A stretched string has a linear density 525 g/m and is under tension 45 N. We send a sinusoidal wave with a frequency 120 Hz and amplitude 8.5 mm along the string from one end. At what average rate does the wave transport energy?

Sol^n,

linear density (μ) $= 525$ g/m

$= \frac{525}{1000}$ kg/m

$= 0.525$ kg/m

Tension (T) $= 45$ N

frequency of wave (f) $= 120$ Hz

Amplitude (a) $= 8.5$ mm

$= 0.0085$ m

velocity of wave (v) $= \sqrt{\frac{T}{\mu}}$

$= \sqrt{\frac{45}{0.525}}$

$= 9.25$ m/s

Average rate at which wave transport energy i.e,

Intensity of wave is,

$I = \frac{1}{2} \times v \times \omega^2 \times a^2 \times \mu$

or $I = \frac{1}{2} \times 9.25 \times (2\pi \times 120)^2 \times (0.0085)^2 \times (0.525)$

$\therefore$ $I = 99.74$ watt.

Hence, intensity of wave is 99.74 watt.

2. Newton's ring are observed in reflected light of wavelength 5.9×10^{-5} cm. The diameter of 10^{th} dark ring is 0.5 cm. Find radius of curvature of this lens and thickness of air film.

Sol^n,

wavelength of reflected light $(\lambda) = 5.9 \times 10^{-5}$ cm
$= 5.9 \times 10^{-7}$ m

Diameter of 10^{th} dark ring $(D_{10}) = 0.5$ cm
$= 0.005$ m

Radius of curvature of lens $(R) = ?$

thickness of air film $(t) = ?$

We know,

For dark ring,

$$\text{Diameter} = 2 \times r_n$$

or $$D_n = 2 \times \sqrt{n \lambda R}$$

or $$(0.005)^2 = 2 \times \sqrt{10 \times 5.9 \times 10^{-7} \times R}$$

$\therefore$ $R = 1.0593$ m.

Hence, Radius of curvature of lens is 1.059 m

For thickness of air film (t):

or $2t = n\lambda$

or $t = \frac{10 \times 5.9 \times 10^{-7}}{2}$

$\therefore t = 2.95 \times 10^{-6}$ m.

Hence, thickness of air film is 2.95×10^{-6}m.

3. What is magnitude of a point charge chosen so that the electric field 50 cm away has maximum magnitude of 2N/C?

Sol^n,

$\Rightarrow$ Electric field (E) = 2 N/C.

magnitude of point charge (q) = ?

Distance (d) = 50 cm

= 0.5 m.

We have,

or $E = \frac{q}{4\pi\epsilon_0 r^2}$

or $2 = \frac{9 \times 10^9 \times q}{(0.5)^2}$

$\therefore q = 5.56 \times 10^{-11}$ C.

4. A 200 mm long tube containing 48 cm^3 of sugar produces an optical rotation of 11° when placed in a sugar saccharimeter. If the specific rotation of sugar solution is 66°. Calculate quantity of sugar contained in the tube in the form of solution.

Sol^n,

Angle of rotation $(\theta) = 11^\circ$

Specific rotation of sugor solution $(S) = 66^\circ$

length of tube (l) = 200 mm

= 20 cm

concentration of active substance $(C) = ?$

volume of sugor (V) = 48 cm^3

We have,

$$S\,l\,C = 10 \cdot \theta$$

or $$C = \frac{10 \times 11^\circ}{20 \times 66}$$

$$\therefore C = \frac{1}{12} \text{ gm/cc}$$

$\therefore$ Quantity of sugor $(m) = C \times V$

$$= \frac{1}{12} \times 48$$

$$= 4 \text{ gm.}$$

5. The time of reverberation of an empty hall with 500 audience in hall is 1.5 sec and 1.4 sec respectively. Find reverberation time with 800 audience in the hall.

Sol^n,

=> We have,

$$T = \frac{0.158\,V}{\alpha \cdot S}$$

For 1.5 sec,

$$1.5 = \frac{0.158\,V}{\alpha S} \quad \text{— (1)}$$

For $t = 1.4$,

$$1.4 = \frac{0.158\ V}{\alpha.S + 500} \quad \text{--- (ii)}$$

Dividing eq^n (i) by (ii)

or $\frac{1.5}{1.4} = \frac{\alpha.S + 500}{S.\alpha}$

or $1.5\alpha.S = 1.4\alpha.S + 700$

$\therefore \alpha S = 7000$

So, from (i),

$$V = \frac{1.5 \times \alpha S}{0.158}$$

$$= \frac{1.5 \times 7000}{0.158}$$

$\therefore V = 66455.7\ m^3$

So,
time of reverberation with 800 audience,

$$T_3 = \frac{0.158\ V}{\alpha S + 800}$$

$$= \frac{0.158 \times 66455.7}{7000 + 800}$$

$\therefore T_3 = 1.346$ sec

6. A circular loop of wire 5 cm of radius carries a current of 100 amp. what is energy density at the centre of the loop?

[redacted]

Tension in the wire $(T) = 500\ N$

We have,

$$V = \sqrt{\frac{T}{\mu}}$$

$$= \sqrt{\frac{T \times l}{m}}$$

$$= \sqrt{\frac{500 \times 0.2}{0.06}}$$

$$\therefore V = 40.82\ m/s$$

8. Calculate wavelength, frequency, speed of wave, maximum particle velocity in the wave represented by $y = 20 \sin\pi(2t - 0.05x)$. values of x and y are in cm.

Soln,

$\Rightarrow$ Given wave is,

$$y = 20 \sin\pi(2t - 0.05x)$$

$$= 20 \sin(2\pi t - 0.05\pi x)$$

Comparing with

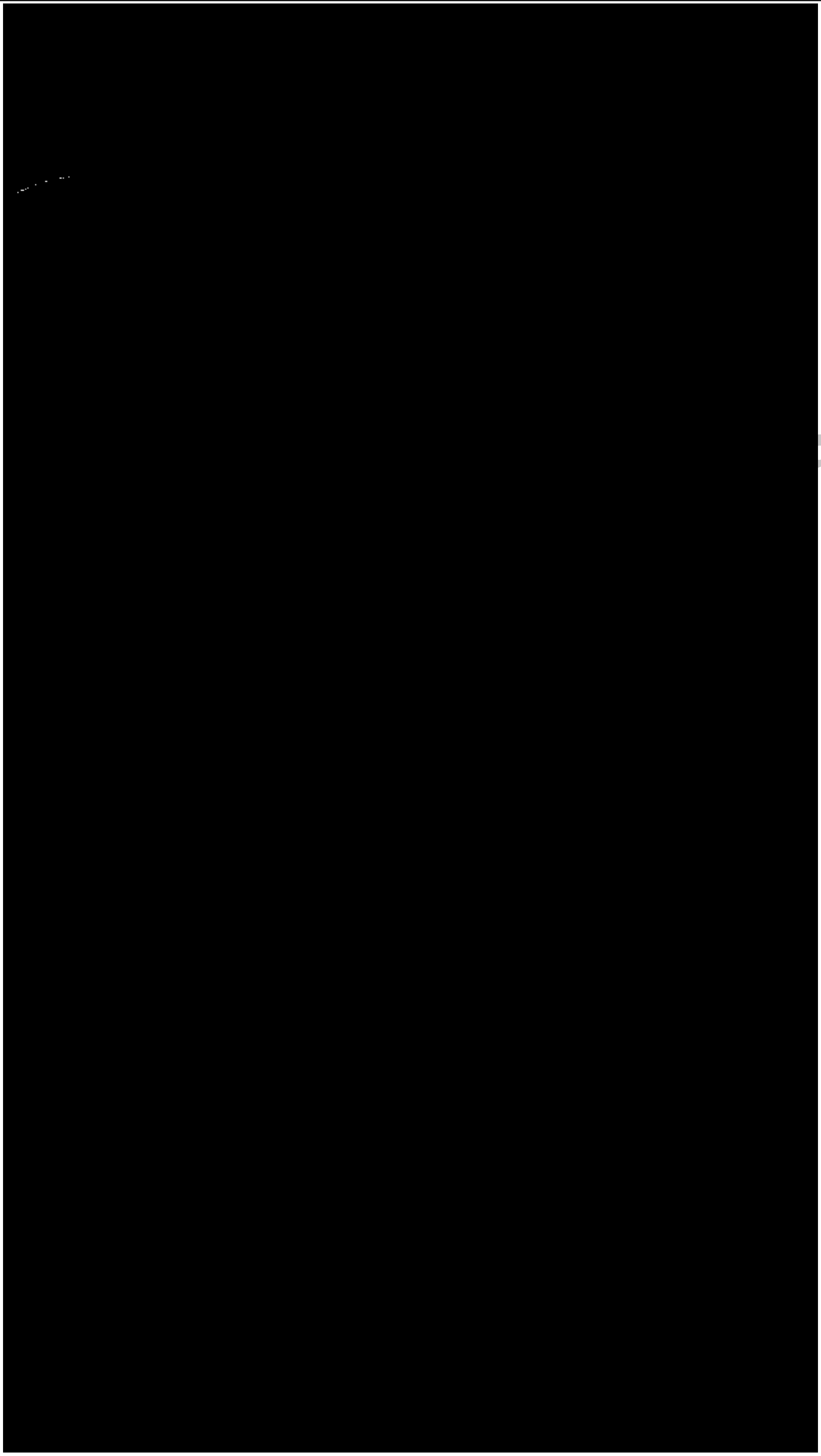
$$y = A \sin(\omega t - kx).$$

we get,

i) $A = 20\ cm$

ii) $k = 0.05\pi$

or) $\frac{2\pi}{\lambda} = 0.05\pi$



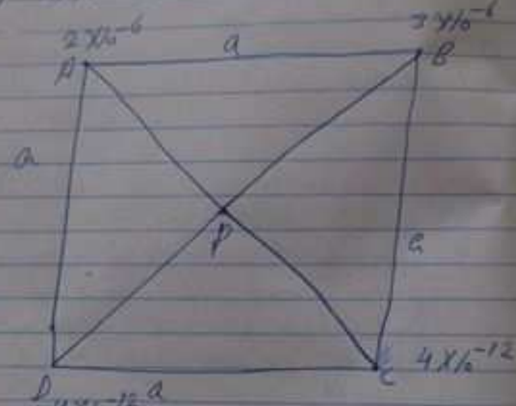
or $f^2 = 60761.52$

∴ $f = 246.5$ Hz

10 Find potential at center of square having charges 2×10^{-6} C, 3×10^{-6} C, 4×10^{-12} C, -4×10^{-12} C at corners

sol^n.

