Introduction

Indeterminany

1) static indeterminancy
   - External static Index
   - Internal static Index

2) Kinematic Indeterminany

Structures

- Plane
- Frames
  - rigid jointed frames
  - Pin-jointed frames
  - Hybrid frames

Structural analysis is to determine the internal forces and the corresponding displacements of all the structural elements as well as those of the entire structural system

- Safety and functioning

Degree of static indeterminancy / redundancy.

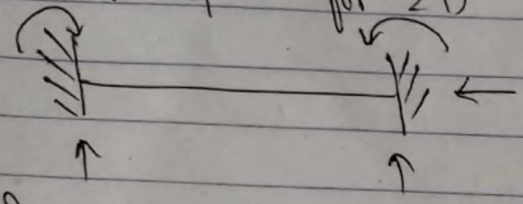
- statically indeterminancy are those structures which cannot be analysed with the help of eq^n of static equilibrium alone

① Degree of static external indeterminancy (D_{se})
② Degree of static internal indeterminancy (D_{si})

$$D_s = D_{se} + D_{si}$$

(1) Support system

$D_{se} = r - eq^n$ for 2D

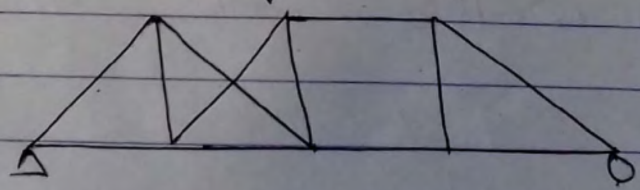


$D_{se} = 6 - 3 = 3$

(2) D_{si}

Consider pin jointed and rigid jointed frames separately

Pin jointed frames
- statically determinate internally
→ If it has min^m no. of members required to preserve its geometry
- if member are more indeterminate
- members required to form triangle + visual inspection



D_{si} = given(member) − required(r×n)
$= 13 - (3 + (j-3) \times 2)$
$= 13 - (3 + (8-3) \times 2)$
$= 0$

2D → $3 + (j-3) * 2$

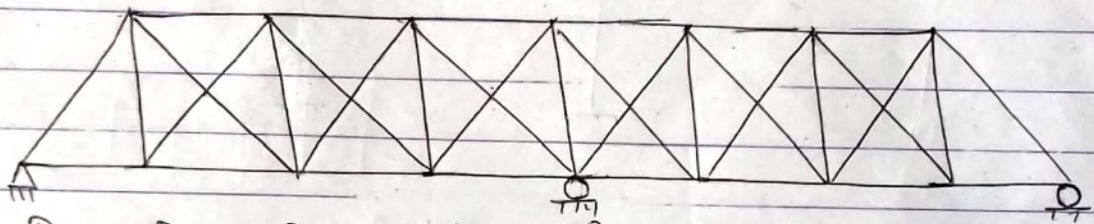
3D → $6 + (j-4) * 3$

Rigid jointed frame.

- Statically determinate internally (Dsi)
- If ther member from an open configuration resembling the structure of tree

$D_{si} = 3C$ ⟶ 2D

$D_{si} = 6C$ ⟶ 3D



$D_s = D_{se} + D_{si} = 6 + 1 = 7$

$D_{se} = 4 - 3 = 1$

$D_{si} = 35 - (3 + (j-3) \times 2)$

$= 35 - (3 + (16-3) \times 2)$

$= 6$

$D_s = (r+m) - j \times 2$

$= (4 + 35) - 16 \times 2$

$= 39 - 32$

$= 7$

Kinematic Indeterminancy

Pin Jointed frame

$D_K = 2j - e \Rightarrow 2D$

$D_K = 3j - e \Rightarrow 3D$

Rigid jointed frame

$D_K = 3j - e \Rightarrow 2D$

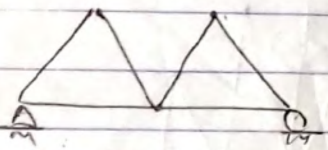
$D_K = 6j - e \Rightarrow 3D$

If the displacement component of its joints can not be deformed by compatibility eqn along

j = no. of joints

e = no of compatibility eqn

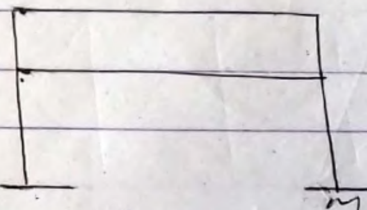
= Constraints imposed by support condition + others factours (



$D_K = 2j - e$

$= 2 \times 5 - 3$

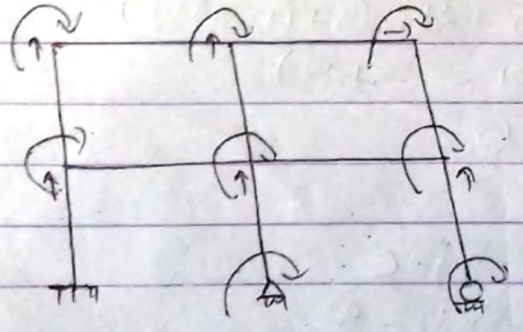
$= 7$



$D_K = 3j - e$

$= 3 \times 6 - 6$

$= 12$



$= 3j - (r + m)$

$= 3 \times 9 - (6 + 10)$

$= 11$ (inextensible)

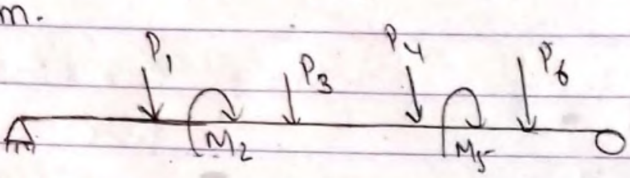
$= 3j - r$

$= 3 \times 9 - 6$

$= 21$ (extensible

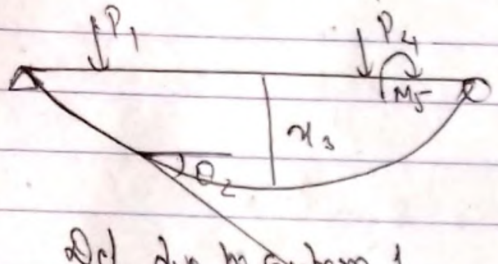
Betti's Theorem (Ennise Beth-1872)

If a linearly elastic structure (follows Hook's law) whose temperature is constant and the support are stable is separately acted upon by two different force systems. Then work done by system 1 in moving through its deformation produced by system 2 equal to System 2 in moving through its deformation caused by system.

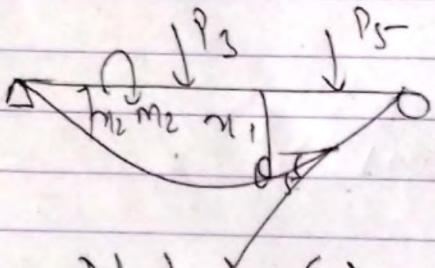


System 2 :- P_3, M_2, P_5

System 1 :- $P_1\ P_4, M_5, P_6$



Def due to System 1



Def due to System 2

$W_1 = F \times \delta = P_1 x_1 + P_4 x_4 + M_5 \theta_5$

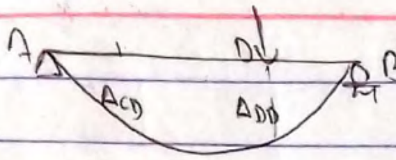
$W_2 = F\delta = P_3 x_3 + P_6 x_6 + M_2 \theta_2$

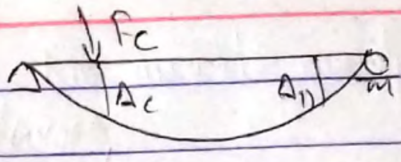
$W_1 = W_2$

Maxwell's Reciprocal Theorem (subset of Betti's law)

Statement

The deformation at point 1 on a linearly elastic structure due to load at point 2 equal to that at point 2 due load 2 equal to that at point 2 due to same level at point 1





$$W_D = \frac{F_D \times \Delta_{DD}}{2}$$

$$W_C = \frac{1}{2} F_C \times \Delta_{CC}$$

$$W_C = \frac{1}{2} F_C \times \Delta_{CC} + F_D \times \Delta_{DC}$$

$$W_D = \frac{1}{2} \times F_D \times \Delta_{DD} + F_C \Delta_{CD}$$

$$W_1 = W_C + W_D$$

$$W_2 = W_C + W_D$$

$$F_D \times \Delta_{DC} = F_C \times \Delta_{CD}$$

$$\Delta_{CD} = \Delta_{DC}$$

Castigliano Theorem

Theorem 1

Partial derivatives of total strain energy in a beam w.r.t the deflection at any point is equal to the load applied at that point

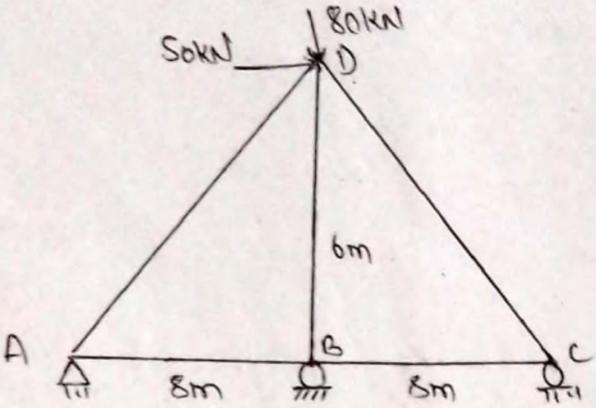
$$\frac{\partial U}{\partial \Delta_1} = W_1$$

Theorem 2

Partial derivative of strain energy of a beam w.r.t a particular force is equal to deflection in that direction of application

$$\frac{\partial U}{\partial W_1} = \Delta_1$$

Find support reaction.

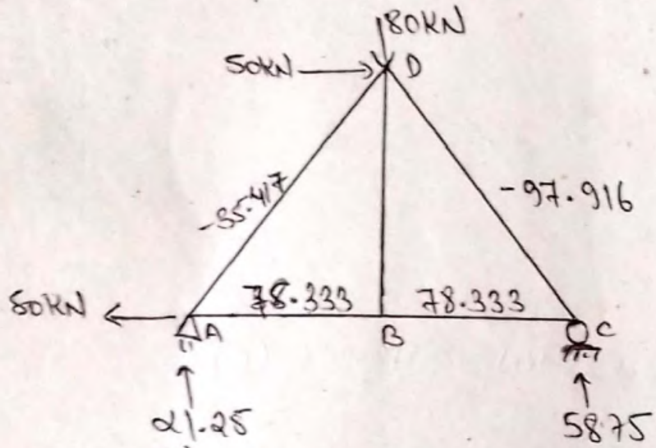


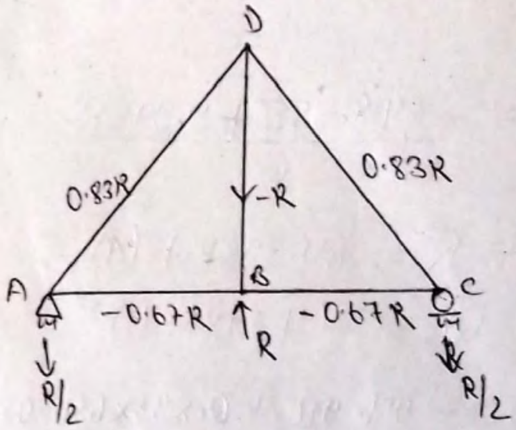
Area of BD = 2000 mm^2

remaining area = 3000 mm^2

Solution

Indeterminancy (D_s) = 4 − 3 = 1





$\Sigma M_A = 0$ (+)

$50 \times 6 + 80 \times 8 - V_C \times 16 = 0$

$\therefore V_C = 58.75$ KN (↑)

$V_A = 21.25$ KN (↑)

Section	L/A	N	$\partial N/\partial R$	$N \frac{\partial N}{\partial R} \times \frac{L}{AE}$
AB	2.67	78.333 − 0.667R	− 0.667	− 139.502 + 1.188R
BC	2.67	78.333 − 0.667R	− 0.667	− 139.502 + 1.188R
CD	3.33	− 97.916 + 0.833R	0.883	− 271.608 + 2.311R
DA	3.33	− 35.417 + 0.833R	0.833	− 98.242 + 2.311R
BD	3	− R	1	3R
				$\Sigma = \frac{-648.855 + 9.998}{E}$

According to castigliano theorem

$\Delta_B = \frac{\partial U}{\partial V_B} = 0$

$U = \frac{N^2 L}{2AE}$

$\Delta_B = \frac{\sum P (\partial P / \partial V_B)}{AE}$

Now.

$\Delta B = \frac{-648.855 + 9.998R}{E} = 0$

or $R = 64.898 kN (\uparrow)$

$V_B = 64.898 kN (\uparrow)$

$F_{BD} = -97.916 + 0.833 \times 64.898 = -43.855 kN = 43.855 (C)$

$V_C = 26.313 kN (\uparrow)$

$F_{DA} = -35.417 + 0.833R = -35.417 + 0.833 \times 64.898 = 18.643 (kN) (T)$

$V_A = 11.211 (kN) (\downarrow)$

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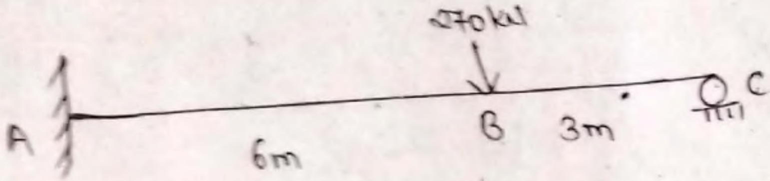
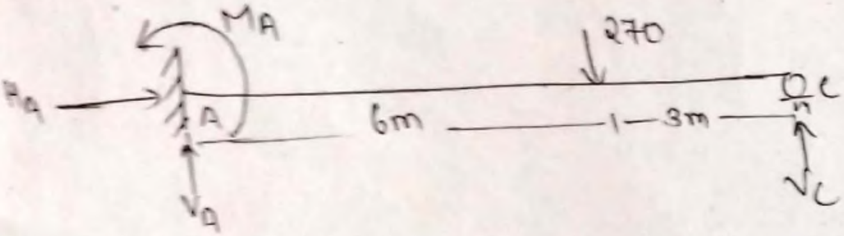
Use Castigliano theorem & find support reaction



Solution



Indeterminancy $(D_s) = r - e = 4 - 3 = 1$

$\Sigma M_A = 0$ (+↺)

$V_C \times 9 - 270 \times 6 + M_A = 0$

$M_A = 1620 - V_C \times 9$ —— (i)

(↑+) $\Sigma V = 0$

$V_C + V_A - 270 = 0$

or $V_A = 270 - V_C$ —— (ii)

According to Castigliano theorem

$\Delta_C = \frac{\partial U}{\partial V_C} \Rightarrow U = \int_0^L \frac{M^2}{2EI} \cdot dx.$

or, $\frac{\partial U}{\partial x} = \frac{1}{2EI} \int_0^L M \frac{dM}{dV_C} \cdot dx$

Section	limit	M	$\partial M/\partial V_c$	$M\frac{\partial M}{\partial V_c}$
CB	0 to 3	$V_c \times x$	x	$x^2 V_c$
BA	$0 \to 6$	$V_c(3+x)-270x$	$3+x$	$V_c x^2 + 6V_c x + 9V_c - 270x^2 - 810x$

$$\Delta_C = \frac{1}{2EI}\left[\int_0^3 x^2 dx + \int_0^6 (V_c x^2 + 6V_c + 9V_c - 270x^2 - 810x)dx\right] = 0$$

or, $\Delta_C = (243V_c - 34020)\frac{1}{EI} = 0$

$V_c = 140 KN (\uparrow)$

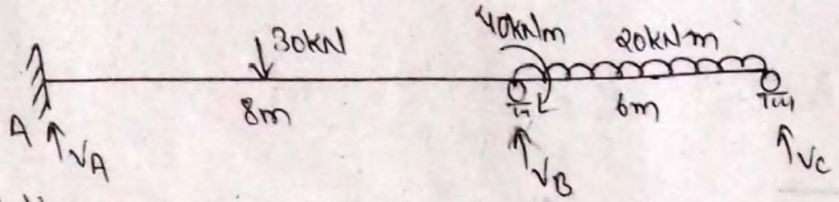
$V_A = 270 - 140 = 130 KN (\uparrow)$

$M_A = 1620 - V_c \times 9$

$= 1620 - 140 \times 9$

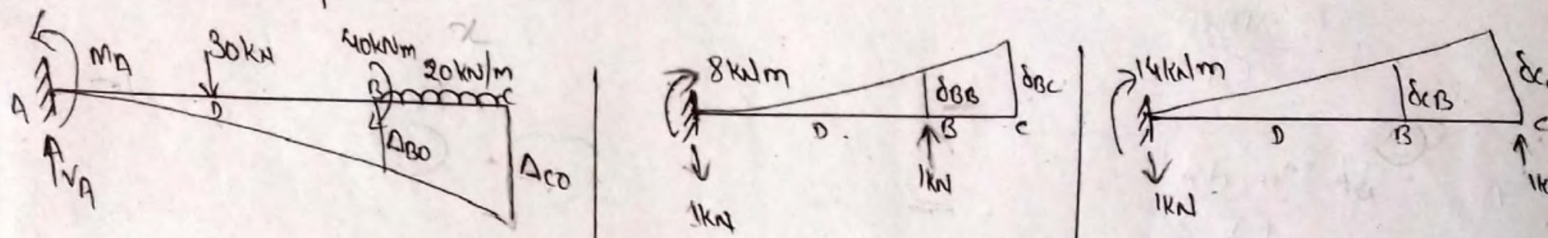
$= 360$ KNm ($\circlearrowleft$)

