn (3) w.r.to time 't'

$$\frac{\partial \psi}{\partial t} = \psi_0 e^{-2\pi i E t/h} \left(-\frac{2\pi i E}{h}\right) \quad --(6)$$

Now, substituting eqn (3), (5) and (6) in eqn (1) we get

$$i\hbar \psi_0 e^{-2\pi i E t/h} \left(-\frac{2\pi i E}{h}\right) = -\frac{\hbar^2}{2m} e^{-2\pi i E t/h} \frac{\partial^2 \psi_0}{\partial x^2} + V \psi_0 e^{-2\pi i E t/h}$$

$$\Rightarrow i\hbar \psi_0 e^{-2\pi i E t/h} \left(-\frac{iE}{\hbar}\right) = -\frac{\hbar^2}{2m} e^{-2\pi i E t/h} \frac{\partial^2 \psi_0}{\partial x^2} + V\psi_0 e^{-2\pi i E t/h}$$

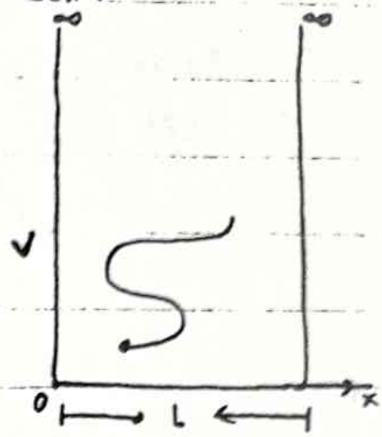
$$\Rightarrow E\psi_0 = -\frac{\hbar^2}{2m} \frac{\partial^2 \psi_0}{\partial x^2} + V\psi_0$$

$$\Rightarrow \frac{\hbar^2}{2m} \frac{\partial^2 \psi_0}{\partial x^2} + E\psi_0 - V\psi_0 = 0$$

$$\Rightarrow \frac{\partial^2 \psi_0}{\partial x^2} + \frac{2m}{\hbar^2} [E - V] \psi_0 = 0 \quad ---(7)$$

This is the time independent form of schrodinger wave equation.

* one dimensional potential well : (particle in a box)



consider a particle of mass m moving inside a box along x-axis (direction) and It is confined to move freely in the region $0 < x < L$

The potential energy 'V' of the particle is infinite on both sides of a box and zero can be assumed between $x = 0$ and $x = L$.

$$\text{i.e} \quad \left.\begin{array}{ll} V = 0 & 0 < x < L \\ V = \infty & 0 \geq x \geq L \end{array}\right\} \quad \text{---(1)}$$

since there is infinite potential at the boundary and particle can not exist outside the box so wave function ψ is zero for $0 \geq x \geq L$

$\therefore$ schrodinger wave equation within the region $0 < x < L$ is

$$\frac{\partial^2 \psi_0}{\partial x^2} + \frac{2m}{\hbar^2}(E - V)\psi_0 = 0 \quad \text{---(2)}$$

but in region $0 < x < L$, $V = 0$

$\therefore$ eqn (2) becomes

$$\frac{\partial^2 \psi_0}{\partial x^2} + \frac{2mE}{\hbar^2}\psi_0 = 0$$

$$\Rightarrow \frac{\partial^2 \psi_0}{\partial x^2} + k^2 \psi_0 = 0 \quad \text{---(3)}$$

where $k^2 = \frac{2mE}{\hbar^2}$ ---(4)

The solution of equation (3) can be writen as

$$\psi_0 = A \sin kx + B \cos kx \quad \text{---(5)}$$

To find the values of A and B we can use boundary condition.

