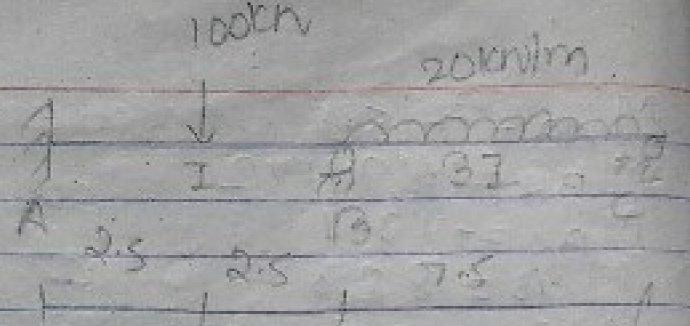
Soln



(i) DKI = 2

(ii) FEM

$FEM_{AB} = -\frac{100\times5}{8} = -62.5$

$FEM_{BA} = \frac{100\times5}{8} = 62.5$

$FEM_{BC} = -\frac{20\times7.5^2}{12} = -93.75$

$FEM_{CB} = 93.75$

(iii) Eq^n.

$M_{AB} = -62.5 + \frac{2EI}{5}(\theta_B) =$

$M_{BA} = 62.5 + \frac{2EI}{5}(2\theta_B) =$

$M_{BC} = -93.75 + \frac{2EI\times3}{7.5}(2\theta_B+\theta_C) =$

$M_{CB} = 93.75 + \frac{2EI\times3}{7.5}(2\theta_C+\theta_B) =$

(iv) Joint Equilibrium

$\Sigma M_B = 0$

$62.5 + 0.8EI\theta_B - 93.75 + 0.53EI2\theta_B + 0.267\theta_C = 0$

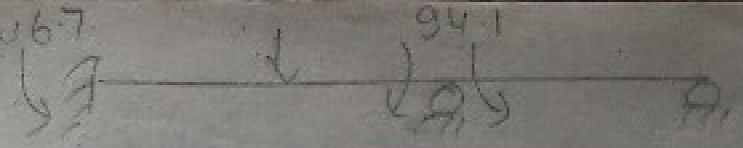
$\therefore 2.39EI\theta_B + \cancel{0.267}\underset{0.801}{}\theta_C = 31.25$

$\Sigma M_C = 0$

$93.75 + \cancel{0.267}\ \cancel{0.53}\overset{1.59}{}EI\theta_C + \cancel{0.267}\overset{0.801}{}EI\theta_B = 0$

Solving,

$\therefore \theta_B = 39.5/EI$



Now, $M_{AB} = -46.7$

$M_{BA} = 94.1$

$M_{BC} = -94.1$

$M_{CB} = 0$

Rxn,

$M_{BA} = -94.1 \quad 46.7$

or, $R_A\times5 - 100\times2.5 = -94.1$ $\quad \therefore R_A = \cancel{31.8}\ 40.52\ kN$