Force Method (find the support r^{xn})



Solution

Indeterminancy = 5−3 = 2



$\Sigma F_y = 0$ (↑)

$V_A = 30 + 20\times6 = 150\,kN$

$\Sigma M_A = 0$ (↻+)

$M_A = 30 * 4 + 40 + 20\times6\times11$

$= 1480\,kNm$

Here,

$\Delta_B = \Delta_{BO} + \delta_{BB} V_B + \delta_{BC} \times V_C$ —— (i)

$\Delta_C = \Delta_{CO} + \delta_{CB} V_B + \delta_{CC} \times V_C$ —— (ii)

Section	Limit	M	m_1	m_2
AD	0→4	$150x - 1480$	$8-x$	$14-x$
DB	0→4	$150(4+x) - 30x - 1480 \Rightarrow 120x - 880$	$-(x+4)+8 \Rightarrow 4-x$	$-(x+4)+14 \Rightarrow 10-x$
CB	0→6	$-10x^2$	0	x

$$\Delta_{BO} = \int_0^L \frac{M m_1 dx}{EI}$$

$$= \int_0^4 \frac{(150x - 1480)(8-x)}{EI} \cdot dx + \int_0^4 \frac{(120x - 880)(4-x)}{EI} \cdot dx$$

$$= -\frac{29120}{EI} - \frac{5760}{EI}$$

$$= -\frac{34880}{EI}$$

$$\Delta_{CO} = \int_0^L \frac{M m_2 dx}{EI}$$

$$= \int_0^4 \frac{(150x - 1480)(14-x)}{EI} \cdot dx + \int_0^4 \frac{(120x - 880)(10-x)}{EI} \cdot dx + \int_0^6 -\frac{10x^3}{EI} \cdot dx$$

$$= -\frac{57440}{EI} - \frac{21120}{EI} - \frac{3240}{EI}$$

$$= -\frac{81800}{EI}$$

$$\Delta_{BB} = \int_0^L \frac{m_1^2 dx}{EI}$$

$$= \int_0^4 \frac{(8-x)^2 dx}{EI} + \int_0^4 \frac{(4-x)^2 \cdot dx}{EI}$$

$$= \frac{448}{3EI} + \frac{64}{EI}$$

$$= \frac{512}{EI}$$

$$\Delta_{CC} = \int_0^L \frac{m_2^2 dx}{EI}$$

$$= \int_0^4 \frac{(14-x)^2 dx}{EI} + \int_0^4 \frac{(10-x)}{EI} \cdot dx + \int_0^6 \frac{x^2 dx}{EI}$$

$$= \frac{1744}{3EI} + \frac{784}{3EI} + 72$$

$$= \frac{2744}{3EI}$$

$$\delta_{BC} = \int_0^l \frac{m_1 m_2 \, dx}{EI}$$

$$= \int_0^4 \frac{(14-x)(8-x)\,dx}{EI} + \int_0^4 \frac{(10-x)(4-x)}{EI} \cdot dx$$

$$= \frac{880}{3EI} + \frac{208}{3EI}$$

$$= \frac{1088}{3EI}$$

Now,

$$-\frac{34880}{EI} + \frac{512}{3EI} \times V_B + \frac{1088}{3EI} \times V_C = 0 \quad \text{——— (iii)}$$

$$-\frac{81800}{EI} + \frac{1088}{3EI} \times V_B + \frac{2744}{3EI} \times V_C = 0 \quad \text{——— (iv)}$$

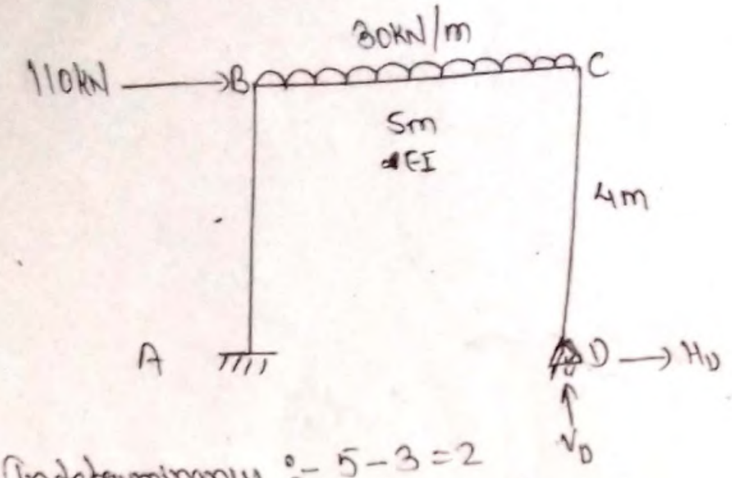
Solving eq$^{\underline{n}}$ (iii) & (iv) we get

$V_B = 91.041$ KN $(\uparrow)$

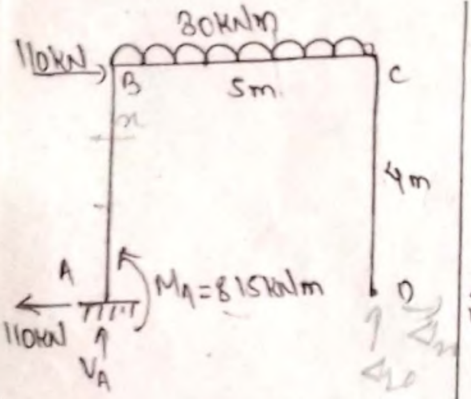
$V_C = 53.333$ KN $(\uparrow)$

$V_A = 5.626$ KN $(\uparrow)$

Horizontal and Vertical rx" at D

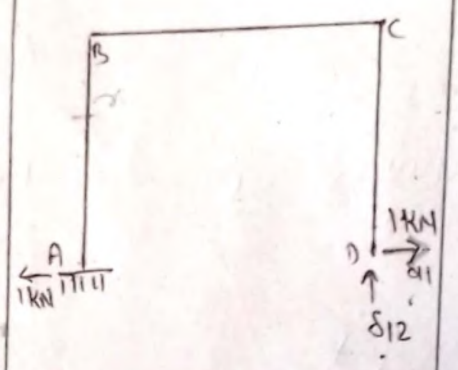


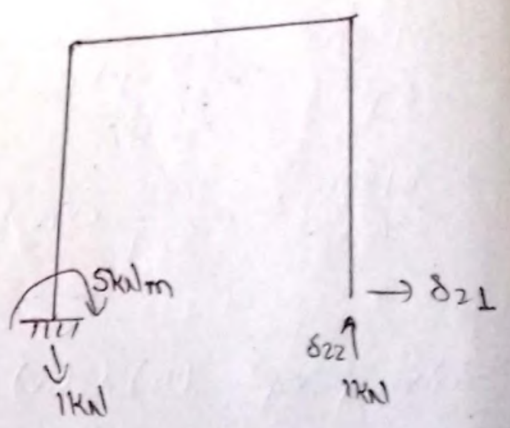
Indeterminacy :- 5-3=2



$V_A = 30\times5 = 150$ KN

$M_A = 110\times4 + 30\times5\times2.5$
$= 815$ KNm





Here,

$\Delta_1 = \Delta_{10} + \delta_{11}\times H_D + \delta_{12}\times V_D = 0$ —— (i)

$\Delta_2 = \Delta_{20} + \delta_{21}\times H_D + \delta_{22}\times V_D = 0$ —— (ii)

Section	Limit	M	m_1	m_2	EI
DC	$0\to4$	0	x	0	EI
CB	$0\to5$	$-15x^2$	4	x	2EI
BA	$0\to4$	$-110x-375$	$4-x$	5	EI

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$$\Delta_{10} = \int_0^L \frac{M m_1}{EI} \cdot dx$$

$$= \int_0^5 \frac{-15x^2 \times 4}{2EI} dx + \int_0^4 \frac{(-110x - 375)(4-x) \cdot dx}{EI}$$

$$= -\frac{16270}{3EI}$$

$$\Delta_{20} = \int_0^L \frac{M m_2 \cdot dx}{EI}$$

$$= \int_0^5 \frac{-15x^3}{2EI} \cdot dx + \int_0^4 \frac{(-110x - 375)\, 5 \cdot dx}{EI}$$

$$= -\frac{104575}{8EI}$$

$$\delta_{11} = \int_0^L \frac{m_1^2 dx}{EI}$$

$$= \int_0^4 \frac{x^2 dx}{EI} + \int_0^5 \frac{4^2 dx}{2EI} + \int_0^4 \frac{(4-x)^2}{EI} dx$$

$$= \frac{64}{3} + 40 + \frac{64}{3}$$

$$= \frac{248}{3EI}$$

$$\delta_{22} = \int_0^L \frac{m_2^2 dx}{EI}$$

$$= \int_0^5 \frac{x^2 dx}{2EI} + \int_0^4 \frac{25 dx}{EI}$$

$$= \frac{725}{6EI}$$

$$\delta_{12} = \int_0^L \frac{m_1 m_2 dx}{EI}$$

$$= \int_0^5 \frac{4x}{2EI} \cdot dx + \int_0^4 5(4-x)\, dx$$

$$= 2 \times \frac{5^2}{2} + 40$$

$$= \frac{65}{EI}$$

Now,

$$\Delta_1 = -\frac{16270}{3EI} + \frac{248}{3EI} H_D + \frac{65 V_D}{EI} = 0 \quad \text{—— (iii)}$$

$$\Delta_2 = -\frac{104575}{EI} + \frac{65 H_D}{EI} + \frac{725 V_D}{6EI} = 0 \quad \text{—— (iv)}$$

Solving eq^n (iii) & (iv)

$$H_D = -33.7188\ \text{kN} (\rightarrow) = 33.718\ \text{kN} (\leftarrow)$$

$$V_D = 126.319 (\uparrow)$$

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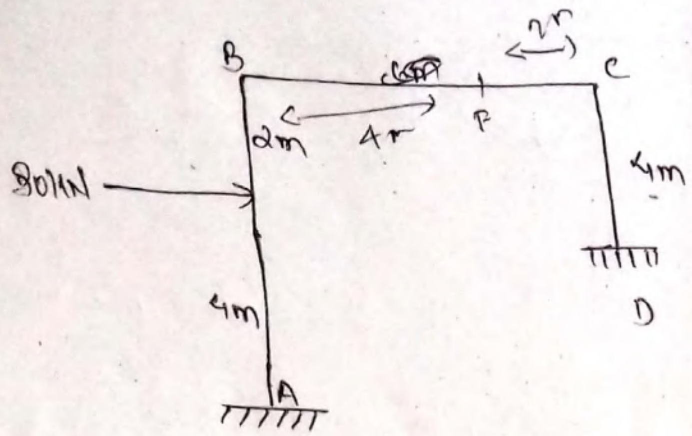
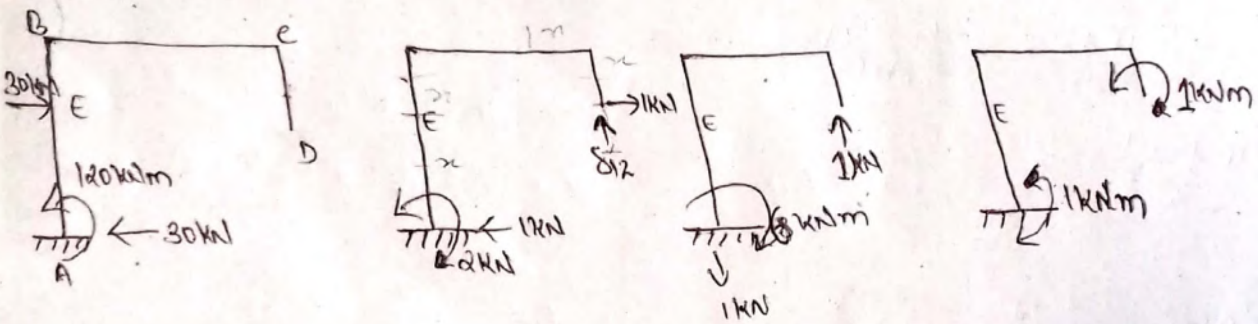
Find Horizontal & vertical reaction at D



Solution,

Indeterminacy = 6 - 3 = 3



$\Delta_1 = \Delta_{10} + \delta_{11} H_D + \delta_{12} \times V_D + \delta_{13} \times M_D = 0$

$\Delta_2 = \Delta_{20} + \delta_{21} H_D + \delta_{22} V_D + \delta_{23} \times M_D = 0$

$\Delta_3 = \Delta_{30} + \delta_{31} H_D + \delta_{32} V_D + \delta_{33} \times M_D = 0$

Section	limit	M	m_1	m_2	m_3
DC	0→4	0	x	0	+1
CB	0→6	0	4	x	+1
BE	0→2	0	$4-x$	6	+1
EAE	0→4	$+30x-120$	$x \mp 2$	6	+1

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$$\Delta_{10} = \int_0^L \frac{M m_1 dx}{EI} = \int_0^4 \frac{(30x-120)(x-2) \cdot dx}{EI} = \Rightarrow \frac{160}{EI}$$

$$\Delta_{20} = \int_0^L \frac{M m_2}{EI} dx = \int_0^4 \frac{(3x-120)\, 6\, dx}{EI} = -\frac{1440}{EI}$$

$$\Delta_{30} = \int_0^L \frac{M m_3}{EI} \cdot dx = \int_0^4 \frac{(3x-120) \times 1\, dx}{EI} = -\frac{240}{EI}$$

$$\delta_{11} = \int_0^L \frac{m_1^2 dx}{EI} = \int_0^4 \frac{x^2 dx}{EI} + \int_0^6 \frac{16\, dx}{EI} + \int_0^2 \frac{(4-x)^2 dx}{EI} + \int_0^4 \frac{(x-2)^2 dx}{EI} = \frac{424}{3EI}$$

$$\delta_{22} = \int_0^L \frac{m_2^2 dx}{EI} = \int_0^6 \frac{x^2 dx}{EI} + \int_0^2 \frac{36 dx}{EI} + \int_0^4 \frac{36\, dx}{EI}$$

$$= \frac{288}{EI}$$

$$\delta_{33} = \int_0^L \frac{m_3^2 dx}{EI} = \int_0^4 \frac{dx}{EI} + \int_0^6 \frac{dx}{EI} + \int_0^2 \frac{dx}{EI} + \int_0^4 \frac{dx}{EI} \Rightarrow \frac{16}{EI}$$

$$\delta_{12} = \int_0^L \frac{m_1 m_2 dx}{EI} = \int_0^6 \frac{4x\, dx}{EI} + \int_0^2 \frac{6(4-x)dx}{EI} + \int_0^4 \frac{(x-2)\, 6}{EI} \Rightarrow \Rightarrow \frac{108}{EI}$$

$$\delta_{13} = \int_0^L \frac{m_1 m_3}{EI} dx = \int_0^6 \frac{4dx}{EI} + \int_0^2 \frac{(4-x)dx}{EI} + \int_0^4 \frac{(x-2)dx}{EI} + \int_0^4 \frac{x\, dx}{EI} \Rightarrow \frac{42}{EI} + \frac{4^2}{2} \Rightarrow \frac{380}{EI}$$

$$\delta_{23} = \int_0^L \frac{m_2 m_3}{EI} dx = \int_0^6 \frac{x\, dx}{EI} + \int_0^2 \frac{6\, dx}{EI} + \int_0^4 \frac{6\, dx}{EI} \Rightarrow \frac{54}{EI}$$

Then,

$$-\frac{160}{EI} + \frac{424}{3EI} H_D + \frac{108}{EI} V_D + \frac{380}{EI} \times M_D = 0 \quad \text{—— (i)}$$

$$-\frac{1440}{EI} + \frac{108}{EI} \cdot H_D + \frac{288}{EI} V_D + \frac{54}{EI} \times M_D = 0 \quad \text{—— (ii)}$$

$$-\frac{240}{EI} + \frac{380}{EI} H_D + \frac{54}{EI} V_D + \frac{16}{EI} M_D = 0 \quad \text{—— (iii)}$$

~~Solving eq^n (i) (ii) & (iii) we get~~

~~$H_D = -5.587\,kN\ (\rightarrow) \Rightarrow 5.587\,kN\ (\leftarrow)$~~

~~$V_D = 7.675\,kN\ (\uparrow)$~~

~~$M_D = 2.721\,kN$ (↻)~~

Solving eq^n (i) (ii) & (iii)

$H_D = 12.91\ (\leftarrow)$

$V_D = 3.49\,kN\ (\uparrow)$

$M_D = 33.89$ (↺)

[Use Matrix to Solve]

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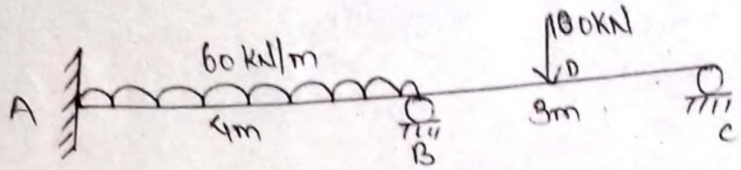
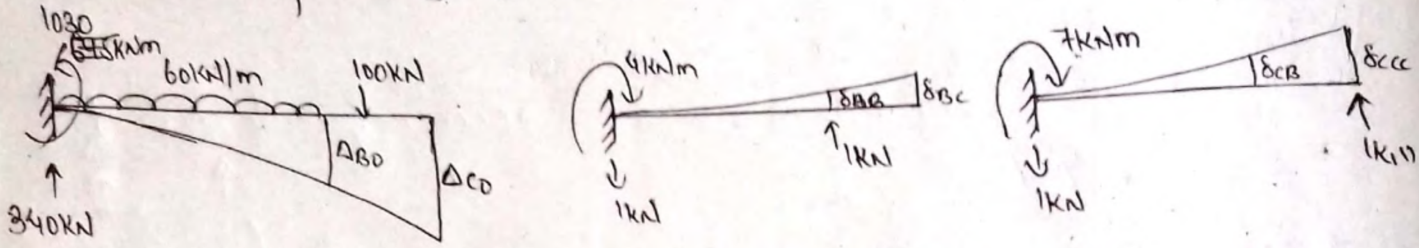
① Find support reactions.