At $x=0$, $\psi_0=0$

$\therefore$ From (5)

$$0 = A\sin 0 + B\cos 0$$

$$\Rightarrow B=0$$

Then eqⁿ (5) becomes

$$\psi_0 = A\sin kx \quad ---(6)$$

Again,

at $x=L$, $\psi_0=0$

$\therefore$ From (6) $\quad A\sin kL = 0$

$$A \neq 0$$

$$\therefore \sin kL = 0$$

$$\Rightarrow \sin kL = \sin n\pi$$

$$\Rightarrow kL = n\pi$$

$$\Rightarrow k = \frac{n\pi}{L} \quad ----(7)$$

From eqⁿ (6) and (7)

$$\psi_0 = A\sin\frac{n\pi}{L}x \quad ---(8) \qquad \left[\text{or } \psi_n(x) = A\sin\frac{n\pi x}{L}\right]$$

From (4) and (7) $\quad \frac{n^2\pi^2}{L^2} = \frac{2mE_n}{\hbar^2}$

$$\Rightarrow E_n = \frac{n^2\pi^2\hbar^2}{2mL^2} \quad ---(9)$$

This gives the total energy of particles

$\therefore$ we can say that for each value of n there is an energy level and corresponding wave function is given by $\psi_n(x)$

The values of E_n is called eigenvalue and corresponding wave function ψ_n is called eigen function. Thus inside the box, the particle can only have descrete energy values specified by E_n.

Note that particle can not come out of the box and also the particle can not have zero energy. It is certain that the particle is somewhere inside the box. Hence there is a probability of finding the particle inside the box. It is given as

$$\int_0^L \psi^* \psi \, dx = 1 \quad \text{-----}(10)$$

It is called normalized wave function

$\therefore$ From (8) and (10)

$$\int_0^L A \sin \frac{n\pi x}{L} \cdot A \sin \frac{n\pi x}{L} \, dx = 1$$

$$\Rightarrow A^2 \int_0^L \sin^2 \frac{n\pi x}{L} \, dx = 1$$

$$\Rightarrow A^2 \int_0^L \left[\frac{1 - \cos 2n\pi x/L}{2} \right] dx = 1$$

$$\Rightarrow A^2 \left[\frac{1}{2} \int_0^L dx - \frac{1}{2} \int_0^L \cos \frac{2n\pi x}{L} \, dx \right] = 1$$

$$\Rightarrow A^2 \left[\frac{L}{2} - \frac{1}{2} \left\{ \sin \frac{2n\pi x}{L} \cdot \frac{L}{2n\pi} \right\}_0^L \right] = 1$$

$$\Rightarrow A^2 \left[\frac{L}{2} - 0 \right] = 1$$

$$\Rightarrow \frac{A^2 L}{2} = 1$$

$$\Rightarrow A^2 = \frac{2}{L}$$

$$\Rightarrow A = \sqrt{2/L} \quad \text{---} \; (11)$$

$\therefore$ eq^n (8) becomes.

$$\psi_n(x) = \sqrt{\frac{2}{L}} \sin \frac{n\pi x}{L} \quad \text{-----} (12)$$

* physical significance of ψ -

The probability of that a particle will be found at a given place in space at given instant of time is characterised by the function ψ. It is called wave function. The function can be either real or complex. The only the quantity having a physical meaning is that the square of its magnitude

$$P = |\psi|^2$$

$$\Rightarrow P = \psi^* \psi$$

The quantity P is the probability density. The probability of finding the particle in a volume $dx\,dy\,dz$ is

$$|\psi|^2 \, dx\, dy\, dz.$$

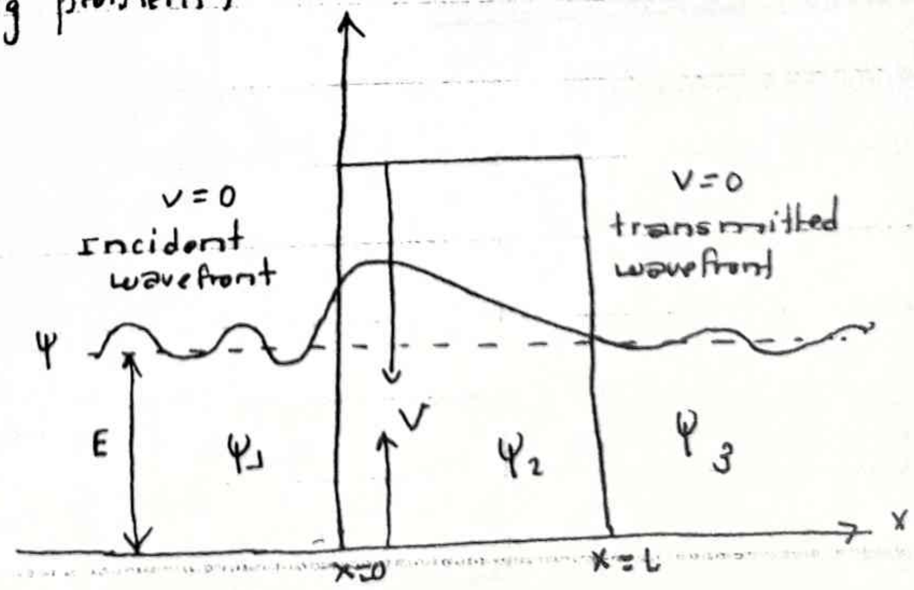
Further the particle is certainly to be found somewhere in space.