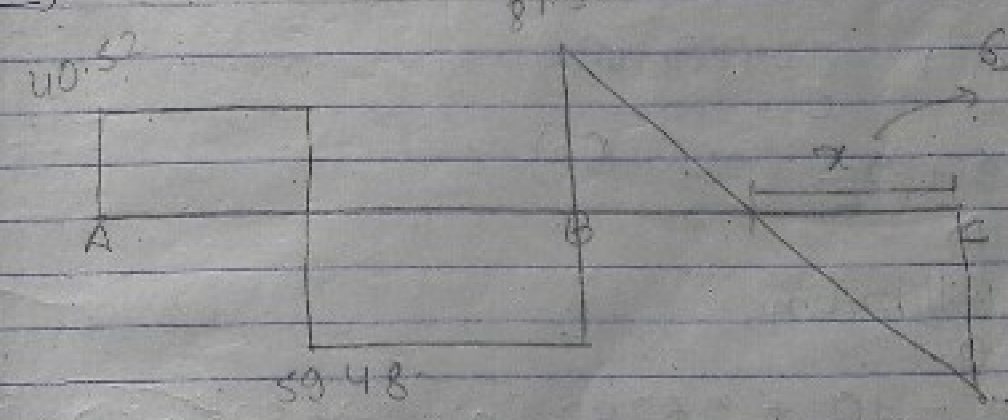
$M_{BC} = -94.1$

or, $R_C\times7.5 - 20\times\frac{7.5^2}{2} = -94.1$

$\therefore R_C = 62.5\ kn$

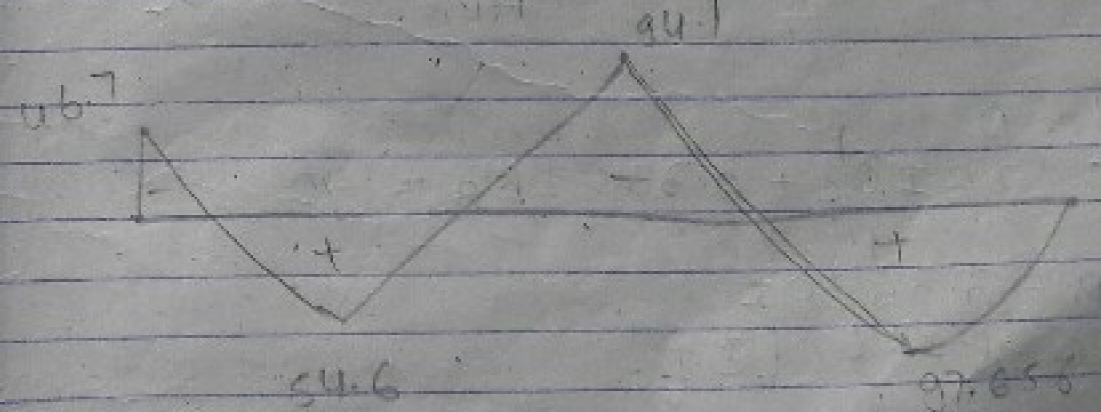
$\therefore R_B = 147.03\ kN$

Now,

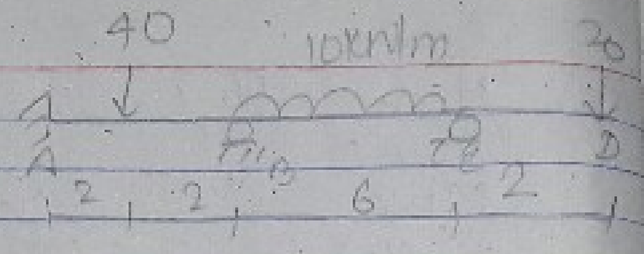


$\frac{62.5}{x} = \frac{87.55}{7.5-x}$

$= 3.12$



Soln



(i) DKI = 2

(ii) FEM

$FEM_{AB} = -\frac{40\times4}{8} = -20$

$FEM_{BA} = \frac{40\times4}{8} = 20$

$FEM_{BC} = -\frac{10\times6^2}{12} = -30$, $FEM_{CB} = 30$

(iii) Eqn

$M_{AB} = -20 + \frac{2EI}{4}\theta_B$

$M_{BA} = 20 + \frac{2EI}{4}\times 2\theta_B$

$M_{BC} = -30 + \frac{2EI}{6}(2\theta_B + \theta_C)$

$M_{CB} = 30 + \frac{2EI}{6}(2\theta_C + \theta_B)$

$M_{CD} = -40$

(iv) Joint Equillibrium

$\Sigma M_B = 0$

$20 + EI\theta_B - 30 + 0.67EI\,2\theta_B + 0.33EI\,2\theta_C = 0$

$\Sigma M_C = 0$

$30 + 0.67EI\theta_C + 0.33EI\theta_B = 40$

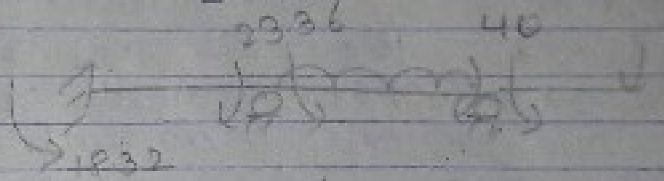
$\therefore \theta_B = 3.36/EI$

$\theta_C = 13.267/EI$

Now, $M_{AB} = \cancel{-23.36}\ -18.32\ \cancel{-16.64}$ $M_{CB} = 40$

$M_{BA} = 23.36$

$M_{BC} = -23.37$



Rxn:

$M_{BA} = -23.36$

$-18.32 + R_A\times4 - 40\times2 = -23.36$

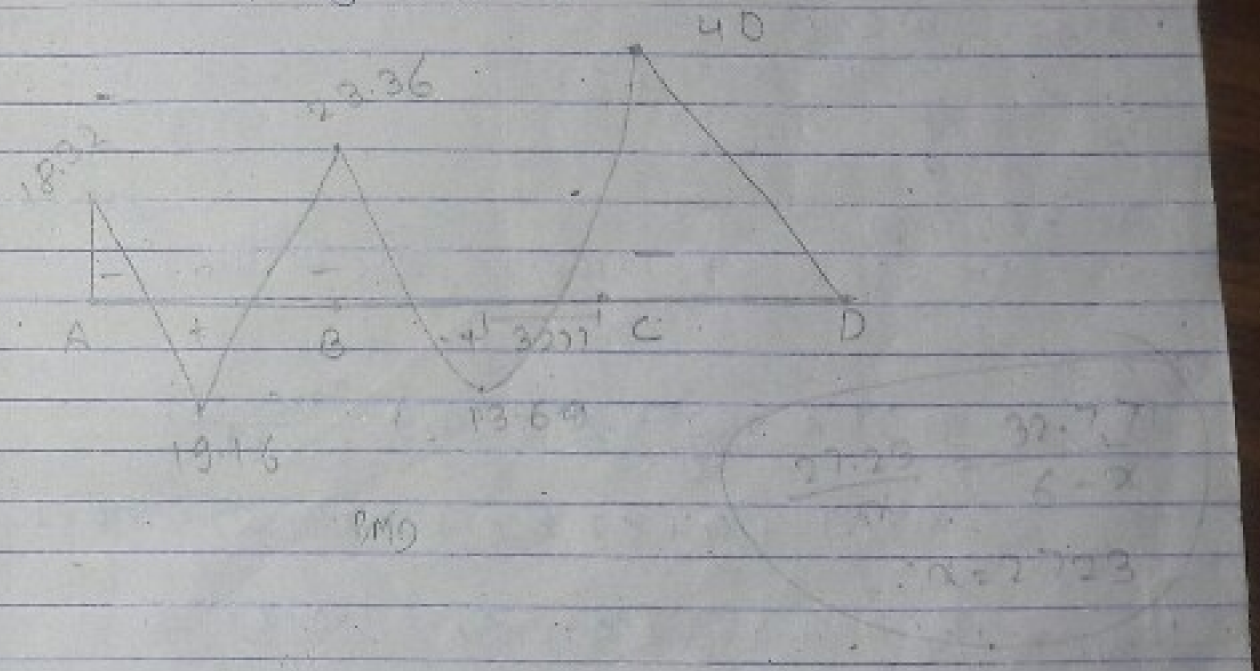
$\therefore R_A = 18.74$

$M_{BC} = -23.36$

or, $R_C\times6 - 20\times8 - 10\times6\times3 = -23.36$

$\therefore R_C = 52.77$

$R_B = 48.49$



A Continous beam ABCD carrying a UDL 5kN/m as shown. Compute rxn & SFD & BMD due to following support settle. Support B is set 0.005m vertically downward. Support C 0.01m vertically downward.

Soln

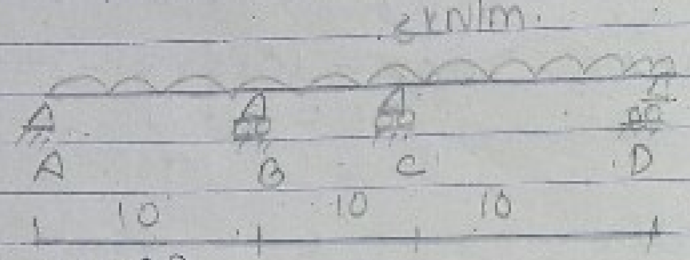
Assume $E = 200$ GPa, $I = 1.35\times10^{-3}\ m^4$

$EI = 27\times10^7 = 27\times10^4\ kNm^2$

Soln

(i) DKI = 4

(ii) FEM



$FEM_{AB} = -\frac{5\times10^2}{12} = -41.67$

$BA = \frac{500}{12} = 41.67$

$BC = -41.67$

$CB = 41.67$, $CD = -41.67$, $DC = 41.67$

(iii) Eqn

$M_{AB} = -41.67 + \frac{2EI}{10}\left(2\theta_A + \theta_B - \frac{3\times0.005}{10}\right)$

$= -41.67 + 0.4EI\theta_A + 0.2\theta_B EI - 3\times10^{-4}EI$

$M_{BA} = 41.67 + \frac{2EI}{10}\left(2\theta_B + \theta_A - \frac{3\times0.005}{10}\right)$

$= 41.67 + 0.4EI\theta_B + 0.2EI\theta_A - 3\times10^{-4}EI$

$M_{BC} = -41.67 + \frac{2\times27\times10^4}{10}\left(2\theta_B + \theta_C - \frac{3\times0.005}{10}\right)$