Solution,

Indeterminancy = 5 - 3 = 2



Here,

$\Delta_B = \Delta_{BO} + \delta_{BB} \times V_B + \delta_{BC} \times V_C$ ——— (i)

$\Delta_C = \Delta_{CO} + \delta_{CB} \times V_B + \delta_{CC} \times V_C$ ——— (ii)

Section	Limit	M_o	m_B	m_C
CD	$0 \to 1.5$	0	0	x
DB	$0 \to 1.5$	$-100x$	0	$(1.5+x)$
BA	$0 \to 4$	$-30x^2 - 100(1.5+x)$	x	$(3+x)$

$$\Delta_{BO} = \int_0^l \frac{M_o \times m_B}{EJ}\,dx$$

$$= \int_0^4 \frac{-30x^3 - 150x - 100x^2}{EJ}\,dx \Rightarrow -\frac{15760}{3EJ}$$

$$\Delta_{CO} = \int_0^l \frac{M_o m_C}{EJ}\,dx$$

$$= \int_0^{1.5} \frac{-100x(1.5+x)}{EJ}\,dx + \int_0^4 \frac{[-30x^2 - 100(1.5+x)](x+3)}{EJ}\,dx$$

$$= -\frac{1125}{4} - \frac{34120}{3EJ} = -\frac{127141}{4EJ} - \frac{139855}{12}$$

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$$\delta_{BB} = \int_0^L \frac{m_B^2 \, dx}{EI}$$

$$= \int_0^4 \frac{x^2 \, dx}{EI} = \frac{64}{3EI}$$

$$\delta_{CC} = \int_0^L \frac{m_C^2 \, dx}{EI}$$

$$= \int_0^{1.5} \frac{x^2 \, dx}{EI} + \int_0^{1.5} \frac{(1.5+x)^2 \cdot dx}{EI} + \int_0^4 \frac{(3+x)^2 \cdot dx}{EI}$$

$$= \frac{343}{3EI}$$

$$\delta_{BC} = \int_0^L \frac{m_B m_C \, dx}{EI}$$

$$= \int_0^4 \frac{(3+x)x \cdot dx}{EI} = \frac{136}{3EI}$$

Now,

$$-\frac{15760}{3EI} + \frac{64}{3EI} V_B + \frac{136}{3EI} V_C = 0 \quad \text{--- (iii)}$$

$$-\frac{139955}{4 \times 3EI} + \frac{136}{3EI} \times V_B + \frac{343}{3EI} V_C = 0 \quad \text{--- (iv)}$$

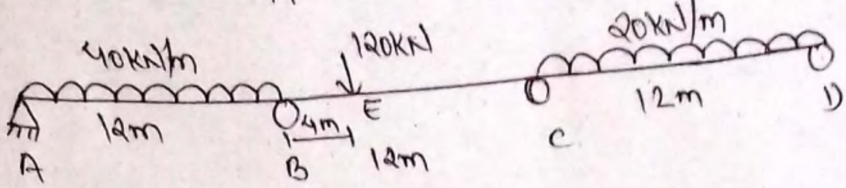
Solving eq$^{\underline{n}}$ (iii) & (iv) we get

$V_B = 187.27$ KN (↑)

$V_C = 27.75$ KN (↑)

$V_A = 124.98$ KN (↑)

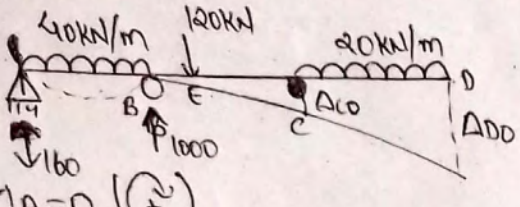
② Find reaction at support



Solution,

indeterminancy = 5 − 3 = 2

Now,

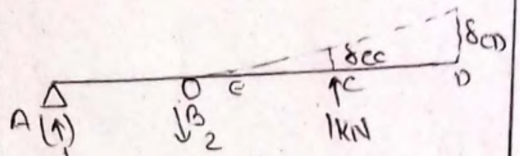


$\Sigma M_A = 0$ (+)

$-V_B \times 12 + 40\times12\times6 + 120\times16 + 20\times12\times30$

$V_B = 1000\,kN\ (\uparrow)$

$V_A = 160\ (\downarrow)$

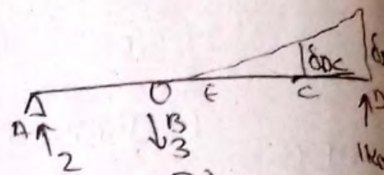


$\Sigma M_A = 0$ (+)

$+V_B \times 12 - 1\times24 = 0$

$V_B = 2\,kN\ (\downarrow)$

$V_A = 1\,kN\ (\uparrow)$



$\Sigma M_A = 0$ (+)

$12\times V_B - 1\times36 = 0$

$V_B = 3\,kN\ (\downarrow)$

$V_A = 2\,kN\ (\uparrow)$

Here,

~~$\Delta_B = \Delta_{BO} + \delta_{BB} \times V_B$~~

$\Delta_C = \Delta_{CO} + \delta_{CC} \times V_C + \delta_{CD} \times V_D$ ——— (i)

$\Delta_D = \Delta_{DO} + \delta_{DC} \times V_C + \delta_{DD} \times V_D$ ——— (ii)

Section	Limit	M_o	m_C	m_D
DC	0 → 12	$-10x^2$	0	x
CE	0 → 8	$\Rightarrow -20\times12(12+x)$ $\Rightarrow -2880 - 240x$	x	$(12+x)$
EB	0 → 4	$\Rightarrow -120x - 240(20+x)$ $\Rightarrow -360x - 4800$	$(8+x)$	$(20+x)$
BA	0 → 12	$\Rightarrow -160x - 20x^2$	x	$2x$

$$\Delta_{C0} = \int_0^L \frac{M_0 m_C \, dx}{EI}$$

$$= \int_0^8 \frac{(-2880-240x)\,x}{EI} + \int_0^4 \frac{(-360x-4800)(8+x)\,dx}{EI} + \int_0^{12} \frac{(-160x-20x^2) \times x}{EI}$$

$$= (-133120 - 222720 - 195840) \times 1/EI$$

$$= -\frac{551680}{EI}$$

$$\Delta_{D0} = \int_0^L \frac{M_0 m_D \, dx}{EI}$$

$$= \frac{1}{EI}\left[\int_0^{12} \frac{-10x^3\,dx}{\cancel{EI}} + \int_0^8 (-2880-240x)(12+x)\,dx + \int_0^4 (-360x-4800)(20+x)\,dx \int_0^{12} (-160x-20x^2)\,2x\right]$$

$$= \frac{1}{EI}\left[-51840 - 501760 - 487680 - 391680\right]$$

$$= \frac{-1432960}{EI}$$

$$\delta_{CC} = \int_0^L \frac{m_C^2 \, dx}{EI}$$

$$= \int_0^8 \frac{x^2\,dx}{EI} + \int_0^4 \frac{(8+x)^2\,dx}{EI} + \int_0^{12} \frac{(x^2)\,dx}{EI}$$

$$= \frac{8^3}{3} + \frac{12^3}{3} + \frac{1216}{3}$$

$$= \frac{1152}{EI}$$

$$\delta_{DD} = \int_0^L \frac{m_D^2 \, dx}{EI}$$

$$= \int_0^{12} \frac{x^2\,dx}{EI} + \int_0^8 \frac{(12+x)^2}{EI} + \int_0^4 \frac{(20+x)^2 \cdot dx}{EI} + \int_0^{12} \frac{(2x)^2\,dx}{EI}$$

$$= \frac{12^3}{3} + \frac{4 \times 12^3}{3} + \cancel{\frac{4368}{3}} + \frac{5824}{3EI} + \frac{6272}{3EI}$$

$$= \frac{6912}{EI}$$

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$$\delta_{CD} = \int_0^L \frac{m_C m_D \, dx}{EJ}$$

$$= \int_0^8 \frac{x(12+x)\,dx}{EJ} + \int_0^4 \frac{(8+x)(20+x)\,dx}{EJ} + \int_0^{12} \frac{2x^2}{EJ}\,dx$$

$$= \frac{1664}{3EJ} + \frac{2658}{3EJ} + \frac{2 \times 12^3}{3}$$

$$= \frac{2592}{EJ}$$

Now,

$$-\frac{551680}{EJ} + \frac{1152}{EJ} \times V_C + \frac{2592}{EJ} V_D = 0 \quad \text{—— (iii)}$$

$$-\frac{1432960}{EJ} + \frac{2592}{EJ} V_C + \frac{6912}{EJ} V_D = 0 \quad \text{—— (iv)}$$

Solving eq$^{\underline{n}}$ (iii) & (iv) we get

$V_C = 79.56 \text{ kN} (\uparrow)$

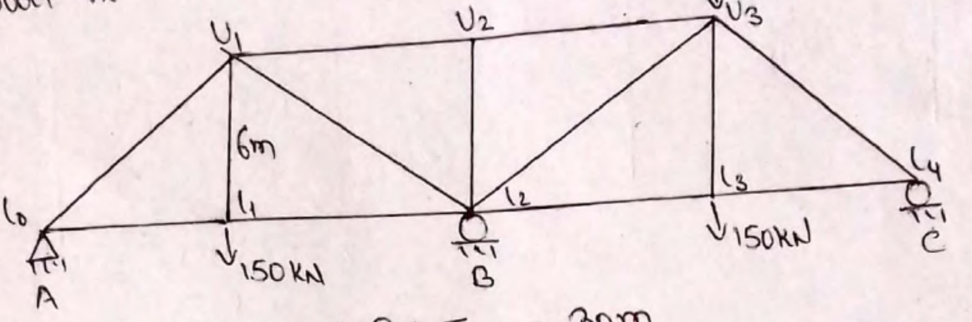
$V_D = 177.481 \text{ kN} (\uparrow)$

$$V_B = \frac{40 \times 12 \times 6 + 120 \times 16 + 20 \times 12 \times 30 - 79.56 \times 24 - 177.481 \times 36}{12}$$

$= 308.437 \text{ kN} (\uparrow)$

$V_A = 274.526 \text{ kN} (\uparrow)$

Find support rx^n

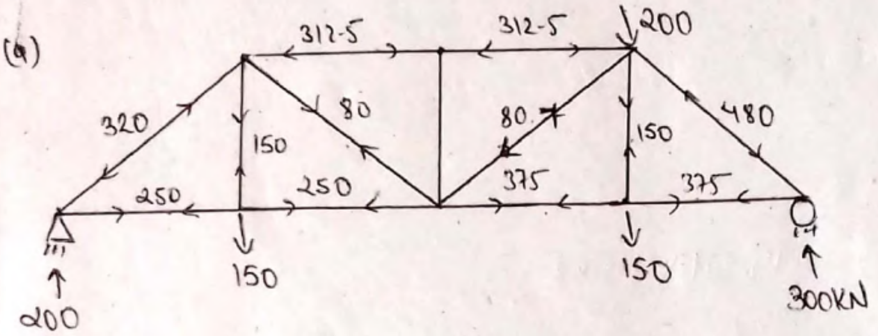


$4 @ 7.5m = 30m$

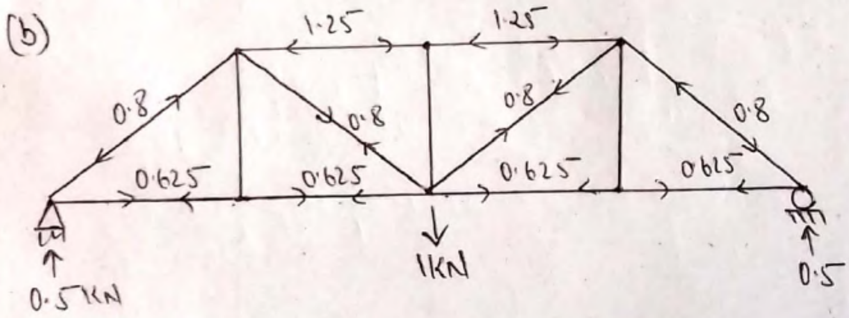
Solution,

In determinancy $= D_{E} + D_{si} \Rightarrow (4-3) + (13 - \cancel{(3 + (j-3)\times 2)}) = 1$

(a)



(b)



$\Delta_B = \Delta_{B0} + \delta_{11} V_B$

Section	(cm^2) AE	(cm) Length	P_0	P_1	$\frac{P_0 P_1 L}{AE}$	$\frac{P_1^2 L}{AE}$
L_0L_1	25	7.50	250	0.625	4687.5	11.72
L_1L_2	25	750	250	0.625	4687.5	11.72
L_2L_3	25	750	375	0.625	703725	11.72
L_3L_4	25	750	375	0.625	7031.25	11.72
U_1U_2	25	750	−312.5	−1.25	11718.75	46.88
U_2U_3	25	750	−312.5	−1.25	11718.75	46.88
U_1L_0	25	960	−320	−0.8	9830.4	24.58
U_1L_1	18.75	600	150	0	0	0
U_1L_2	18.75	960	80	0.8	3278.8	32.77
U_2L_2	18.75	600	0	0	0	0

$V_3 L_2$	25	960	−80	0.8	−3276.8	32.75
$V_3 L_3$	18.75	600	150	0	0	0
$V_3 L_4$	18.75	960	−480	−0.8	14745.6	24.58

$\Sigma = \frac{71451}{E}$ $\Sigma = \frac{255.34}{E}$

Here,

$\Delta_B = \Delta_{B0} + \delta_{BB} \times V_B = 0$

or, $V_B = \frac{-71451}{255.34}$

$= -279.826 \text{ kN} (\downarrow)$

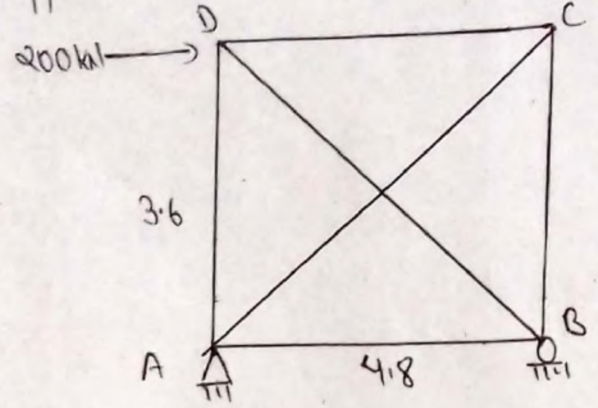
$= +279.826 (\uparrow)$

$V_C = \frac{(200+150) \times 7.5 \times 3 + 180 \times 7.5 - 279.826 \times 7.5 \times 2}{7.5 \times 4}$

$= \cancel{42.69 \text{ kN} (\uparrow)}$ $160.087 (\uparrow) \text{ kN}$

$V_A = 60.087 \text{ kN} (\uparrow)$

Find support reaction.