* potential barrier

(The barrier penetrating problem)

(Tunneling effect)

consider a beam of particles of kinetic energy E incident from the left on a potential barrier of height V and



width 'L', with $V > E$ and on the sides of the barrier $V=0$ which means that no forces acts upon the particles, there.

The potential is described as

$V = 0$, $x < 0$, region I

$V = V$, $0 < x < L$ region II

$V = 0$, $x > L$ region III

Let ψ_1, ψ_2 and ψ_3 be the respective wave function in region I, II and III as indicating in the figure.

The corresponding schrodinger wave equations are

In region I,

$$\frac{\partial^2 \psi_1}{\partial x^2} + \frac{2m}{\hbar^2}(E-V)\psi_1 = 0$$

but $V=0$

$$\therefore \quad \frac{\partial^2 \psi_1}{\partial x^2} + \frac{2mE}{\hbar^2}\psi = 0$$

$$\Rightarrow \quad \frac{\partial^2 \psi_1}{\partial x^2} + \alpha^2 \psi = 0 \quad ---(1)$$

where $\alpha = \frac{2mE}{\hbar^2}$ $--(2)$

similarly, in region II,

$$\frac{\partial^2 \psi_2}{\partial x^2} + \frac{2m}{\hbar^2}(E-V)\psi_2 = 0$$

$$\Rightarrow \quad \frac{\partial^2 \psi_2}{\partial x^2} - \beta^2 \psi_2 = 0 \quad --(3)$$

where, $\beta^2 = \frac{2m}{\hbar^2}(V-E)$ $---(4)$

In region III

$$\frac{\partial^2 \psi_3}{\partial x^2} + \frac{2m}{\hbar^2}(E-V)\psi_3 = 0$$

but $V=0$,

$$\therefore \frac{\partial^2 \psi_3}{\partial x^2} + \frac{2mE}{\hbar^2}\psi_3 = 0$$

$$\Rightarrow \frac{\partial^2 \psi_3}{\partial x^2} + \alpha^2 \psi_3 = 0 \quad ----(5)$$

The solution of eq^n (1), (3) and (5) can be written as

$$\psi_1 = Ae^{i\alpha x} + Be^{-i\alpha x}$$
$$\psi_2 = Fe^{-\beta x} + Ge^{\beta x}$$
$$\psi_3 = Ce^{i\alpha x} + De^{-i\alpha x}$$

where A is the amplitude of incident wave on the barrier from left. B is the amplitude of reflected wave in region I. F is the amplitude of the wave penetrating the barrier in region II, G is the amplitude of reflected wave in region II. C is the amplitude of transmitted wave and D is the amplitude of non-existing reflected wave in the region III.

since the probability density associated with the wave function is proportional to the square of the amplitude of that function, we define the barrier transmission coefficient as

$$T = \frac{|C|^2}{|A|^2}$$

And the reflected coefficent for the barrier surface at $x=0$ is

$$R = \frac{|B|^2}{|A|^2}$$

Date . .

The ratio $\frac{|C|^2}{|A|^2}$ is also called penetrability of the barrier. It represents the probability that a particle incidence on the barrier from one side will appear on the other side. such probability is zero classically. But it is finite quantity in quantum mechanics. we thus conclude that if the particle with energy E incident on the thin barrier of hight greater than E, there is a finite probability of the particle penetrating the barrier.

This phenomena is called Tunneling effect.

* semiconductor and superconductivity:
(chapter-10)

Date . .