From figure,



$AC = \sqrt{a^2+a^2}$

$= a\sqrt{2}$

$AP = \dfrac{AC}{2}$

$= a/\sqrt{2}$

$AP = BP = PD = PC$

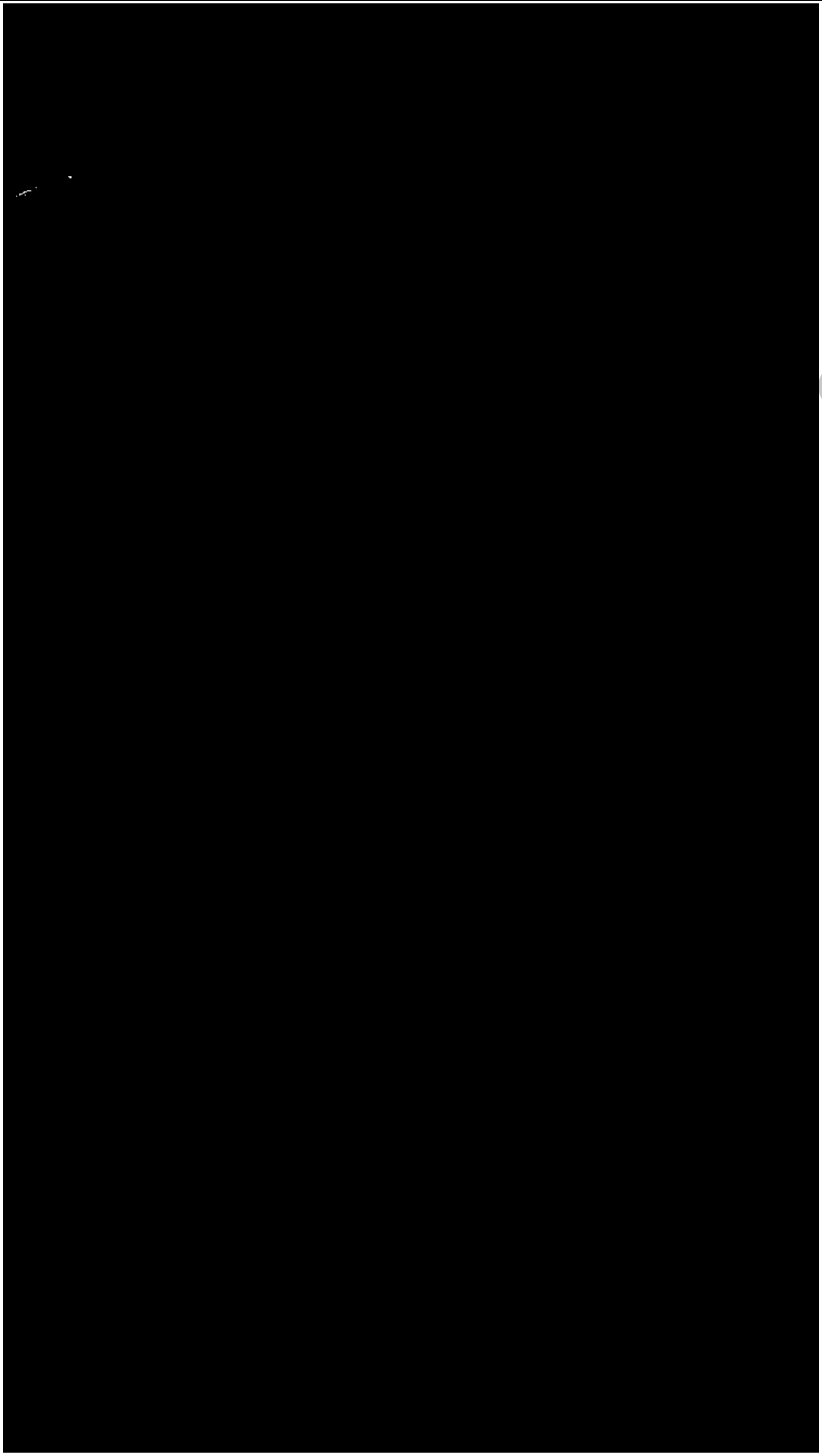
Total potential at P is.

$$V = \frac{1}{4\pi\epsilon_0}\left[\frac{q_1}{AP} + \frac{q_2}{BP} + \frac{q_3}{CP} + \frac{q_4}{PD}\right]$$

$$= \frac{9\times10^9}{a/\sqrt{2}} \times \left[2\times10^{-6} + 3\times10^{-6} + 4\times10^{-12} - 4\times10^{-12}\right]$$

$$= \frac{9\times10^9}{a/\sqrt{2}} \times 5\times10^{-6}$$

∴ $V = \dfrac{4.5\times10^4}{\sqrt{a}}$ Volts.



Sol^n,

Radius $(r) = 5.1 \times 10^{-11}$ m

frequency $(f) = 6.8 \times 10^{15}$ rev/sec

$= 6.8 \times 10^{15} \times 2\pi \cdot \frac{1}{sec}$

$= 4.273 \times 10^{16}$ Hz

magnetic field at center $(B) = ?$

We know,

$$B = \frac{\mu_0 I}{2r}$$

$$= \frac{\mu_0 (q/t)}{2r}$$

$$= \frac{\mu_0}{2r} \times ne \times f$$

$$= \frac{4\pi \times 10^{-7}}{2 \times 5.1 \times 10^{-11}} \times 1 \times 1.6 \times 10^{-19} \times 4.27 \times 10^{16}$$

$$\therefore B = 13.43 \text{ T}$$

13. What distance should the plate each of area $0.2\text{ m} \times 0.1\text{ m}$ of an air capacitor be placed in order to have same capacitance as a spherical conductor of radius 0.5 m ?

Sol^n,

~~For parallel~~

For parallel plate capacitor,

$$C = \frac{\epsilon_0 A}{d}$$

So, $$C = \frac{\epsilon_0 \times (0.1 \times 0.2)}{d} \quad \text{--- (1)}$$

[redacted]

$$\alpha = \omega^2 \theta$$
$$= \left(\frac{2\pi}{T}\right)^2 \times \frac{\pi}{2}$$
$$= \left(\frac{2\pi}{0.5}\right)^2 \times \frac{\pi}{2}$$
$$\therefore \alpha = 248 \text{ rad/sec}^2$$

15. What is initial rate of increase of current and final saturation current in LR circuit with $L = 15\,mH$, $R = 24\,\Omega$ and emf $= 10\,V$?

Sol^n,

$\Rightarrow$ $R = 24\,\Omega$

$E = 10\,V$

$L = 15\,mH$

$= 15 \times 10^{-3}\,H.$

For increase of current,

$$I = I_0 \left(1 - e^{-\frac{Rt}{L}}\right)$$

Here,

$$I_0 = \frac{E}{R}$$
$$= \frac{10}{24}$$
$$= 0.4167\ A$$

So,

$$\frac{dI}{dt} = 0.4167 \times (-)\left(\frac{-R}{L}\right)\left(e^{\frac{-Rt}{L}}\right)$$

$\frac{dI}{dt} = 0.4167 \times \left(\frac{24}{15 \times 10^{-3}}\right) \left(e^{\frac{-R \times 0}{L}}\right)$

$\therefore \frac{dI}{dt} = 666.72$ A/sec.

16. A LC circuit is converted into an LCR circuit inserting a resistance of 10Ω. calculate percentage change in frequency in this process.

sol^n,

$\Rightarrow$ Inductance (L) = 10 mH
$= 10 \times 10^{-3}$ H

capacitance, C = 10 μF
$= 10 \times 10^{-6}$ F

$R = 10\ \Omega$

For LC circuit,

$$f = \frac{1}{2\pi}\sqrt{\frac{1}{LC}}$$

For LCR circuit,

$$f' = \frac{1}{2\pi}\sqrt{\frac{1}{LC} - \left(\frac{R}{2L}\right)^2}$$

So,

$$f = \frac{1}{2\pi}\sqrt{\frac{1}{10^{-3} \times 10^{-6}}}$$

$= 5032.92$ Hz

$$f' = \frac{1}{2\pi}\sqrt{\frac{1}{10^{-3} \times 10^{-6}} - \left(\frac{10}{2 \times 10^{-3}}\right)^2}$$

$\therefore$ $f' = 4969.61$ Hz

so,

$\% \text{ change in frequency} = \frac{f - f'}{f} \times 100 \%$

$= \frac{5032.92 - 4969.61}{5032.92}$

$= 1.26 \%$

17. A parallel plate capacitor has capacitance of 100×10^{-12} F, a plate area of 100 cm^2 mica is used as dielectric. At 50 volts p.d calculate electric field intensity and magnitude of induced charged.

Soln,

=> capacitance (C) = 100×10^{-12} F

Area (A) = 100 cm^2

$= (0.00001)\ m^2$

potential diff (V) = 50 V

Electric field intensity (E) = ?

We have,

$C = \frac{\epsilon_0 A}{d}$

or $d = (0.0001) \times \frac{8.85 \times 10^{-12}}{100 \times 10^{-12}}$

$\therefore$ $d = 8.85 \times 10^{-4}$ m

Electric field intensity $(E) = \frac{q}{\epsilon_0 A}$

$$= \frac{CV}{\epsilon_0 A}$$

$$= \frac{100 \times 10^{-12} \times 50}{8.85 \times 10^{-12} \times 0.0001}$$

$$\therefore E = 5.65 \times 10^{6} \ N/C$$

Also,

magnitude of induced charged $(q) = CV$

$$= 50 \times 100 \times 10^{-12}$$

$$= 5 \times 10^{-9} \ V.$$

18. What is highest order speed which may be seen with monochromatic light of wavelength 6000 A° by means of a diffracting grating with 5000 lines /cm.

Soln,

$\Rightarrow$ $\lambda = 6000 \ A°$

$= 6 \times 10^{-7} \ m$

$= 6 \times 10^{-5} \ cm$

$N = (1/5000)$ lines/cm

We have,

$(a+b) \sin\theta = n\lambda$

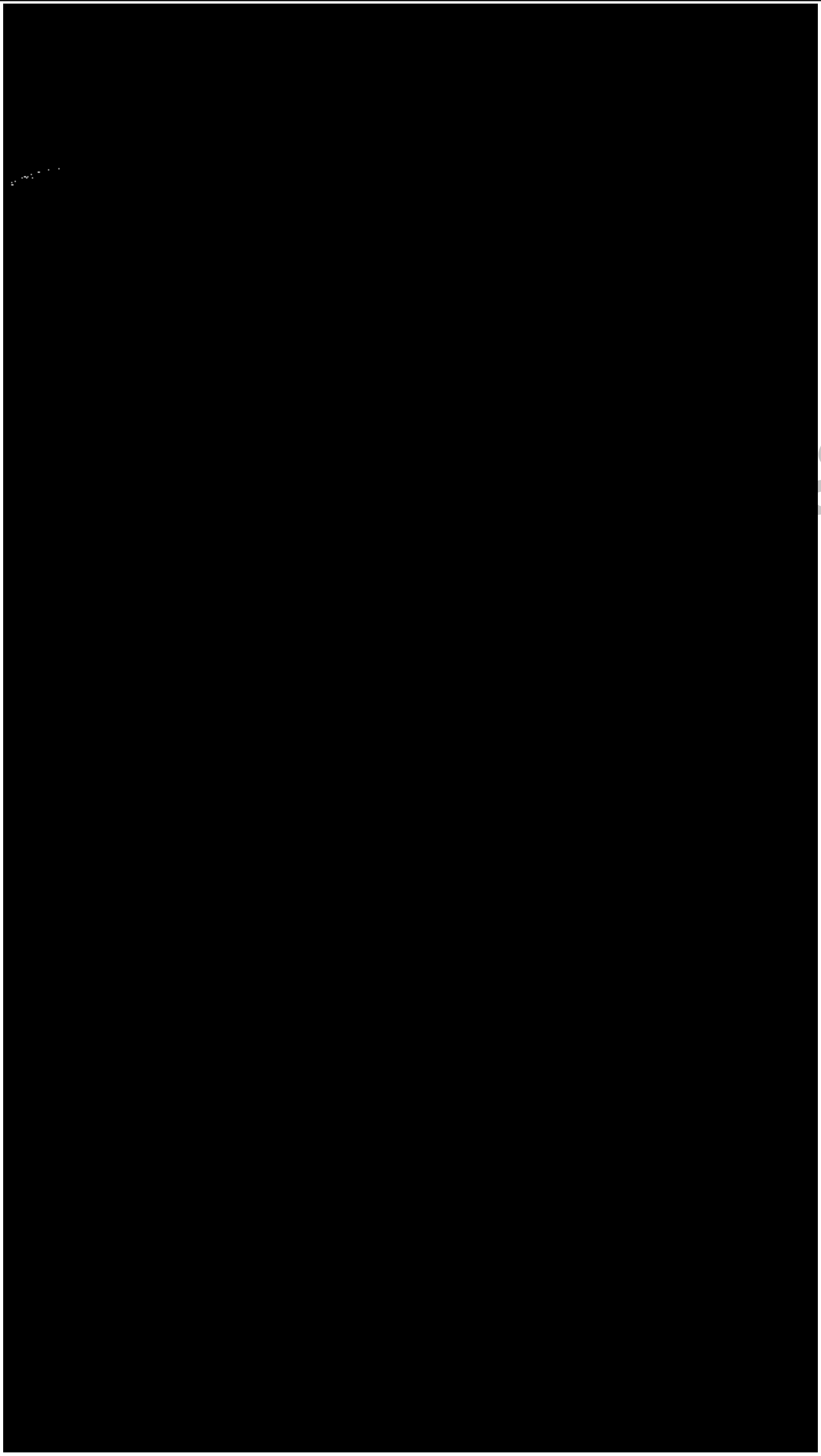
or $\frac{1}{5000} \times \sin\theta = n \times 6 \times 10^{-5}$