$A = 0.002\ m^2$

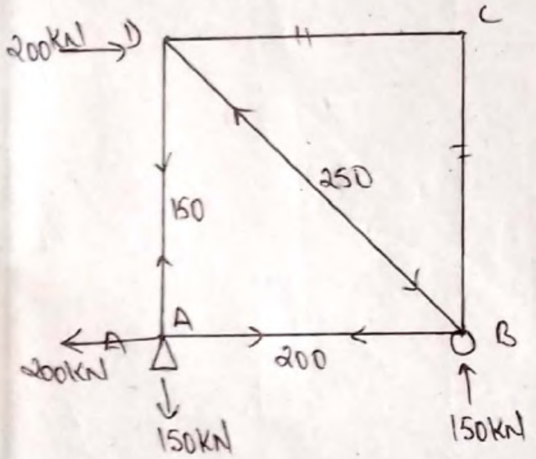
$E = 240 \times 10^{-6}\ kN/m^2$

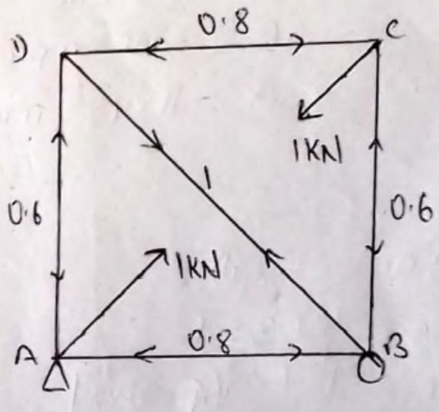
Solution,

indeterminary $= D_{se} + D_{si}$

$D_{se} = 3 - 3 = 0$

$D_{si} = 6 - (3 + (4-3) \times 2) = 1$





$\Delta_B = \Delta_0 + \delta_{11} \times F_{CA} = 0$

$$\Delta_0 = \frac{P_0 P_1 L}{AE} = \frac{150 \times (-0.6) \times \cancel{0.002}\ 3.6}{0.002 \times 240 \times 10^{+6}} + \frac{200 \times (-0.8) \times 4.8}{0.002 \times 240 \times 10^{6}} + \frac{-250 \times 1 \times 6}{0.002 \times 240 \times 10^{6}}$$

$$= -\frac{\cancel{27}}{\cancel{8000}} - 4.143 \times 10^{-3}$$

$$\delta_{11} = \frac{P_1^2 L}{AE} = \frac{1}{48 \times 10^4}\left[-0.8^2 \times 4.8 - 0.36 \times 3.6 - 0.64 \times 4.8 - 0.36 \times 3.6 + 6\right]$$

$$= -\cancel{5.7} \times 10^{-6}$$

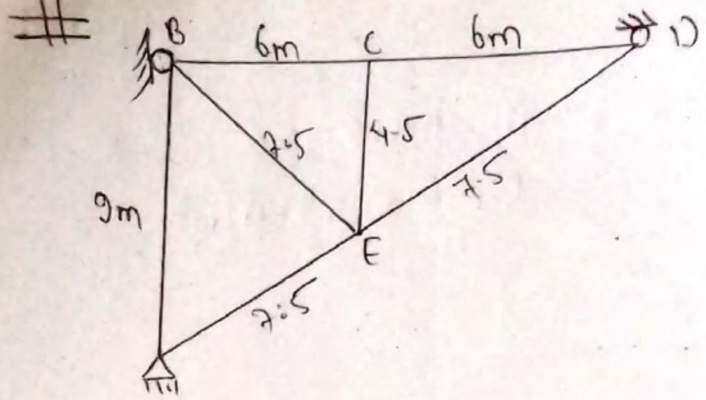
$$F_{CA} = \frac{-\Delta_0}{\delta_{11}} \Rightarrow \frac{-726.842}{\cancel{580.645}}\ kN\ (T)$$

$$= 726.842\ kN(C)$$

068.96/E | 4260/E

0
0
0
0
0
28

#



(a) Temp of bar BC increased.

(b) AE is fabricated 7.5mm too short.

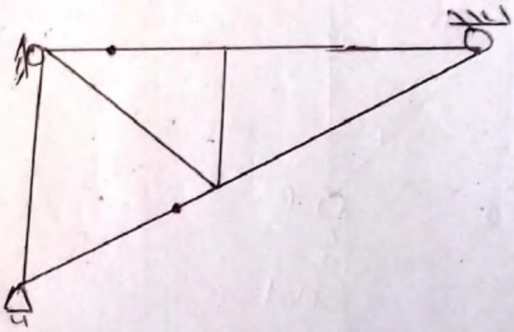
(c) Support B is constructed 12mm to right of its intended position.

(d) support D is constructed 6mm too high

(e) All bars has $A = 12.5\ cm^2$

$E = 2\times10^5$ mpa

$\alpha = 11\times10^{-6}$ mm/°c



Solution

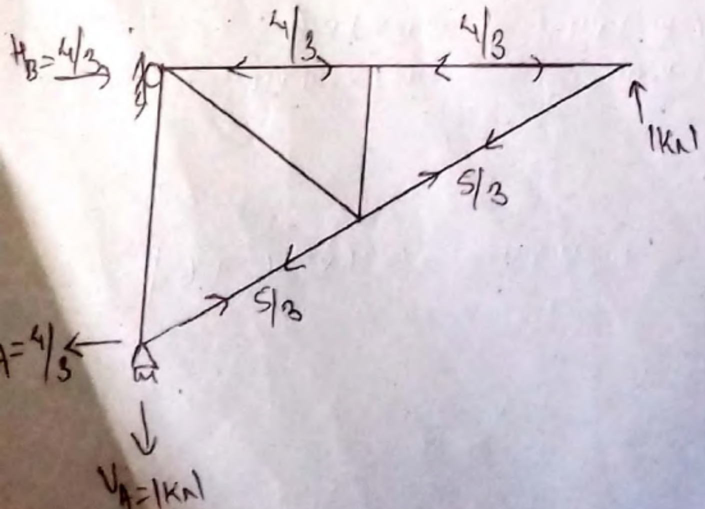
indeterminacy $\rightarrow Ds = Dse + Dsi$

$Dse = 4-3 = 1$

$Dsi = 7-(3+(5-3)\times2]$

$= 0$

$\Delta_D = \Delta_{D0} + \delta_{11}\times V_D = 6mm$



$$\delta_{11} = \frac{\Sigma P_1^2 L}{AE}$$

$$= \frac{63000}{AE}$$

Now,

$$1 \times \Delta_{D0} + 4/3 \times 12mm = 5/3 \times (-7.5) - 4/3 \times 1.848$$

$$\Delta_{D0} = -30.946$$

$$= 30.946(\downarrow)$$

$$\therefore V_D = 146.68kN\ (\uparrow)$$

$$F_{CD} = \frac{4}{3} \times 146.68 = 195.573\ kN\ (C)$$

$$F_{BC} = 4/3 \times 146.68 = 195.573\ kN\ (C)$$

$$F_{ED} = 5/3 \times 146.68 = 244.467\ kN\ (T)$$

$$F_{AE} = 5/3 \times 146.68 = 244.467\ kN\ (T)$$

$$H_A = 4/3 \times 146.68 = 195.573\ kN\ (\leftarrow)$$

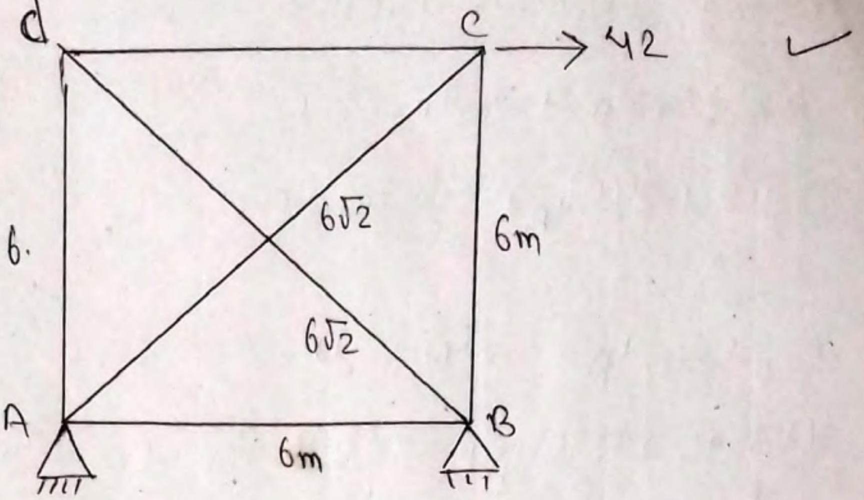
$$V_A = 146.68\ kN\ (\uparrow)$$

$$H_B = 4/3 \times 146.68 = 195.573\ kN\ (\rightarrow)$$

Find member forces.

$A = 10\,cm^2 \Rightarrow 0.001\,m^2$

$E = 240 \times 10^6\ kN/m^2$



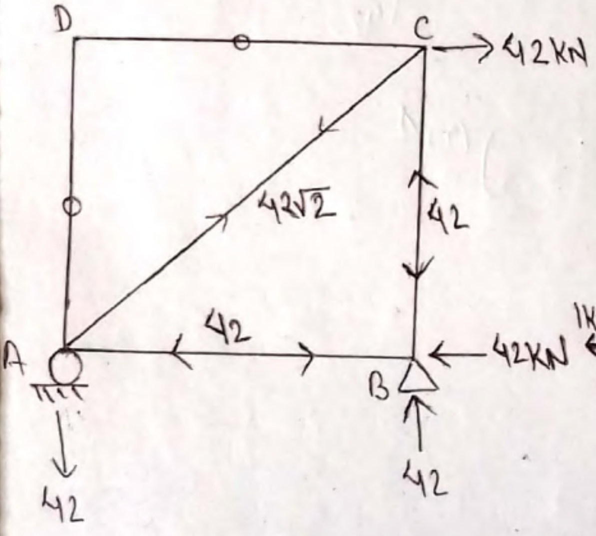
Solution:

Indeterminacy $(D_s) = D_{si} + D_{se}$

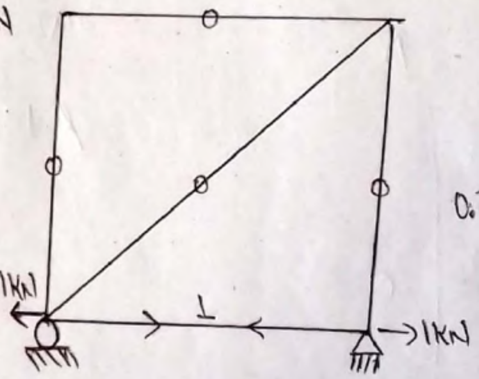
$D_{se} = 4 - 3 = 1$

$D_{si} = 6 - [3 + (4-3)\times 2] = 1$

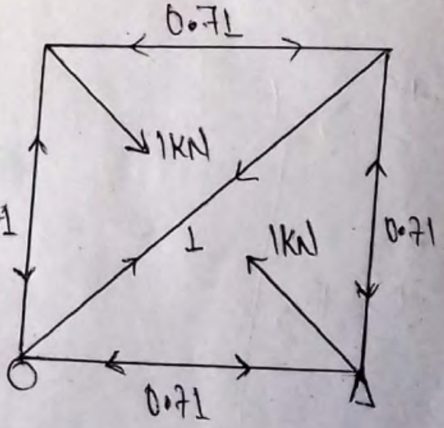
Now.



(i) Member force due to external loading



(ii) P_1 forces



(iii) P_2 forces

Member	length (m)	l/A	P_0	P_1	P_2	Δ_{10} $\frac{P_0P_1L}{AE}$	Δ_{20} $\frac{P_0P_2L}{AE}$	δ_{11} P_1^2L/AE	δ_{22} P_2^2L/AE	δ_{12} $\frac{P_1P_2L}{AE}$
AB	6	6000	−42	1	−0.71	−252000	178920	6000	3024.6	4260
BC	6	6000	−42	0	−0.71	0	178920	0	3024.6	0
CD	6	6000	0	0	−0.71	0	0	0	3024.6	0
DA	6	6000	0	0	−0.71	0	0	0	3024.6	0
AC	$6\sqrt{2}$	8485.28	$42\sqrt{2}$	0	1	0	503999.92	0	8485.28	0
BD	$6\sqrt{2}$	8485.28	0	0	1	0	0	0	8485.28	0
						$\frac{-252000}{E}$	$\frac{861839.92}{E}$	$\frac{6000}{E}$	$\frac{29068.96}{E}$	$\frac{4260}{E}$

Using compatibility eq^n; we get

$$\Delta_{10} + \delta_{11} H_A + \delta_{12} F_{BD} = 0$$

$$\text{or, } -\frac{252000}{E} + \frac{6000}{E} H_A + \frac{4260}{E} F_{BD} = 0 \quad \text{---} (i)$$

&

$$\Delta_{20} + \delta_{12} H_A + \delta_{22} F_{BD} = 0$$

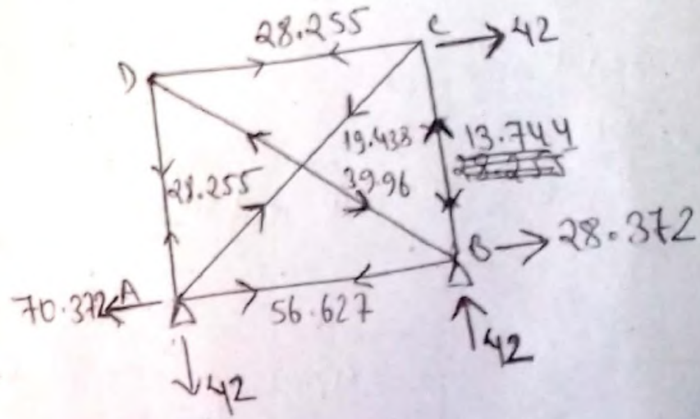
$$\frac{86183.92}{E} + \frac{4260}{E} H_A + \frac{29068.96}{E} F_{BD} = 0 \quad \text{---} (ii)$$

Solving eq^n (i) & (ii)

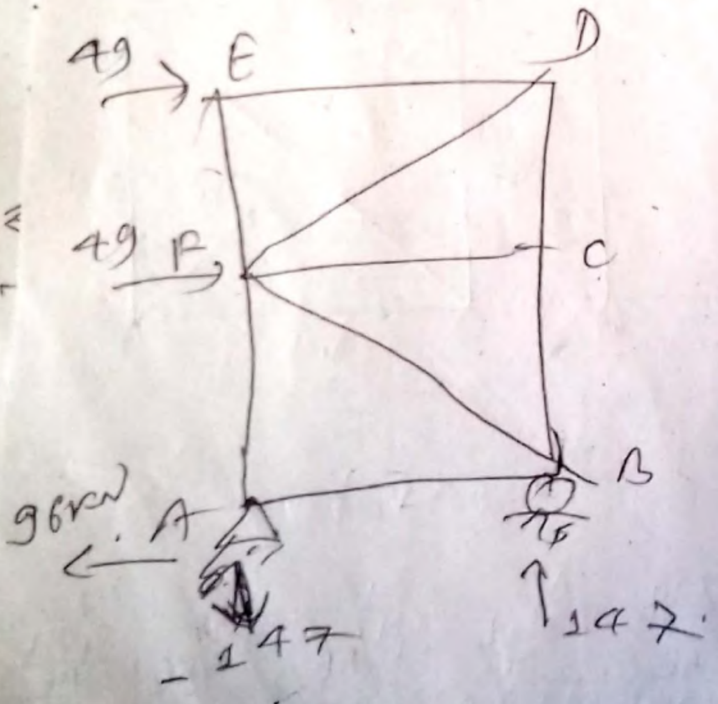
$H_A = 70.372$ KN ($\leftarrow$)

$F_{BD} = -39.96$ KN (T)

$= 39.96$ KN (C)



At joint



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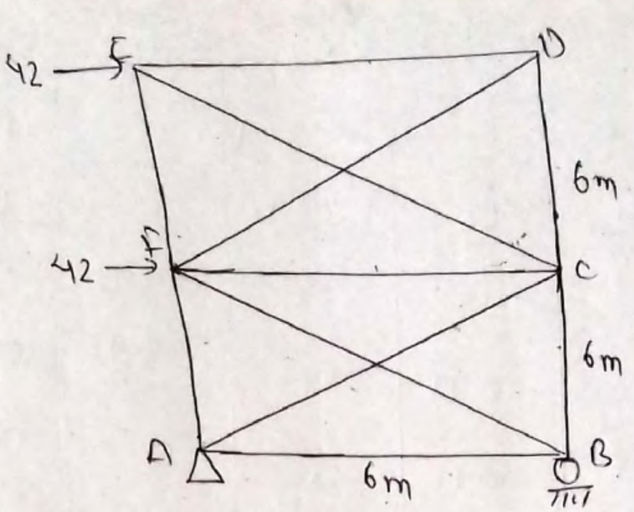


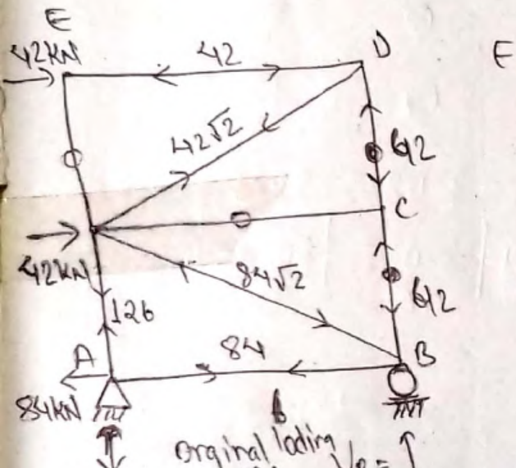
Horizontal & vertical rigidity EA
inclined member = 2EA
Find member forces.