* Semi-conductor:

The substance whose conductivity lies between those of conductor and insulator are called semi-conductors. For example Germinium and silican are the semi-conductors. They conduct electricity when an electric field is applied. The distinguishing feature about a semi-conductor is that conductivity increases as the temperature is increased.

* valence band -

The range of energy of valence electron is called valence band.

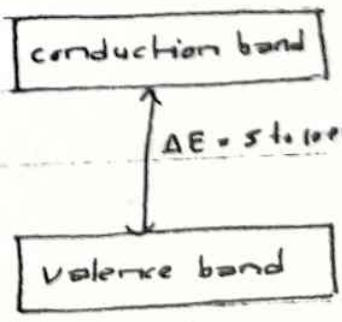
* conduction band:

when the electron in valence band get sufficient energy to become free from the atom, It reaches the higher band as conduction band. Therefore the range of energy of conduction electrons (free electrons) is called conduction band.

The valence band is separated by the conduction band by a certain energy gap called forbidden energy gap denoted by ΔE.

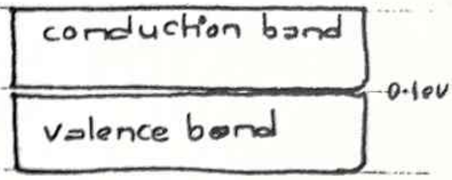
* Insulator, conductor and semi-conductor in terms of band theory:

* Insulator: In case of insulator valence band is completely full and conduction band is completely empty. And there is a large energy gap of an order of 5-10 ev between them.



Insulator.

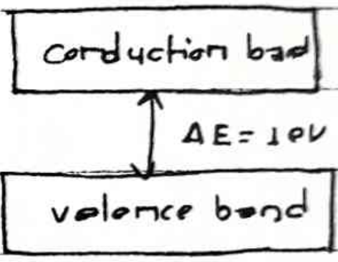
* conductor:



* conductor

In case of conductor there is overlaping between valence band and conduction band. The energy gap is very small of an order of 0.1 eV. Therefore the electron in the valence band can easily jump to the conduction band and the conductivity is very high.

* Semi-conductor:



semi-conductor.

In case of semi-conductor, the valence band is completely full and conduction band is completely empty.
There is small energy gap of 1eV between the valence band and conduction band. Thus at 0°K temperature semi-conductor behaves as an insulator. At room temperature, 1 eV energy becomes available from the atmosphere so that electrons from valence band jump to the conduction band and conductivity of the semi-conductor increases.

* Types of semi-conductor:

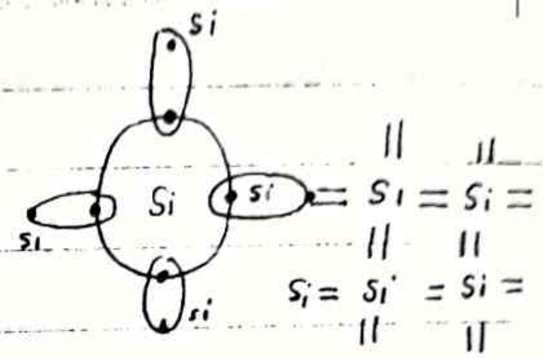
(1) Intrinsic semi-conductor:

Intrinsic semi-conductor is pure semi-conductor of four valence electron system. The valence electrons of an atom form covalent bonds with the four neighbouring atoms so as to seek eight electrons in the valence shell.

In this way all the valence electrons are located in the covalent bonds and no electrons is set free. As a

Date

result, the valence band is completely full and conduction band is completely empty.



At room temperature the valence band is completely full and conduction band is completely empty. There is a forbidden gap of 1eV. This energy becomes available from the su-roundings: Therefore some of the electrons from the covalent bond may comeout from thermal agitation and reach to the conduction band. A e vacancy is thus created in the valence band. This vacancy is called hole. The moment of hole is opposite to that of electron. Therefore the hole is called positive charge carrier. The conduction of electricity is due to both free electrons and the holes.

As the temperature increased, the valence electrons becomes free in more numbers and conductivity of the semi-conductor increases accordingly. Also as the crystal is neutral, the number of free electron is equal to the number of free holes.