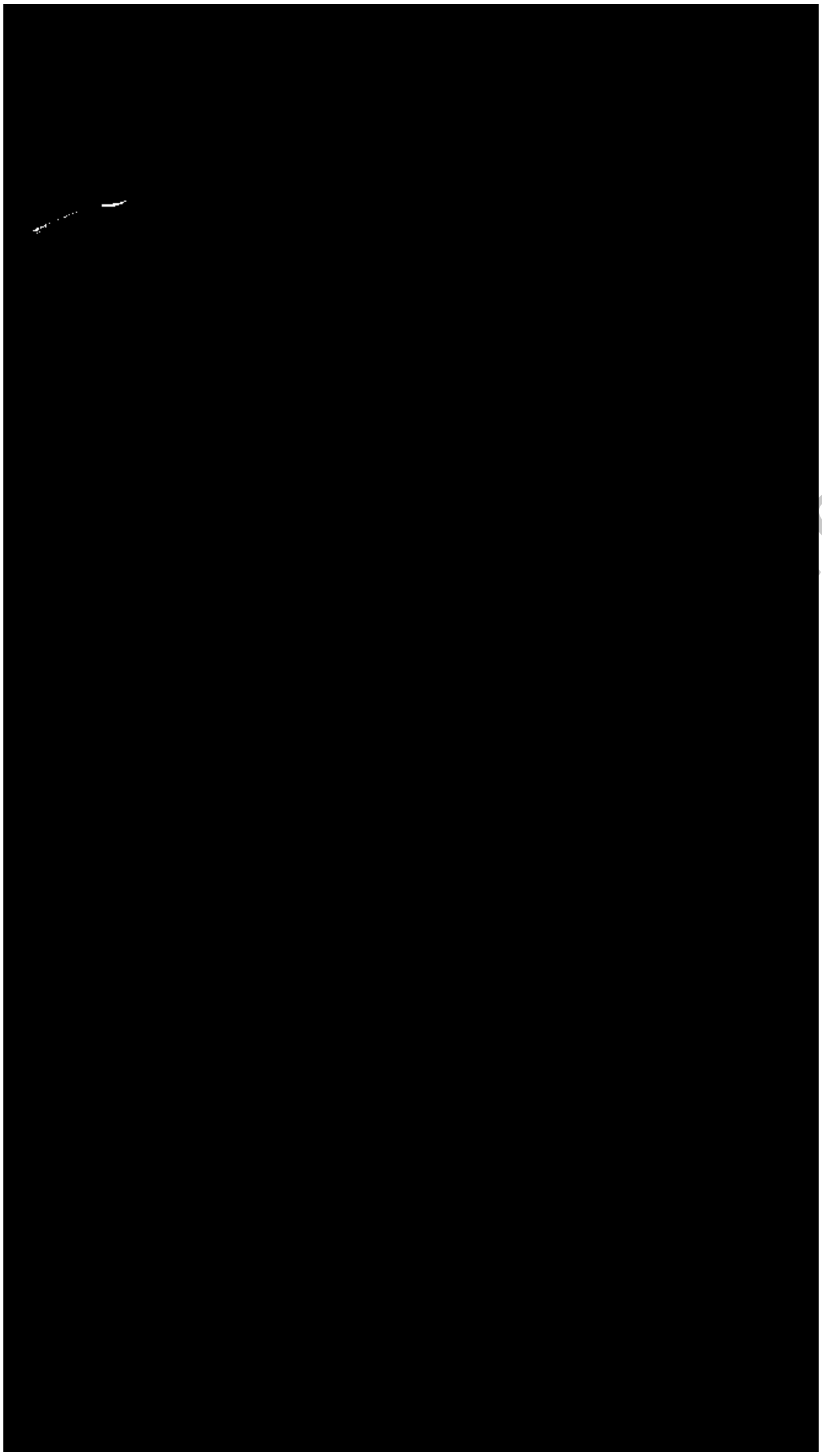
For highest order,

$\sin\theta = 1,$

or $n = \frac{1}{5000 \times 6 \times 10^{-5}}$

[redacted]





[redacted]

and magnetic force on electron.

Soln.

$\Rightarrow$ $E_y = 300 \sin\left[\omega t - \frac{\omega x}{c}\right]$.

Comparing with,

$E_y = E_m \sin\left[\omega t - kx\right]$ — ①

we get,

$E_m = 300$ V/m.

$\omega = 1$

$\therefore$ $kx = \frac{\omega x}{c}$

or $\frac{2\pi}{\lambda} = \frac{\omega}{c}$

or $\frac{2\pi}{\lambda} = \frac{2\pi f}{c}$

$\therefore$ $\lambda = \frac{c}{f}$.

Diff ① w.r.t. x,

$\frac{dE}{dx} = E_m \cdot \left(\frac{\omega}{c}\right) \cos\left[\omega t - \frac{\omega x}{c}\right]$.

~~Diff ① w.r.t. t,~~

$\cancel{\frac{dB}{dt} = E_m}$

Also, $\frac{E_m}{B_m} = c$

or $\frac{300 \times \cancel{9} \times}{2 \times 10^7} = B_m$

$\therefore \quad B_m = 1.5 \times 10^{-5}$ T

25. At some distance from transmitter of radio station, magnetic field of electromagnetic wave emitted by radio station is found to be 1.6×10^{-4} T. If frequency of broadcast is 1020 KHZ, find speed, wavelength, maximum electric field of electromagnetic wave.

Sol^n,

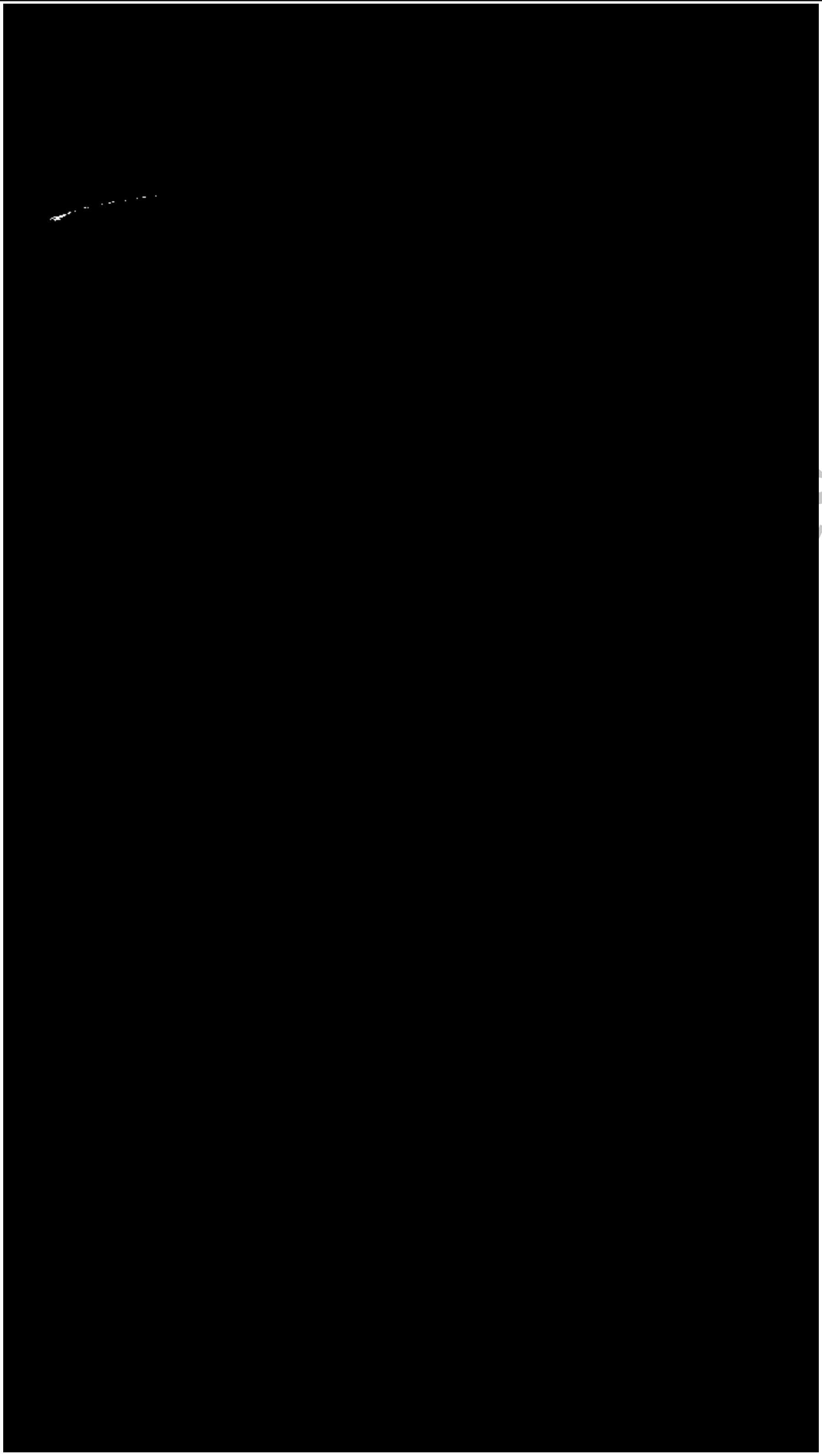
$\Rightarrow \quad f = 1020$ KHz
$\quad = 1020000$ Hz

$B_m = 1.6 \times 10^{-4}$ T.

speed (c) $= \frac{1}{\sqrt{\mu_0 \epsilon_0}}$
$= 3 \times 10^8$ m/s.

wavelength (λ) $= \frac{c}{f}$
$= \frac{3 \times 10^8}{1020000}$
$= 294.12$ m

electric field (E_m) $= c \cdot B_m$
$= 3 \times 10^8 \times 1.6 \times 10^{-4}$
$= 4.80 \times 10^4$ V/m.



$$= \frac{-1.6 \times 10^{-19}}{8.85 \times 10^{-12}}$$

$\therefore\ E = -1.8 \times 10^{-8}\ N/C.$

-ve sign shows that electric field points radially ~~inward~~ inward.

27. A particle is moving in one dimensional box at infinite probability of finding the particle within range 1 A° at centre of box when it is in lowest energy state.

Sol^n,

=). $L = 1\ A^\circ$

$P = ?$

At ground state, $n = 1$,

At center, $x = \frac{L}{2}$

We have,

$$\psi(x) = \pm\sqrt{\frac{2}{L}}\ \sin\left(\frac{\pi}{L} \cdot \frac{L}{2}\right)$$