Soln

Indeterminancy :- $D_{se} + D_{si}$

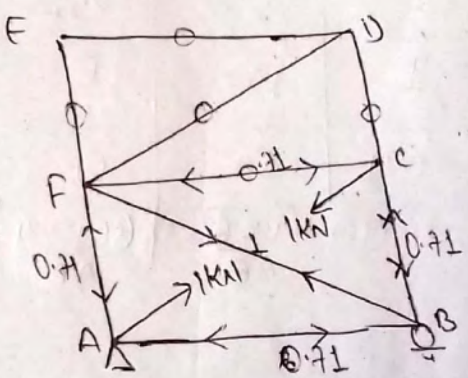
$$D_{se} = 3-3 = 0$$

$$D_{si} = 11-(3+(6-3)\times 2)$$
$$= 11-9 = 2$$

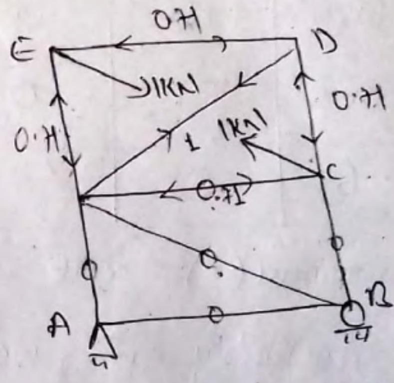


Original loading (P_0)

$V_A = 126$ kN $V_B = 126$ kN



[P_1 forces]



[P_2 forces]

$\Sigma M_B = 0$

$$6\times V_A + 6\times 42 + 12\times 42 = 0$$
$$V_A = -126 \text{ kN}$$
$$= 126 \text{ kN} (\downarrow)$$

Member	Length (m)	P_0	P_1	P_2	(Δ_{10}) P_0P_1L/AE	(Δ_{20}) P_0P_2L/AE	(δ_{12}) P_1P_2L/AE	(δ_{11}) P_1^2L/AE	δ_{22} P_2^2L/AE
AB	6	84	-0.71	0	-357.84	0	0	3.025	0
BC	6	-42	-0.71	0	+178.92	0	0	3.025	0
CD	6	-42	0	-0.71	0	178.92	0	0	3.025
DE	6	-42	0	-0.71	0	178.92	0	0	3.025
EF	6	0	0	-0.71	0	0	0	0	3.025
FA	6	126	-0.71	0	-536.76	0	0	3.025	0
• BF	$6\sqrt{2}$	$-84\sqrt{2}$	1	0	-504	0	0	$3\sqrt{2}$	0
• AC	$6\sqrt{2}$	0	1	0	0	0	0	$3\sqrt{2}$	0
• FD	$6\sqrt{2}$	$42\sqrt{2}$	0	1	0	252	0	0	$3\sqrt{2}$
• CE	$6\sqrt{2}$	0	0	1	0	0	0	0	$3\sqrt{2}$
CF	6	0	-0.71	-0.71	0	0	3.025	3.025	3.025
				Sum ⇒	$-\frac{1219.68}{AE}$	⇒ $\frac{609.84}{AE}$	⇒ $\frac{3.025}{AE}$	$\frac{20.585}{AE}$	$\frac{20.585}{AE}$

inclined member (•) = 2AE

Using compatibility eq^n, we get

$$-\Delta_{AC} = \Delta_{10} + \delta_{11} \times F_{AC} + \delta_{12} F_{CE} = 0$$

$$\Delta_2 = \Delta_{20} + \delta_{21} \times F_{AC} + \delta_{22} F_{CE} = 0$$

or, $-1219.68 + 20.585 F_{AC} + 3.025 F_{CE} = 0$ — (i)

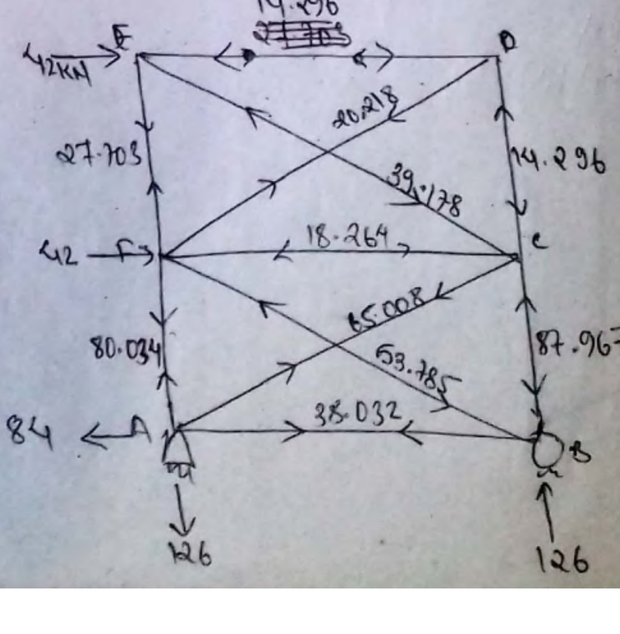
$609.84 + 3.025 F_{AC} + 20.585 F_{CE} = 0$ — (ii)

Solving eq^n (i) & (ii) we get

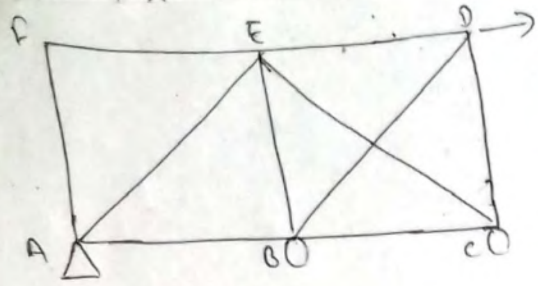
$F_{AC} = 65.008$ KN (T)

$F_{CE} = -39.178$ KN (T)

$= 39.178$ KN (C)



#



All inclined members 2mm too long
All vertical " decreased by 15°C
Area = 40cm² = 0.004 m²
E = 240×10^6 KN/m²
$\alpha = 11\times10^{-6}$ mm/°c.
AE = 96×10^4 KN

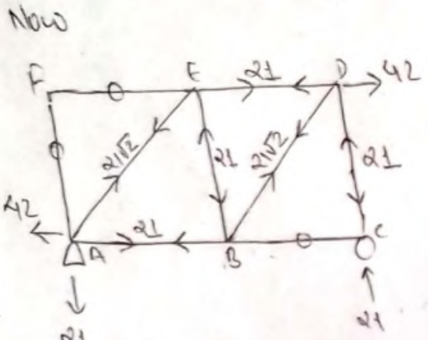
Sol^n

Indeterminancy (D_s) = $D_{se} + D_{si}$

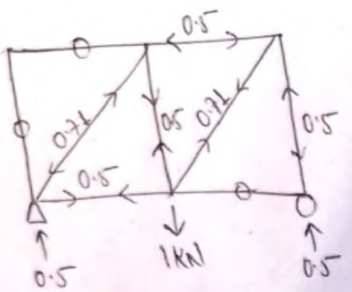
$D_{se} = 4-3 = 1$

$D_{si} = 10-(3+(6-3)\times2) = 1$

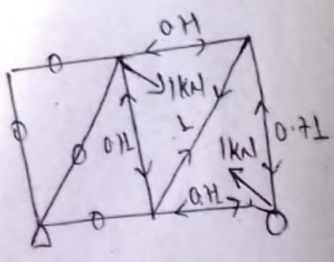
Now



(loading P_0)



(P_1 - force)



(P_2 - force)

$(\Delta l)_t = l\alpha\Delta t = 6000\times11\times10^{-6}\times15 = 0.99mm = 9.9\times10^{-4}$

Member	length (m)	P_0	P_1	P_2	$\frac{L}{AE}$	$\frac{P_0P_1L}{AE}$ (Δ_{10})	$\frac{P_0P_2L}{AE}$	$\frac{P_1^2L}{AE}$	$\frac{P_2^2L}{AE}$	$\frac{P_1P_2L}{AE}$	Δl	Δt	$P_1\Delta l$	$P_1\Delta t$	$P_2\Delta l$	$P_2\Delta t$
AB	6	21	0.5	0	6.25×10^{-6}	6.56×10^{-5}	0	1.56×10^{-6}	0	0	0	0	0	0	0	0
BC	6	0	0	-0.71	"	0	0	0	3.15×10^{-6}	0	0	0	0	0	0	0
CD	6	-21	-0.5	-0.71	"	6.56×10^{-5}	9.318×10^{-5}	1.56×10^{-6}	"	2.22×10^{-6}	0	-9.9×10^{-4}	[illegible]	4.95×10^{-4}	0	$+7.029\times10^{-4}$
DE	6	21	-0.5	-0.71	"	-6.56×10^{-5}	-9.318×10^{-5}	1.56×10^{-6}	"	"	0	0	0	0	0	0
EF	6	0	0	0	"	0	0	0	0	0	0	0	0	0	0	0
FA	6	0	0	0	"	0	0	0	0	0	0	-9.9×10^{-4}	0	0	0	0
AE	$6\sqrt{2}$	$21\sqrt{2}$	-0.71	0	8.84×10^{-6}	-1.88×10^{-4}	0	3.15×10^{-6}	0	0	0.002	0	$+1.42\times10^{-3}$	0	0	0
BD	$6\sqrt{2}$	$21\sqrt{2}$	0.71	1	8.84×10^{-6}	$+1.88\times10^{-4}$	2.625×10^{-4}	3.15×10^{-6}	8.84×10^{-6}	6.276×10^{-6}	0.002	0	$+1.42\times10^{-3}$	0	0.002	0
CE	$6\sqrt{2}$	0	0	1	8.84×10^{-6}	0	0	0	8.84×10^{-6}	0	0.002	0	0	0	0.002	0
BE	6	-21	0.5	-0.71	6.25×10^{-6}	-6.56×10^{-5}	9.318×10^{-5}	1.56×10^{-6}	3.15×10^{-6}	-2.22×10^{-6}	0	-9.9×10^{-4}	0	-4.9[illegible]		7.029×10^{-4}
						0	3.56×10^{-4}	1.25×10^{-5}	2.39×10^{-5}	8.4×10^{-6}			0	0	0.004	1.405×10^{-3}
						Δ_{10}	Δ_{20}	δ_{11}	δ_{22}	δ_{12}					Δ_{2l}	Δ_{2t}

Using com[illegible]

$A_1 = A_{10}+$ [illegible]

or, 1.25×10[illegible]

$A_2 = A_{20}$ [illegible]

$\Rightarrow 3.56\times$[illegible]

$\Rightarrow 8.4\times$[illegible]

Solving [illegible]

$V_B = 2$[illegible]

$F_{CE} =$ [illegible]

$=$ [illegible]

using compatibility eq^n

$$\Delta_1 = \Delta_{10} + \delta_{11} V_B + \delta_{12} F_{CE} + \Delta_{1l} + \Delta_{1t} = 0$$

or, $1.25\times10^{-5} V_B + 8.4\times10^{-6} F_{CE} = 0$ — (I)

$$\Delta_2 = \Delta_{20} + \delta_{12} V_B + \delta_{22} F_{CE} + \Delta_{2l} + \Delta_{2t} = 0$$

$\Rightarrow 3.56\times10^{-4} + 8.4\times10^{-6} V_B + 2.39\times10^{-5} F_{CE} + 0.004 + 1.405\times10^{-3} = 0$

$\Rightarrow 8.4\times10^{-6} V_B + 2.39\times10^{-5} F_{CE} = -5.761\times10^{-3}$ — (II)

Solving eq^n (I) & (II) we get

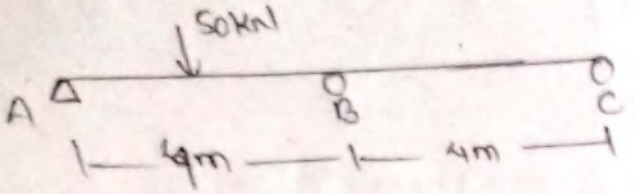
$V_B = 212.07\,kN\,(\downarrow)$

$F_{CE} = -315.58\,kN\,(T)$

$= 315.58\,kN\,(C)$

Δ_l	$P_2\Delta_t$
0	0
0	0
	$+7.029\times10^{-4}$
	0
	0
	0
	0
002	0
002	0
	7.029×10^{-4}
004	1.405×10^{-3}
Δ_{2l}	Δ_{2t}

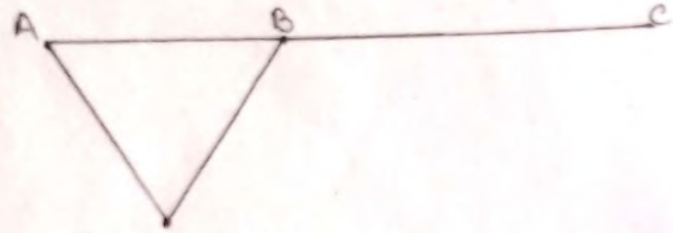
Three Moment Equation

① 

Solution,

i) Free moment diagram

(Assume all support simply supported section ⟷ section BMD)



$\frac{Pab}{l} = \frac{50 \times 2 \times 2}{4} = 50kNm$

Area of BMD left of B is

$A_1 = \frac{1}{2} \times 50 \times 4 = 100$

$a_1 = 2m$

Area of BMD right ~~left~~ of B is

$A_2 = 0$

$a_2 = 0$

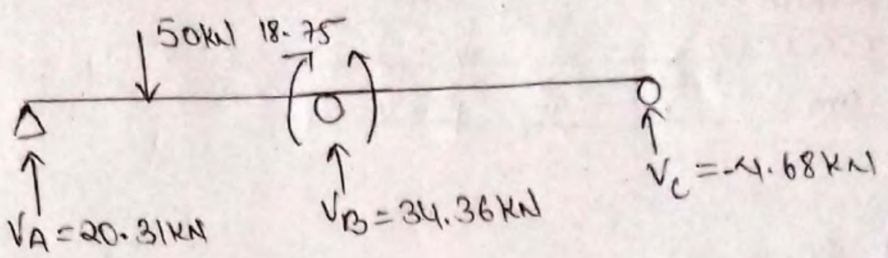
Using 3-moment Equation

$$M_A\left(\frac{L_1}{I_1}\right) + 2M_B\left(\frac{l_1}{I_1} + \frac{l_2}{I_2}\right) + M_C\left(\frac{L_2}{I_2}\right) = -\frac{6A_1a_1}{L_1I_1} - \frac{6A_2a_2}{L_2I_2}$$

$$0 + 2M_B\left(\frac{4}{I} + \frac{4}{I}\right) = \frac{-6 \times 100 \times 2}{4I}$$

$M_B = -18.75$ kNm (sag)

$= 18.75$ kNm (hogging)



$(\curvearrowright +)\ \Sigma M_B = 0$ right

$-V_C \times 4 - 18.75 = 0$

$V_C = -4.68kN\ (\uparrow)$

$= 4.68kN\ (\downarrow)$

$\Sigma M_B = 0$ (left) $(\curvearrowright +)$

$V_A \times 4 - 50 \times 2 + 18.75 = 0$

$V_A = 20.3125\ kN \uparrow$

$\Sigma F_y = 0\ (\uparrow +)$

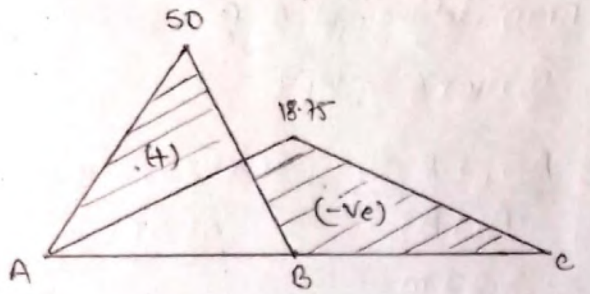
$V_A + V_B + V_C = 50$

or $V_B = 50 + 4.68 - 20.3125$

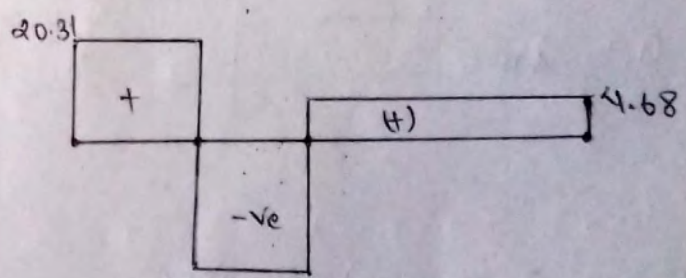
$= \cancel{25.0075kN}(\uparrow)$

$= 34.3675kN(\uparrow)$

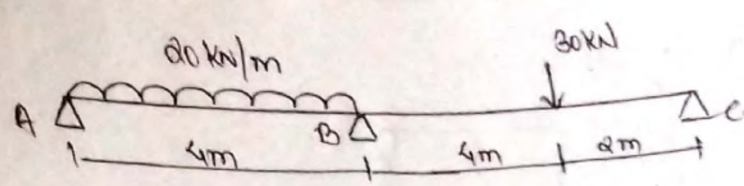
BMD



SFD

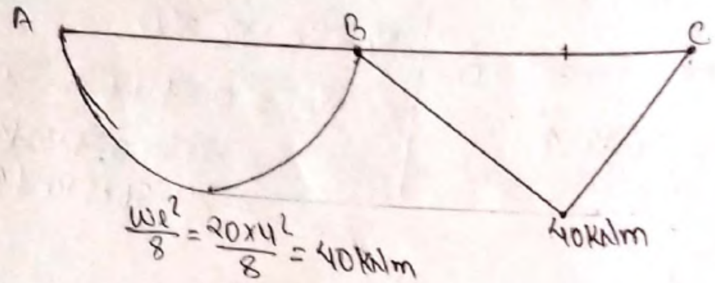


(17)



Sol^n

i) Free moment Diagram.