(2) Extrinsic semi-conductor:

When some impurity atoms are mixed with a pure semi-conductor crystal, we get extrinsic semi-conductor. The process of adding impurity atoms is called dopping. The dopping agents are of two types (I) Donar agent and (II) Acceptor agents. Donar agents provide free electrons and is pentavalent atoms like As, Sb and acceptor agents provide free holes and is trivalent atoms like Indium (In) or Gallium (Ga).

According to the nature of impurity atoms the extrinsic semi-conductor is of two types-

* N-type extrinsic semi-conductor:

The four valence electrons of pentavalent impurity atom form co-valent bond with four neighbouring host atom. The fifth valence electron of the impurity atom does not take part in co-valent bonding and it thus almost free. This electron can be conveyed to the conduction band marely by small energy. The number of free electrons in the conduction band thus depends on the number of impurity atoms added. Also there are some free holes and free electrons formed by the thermal agitation. In all, the free electrons are in majority and free holes are in minority. Therefore it is called N-type semi-conductor.

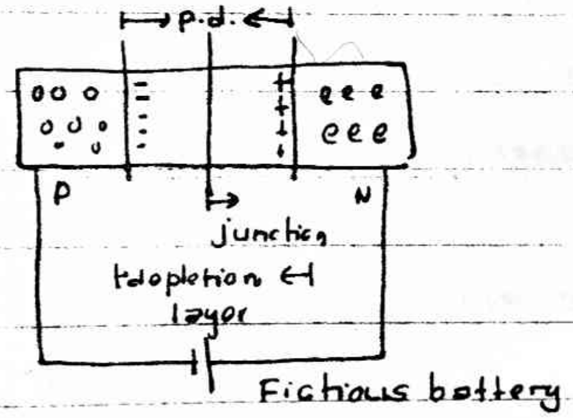
* P-type extrinsic semi-conductor:

The valence electron of trivalent impurity atom for covalent bond with the neighbouring atoms. In this way three bonds are formed by shearing equal electrons. However in the fourth bond electron is sheared by the host atom only and so this bond is devoid of an electron. This bond is filled by an electron from other covalent bond. The bond which releases electron acquires a free hole. In this way for each impurity atom one hole is created in the valence band. However, there are some free electrons in conduction band and equal numbers of free holes in valence band produced by thermal agitation. In all, holes are majority and free electrons are in minority. Therefore it is called p-type semi-conductor.

* P-N junction:

when a p-type semi-conductor is kept in contact with N-type semi-conductor so as to form a single piece of crystal, It is called P-N junction or junction diode. The surface which separates two types of semi-conductor is called junction.

when the crystal is formed electrons from N-type diffuse into p-type and recombine the holes. Therefore, round the junction there is certain region which contains no free charge carriers. (no electrons in N side and no holes in p side) This is called depletion layer.



The p-type region has positive holes as majority carriers and N-type region has negative electrons are as majority carriers. In addition there are few minority charge carriers in each region. Thus at the junction there is decreasing hole concentration from left to right which makes hole to diffuse from p-side to N-side. similarly electrons diffuse from right to left across the junction. Holes leaving and electrons entering the p-side make, it negative. similarly holes entering and electrons leaving the N-region makes It positive. Thus there is net negative charges on the p-side of the junction and net positive

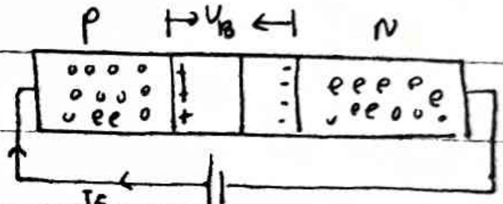
Date

charge on the N-side. This produces an electric field across the junction. The potential difference increases with the electrons diffuse into p- side and hole diffuse into N- side. For a certain potential difference across the junction, the flow of electrons stops. This is called barrier potential. (V_B)

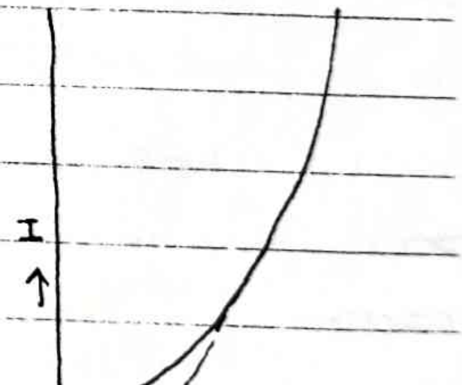
$V_B = 0.3V$ for Ge

$V_B = 0.7V$ for si

* Forward biasing:



when p-type of semi-conductor is connected to the positive terminal and N-type is connected to the negative terminal of battery, the junction diode is called forward biased.