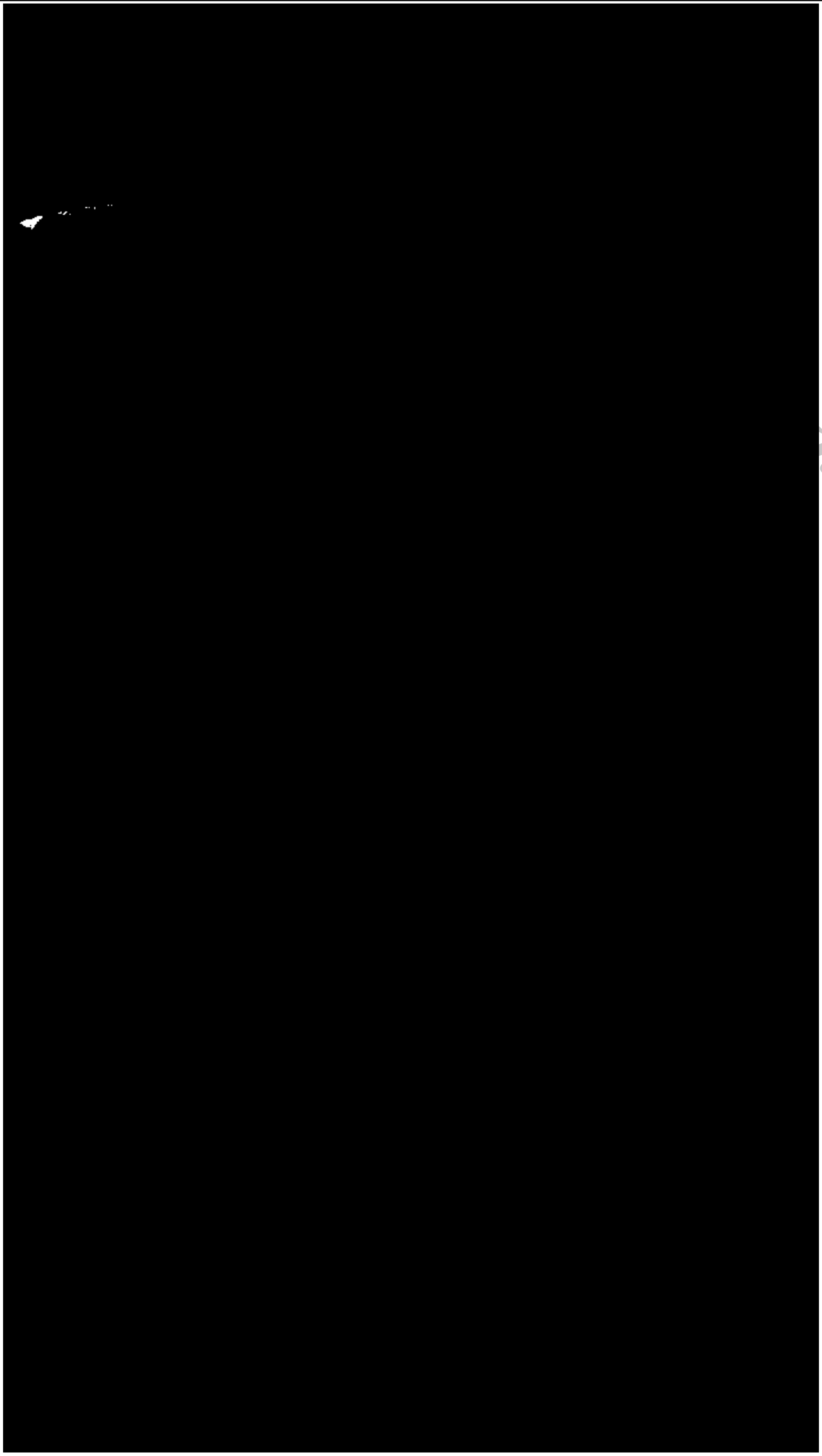
So,

$$\psi(x) = \pm\sqrt{\frac{2}{L}}.$$

Now,

$$P = \int_{x_1}^{x_2} |\psi|^2\, dx$$

$$= \frac{2}{L}\int_{x_1}^{x_2} dx$$



$\therefore \quad N = 5700$ lines/cm

Hence, number of lines per cm is 5700.

29 calculate possible order of spectra with a plane transmission grating having 18000 lines per inch when wavelength of light 4500 A° is used.

Sol^n,

$$\lambda = 4500\ A^\circ$$
$$= 4.5 \times 10^{-5}\ m$$

possible order $(n) = ?$

grating $(N) = 18000$ lines/inch.

$$(a+b) = \frac{1 \text{ inch}}{18000}$$
$$= \frac{2.54}{18000}\ cm$$
$$= 1.4111 \times 10^{-4}\ cm$$

So,

$$(a+b)\sin\theta = n\lambda$$

For $n = 1$

$$(a+b)\sin\theta = \lambda$$

or $1.4111 \times 10^{-4} \times \sin\theta = 4.5 \times 10^{-5}$

or $\sin\theta = 0.318$

$\therefore \quad \theta_1 = 18.59^\circ$

For $n = 2$,

$$(a+b)\sin\theta = 2\lambda$$

or $1.411 \times 10^{-4} \times \sin\theta = 2 \times 4.5 \times 10^{-5}$

or $\sin\theta = 0.6378$

$\therefore\ \theta_2 = 39.63°$

For $n = 3$,

$1.411 \times 10^{-4} \times \sin\theta = 3 \times 4.5 \times 10^{-5}$

or $\sin\theta = 0.9567$

$\therefore\ \theta_3 = 73.09$

For $n = 4$,

or $1.411 \times 10^{-4} \times \sin\theta = 4 \times 4.5 \times 10^{-5}$

or $\sin\theta = 1.275$

> 1 (impossible).

Hence,

possible order of spectra are 1, 2, 3.

30. How many time constants must we wait for the current in LR circuit to build up to within 0.1 % of its equilibrium value?

Sol^n,

or, According to question,

$$I = I_0 - \frac{0.1}{100} \times I_0$$

$$I = 0.999\ I_0$$

We have,

$$I = I_0\left(1 - e^{\frac{-Rt}{L}}\right)$$

or $$0.999 = \left(1 - e^{\frac{-Rt}{L}}\right)$$

or $$-0.0010 = \left(-e^{-Rt/L}\right)$$

or $\ln(0.0010) = \frac{-Rt}{L}$

or, $-6.907 = \frac{-t}{\tau}$

or $\frac{t}{\tau} = 6.907$

Hence, time constant is 6.907

31. Other probable important Questions.

A stretched string has a linear mass density of 5 gm/cm and tension of 10 N. A wave on this string has an amplitude of 0.12 mm and a frequency of 100 Hz. and is traveling in -ve x-direction. Write wave eqn with appropriate units.

Soln,

=> linear mass density $(\mu) = 5$ gm/cm

$= \frac{5 \times 10^{-3}}{10^{-2}}$ kg/m

$= 0.5$ kg/m

Tension $(T) = 10$ N.

So,

velocity $(v) = \sqrt{\frac{T}{\mu}}$

$= \sqrt{\frac{10}{0.5}}$

$= 4.47$ m/sec.

Amplitude $(a) = 0.12$ mm

$= 0.00012$ m.

Frequency $(f) = 100$ Hz.

Since, $v = f\lambda$

$\therefore \quad \lambda = \dfrac{4.47}{100}$

$\therefore \quad \lambda = 0.0447$ m

So,

$k = \dfrac{2\pi}{\lambda}$

$= \dfrac{2\pi}{0.0447}$

$\therefore \quad k = 141 \ m^{-1}$

The equation of wave travelling in $-ve$ x direction is,

$y = a \sin(\omega t + kx)$

$= 0.00012 \left[\sin(2\pi f)t + (141)x\right]$

$= 0.00012 \sin\left[628 t + 141 x\right]$

is required eq^n.

32. A wave of frequency 500 cycles/sec has a phase velocity of 350 m/s. How far apart are two points 60° out of phase. What is phase difference between two displacements at certain points at time of 10^{-3} sec apart.

Sol^n,

$\Rightarrow$. $f = 500$ cycles/sec.

$v = 350 \ m/s$

Since, $v = f \times \lambda$

$\lambda = \frac{v}{f}$

$= \frac{350}{500}$

$\therefore \lambda = 0.7 \ m$

i) We have, phase diff 2π = path diff λ

phase diff 1 = path diff $\frac{\lambda}{2\pi}$

phase diff 60° = path diff $\frac{\lambda}{2\pi} \times 60^\circ$

$\therefore$ path diff $= \frac{0.7 \times 60}{2 \times 180}$

$\doteq 0.116 \ m$

ii) time period $(T) = \frac{1}{f}$

$= \frac{1}{500}$

$= 2 \times 10^{-3}$ sec.

For time period T, phase diff 2π

For time period 1 sec, phase diff $\frac{2\pi}{T}$

For time period 10^{-3} sec, phase diff $= \frac{2\pi}{T} \times 10^{-3}$

$\therefore$ Phase diff $= \dfrac{2\pi}{2\times10^{-3}} \times 10^{-3}$

$= \pi$ rad

33. A string 2.72 m long has a mass of 263 gm. The tension in the string is 36.1 N. What is the frequency of traveling waves of amplitude 2.5 mm in order that average transmitted power is 85.5 W?

Sol^n,

$\Rightarrow$ length of string $(l) = 2.72$ m

mass of string $(m) = 263$ gm

$= 0.263$ kg

tension in string $(T) = 36.1$ N

frequency $(f) = ?$

amplitude $(a) = 2.5$ mm

$= 0.00250$ m

power $(P) = 85.5$ W

We have,

$$v = \sqrt{\frac{T}{\mu}}$$

$$= \sqrt{\frac{T \times l}{m}}$$

$$= \sqrt{\frac{36.1 \times 2.72}{0.263}}$$

$$= 19.32 \text{ m/s}$$

So,

$$P = \frac{1}{2} A \times a^2 \times \mu \times v \times \omega^2$$

or $85.5 = \frac{1}{2} \times \rho \times (0.0025)^2 \times 19.32 \times \left(\frac{0.263}{2.72}\right) \times (2\pi f)^2$

or, $f = 609$ Hz

34. In Newton's ring experiment, radius of 2^{nd} and 9^{th} rings are 0.22 cm and 0.37 cm respectively. Find diameter of 17^{th} dark ring.

Soln.

$\Rightarrow$ Given,

Radius of 2^{nd} ring $(r_2) = 0.22$ cm

9^{th} $(r_9) = 0.37$ cm

Diameter of 17^{th} dark ring $(D_{17}) = ?$

We have,

$$\lambda = \frac{D_n^2 - D_m^2}{4(n-m)R}$$

or $$\lambda = \frac{D_9^2 - D_2^2}{4(9-2)R}$$

$$\therefore \lambda R = \frac{D_9^2 - D_2^2}{28}$$

Also,

$$D_{17}^2 = 4n\lambda R$$

$$= 4 \times 17 \times \frac{D_9^2 - D_2^2}{28}$$

$$= \frac{17}{7}\left(4r_9^2 - 4r_2^2\right)$$

$$= \frac{17}{7} \times 4\left(0.37^2 - 0.22^2\right)$$

or $D_{17}^2 = 0.859$ cm
$\therefore$ $D_{17} = 0.927$ cm
Hence, diameter is 0.927 cm.

35. Find specific rotation of a sample of sugar solution if plane of polarization is turned through 46°. The length of tube containing 25 % solution is 25 cm.

Sol^n,

$\Rightarrow$ $\theta = 46°$

$l = 25$ cm

$c = 25$ %

$= \frac{1}{4}$ gm/cc

we have,

$$S = \frac{10\ \theta}{lc}$$

$$= \frac{10 \times 46}{25 \times 0.25}$$

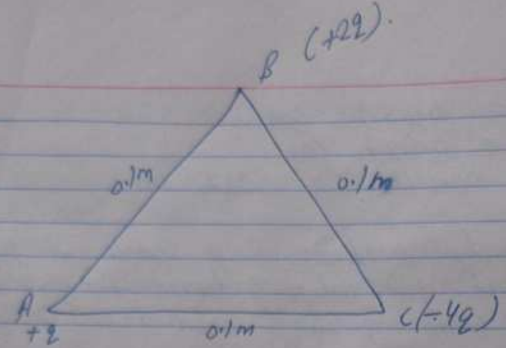
$$\therefore\ S = 73.60°$$

36. Three charges +q, +2q and −4q are placed at the three vertices of an equilateral triangle of side 10 cm. What is mutual potential energy of the system of the charges.

Sol^n,

distance (a) = 10 cm

= 0.1 m



Total energy of configuration is

$U = U_{AB} + U_{BC} + U_{CA}.$

$$= \frac{1}{4\pi\epsilon_0}\left[\frac{q_1 q_2}{a} + \frac{q_2 q_3}{a} + \frac{q_3 q_1}{a}\right]$$

$$= \frac{1}{4\pi\epsilon_0 a}\left[q.2q + 2q.(-4q) + (-4q)(+q)\right]$$

or $U = -\frac{10 q^2}{4\pi\epsilon_0 a}$

or $U = -\frac{10 \times 9\times10^9 \times q^2}{0.1}$

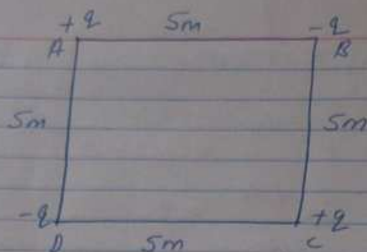
$\therefore U = -9.1 \times 10^{+11} \times q^2$ Joule.