$\frac{wl^2}{8} = \frac{20\times4^2}{8} = 40$ kN/m

~~Using~~

Area of BMD left of B

$A_1 = \frac{2}{3}\times40\times4 = 106.67$

$a_1 = 2m$

Area of BMD ~~left~~ right of B

$A_2 = \frac{1}{2}\times40\times6 = 120$

$a_1 = \frac{A_1\bar{x}_1 + A_2\bar{x}_2}{A_1+A_2} = \frac{80\times4\times2/3 + 40\times4+2/3}{80+40}$

$= 3.33m$

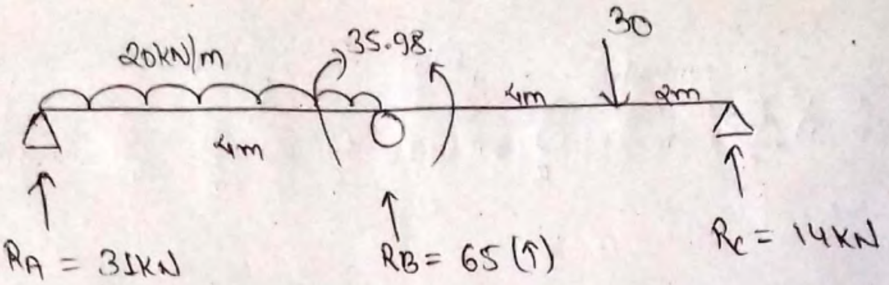
Using 3-moment equation.

$M_A\left(\frac{L_1}{I_1}\right) + 2M_B\left(\frac{L_1}{I_1}+\frac{L_2}{I_2}\right) + M_C\left(\frac{L_2}{I_2}\right) = -\frac{6A_1a_1}{L_1I_1} - \frac{6A_1a_2}{I_2L_2}$

$2M_B(4+6) = -\frac{6\times106.67\times2}{4} - \frac{6\times120\times3.33}{6}$

$20M_B = -719.61$

$M_B = -35.9805$ kN



$\curvearrowright \Sigma M_A = 0$ (right)

$-6 \times R_C + 30 \times 4 - 35.98 = 0$

$\therefore R_C = 14$ kN $(\uparrow)$

$\curvearrowright \Sigma M_B = 0$ (left)

$4 \times R_A - 20 \times 4 \times 2 + 35.98 = 0$

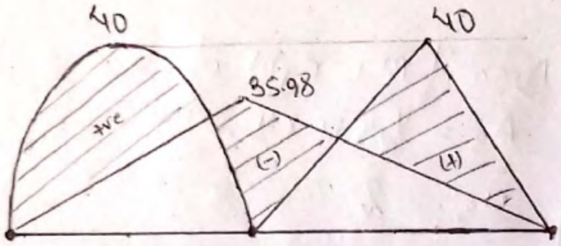
$\therefore R_A = 31$ kN $(\uparrow)$

$\Sigma F_y = 0$

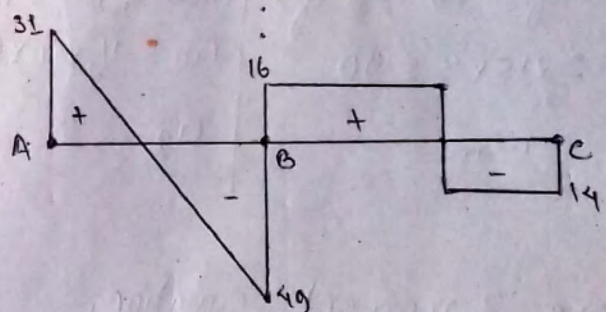
$R_A + R_B + R_C = 30 + 20 \times 4$

$\therefore R_B = 65$ kN $(\uparrow)$

BMD and SFD Diagram

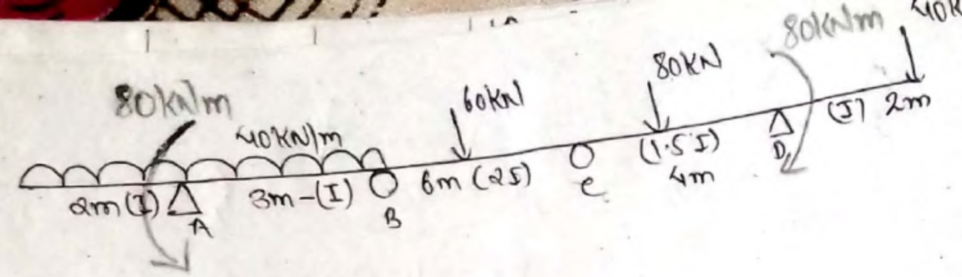


BMD



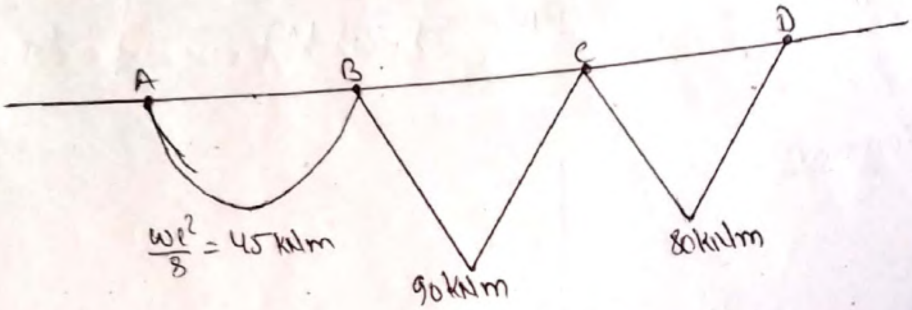
SFD

(3)



Solution,

1) Fixed moment Diagram



Area of BMD left of B	Area of BMD right of B	Area of BM right of C
$A_1 = \frac{2}{3} \times 45 \times 3 = 90$	$A_2 = \frac{1}{2} \times 90 \times 6 = 270$	$A_3 = \frac{1}{2} \times 80 \times 4 = 160$
$a_1 = 1.5m$	$a_2 = 3$	$a_3 = 2$

Using 3 moment equation,

$$M_A\left(\frac{L_1}{I_1}\right) + 2M_B\left(\frac{L_1}{I_1} + \frac{L_2}{I_2}\right) + M_C\left(\frac{L_2}{I_2}\right) = -\frac{6A_1a_1}{L_1I_1} - \frac{6A_2a_2}{L_2I_2}$$

or, $-80 \times \frac{3}{I} + 2M_B\left(\frac{3}{I} + \frac{6}{2I}\right) = -\frac{6 \times 90 \times 1.5}{3I} - \frac{6 \times 270 \times 3}{6 \times 2I} - M_C\left(\frac{6}{2I}\right)$

or, $-240 + 12M_B + 3M_C = -675$

$4M_B + M_C = -145$ —— (i)

Also, Again using 3 moment equation (for BCD)

~~Area of BMD~~ $M_B\left(\frac{6}{2I}\right) + 2M_C\left(\frac{6}{2I} + \frac{4}{1.5I}\right) - 80\left(\frac{4}{1.5I}\right) = -\frac{6 \times 270 \times 3}{6 \times 2I} - \frac{6 \times 160 \times 2}{4 \times 1.5I}$

or, $3M_B + \frac{34}{3}M_C - \frac{640}{3} = -725$

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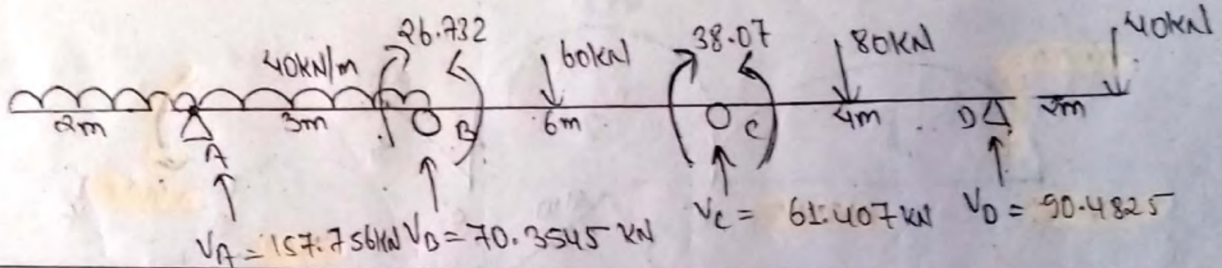
or, $3M_B + \frac{34}{3}M_C = -\frac{1535}{3}$

or $9M_B + 34M_C = -1535$ —— (ii)

Solving eq$^{\underline{n}}$ (i) and (ii) we get

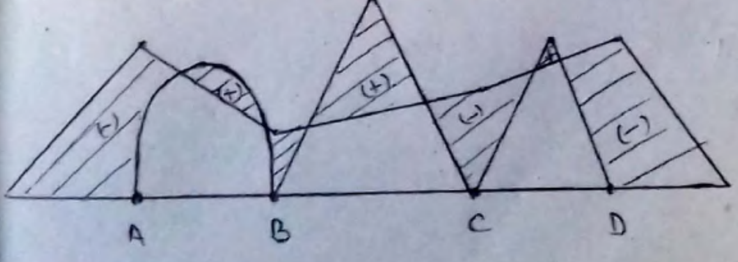
$M_B = -26.732$ KNm (sag)
$= 26.732$ KNm (Hogging)

$M_C = -38.07$ KNm (sag)
$= 38.07$ KN m (Hog)

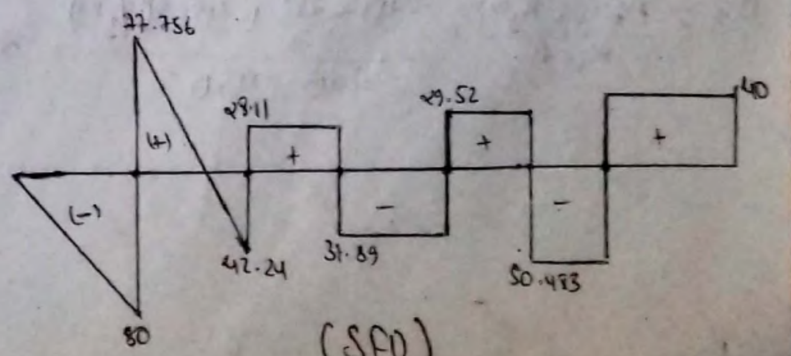


↻+ $\Sigma M_C = 0$ right $-V_D \times 4 + 80 + 80 \times 2 - 38.07 = 0$ $\therefore V_D = 50.4825$ KN (↑)	↻+ $\Sigma M_B = 0$ left $V_A \times 3 - 80 - \cancel{40 \times 3 \times 1.5 + 26.732} = 0$ $\therefore V_A = 77.756$ KN (↑)	↻+ $\Sigma M_B = 0$ ~~right~~ $80 - 50.48 \times 10 + 80 \times 8 + 60 \times 3 - V_C \times 6 - 26.732 = 0$ $\therefore V_C = 61.407$ KN (↑)
↻+ $\Sigma M_C = 0$ (right) $40 \times 6 - V_D \times 4 + 80 \times 2 - 38.07 = 0$ $\therefore V_D = 90.4825$ KN (↑)	↻+ $\Sigma M_B = 0$ (left) $V_A \times 3 + 26.732 - 40 \times 5 \times 2.5 = 0$ $\therefore V_A = 157.756$ KN	↻+ $\Sigma M_B = 0$ (right) $40 \times 12 - 90.4825 \times 10 + 80 \times 8 + 60 \times 3 - V_C \times 6 - 26.732 = 0$ $\therefore V_C = 61.407$ KN (↑)

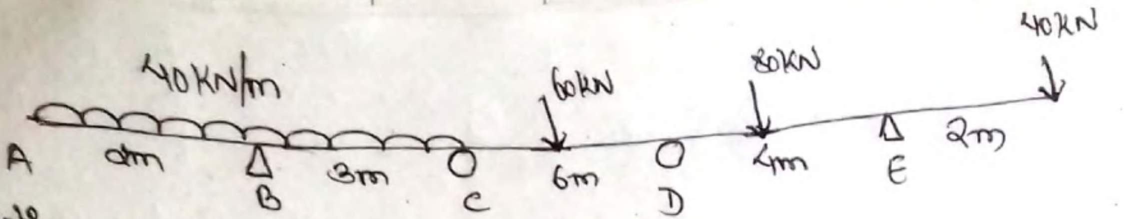
BMD & SFD Diagram



(BMD)

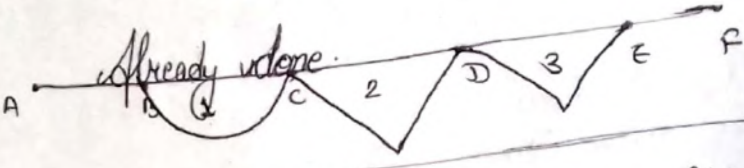


(SFD)

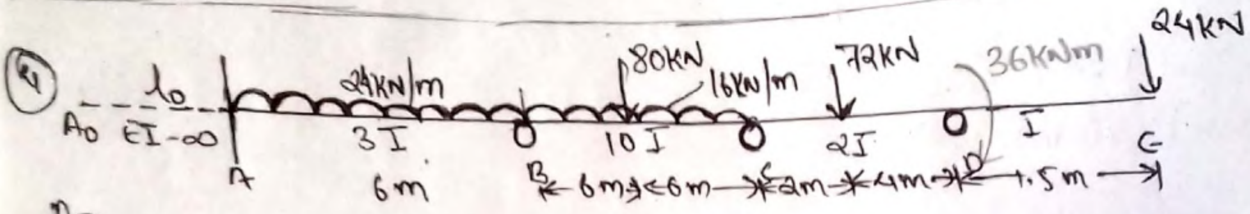


Solution

fixed moment diagram

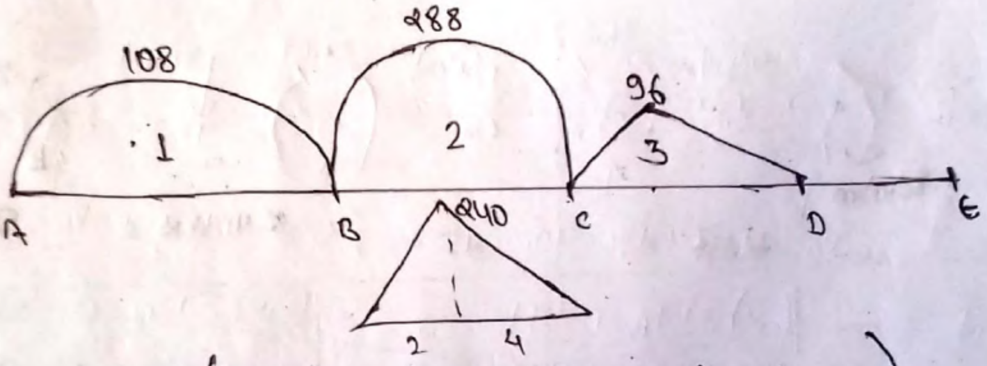


$A_1 =$



Soln fixed

Since, the support A is fixed add a support A_0 left to it.