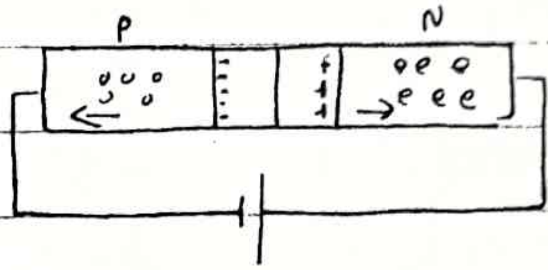


when applied voltage exceeds barrier potential the majority charge carriers starts crossing the junction and forward current increases i.e junction diode provides low resistance to the majority charge carriers.

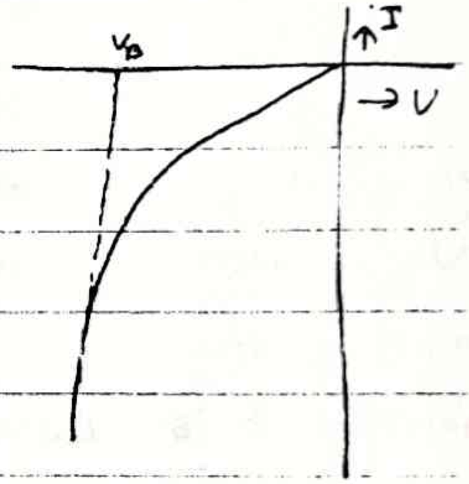
when applied voltage is zero, forward current is zero. when the forward voltage is increased the forward current increases gradually. At certain voltage current increases rapidly. The voltage which separates the forward high current and forward low current is called knee voltage. (V_K)

The knee voltage is nearly equal to barrier voltage.

* Reverse biasing:



when p-type of junction diode is connected to the negative terminal and N-type of junction is connected to the positive terminal of the battery then the junction diode is called reversed biased.



when applied voltage is increased, the majority charge carriers are pulled away from the junction so the thickness of the depletion layer increases. It means the junction diode provides high resistance to the majority charge carriers.

when reverse voltage is zero, there is no current in the circuit. when the reverse voltage is zero increased there is small current in the reverse direction. This is due to the minority charge carriers.

when the reverse biasing goes on increasing the current in the circuit increases in large amount at once. This is called junction breakdown.

* Metal semi-conductor junction:

To overcome the difficulties face in PN-junction diode a new diode is introduced, which is metal semi-conductor junction. In PN junction there generates a barrier potential which delay to conduct electricity in forward biased i.e applied voltage should be greater than the barrier potential, the least current flow in reverse biased condition due to minority charge carriers.

When two materials come into contact the energies at their surface are initially equal. The energy difference between the fermi level and surface (vacuum level) is called work function ϕ. If an electron is at fermi-level, the work function is the energy required to completely remove the electron from the material.

Consider a metal, workfunction ϕ_m in contact with an N-type semi-conductor with work function ϕ_s.

Let $\phi_m > \phi_s$

The fermi level of the semi-conductor is higher than the fermi-level of the metal. The electrons will flow from the conduction band of the semi-conductor to the metal and the metal will become negatively charged. There is no potential barrier between the metal and semi-conductor i.e will not delay the electricity.

* superconductivity:

The electrical resistivity of some metals and alloys drops suddenly to zero when the specimen is cooled to a sufficient low temperature. This phenomenon is called superconductivity. And the substances which shows that phenomenon called superconductors. At a certain temperature called the critical

Date

temperature, the specimen undergoes a phase transition from a state of normal electrical resistivity to a super conducting state.

For example for below a temperature of about 4K, mercury suddenly lost all resistivity and becomes a perfect conductor called superconductor.