37. Find work done on assembling four charges $+q$, $-q$, $+q$ and $-q$ on the corners of a square of side 5 m. $[q = 2\times10^{-8}\ C]$

Sol^n,

$q = 2\times10^{-8}\ C$

$r = 5\ m.$



$$U = \frac{1}{4\pi\epsilon_0}\left[\frac{q_1q_2}{a} + \frac{q_2q_3}{a} + \frac{q_3q_4}{a} + \frac{q_4q_1}{a}\right]$$

$$= \frac{1}{4\pi\epsilon_0 a}\left[+q\times -q + (-q)(+q) + (+q)(-q) + (+q)(-q)\right]$$

$$= \frac{1}{4\pi\epsilon_0 a}\left[-4q^2\right]$$

$$= -\frac{9\times 10^9 \times 4 \times (2\times10^{-8})^2}{5}$$

$$\therefore\ U = 2.88\times10^{-6}\ J$$

38. A capacitor of capacitance C is charged through a resistor R. Calculate the time at which potential across the resistor is equal to potential across the capacitor.

Sol^n,

$$V_R = V_C = IR = \frac{q}{C}$$

[redacted]

40. If 10 mH inductor and two capacitor of 5 μF and 2 μF are given, find two resonant frequencies that can be obtained by connecting these elements in different ways.

Soln,

$\Rightarrow$ $L = 10\ mH$

$C_1 = 5\ \mu F$

$= 5 \times 10^{-6}\ F$

$C_2 = 2\ \mu F$

$= 2 \times 10^{-6}\ F$

1) In series connection,

$$C = \frac{C_1 C_2}{C_1 + C_2}$$

$$= \frac{5 \times 2}{5 + 2}$$

$$= 1.43\ \mu F$$

$$= 1.43 \times 10^{-6}\ F$$

$$\therefore f_r = \frac{1}{2\pi\sqrt{LC}}$$

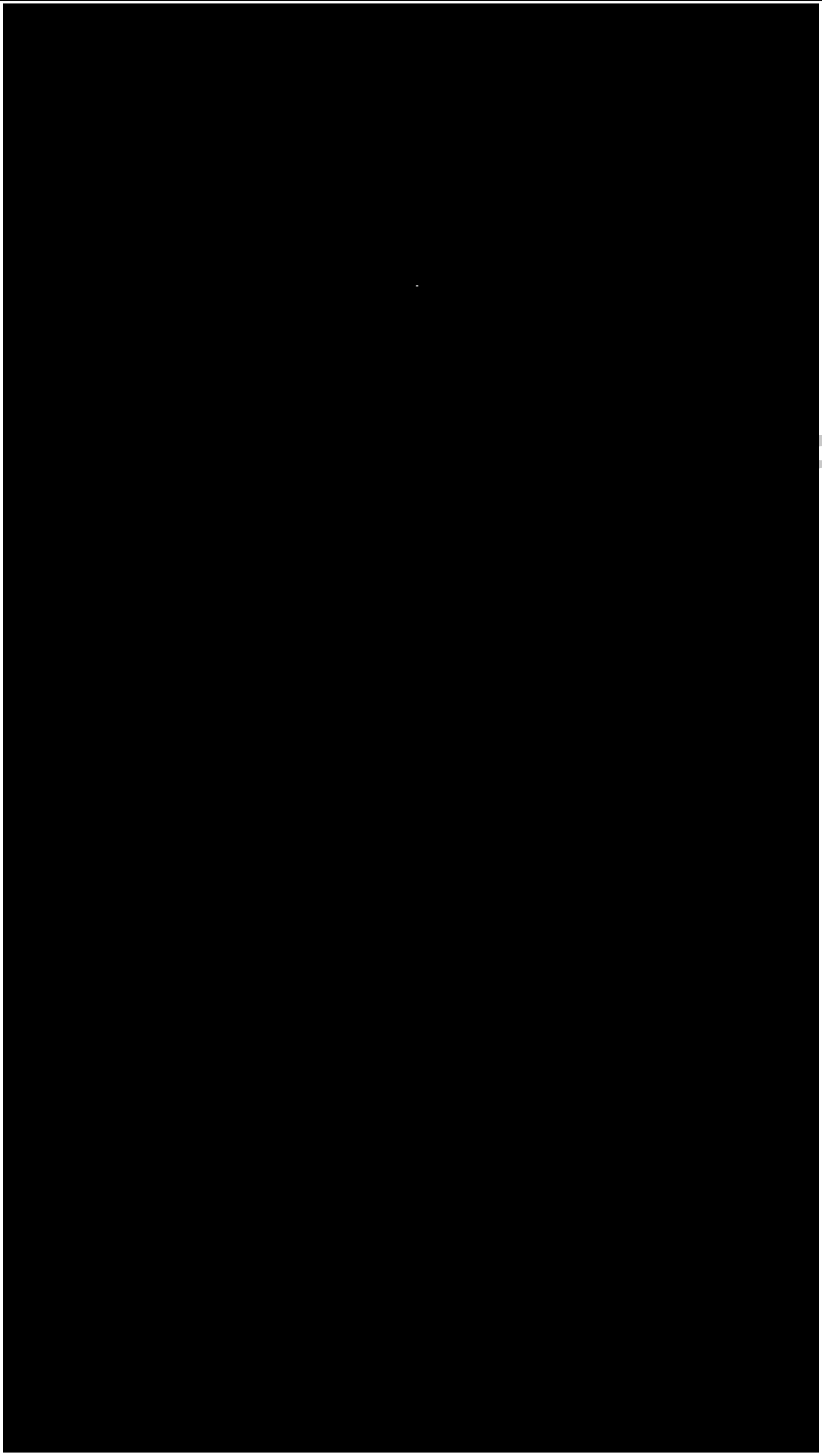
$$= \frac{1}{2\pi} \cdot \frac{1}{\sqrt{10 \times 10^{-3} \times 1.43 \times 10^{-6}}}$$

$$\therefore f_r = 1.33\ KHZ.$$

) In parallel connection,

$$C = 5 + 2 = 7\ \mu F = 7 \times 10^{-6}\ F$$

$$\therefore f_r = \frac{1}{2\pi\sqrt{LC}}$$



$$S = \frac{\theta}{l C}$$

$$S = \frac{\theta}{l \frac{m}{V}} \Rightarrow S = \frac{\theta V}{l m}$$

$$\Rightarrow m = \frac{\theta V}{l S} = \frac{11 \times 48}{2 \times 66} = 4 \text{ gm}$$

5. The time of reverberation of an empty hall with 500 audience in hall is 1.5 sec and 1.4 sec respectively. Find reverberation time with 800 audience in hall.

$\Rightarrow$ For $T = 1.5$ sec, $\quad 1.5 = \frac{0.158}{\alpha S} V$ --- (1)

for $T = 1.4$ sec, $\quad 1.4 = \frac{0.158 V}{\alpha S + 500}$ --- (2)

eq^n (1) $\div$ eq^n (2) $\Rightarrow \quad \frac{1.5}{1.4} = \frac{\alpha S + 500}{\alpha S}$

or $\quad 1.5 \alpha S = 1.4 \alpha S + 1.4 \times 500$

$\Rightarrow \alpha S = 7000$ --- (3)

from eq^n (1) & (3),

$$V = \frac{1.5 \times 7000}{0.158} = 66455.7 \text{ m}^3$$

Now, reverberation time with 800 audiances

$$T_3 = \frac{0.158 V}{\alpha S + 800} = \frac{0.158 \times 66455.7}{7000 + 800}$$

$$= 1.346 \text{ sec}$$

6. A circular loop of wire 5 cm of radius carries a current of 100 amp. What is the energy density at the center of the loop?

$\Rightarrow$ $I = 100\,amp$, $n = 1$

$r = 5\,cm = 0.05\,m$

May. Energy density at center of loop $(U_B) = ?$

$$B = \frac{\mu_0 n I}{2r} = \frac{4\pi\times10^{-7}\times100}{2\times0.05} = 1.256\times10^{-3}\,T$$

& $$U_B = \frac{B^2}{2\mu_0} = \frac{(1.256\times10^{-3})^2}{2\times4\pi\times10^{-6}}$$

$$\therefore U_B = 0.006283 \text{ Joule}/m^3$$

7. Calculate speed of transverse wave in a rope of length 20 cm having 60 gm of wire under tension of 500 N.

$\Rightarrow$ $V = ?$

$l = 20\,cm = 20\times10^{-2}\,m$

$m = 60\,gm = 60\times10^{-3}\,kg$

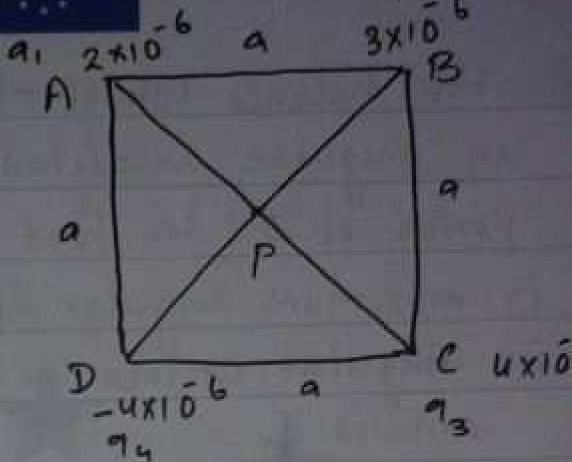
$T = 500\,N$

$$V = \sqrt{\frac{T}{\mu}} = \sqrt{\frac{T}{m/l}} = \sqrt{\frac{Tl}{m}} = \sqrt{\frac{500\times20\times10^{-2}}{0.06}}$$

$$= 40.82\,m/s$$

8. Find the potential at center of square having charges $2\times10^{-6}C$, $3\times10^{-6}C$, $4\times10^{-6}C$, $-4\times10^{-6}C$ at corners.

⇒



from the figure

$AC = \sqrt{a^2+a^2} = \sqrt{2a^2}$

$= a\sqrt{2}$

$\therefore AP = PC = DP = BP = \frac{a\sqrt{2}}{2}$

$= \frac{a}{\sqrt{2}}$

$\therefore$ Total Potential at P is

$$V = \frac{1}{4\pi\epsilon_0}\left[\frac{q_1}{AP} + \frac{q_2}{BP} + \frac{q_3}{CP} + \frac{q_4}{DP}\right]$$

$$= \frac{9\times10^9}{\frac{a}{\sqrt{2}}}\left[2\times10^{-6} + 3\times10^{-6} + 4\times10^{-6} - 4\times10^{-6}\right]$$

$$= \frac{4.5\times10^4}{a} \text{ Volts}$$

9. Calculate energy in electron volt of an electron of wavelength 3×10^{-12} m.

⇒ $h = 6.62\times10^{-34}$ Js

$\lambda = 3\times10^{-12}$ m

$E = ?$

$$E = \frac{1}{2}mv^2 = \frac{1}{2}m\left(\frac{h}{m\lambda}\right)^2 = \frac{1}{2}\frac{h^2}{m\lambda^2}$$

$$= \frac{1}{2}\frac{(6.62\times10^{-34})^2}{9.1\times10^{-31}\times(3\times10^{-12})^2}$$

$$= 2.945\times10^{16} \text{ Joule} = \frac{2.945\times10^{16}}{1.6\times10^{-19}} \text{ eV}$$

$E = 1.84\times10^{35}$ eV

10. The balance wheel of watch oscillates with an angular amplitude of π radian and period of 0.5 sec Find

i) maximum angular speed of wheel. v_{max}

ii) Angular acceleration of wheel when displacement is $\frac{\pi}{2}$.

$\Rightarrow$ $\theta_{max} = \pi$ rad

$T = 0.5$ sec

(i) $\therefore v_{max} = \omega \theta_{max}$

$$= \frac{2\pi}{T} \times \pi = \frac{2\pi}{0.5} \times \pi = 39.44 \text{ rad/sec}$$

(ii) Angular accⁿ at $\theta = \frac{\pi}{2}$ rad is

$$\alpha = \omega^2 \theta$$

$$= \omega^2 \times \frac{\pi}{2} = \left(\frac{2\pi}{T}\right)^2 \cdot \frac{\pi}{2} = 248 \text{ rad/sec}^2$$

11. A parallel plate capacitor has capacitance of 100×10^{-12} F, A plate of area 100 cm^2 is used as dielectric. At 50 Volt P.d, Calculate electric field intensity and magnitude of induced charge.