(free End moment diagram)

$A_1 = 2/3 \times 10.8 * 6 = 432$

$a_1 = 3m$

$A_2 = 2/3 * 288 * 12 = 2340$] UDL

$a_2 = 6$

$A_2 = 1/2 \times 240 \times 12 = 1446$

$a_2 = 6$

$A_3 = 1/2 \times 96 * 6 = 288$

$$a_3 = \frac{A_i x_i + A_j x_j}{A_i + A_j} = \frac{240 * \frac{2 \times 2}{3} + 480 \times (4/3 + 2)}{240 + 480} = 2.67$$

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Applying 3 moment equation at (A_0AB).

$$M_{A0}\left(\frac{l_0}{I_0}\right)+2M_A\left(\frac{l_0}{I_0}+\frac{l_1}{I_1}\right)+M_B\left(\frac{l_1}{I_1}\right)=-\frac{6A_0a_0}{l_0I_0}-\frac{6A_1a_1}{l_1I_1}+\cancel{\frac{6A_2a_2}{l_2I_2}}\ \frac{6Eh_0}{l_0}\ \frac{6Eh_1}{l_1}$$

$$0+2M_A\left(\frac{l_0}{\infty}+\frac{6}{3I}\right)+M_B\left(\frac{6}{3I}\right)=-\frac{6*432\times3}{6\times3I}$$

or, $4M_A+2M_B=-432$ — (i)

Applying 3 moment equation in ABC

$$M_A\left(\frac{6}{3I}\right)+2M_B\left(\frac{6}{3I}+\frac{12}{10I}\right)+M_c\left(\frac{12}{10I}\right)=-\frac{6\times432*3}{6\times3I}-\left(\frac{6\times2340\times6}{10I\times12}+\frac{6\times1446*6}{10I\times12}\right)$$

or, $2M_A+6.4M_B+1.2M_C=\cancel{-1783.8}$ -1567.8 — (ii)

Applying 3moment equation in BCD

$$M_B\left(\frac{12}{10I}\right)+2M_C\left(\frac{12}{10I}+\frac{6}{2I}\right)\cancel{+}\ 3\frac{6\times1.5}{5}-\left(\frac{6*2340*6}{10I*12}+\frac{6\times1446*6}{10I*12}\right)-\frac{6\times288\times2.67}{6*2I}$$

$1.2M_B+8.4M_C=-1466.28$ — (iii)

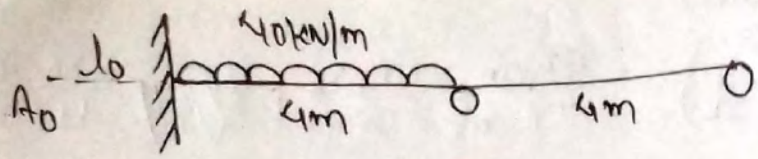
Solving eq^n (i), (ii) & (iii) we get

$\cancel{\boxed{M_A=16 \quad -182.907 \quad -483.415}}$

$M_A=$ 1.239 KNm

$M_B=$ −218.478 KNm

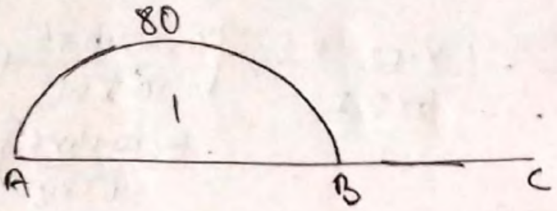
$M_C=$ −143.345 KNm



support B settles by 10mm = 0.01m

E = 15 kN/mm²

I = 0.4×10^{9} mm⁴



$A_1 = 2/3 \times 4\times 80 = 213.33$

$a_1 = 2$

Applying 3moment equation (A_0AB)

$$M_{A0}\left(\frac{l_0}{I_0}\right) + 2M_A\left(\frac{l_0}{I_0}+\frac{l_1}{I_1}\right) + M_B\left(\frac{l_2}{I_1}\right) = -\frac{6A_0a_0}{l_0I_0} - \frac{6A_1a_2}{l_1I_1} + \left(\frac{6E\cancel{h_A}}{l_1}^{0} + \frac{6Eh_B}{l_1}\right)$$

$$\text{or, } 0 + 2M_A\left(0+\frac{4}{I}\right) + M_B\left(\frac{4}{I}\right) = 0 - \frac{6*213.33\times2}{4\times I} - \frac{6*E*(0.01)}{4}$$

$$\text{or, } 8M_A + 4M_B = 0.4\times10^{+9}\left[\frac{-639.99}{0.4\times10^{+9}} - \frac{6*15\times0.01}{4}\right]$$

$$\text{or } 8M_A + 4M_B = -639.99 \quad \text{— (i)}$$

Applying 3moment equation at (ABC)

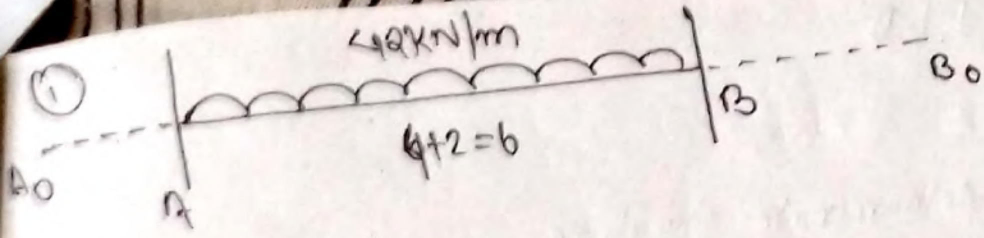
$$M_A\left(\frac{4}{I}\right) + 2M_B\left(\frac{4}{I}+\frac{4}{I}\right) + \cancel{2M_C\left(\frac{4}{I}\right)}^{0} = -\frac{639.99}{0.4\times10^{9}} + \frac{6\times15\times(0.01)}{4}$$

$$4M_A + 16M_B = -639.99 \quad \text{— (ii)}$$

$M_A = -68.57$ kNm

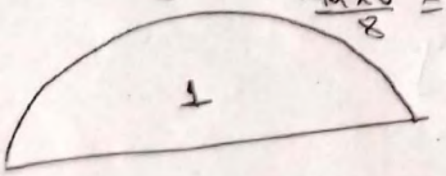
$M_B = -22.856$ kNm

(1)



Free end moment diagram

$\frac{42\times6^2}{8} = 189$



$A_2 = 2/3 * 189 * 6 = 756$

$a_2 = 3$

Applying 3 moment equation at A_0AB

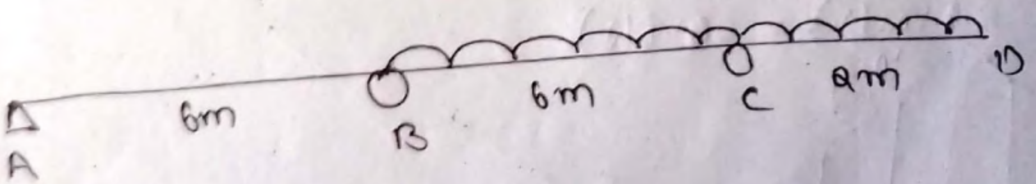
$$M_{A0}\left(\frac{L_1}{I_1}\right) + M_A\left(\frac{L_1}{I_1}+\frac{L_2}{I_2}\right) + M_B\left(\frac{L_2}{I_2}\right) = -\frac{6A_1a_1}{l_1I_1} - \frac{6A_2a_2}{l_2I_2}$$

$$0 + M_A\left(0+\frac{6}{I}\right) + M_B(0) = -\frac{\cancel{6}*756\times3}{\cancel{6}*I}$$

or, $6M_A = -2268$

$M_A = -378\ kNm$

(2)



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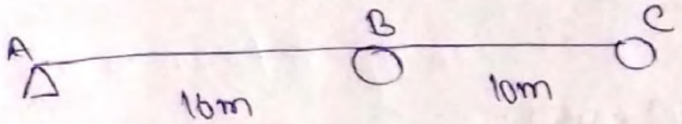
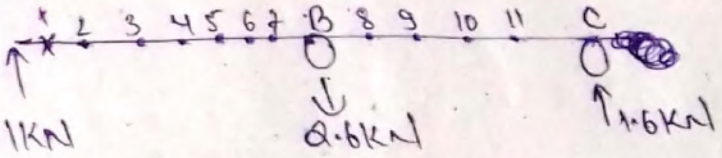
ILD

Draw ILD for R_A [use am section.



Solution,



$\Sigma M_C = 0$ (+)

$1 * 26 + R_B * 10 = 0$

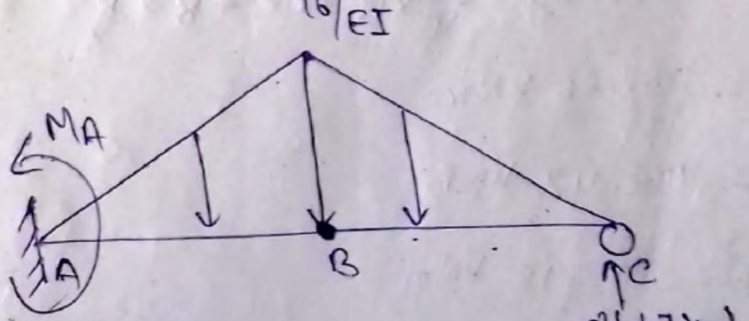
or $R_B = -\frac{26}{10} = -2.6$ KN

$\Sigma F_y = 0$ (↑+)

$1 + R_B + R_C = 0$

$\therefore R_C = 2.6 - 1 = 1.6$ KN

Conjugate Beam



$\Sigma M_B = 0$ (right) (+)

$R_C * 10 + \frac{1}{2} \times 16 * 10 * \frac{10}{3} = 0$

$\therefore R_C = \frac{26.67}{EJ}$ KN

$\Sigma F_y = 0$ (↑+)

$R_A + R_C = \frac{1}{2} * 16 * 26$

$R_A = \frac{181.63}{EJ}$ KN

$\Sigma M_A = 0$ (+)

$-M_A - 26.67 * 26 + 0.5 * 16 \times 16 * \frac{16 \times 2}{3} + 0.5 * 16 \times 10 * \left(\frac{10}{3} + 16\right) = 0$

$M_A = \frac{2218.58}{EJ}$ KNm

$M_A = 2218.58\,kNm$

$M_1 = -2218.58 + 181.63 * 2 - \frac{16}{16} * 2 \times \frac{1}{2} * 2 \times \frac{2}{3}$

$= -1853.98\,kN$

$M_2 = -2218.58 + 181.67 \times 4 - \frac{16}{16} * 4 * \frac{1}{2} * 4 * \frac{4}{3}$

$= -1502.727$

$M_3 = -1164.8\,kNm$

$M_4 = -880.873\,kNm$

$M_5 = -568.946\,kNm$

$M_6 = -327.02\,kNm$

$M_7 = -133.093\,kNm$

For portion BC, taking moment from right side.

$M_8 = 26.67 * 8 - \frac{16}{10} \times 8 * 0.5 * 8 * \frac{8}{3}$

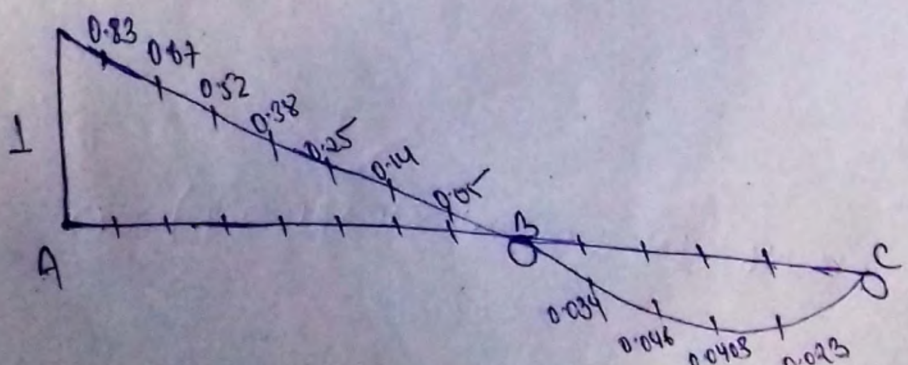
$= 76.826\,kNm$

$M_9 = 102.42\,kNm$

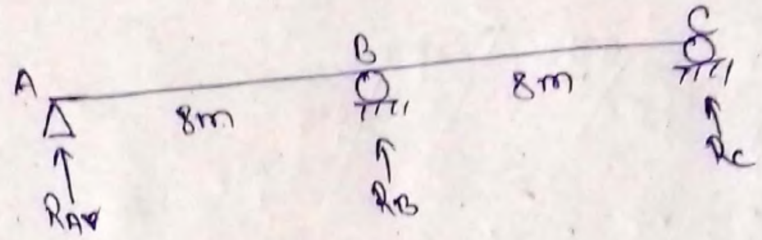
$M_{10} = 89.613\,kNm$

$M_{11} = 51.2067\,kNm$

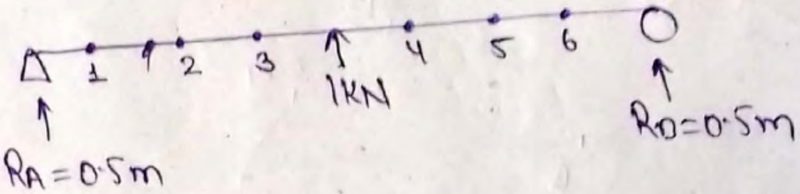
Ordinate of ILD for RA can be obtained by divided with $M_A = -2218.58$



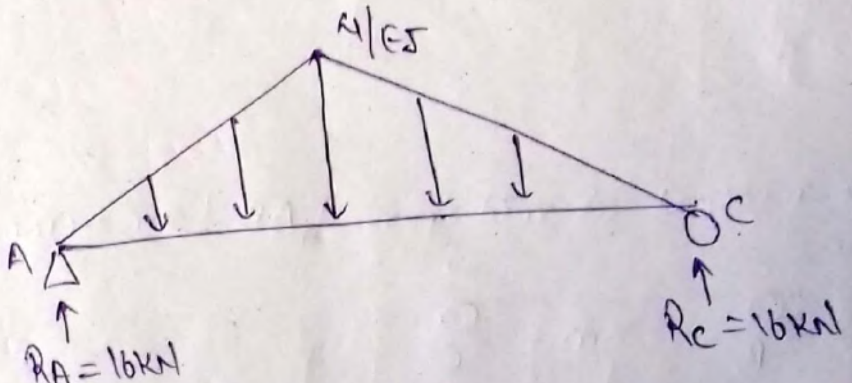
Draw ILD for RBV (use section 8m)



Solution



Conjugate Beam



$\Sigma M_A = 0$ (+)

$-R_c * 16 + 0.5 * 4 * 16 * \frac{16}{2} = 0$

$R_c = 16KN$

$\therefore R_A = 16 KN$

$M_A = 0$

$M_1 = 16 * 2 - \frac{4}{8} * 2 * \frac{1}{2} * 2 * \frac{2}{3} = \cancel{31.467 KNm}$ 31.33 KNm

$M_2 = \cancel{59.733 KNm}$ 58.66 KNm

$M_3 = \cancel{81.6 KNm}$ 78 KNm

$M_B = \cancel{93.86 KNm}$ 85.33 KNm

Now, taken moment from right side.

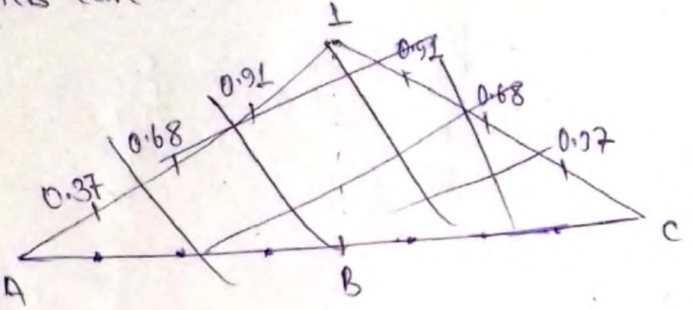
$M_4 = 16 \times 6 - \frac{4}{8} * 6 * 0.5 * 6 * 6 * \frac{1}{3} = \cancel{87.2 KNm}$ 78. KNm

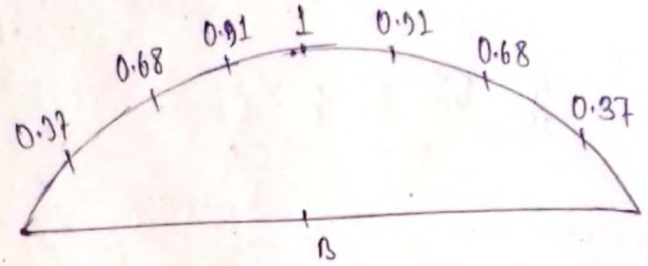
$M_5 = \cancel{55.467 KNm}$ 58.66KNm.