$= -41.67 + 108000\theta_B + 54000\theta_C - 81$

$M_{CB} = 41.67 + \frac{2\times27\times10^4}{10}\left(2\theta_C + \theta_B - \frac{3\times0.005}{10}\right)$

$= 41.67 + 54000\theta_B + 108000\theta_C - 81$

$M_{CD} = -41.67 + \frac{2\times27\times10^4}{10}\left(2\theta_C + \theta_D + \frac{3\times0.01}{10}\right)$

$= -41.67 + 108000\theta_C + 54000\theta_D + 162$

Then

(iv) Joint Equilibrium

$M_A = 0$

$M_{AB} = 0$

or, $-41.67 + 0.4\times27\times10^4\times\theta_A + 0.2\theta_B\times27\times10^4 - 3\times10^{-4}\times27\times10^4 = 0$

or, $108000\theta_A + 54000\theta_B - 81 - 41.67 = 0$

$\theta_A = \frac{122.67 - 54000\theta_B}{108000}$

$\Sigma M_B = 0$

$M_{BA} + M_{BC} = 0$

or, $41.67 + 54000\theta_A + 108000\theta_B - 81 - 41.67 + 108000\theta_B + 54000\theta_C - 81 = 0$

or, $54000\left(\frac{122.67 - 54000\theta_B}{108000}\right) + 216000\theta_B + 54000\theta_C - 81 = 0$

or, $61.335 - 27000\theta_B + 216000\theta_B + 54000\theta_C - 81 = 0$

$\therefore 189000\theta_B + 54000\theta_C = 19.665$

$\Sigma M_C = 0$

$M_{CB} + M_{CD} = 0$

or, $41.67 + 54000\,\theta_B + 108000\,\theta_C - \overset{-81}{\cancel{81}} - 41.67 +$
$\cancel{108000}\,\theta_C + \overset{54}{\cancel{108}}000\,\theta_D + \overset{+162}{\cancel{81}} = 0$

or, $54000\,\theta_B + \underset{216}{\cancel{162}}000\,\theta_C + \underset{54}{\cancel{108}}000\,\theta_D = \overset{-81}{\cancel{0}}$

$\Sigma M_D = 0$

$M_{DC} = 0$

or, $41.67 + \overset{108000}{\cancel{54000}}\,\theta_D + \overset{54000}{\cancel{108000}}\,\theta_C + \overset{162}{\cancel{81}} = 0$

$\therefore 54000\,\theta_C + 108000\,\theta_D = -\overset{203.67}{\cancel{122.67}}$