$\Rightarrow$ $C = 100 \times 10^{-12}$ F

$A = 100 \text{ cm}^2 = 100 \times 10^{-4} \text{ m}^2$

$V = 50$ V

$E = ?$

we have, $C = \frac{\epsilon_0 A}{d}$

$$\therefore d = \frac{\epsilon_0 A}{C} = \frac{8.85 \times 10^{-12} \times 100 \times 10^{-4}}{100 \times 10^{-12}}$$

$$\therefore d = 8.85 \times 10^{-4} \, m.$$

$$E = \frac{q}{\epsilon_0 A} \quad (\text{From Gauss law})$$

$$= \frac{CV}{\epsilon_0 A} = \frac{100 \times 10^{-12} \times 50}{8.85 \times 10^{-12} \times 100 \times 10^{-4}}$$

$$= 5.65 \times 10^{6} \, N/C$$

$\therefore$ Magnitude of induced charge $(q) = CV = 50 \times 100 \times 10^{-}$

$= 5 \times 10^{-9}$ V

12. obtain the charging constant of a Capacito in RC Circuit such that current through the resistor is decreased by 50% of it's peak value in 5 sec.

$\Rightarrow$ $t = 5$ sec

$$I = I_0 - 50\% \text{ of } I_0 = I_0 - \frac{50}{100} I_0$$

$$= 0.5 I_0$$

$$q = q_0 e^{-t/RC}$$

$$I = I_0 e^{-t/RC}$$

$$0.5 \cancel{I_0} = \cancel{I_0} e^{-t/RC}$$

Taking log,

$$\log_e (0.5) = -\frac{5}{RC}$$

or $RC = \frac{-5}{\ln(0.5)} = 7.22$ sec

$\therefore$ charging time constant $RC = 7.22$ sec.

13. An electron moving in a wave has wave function $\psi(x) = 2\sin 2\pi x$. Find probability of electron finding in the region $x = 0.25$ to 0.50 m.

$\Rightarrow \psi(x) = 2\sin 2\pi x$

$x = 0.25$ to 0.50 m

We have $P = \int_{0.25}^{0.5} |\psi|^2 dx$

$$= \int_{0.25}^{0.5} 4\sin^2 2\pi x \, dx$$

$$= 4\int_{0.25}^{0.5} \left(\frac{1-\cos 4\pi x}{2}\right) dx$$

$$= \frac{4}{2}\int_{0.25}^{0.5} dx - \frac{4}{2}\int_{0.25}^{0.5} \cos 4\pi x \, dx$$

$$= 2\left[x\right]_{0.25}^{0.5} - 2\times\frac{1}{4}\left[\sin 4\pi x\right]_{0.25}^{0.5}$$

$$= 0.5 - 0$$

$\therefore P = \frac{1}{2}$

$\therefore$ Probability $= \frac{1}{2}$.

14. A Copper Strip of 2 cm wide and 1 mm thick is placed in a magnetic field 1.5 T. If Current of 200 Amp is set up in the Strip, Calculate the hall voltage, hall mobility if no. of electrons per unit volume is $8.4 \times 10^{28}\ m^{-3}$ and resistivity is $1.72 \times 10^{-8}\ \Omega m$.

$\Rightarrow$ $d = 2\ cm = 2 \times 10^{-2}\ m$

$t = 1\ mm = 10^{-3}\ m$

$B = 1.5\ T$

$I = 200\ amp$

$n = 8.4 \times 10^{28}\ m^{-3}$

$\rho = 1.72 \times 10^{-8}\ ohm/m$

$$V_H = \frac{BI}{net} = \frac{200 \times 1.5}{8.4 \times 10^{28} \times 1.6 \times 10^{-19} \times 10^{-3}} = 2.23 \times 10^{-6}\ V$$

$$\sigma = ne\mu$$

$$\mu = \frac{\sigma}{ne} = \frac{1}{\rho ne} = \frac{1}{1.72 \times 10^{-8} \times 8.4 \times 10^{28} \times 1.6 \times 10^{-19}}$$

$$= 231.12\ m^2 V^{-1} s^{-1}$$

15. Normalize one dimensional wave eq^n

$$\psi(x) = A \sin\left(\frac{n\pi}{a}\right) \qquad 0 < x < a$$

where A is normalizing Constant.

$\Rightarrow$ We have $\int_0^a \psi \psi^* dx = 1$

or $$A^2 \int_0^a \sin^2\left(\frac{n\pi x}{a}\right) dx = 1$$

or $$A^2 \int_0^a \frac{1}{2}\left[1 - \cos 2\left(\frac{n\pi x}{a}\right)\right] dx = 1$$

or $$\frac{A^2}{2}[x]_0^a - \frac{A^2}{2}\left[\sin\left(\frac{2n\pi x}{a}\right)\right]_0^a = 1$$

or $$\frac{A^2}{2} \times a - 0 = 1$$

or $$\frac{A^2}{2} a = 1$$

or $$A^2 = \frac{2}{a}$$

$$A = \sqrt{\frac{2}{a}}$$

$\therefore$ Normalized wave function is

$$\sqrt{\frac{2}{a}} \sin\left(\frac{n\pi x}{a}\right)$$

16. A Capacitor of Capacitance C is charged through a resistor. Calculate time ~~Constant~~ at which Potential across the resistor is equal to Potential across the Capacitor.

$\Rightarrow$ $$V_R = V_C$$

$$IR = \frac{q}{C}$$

$$R I_0 e^{-\frac{t}{RC}} = \frac{q_0}{C}\left(1 - e^{-\frac{t}{RC}}\right)$$

or $$e^{-\frac{t}{RC}} = 1 - e^{-\frac{t}{RC}} \qquad \left[\because I_0 R = \frac{q_0}{C}\right]$$

$$\Rightarrow \quad 2e^{-\frac{t}{RC}} = 1$$

$$\Rightarrow \quad e^{\frac{t}{RC}} = 2$$

$$\therefore t = \ln(2) \times RC$$

$$= 0.693\,RC.$$

17. Find the specific rotation of sample of sugar solution if plane of polarization is turned through 46°. The length of tube containing 25% of solution is 25 cm

$\Rightarrow \theta = 46°$

$l = 25\,cm = 2.5\,dm$

$C = 25\% = \frac{25}{100} = \frac{1}{4}\,gm/cc$

we have

$$S = \frac{\theta}{lC} = \frac{46°}{2.5 \times \frac{1}{4}} = 73.60 \text{ degree } dm^{-1}\,gm^{-1}\,cc$$

18. Calculate possible order of spectra with plane transmission gratting having 1800 lines per inch when wavelength of light 4500 A° is used.

$\Rightarrow \lambda = 4500\,A° = 4500 \times 10^{-10}\,m$

$n = ?$

$N = 1800$ lines/inch

$$(a+b) = \frac{2.54}{1800}\,cm = \frac{2.54}{1800} \times 10^{-2}\,m$$

So. $(a+b) \sin\theta = n\lambda$

for $n=1$

$(a+b)\sin\theta_1 = \lambda$

$\sin\theta_1 = \dfrac{\lambda}{a+b}$

$\theta_1 = \sin^{-1}\left(\dfrac{\lambda}{a+b}\right) = \sin^{-1}\left(\dfrac{4500 \times 10^{-10}}{2.54 \times 10^{-2}} \times 1800\right)$

$= 18.59°$ //

for $n=2$

$\theta_2 = \sin^{-1}\left(\dfrac{2\lambda}{a+b}\right) =$

$= 39.63°$

for $n=3$,

$\theta_3 = \sin^{-1}\left(\dfrac{3\lambda}{a+b}\right)$

$= 73.09$

for $n=4$

$\theta_4 = \sin^{-1}\left(\dfrac{4\lambda}{a+b}\right)$

$=$

& $\sin\theta_4 = 1.275 > 1$

which is impossible

$\therefore$ Possible order of spectra are 1, 2, 3.

Q:. A Particle moving in one dimensional box at infinite Probability of finding the Particle with in range 1 A° at Center of When it is in lowest energy state. . . .

$\Rightarrow$ $L = 1 A°$

$P = ?$

at ground state $n = 1$

at Center $x = \frac{L}{2}$

we have

$$\psi(x) = \pm\sqrt{\frac{2}{L}} \sin\left(\frac{\pi}{L}\cdot\frac{L}{2}\right)$$

So $\psi(x) = \pm\sqrt{\frac{2}{L}}$

Now, $P = \int_{x_1}^{x_2} |\psi|^2 dx$

$$= \frac{2}{L}\int_{x_1}^{x_2} dx$$

$$= \frac{2}{L}(x_2 - x_1)$$

$$= \frac{2}{10} \times 1$$

$$= \frac{1}{5} \; //$$

Numericals:→

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2012 fall | 2008 fall — In Bohr's model of H-atom, electrons around the nucleus in a path of radius 5.1×10^{-11} m at frequency of 6.8×10^{15} rev/sec. Find the value of 'B' at center of coil.

$$\Rightarrow B = \frac{\mu_0 I}{2a} = \frac{\mu_0 \left(\frac{q}{t}\right)}{2R} = \frac{\mu_0 \frac{e}{t}}{2R} = \frac{\mu_0 e f}{2R}$$

$$= \frac{4\pi\times10^{-7}\times1.6\times10^{-19}\times6.8\times10^{15}}{2\times5.1\times10^{-11}} = 13.4\ T.$$

2007 fall — Calculate the flux density of magnetic field due to the circular coil of radius 10 cm at axial point at distance 20 cm from the center of coil. $a = 10$ cm, $x = 20$ cm

$$\Rightarrow B = \frac{\mu_0 I a^2}{2(a^2+x^2)^{3/2}} = \frac{4\pi\times10^{-7}\times I\,(10\times10^{-2})^2}{2\{(10\times10^{-2})^2+(20\times10^{-2})^2\}^{3/2}}$$

$$= (\quad)\times I \text{ Tesla.}$$

2005 spring — The current in RL ckt drops from 1.0 Amp to 10 mA within the first sec following the removal of the battery from the ckt. If the value of inductance is 1 H, find resistance of the ckt.

$I = 10$ mA $= 10\times10^{-3}$ Amp, $I_0 = 1$ amp, $t = 1$ sec (first sec)

$L = 1$ H, $R = ?$

For decay of current, $I = I_0 e^{-\frac{R}{L}t}$

$$\log_e \frac{I}{I_0} = -\frac{R}{L}t$$

$$\Rightarrow R = \frac{-L}{t}\log_e\frac{I}{I_0} = \qquad = (\quad)\text{ ohm}$$

2012 fall / 2005 fall

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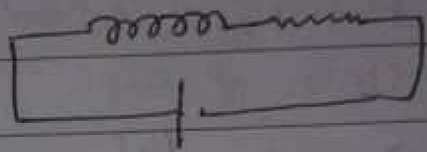
What is the initial rate of increase of current and final saturation current in R-L ckt with $L = 15\,mH$, $R = 24\,\Omega$ and $emf = 10\,V$?

$\Rightarrow \frac{dI}{dt} = ?$ when $t = 0$



Final saturation current $I_0 = ?$

using Krichoff's law, $E - L\frac{dI}{dt} = R.$

1st case.

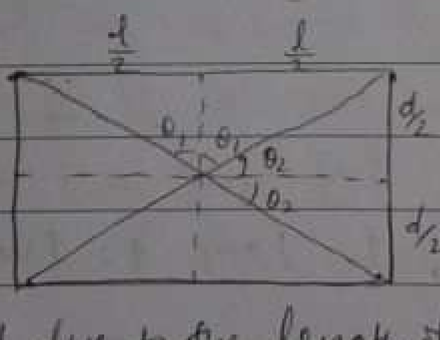
$$\frac{dI}{dt} = \frac{E - IR}{L}$$

at $t = 0$, $I = 0$ $\Rightarrow \frac{dI}{dt} = \frac{E}{L} = \frac{10}{15\times10^{-3}} = (\quad) As^{-1}$

2nd case, $I_0 = \frac{E}{R} = \frac{10}{24} = 0.416$ Amp

2007 Spring Show that 'B' at the center of rectangle of length l and width d carrying current I is given by $B = \frac{2\mu_0 I}{\pi}\left(\frac{l^2+d^2}{ld}\right)^{1/2}$.