$M_6 = \cancel{30.937 KNm}$ 31.33 KNm

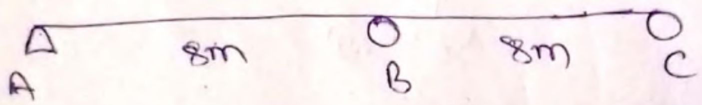
$M_C = 0$

Ordinate of ILD for R_B can be calculated by dividing by $M_B = 85.38$ kNm

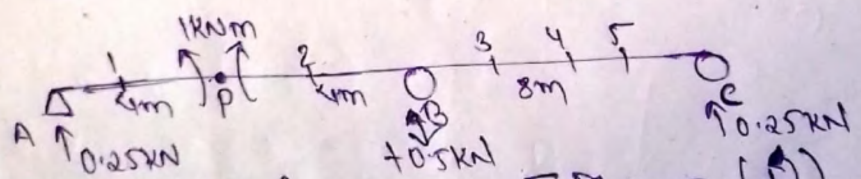




③ Draw ILD for moment at mid span of AB (2m section)



Solution



$\Sigma M_P = 0$ (left)

$R_A * 4 - 1 = 0$

$R_A = 0.25$ KN

$\Sigma M_C = 0$

$R_A * 16 + R_B * 8 + 0 = 0$

Or, $R_B = \frac{-0.25 \times 16}{8}$

$= +0.5 (\downarrow)$

$\Sigma f_y = 0$

$R_A + R_C + R_C = 0$

$R_C = 0.5 - 0.25$

$= 0.25$ KN

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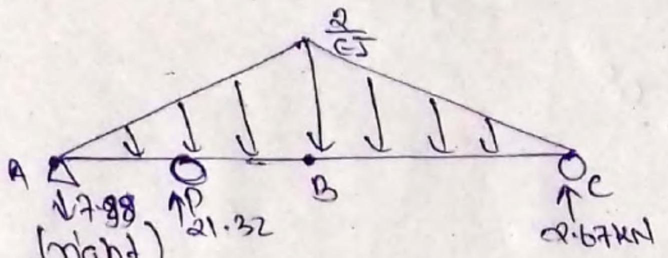
Conjugate beam



$\Sigma M_B = 0$ (right)

$R_C * 8 - 0.5 * 8 * 2 * \frac{8}{3} = 0$

$\therefore R_C = \frac{2.67}{EJ}$ KN

$\Sigma M_A = 0$ (fn)

$-R_C * 16 - R_P * 4 + 0.5 * 2 \times 8 * \frac{8 \times 2}{3} + 0.5 * 2 * 8 (8 + 8/3) = 0$

or, $-2.67 * 16 - 4R_P + 42.67 + 85.33 = 0$

or, $R_P = 21.32$ KN

$\therefore R_A = -7.99$ KN
$= 7.99$ KN (↓)

Calculation of moment

$M_A = 0$

$M_1 = -7.99 * 2 - \frac{2}{8} * 2 * 0.5 * 2 \times \frac{2}{3} = -16.31$ KNm

$M_P = -34.62$

$M_2 = -56.94 + 21.32 * 2 = -14.3$ KNm

$M_4 = M_B = 0$

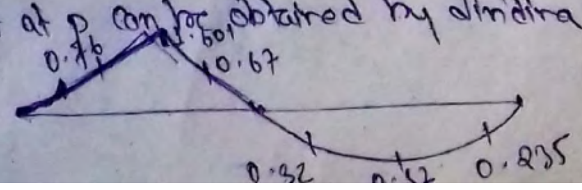
Moment about right

$M_3 = 2.67 * 6 - \frac{2}{8} * 6 * 0.5 \times 6 * 6/3 = 7.02$ KNm (↓)

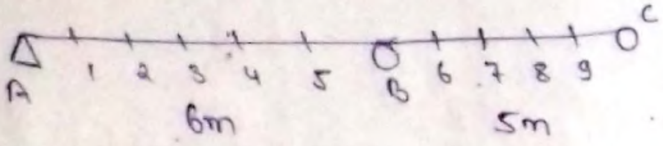
$M_4 = 8.013$ KNm (↓)

$M_5 = 5.01$ KNm (↓)

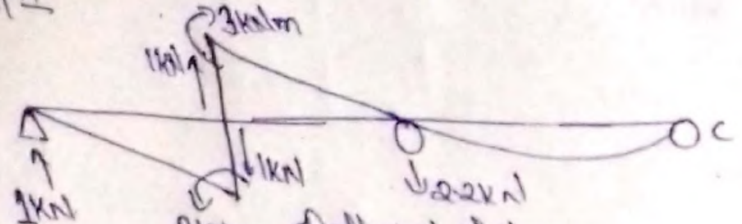
Ordinate of ILD for moment at P can be obtained by dividing by value shear force at P = 21.32



ILD for shear force at 3



Sol^n



Deflected shape.

$R_A = 1kN$

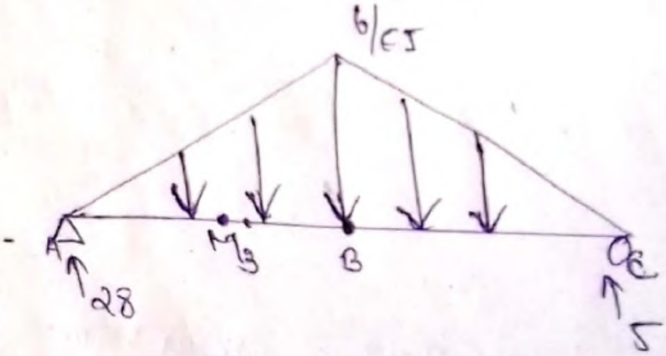
$\Sigma M_c = 0$

$1*11 + R_B \times 5 = 0$

$R_B = -11/5 = -2.2KN$

$R_c = 1.2kN$

Conjugate beam



$\Sigma M_B = 0$ (right)

$R_c * 5 - 0.5 * 6 \times 5 * \frac{5}{3} = 0$

$R_c = \frac{5}{EJ} kN$

$\Sigma F_y = 0$

$R_A = \frac{1}{2} \times 6 * 11 - 5 = 33 - 5 = \frac{28}{EJ}$

Considering portion AB from moment equilibrium condition, we get

$\left(\frac{28}{EJ} * 6\right) - M_3' - \left(\frac{1}{2} * 6 * \frac{6}{EJ} * 2\right) = 0$

$\therefore M_3 = \frac{132}{EJ}$

Calculation of moment

$M_1 = 28 \times 1 - 0.5 * 1 * 1 * 1/3 = 27.833$

$M_2 = 28 \times 2 - 0.5 * 2 * 2 \times 2/3 = 54.667$

$M_{3L} = 28 \times 3 - 1/2 * 3 * 3 * 3/3 = 79.5$

$M_{3R} = 79.5 - 132 = -52.5$

$M_4 = 28 * 4 - 1/2 * 4 * 4 \times 4/3 - 132 = \frac{-30.67}{EJ}$

$M_5 = 28 \times 5 - 1/2 * 5 \times 5 * 5/3 - 132 = -12.83$

Calculation moment from ~~end~~ right

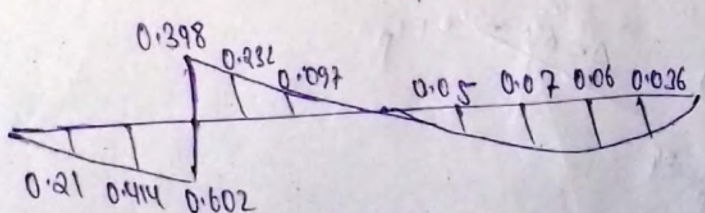
$M_6 = 5 \times 4 - 1/2 * 4 * 4.8 * 4/3 = 7.2$

$M_7 = 5 \times 3 - 1/2 \times 3 * 3.6 * 3/3 = 9.6$

$M_8 = 5 * 2 - 1/2 * 2 * 2.4 * 2/3 = 8.4$

$M_9 = 5 * 1 - 0.5 * 1 \times 1.2 * 1/3 = 4.8$

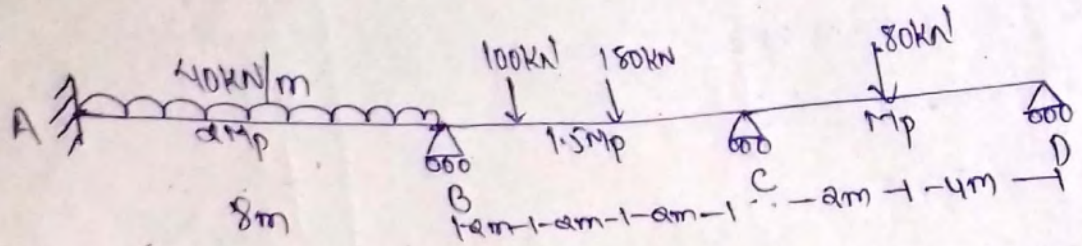
The ordinate of ILD for shear force at B is obtained by dividing by $\frac{132}{EJ}$



ILD

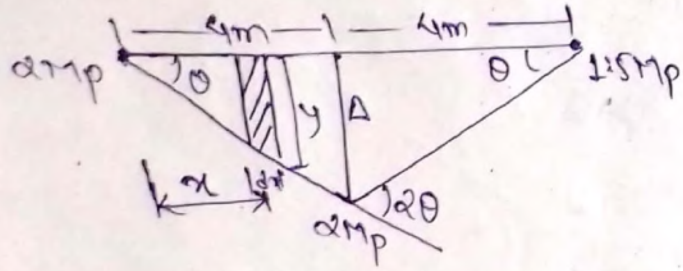
Plastic Method *

1)



Sol^n

1) Beam Mechanism in AB



Equating internal workdone to external workdone, we get

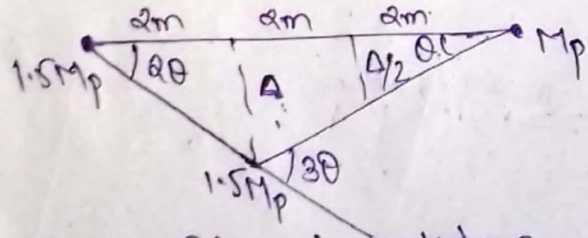
$$2M_p\theta + 2M_p * 2\theta + 1.5M_p\theta = 40 * 8 * \tfrac{1}{2} * 4\theta$$

$$\boxed{\text{or} = 2\int_0^4 wy\,dx = 2w\int_0^4 \theta x\,dx}$$

$$\therefore M_p = 85.33 \text{ kNm}$$

② Beam Mechanism in BC

Case I



$$\theta = \frac{\Delta}{4}$$

Internal workdone = External workdone.

$$1.5M_p 2\theta + M_p\theta + 1.5M_p * 3\theta = 100 * \Delta + 180 * \Delta/2$$

$$8.5\theta M_p = 100 * 4\theta + 180 * 2\theta$$

$$M_p = 82.352 \text{ kNm}$$

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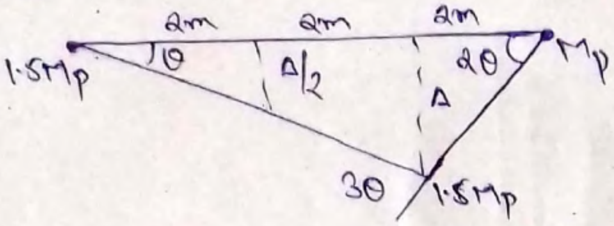
Case II



$2\theta = \frac{\Delta}{2}$

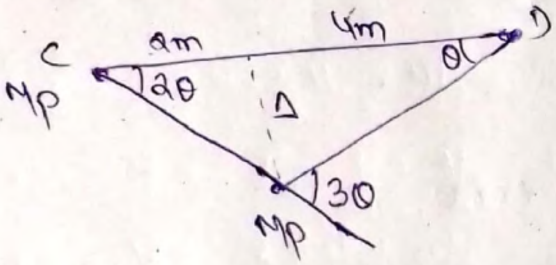
Internal workdone = External workdone

$1.5M_p * \theta + 3\theta * 1.5M_p + 2\theta * M_p = 100 * 2\theta + 150 * 4\theta$

$8M_p\theta = 800$

$M_p = 100$ KNm

(4) Beam Mechanism in CD



Internal workdone = External workdone

$2 * M_p\theta + 3\theta M_p = 80 * \Delta$

$5\theta M_p = 80 * 4\theta$

$M_p = 64$ KNm

Hence, mechanism 3 is the real mechanism & required value is $M_p = 100$ KNm

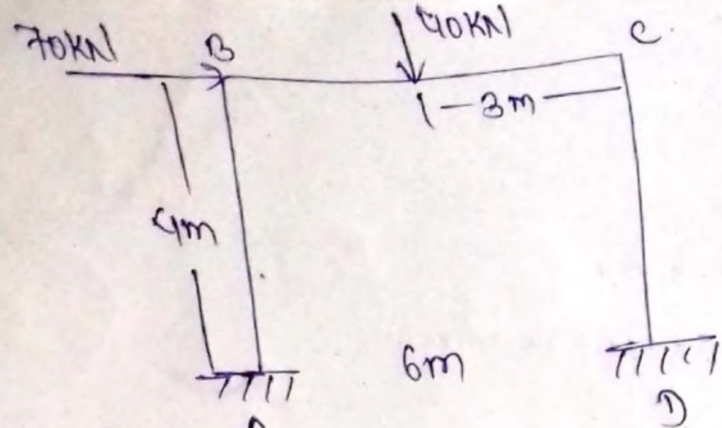
The plastic moment capacity for various spans

$AB = 2M_p = 200$ KNm

$BC = 1.5M_p = 150$ KNm

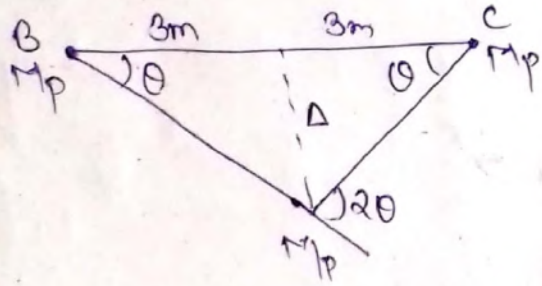
$CD = M_p = 100$ KNm

(11)



① Beam Mechanism

For BC:



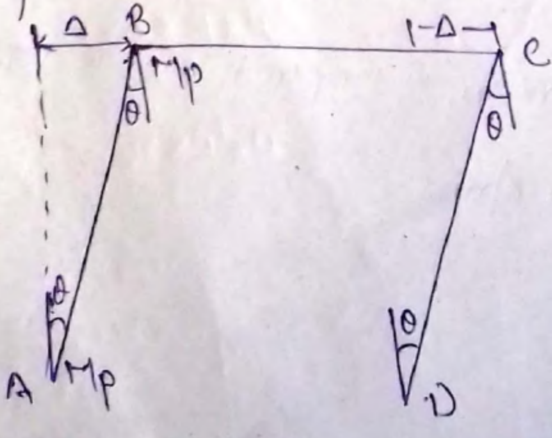
Internal workdone = External workdone

$$M_p * \theta + 2\theta * M_p + \theta M_p = 40 * 3\theta$$

$$4 M_p \theta = 120\theta$$

$$M_p = 30 \text{ KNm}$$

② Sway Mechanism



internal workdone = external workdone

$$\cancel{M_p\theta} M_p\theta + M_p\theta + M_p\theta = 70 * \Delta$$

$$4 M_p \theta = 70 * 4\theta$$

$$M_p = 70 \text{ KNm}$$

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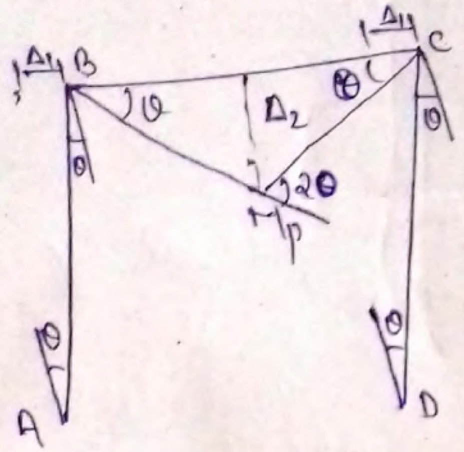
(3) Combine Mechanism



$$M_p\theta + 2\theta M_p + 2\theta M_p + M_p\theta = 70*\Delta_1 + 40*\Delta_2$$

$$6\theta M_p = 70*4\theta + 40*3\theta$$

$$M_p = 66.67 \text{ KN/m}$$

From (1) (2) & (3) we conclude
$M_p = 70$ KN/m, ~~combine~~ sway mechanism is real mechanism

The plastic moment capacity for various span

AB = BC = CD = 70KN/m