$\Rightarrow$



Mag. field due to the length of one side,

$$B_1 = \frac{\mu_0 I(\sin\theta_1 + \sin\theta_1)}{4\pi a} = \frac{2\mu_0 I \sin\theta_1}{4\pi\frac{d}{2}}$$

Here for length, $a = \frac{d}{2}$ is distance betn wire and center of the rectangle.

$$\therefore B_1 = \frac{\mu_0 I \sin\theta_1}{\pi d}$$

$$= \frac{\mu_0 I}{\pi d} \frac{\frac{l}{2}}{\sqrt{\left(\frac{l}{2}\right)^2 + \left(\frac{d}{2}\right)^2}} \quad \left[\because \sin\theta = \frac{P}{h}\right]$$

$$= \frac{\mu_0 I l}{\pi d \sqrt{l^2 + d^2}}$$

For both length, $B_l = 2B_1 = \dfrac{2\mu_0 I l}{\pi d\sqrt{l^2+d^2}}$

Similarly for both width, $B_w = \dfrac{2\mu_0 I d}{\pi l \sqrt{l^2+d^2}}$

$\therefore$ Total field at the Center $B = B_l + B_w$

$$= \frac{2\mu_0 I}{\pi\sqrt{l^2+d^2}}\left(\frac{l}{d} + \frac{d}{l}\right)$$

$$= \frac{2\mu_0 I}{\pi} \frac{(l^2+d^2)^{1/2}}{ld} \quad \text{proved}$$

Q: A Straight wire segment of length L carries a current I. Show that the magnetic field 'B' associated with this segment, at a distance R from the segment along perpendicular bisector is given by $B = \dfrac{\mu_0 I}{2\pi R} \dfrac{L}{(L^2+4R^2)^{1/2}}$. Show that expression reduces to expected result as $L \to \infty$.

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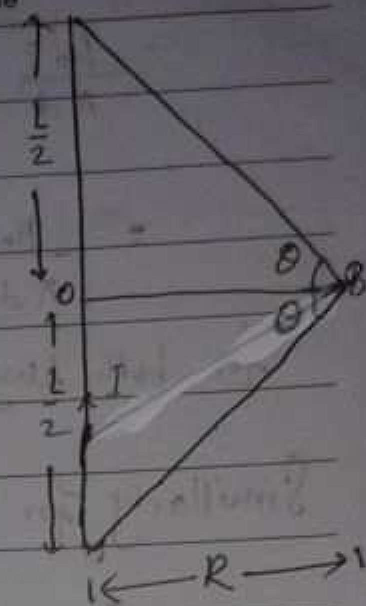
Field at B is given by



$$B = \frac{\mu_0 I(\sin\theta + \sin\theta)}{4\pi R}$$

$$= \frac{\mu_0 I\, 2\sin\theta}{4\pi R} = \frac{\mu_0 I \sin\theta}{2\pi R}$$

$$= \frac{\mu_0 I}{2\pi R} \frac{\frac{L}{2}}{\sqrt{R^2 + (\frac{L}{2})^2}} \quad [\because \sin\theta = \frac{P}{h}]$$

$$= \frac{\mu_0 I L}{2\pi R(4R^2 + L^2)^{1/2}}$$

at $L \to \infty$, $4R^2$ can be neglected as compared to L^2.

So, $B = \frac{\mu_0 I}{2\pi R} \frac{L}{\sqrt{L^2}} = \frac{\mu_0 I}{2\pi r}$ which is the required result.

Q. A straight section of wire of length L carries a current I (a) Show that magnetic field associated with the segment at P, a perpendicular distance D from one end of wire is given by

$$B = \frac{\mu_0 I}{4\pi D} \frac{L}{(L^2 + D^2)^{1/2}}.$$

(b) Show that magnetic field is zero at the point Q along the line of the wire.

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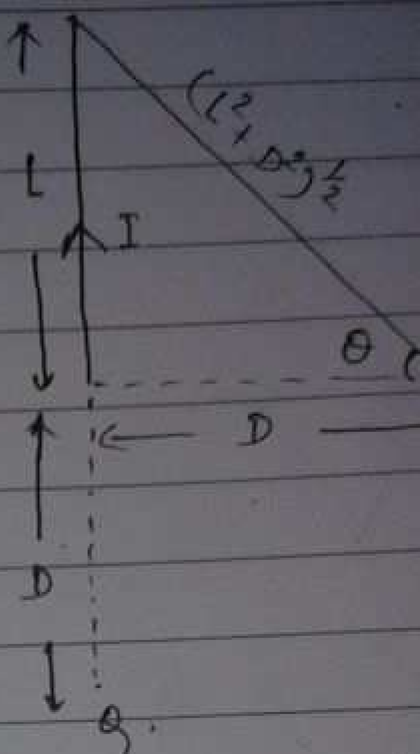
Sol^n :→



(a) At Pt. P,

$$B = \frac{\mu_0 I}{4\pi D}(\sin\theta + \sin 0)$$

$$= \frac{\mu_0 I}{4\pi D}\sin\theta$$

$$= \frac{\mu_0 I}{4\pi D}\frac{L}{(L^2+D^2)^{1/2}} \text{ //.}$$

(b) At point Q, $\theta = 180$

$$\therefore B = \frac{\mu_0 I}{4\pi D}(\sin 180 + \sin 180)$$

$$= 0 \text{ //.}$$

✡ ✡ ✡ ✡ ✡

Numericals:→

2005 fall — A 1.5 mf Capacitor is charged to a potential of 25 V, the charging battery is now disconnected and the Capacitor is now joined to 10 mH Coil. What is the peak value of L.C. oscillation?

$\Rightarrow$ $C = 1.5\,mf = 1.5 \times 10^{-3} F$, $V = 25\,V$, $L = 10\,mH$
$= 10 \times 10^{-3} H$

For maximum Current, $U_B = U_E$

$$\Rightarrow \frac{1}{2} L I_m^2 = \frac{1}{2} C V^2$$

$$\Rightarrow I_m = \sqrt{\frac{CV^2}{L}} = V\sqrt{\frac{C}{L}} = 25\sqrt{\frac{1.5 \times 10^{-3}}{10 \times 10^{-3}}}$$

$$= 25\sqrt{15} \text{ Amp.}$$

2008 spring — A ckt has $L = 10\,mH$, $C = 1.0\,\mu F$. How much resistance must be inserted in the ckt to reduce the (undamped) resonant frequency by 0.01%?

$\Rightarrow$ undamped resonat frequency $= f = \frac{1}{2\pi\sqrt{LC}}$

and damped resonant frequency $f' = \frac{1}{2\pi}\sqrt{\frac{1}{LC} - \left(\frac{R}{2L}\right)^2}$

Here, $\frac{F - F'}{F} = \frac{0.01}{100} = 10^{-4}$

$$\Rightarrow 1 - \frac{F'}{F} = 10^{-4}$$

$$\Rightarrow 1 - \frac{\sqrt{\frac{1}{LC} - \frac{R^2}{4L^2}}}{\sqrt{\frac{1}{LC}}} = 10^{-4}$$

$$\Rightarrow 1 - \sqrt{1 - \frac{R^2C}{4L}} = 10^{-4}$$

$$\Rightarrow \sqrt{1 - \frac{R^2C}{4L}} = 0.9999$$

$$\Rightarrow 1 - \frac{R^2C}{4L} = 0.9998$$

$$\Rightarrow \frac{R^2C}{4L} = 0.0002$$

$$\Rightarrow R^2 = \frac{0.0002 \times 4 \times L}{C}$$

$$\Rightarrow R = \sqrt{\frac{81 \times 10^{-4}}{C}} = 2.8 \text{ ohm}.$$

2008 fall. $L = 5mH$, $C = 1\mu F$, $\frac{F-F'}{F} = 5\% = \frac{5}{100}$

$R = ?$ (Convert C in to Farad)

2007 spring $L = 5mH$, $C = 2\mu F$, $\frac{E-E'}{E} = 5\%$, $R = ?$

2007 fall Sunlight strikes the earth outside it's atmosphere with an intensity of $2\ cal/cm^2\ min$. Calculate E_m and B_m for sunlight.

$\Rightarrow$ we have $S = \frac{E_m B_m}{\mu_0}$

Average intensity $\bar{S} = \frac{E_m B_m}{2\mu_0} = \frac{1}{2\mu_0}(C B_m^2)$

Here $\bar{S} = 2\ Cal/cm^2\text{-}min$

$$= \frac{2 \times 4.2}{(1 \times 10^{-2})^2 \times 60}\ watt/m^2 = 1.4 \times 10^3\ watt/m^2$$

Now, $\bar{S} = \frac{1}{2\mu_0}(C B_m^2)$

$$\therefore B_m = \sqrt{\frac{2\mu_0 \bar{S}}{C}} = \sqrt{\frac{2 \times 4\pi \times 10^{-7} \times 1.4 \times 10^3}{3 \times 10^8}}$$

$$= 3.4 \times 10^{-6}\ T$$

$$E_m = C B_m = 3 \times 10^8 \times 3.4 \times 10^{-6}$$

$$= 1020\ V/m.$$

2003 An observer 1.8 m from an isotropic point light source whose power is 250 watt. Calculate the rms values of electric and magnetic fields due to the source at position of observer.

$\Rightarrow r = 1.8$ m

$P = 250$ watt

$E_m = ?$, $B_m = ?$

$E_{rms} = \frac{E_m}{\sqrt{2}} = ?$, $B_{rms} = \frac{B_m}{\sqrt{2}} = ?$

$$\bar{S} = \frac{P}{A} = \frac{P}{4\pi r^2} = \frac{E_m B_m}{2\mu_0}$$

$$\therefore E_m B_m = \frac{2\mu_0 P}{4\pi r^2}$$

$$\therefore C B_m^2 = \frac{2\mu_0 P}{4\pi r^2}$$

$$\therefore \quad B_m = \sqrt{\frac{2\mu_0 P}{4\pi r^2 C}} = \sqrt{\frac{2\times 4\pi\times10^{-7}\times 250}{4\pi\times(1.8)^2\times3\times10^8}} = (\quad) T$$

$$E_m = C B_m = \quad = \quad V/m$$

$$B_{rms} = \frac{B_m}{\sqrt{2}} = \quad = \quad T$$

$$E_{rms} = \frac{E_m}{\sqrt{2}} = \quad = \quad V/m.$$

2005 spring A. Source of radiation is 400 m away from given position. What will be the peak values of electric and magnetic field vectors related to the wave if power of source is 4 kW?

$\Rightarrow r = 400\ m, \quad P = 4\ kW = 4000\ W, \quad E_m = ?, \quad B_m = ?$

2010 A parallel plate capacitor with circular plates each of radius 5 cm, if rate of change of electric field is 10^{12} V/m-s then find the displacement current.

$\Rightarrow r = 5\ cm = 5\times10^{-2}\ m$

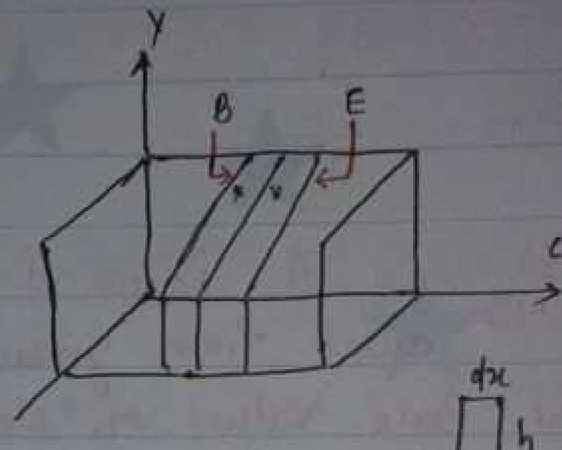
$$\frac{dE}{dt} = 10^{12}\ V\,m^{-1}s^{-1}$$

$$\text{Then, } I_d = \epsilon_0 A \frac{dE}{dt} = \epsilon_0 \pi r^2 \frac{dE}{dt}$$

$$= 8.85\times10^{-12}\times3.14\times(5\times10^{-2})^2\times10^{12}$$

$$= (\quad)\ \text{Amp}$$

Q^n Prove that for any point in an electro-magnetic wave, the density of energy stored in electric field equals to that stored in magnetic field.



The density of energy stored in electric field is, $U_E = \frac{1}{2}\epsilon_0 E^2$ --- ①

The density of energy stored in magnetic field is

$$U_B = \frac{B^2}{2\mu_0} \quad \text{---- ②}$$

The fields for plane wave are

$$E = E_m \sin(kx - \omega t)$$

$$B = B_m \sin(kx - \omega t)$$

eq^n ① ÷ eq^n ② ⇒

$$\frac{U_E}{U_B} = \frac{\frac{1}{2}\epsilon_0 E_m^2 \sin^2(kx-\omega t)}{\frac{1}{2\mu_0} B_m^2 \sin^2(kx-\omega t)} = \mu_0\epsilon_0\left(\frac{E_m}{B_m}\right)^2$$

$$= \frac{1}{c^2} \times c^2 = 1$$

$\Rightarrow U_E = U_B$ proved

(3) What is the maximum electric field ?

$\Rightarrow$ $f = 1020 \, kHz = 1020 \times 10^{3} \, Hz$

$B_m = 1.6 \times 10^{-4} \, T$

(1) $c = \frac{1}{\sqrt{\mu_0 \epsilon_0}} = \frac{1}{\sqrt{4\pi \times 10^{-7} \times 8.85 \times 10^{-12}}} = 3 \times 10^{8} \, m/$

(2) $\lambda = \frac{c}{f} = \frac{3 \times 10^{8}}{1020 \times 10^{3}} = 294.12 \, m$

(3) $E_m = c B_m = 3 \times 10^{8} \times 1.6 \times 10^{-4} = 4.8 \times 10^{4}$ volts

Numericals :→

2012 Fall

Calculate the energy in electron volt of an electron of wave of $\lambda = 3\times10^{-2}$ m. Given $h = 6.62\times10^{-34}$ Js.

⇒ Moving electron has energy $E = \frac{1}{2}mv^2$

$$= \frac{1}{2}m\left(\frac{h}{m\lambda}\right)^2$$

$$= \frac{h^2}{2m\lambda^2}$$

Here $m = 9.1\times10^{-31}$ kg,

$$\therefore E = \frac{(6.62\times10^{-34})^2}{2\times9.1\times10^{-31}\times(3\times10^{-2})^2} = 0.27\times10^{-23} \text{ Joules}$$

$$\therefore E = \frac{0.27\times10^{-23}}{1.6\times10^{-19}} \text{ eV} = 1.68\times10^{-15} \text{ eV}$$

Q^n. What would be the wavelength of quantum radiant energy emitted, if an electron transmitted in to radiation and converted in to one quantum.

⇒ According to planck, Energy of 1 quantum is

$$E = h\nu$$

$$\therefore mc^2 = h\nu$$

$$\therefore \frac{hc}{\lambda} = mc^2 \Rightarrow \lambda = \frac{h}{mc} = \frac{6.6\times10^{-34}}{9.1\times10^{-31}\times3\times10^{8}}$$

$$= 0.0244\times10^{-10} \text{ m}$$

$$= 0.0244 \text{ Å}$$

Qn Calculate the velocity and de Broglie wave. length of a proton of energy 10^5 eV.

given, $m_p = 1.66\times10^{-24}$ gm.

Charge on the electron = 4.8×10^{-10} esu

$h = 6.62\times10^{-27}$ erg. sec.

⇒ The energy of proton, $E = \frac{1}{2}m_p v^2 = 10^5\ eV = 10^5\times1.6\times10^{-12}$ ergs

$$\Rightarrow v^2 = \frac{2\times10^5\times1.6\times10^{-12}}{m_p} = \frac{2\times10^5\times1.6\times10^{-12}}{1.66\times10^{-24}}$$

$$\Rightarrow v = 4.47\times10^{6}\ cm/sec$$

Now, $\lambda = \frac{h}{mv} = \frac{h}{\sqrt{2mE}}$, because $\frac{1}{2}mv^2 = E$

$$\Rightarrow mv = \sqrt{2mE}$$

$$\Rightarrow \lambda = \frac{h}{\sqrt{2mE}}$$

$$= \frac{6.62\times10^{-34}}{\sqrt{2\times1.6\times10^{-24}\times10^5\times1.6\times10^{-12}}} = 9.3\times10^{-12}\ cm$$

Qn Find the energy of neutron in units of eV whose de Broglie wavelength is 1 Å.

$$\Rightarrow E = \frac{h^2}{2m\lambda^2} = \frac{6.6\times10^{-34}}{2\times1.67\times10^{-27}\times(10^{-10})^2} = 13.01\times10^{-21}\ J$$

$$E = \frac{13.01\times10^{-21}}{1.6\times10^{-19}}\ eV = 8.12\times10^{-2}\ eV$$

$m_n = 1.67\times10^{-27}$ kg

Qn: Compute de Broglie's wavelength of 10^{11} keV neutrons. Mass of neutron may be taken as 1.675×10^{-27} kg.

$$\Rightarrow \frac{1}{2}mv^2 = 10^{11}\ keV = 10^{11}\times1000\ eV = 10^{14}\times1.6\times10^{-19}\ J = 1.6\times10^{-5}\ J$$

$$\Rightarrow v = \left(\frac{2\times1.6\times10^{-5}}{m}\right)^{1/2} = \left(\frac{3.2\times10^{-5}}{1.675\times10^{-27}}\right)^{1/2} = (\quad)\ m/s.$$

Q: Find the energy of an electron moving in one dimension in an infinitely high potential box of width 1 Å.

($m = 9.1\times10^{-31}$ kg, $h = 6.63\times10^{-34}$ Js)

⇒ we have $E = \dfrac{n^2h^2}{8ma^2}$ [∵ L = a]

$n = 1, 2, 3 \ldots$

at $n = 1$, $E = \dfrac{h^2}{8ma^2} = \dfrac{(6.63\times10^{-34})^2}{8\times9.1\times10^{-31}\times(10^{-10})^2}$

$= 9.1\times10^{-19}$ J $= \dfrac{9.1\times10^{-19}}{1.6\times10^{-19}} = 5.68$ eV

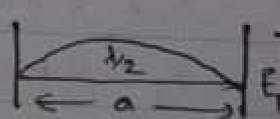
Q: For a free particle show that Schrodinger wave eqn leads to de Broglie relation.

⇒ The time independent Schrodinger wave eqn is given by

$$E_n = \frac{n^2h^2}{8ma^2}$$ [L is replaced by a]

$$\Rightarrow E_1 = \frac{h^2}{8ma^2}$$

in this case $a = \dfrac{\lambda}{2}$ [∵ ψ



$$\therefore E_1 = \frac{4h^2}{8m\lambda^2} = \frac{h^2}{2m\lambda^2} \quad \text{--- (1)}$$

Also, $E_1 = \frac{1}{2}mv^2 = \frac{1}{2}\frac{m^2v^2}{m} = \frac{1}{2}\frac{p^2}{m}$ --- (2)

from eqn (1) & (2), $\dfrac{h^2}{2m\lambda^2} = \dfrac{1}{2}\dfrac{p^2}{m}$

$$\Rightarrow \frac{h^2}{\lambda^2} = p^2$$

$$\Rightarrow p = \frac{h}{\lambda}$$

$$\Rightarrow \lambda = \frac{h}{p} = \frac{h}{mv} \quad \underline{\text{Proved}}$$

Q^n. Calculate the value of the difference bet the energy of a hydrogen molecule having back and forth motion on the path of 1 cm long when n = 2 and n = 3 at 298 K.

$\Rightarrow L = a = 1\,cm = 0.01\,m$

Mass of hydrogen molecule $= \dfrac{2}{6.023 \times 10^{23}}$ $\left[\because \text{Mass of 1 molecule} = \dfrac{\text{Molecular w}}{N_A}\right]$

$$m = 0.33 \times 10^{-26}\,kg$$

$$\Rightarrow E_3 - E_1 = \frac{3^2 h^2}{8ma^2} - \frac{2^2 h^2}{8ma^2} \qquad \left[\because E = \frac{n^2 h^2}{8ma}\right]$$

$$= \frac{5h^2}{8ma^2} = \frac{5 \times (6.62 \times 10^{-34})^2}{8 \times 0.33 \times 10^{-26} \times (0.01)^2}$$

$$= 83.1 \times 10^{-38}\,J = \frac{83.1 \times 10^{-38}}{1.6 \times 10^{-19}}\,eV = 5.2 \times 10^{-18}\,eV$$

1) A stretched string has linear density 525 gm/m and is under tension 45 N. We send sinusodial wave with frequency 120 Hz and amplitude 8.5 mm along the string from one end. At what average rate does the wave transport energy?

$\Rightarrow$ Linear density (μ) = 525 gm/m $= \frac{525}{1000}$ kg/m

$T = 45\,N$

$f = 120\,Hz$

$a = 8.5\,mm = 8.5\times10^{-3}\,m$

velocity of wave $(v) = \sqrt{\frac{T}{\mu}} = \sqrt{\frac{45}{0.525}} = 9.25$ m/s

Average rate at which wave transport energy i.e

[Intensity $(I) = \frac{1}{2} v \omega^2 a^2 \mu$ $\quad [\because \mu = \frac{m}{l}]$

$= \frac{1}{2} \times 9.25 \times (2\pi\times120)^2 + (0.0085)^2 \times 0.525$

$= 99.74$ watt.

2) Newtons rings are obsesved in reflected light of wavelength 5.9×10^{-5} cm. The diameter of 10^{th} dark ring is 0.5 cm. Find radius of curvature of this lens and thickness of air film.

$\Rightarrow$ $\lambda = 5.9\times10^{-5}$ cm $= 5.9\times10^{-7}$ m

for 10^{th} dark ring $(D_{10}) = 0.5$ cm $= 0.005$ m

$R = ?$

$t = ?$

for dark ring $\quad D_n^2 = 4n\lambda R$

$$\therefore R = \frac{D_{10}^2}{4\times10\times5.9\times10^{-7}} = \frac{(0.005)^2}{4\times10\times5.9\times10^{-7}}$$

$$= 1.0593 \text{ m}$$

Also, $2t = n\lambda$

$$t = \frac{10\times5.9\times10^{-7}}{2} = 2.95\times10^{-6} \text{ m.}$$

3. What Magnitude of point charge choosen so that the electric field 50 cm away has magnitude of 2 N/C?

$\Rightarrow$ $E = 2$ N/C

$r = 50$ cm $= 0.5$ m

$q = ?$

$$E = \frac{1}{4\pi\epsilon_0}\frac{q}{r^2} = \frac{9\times10^9\times q}{(0.5)^2}$$

$$\therefore q = \frac{2\times(0.5)^2}{9\times10^9} = 5.56\times10^{-11} \text{ C.}$$

4. A 200 mm long tube Containing 48 cm³ of Sugar ~~of~~ Produces an optical rotation of 11° when placed in a Sugar Saccharimeter (polarimeter). If specific rotation of sugar solution is 66°. Calculate quantity of sugar contained in tube in the form of solution.

$\Rightarrow$ $\theta = 11°$

$S = 66$ deg dm^{-1} gm^{-1} cc

$l = 200$ mm $= 20$ cm $= 2$ dm

$V = 48$ cm³

$m = ?$

$$S = \frac{\theta}{l\,c}$$

$$S = \frac{\theta}{l\,\frac{m}{V}} \Rightarrow S = \frac{\theta V}{l m}$$

$$\Rightarrow m = \frac{\theta V}{l S} = \frac{11 \times 48}{2 \times 66} = 4 \text{ gm}$$

5. The time of reverberation of an empty hall with 500 audience in hall is 1.5 sec and 1.4 sec respectively. Find reverberation time with 800 audience in hall.

$\Rightarrow$ For $T = 1.5$ sec, $\quad 1.5 = \frac{0.158}{\alpha s} V$ ... (1)

for $T = 1.4$ sec, $\quad 1.4 = \frac{0.158 V}{\alpha s + 500}$ ... (2)

eq^n (1) $\div$ eq^n (2) $\Rightarrow$ $\quad \frac{1.5}{1.4} = \frac{\alpha s + 500}{\alpha s}$

or $1.5 \alpha s = 1.4 \alpha s + 1.4 \times 500$

$\Rightarrow \alpha s = 7000$ ... (3)

from eq^n (1) & (3),

$$V = \frac{1.5 \times 7000}{0.158} = 66455.7 \text{ m}^3$$

Now, reverberation time with 800 audiances

$$T_3 = \frac{0.158 V}{\alpha s + 800} = \frac{0.158 \times 66455.7}{7000 + 800}$$

$$= 1.346 \text{ sec}$$