By solving

$\theta_A = 8.632 \times 10^{-4}$

$\theta_B = 5.45 \times 10^{-4}$

$\theta_C = -4.56 \times 10^{-5}$

$\theta_D = -1.863 \times 10^{-3}$

Now,
then

$M_{AB} = 0$

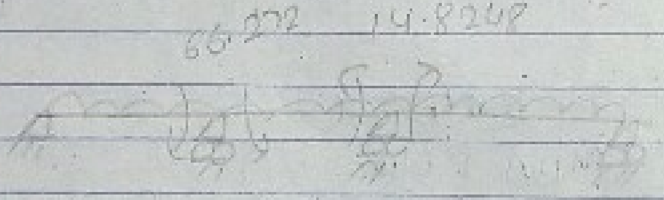
$M_{BA} = 66.1428$

$M_{BC} = -66.272$

$M_{CB} = -14.8248$

$M_{CD} = 14.8032$

$M_{DC} = 0$



Now, $M_{BA} = -66.1428$

or, $R_A \times 10 - 5 \times 10 \times 5 = -66.1428$

$\therefore R_A = 18.385$

$M_{CD} = 14.825$

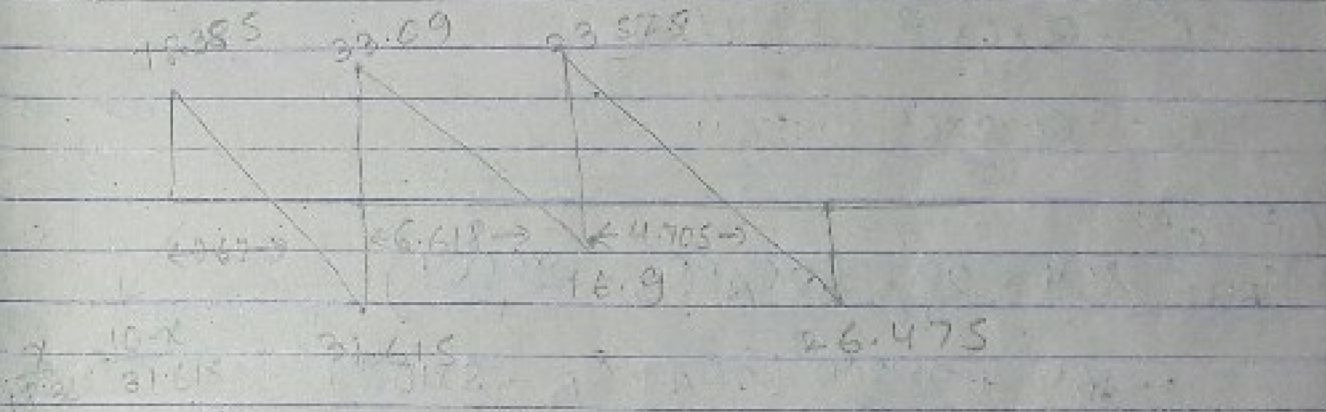
or, $R_D \times 10 - 250 = 14.825$

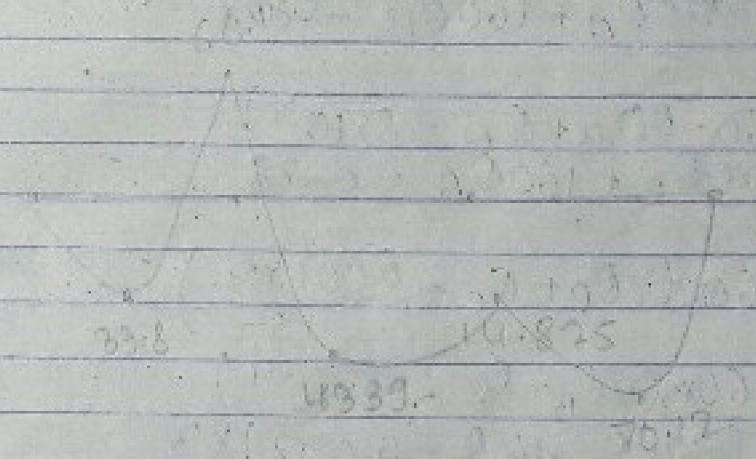
$\therefore R_D = 26.48$

$M_{CB} = 14.825$

or, $18.385 \times 20 - 5 \times 20 \times 10 + R_B \times 10 = 14.825$

$\therefore R_B = 64.7125$, $R_C = \cancel{62.078}\ 40.925$





$\therefore \theta_B = -0.067$ $\theta_A = 0.304$

$\theta_C = 0.079$

$\theta_D = -0.243$

Now

$M_{AB} = 0$

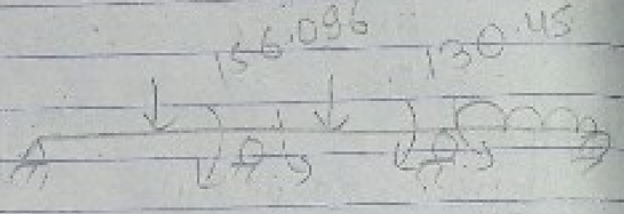
$M_{BA} = 156.672$

$M_{BC} = -156.096$

$M_{CB} = \cancel{151.424}\ 129.984$

$M_{CD} = -130.45$

$M_{DC} = -0.3146 \approx 0$



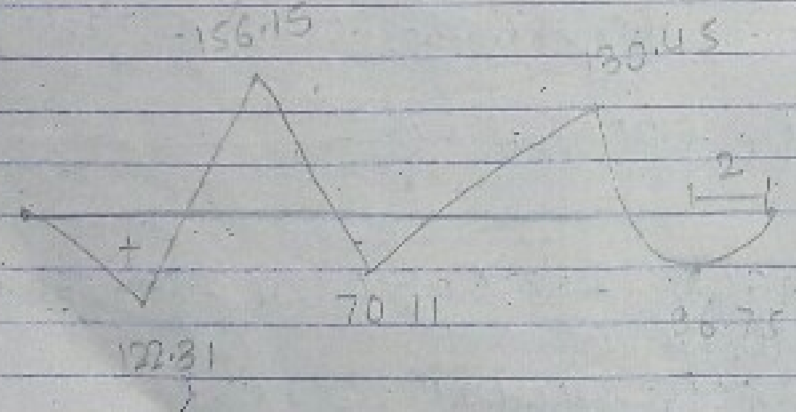
Then

$R_A = 13.59$

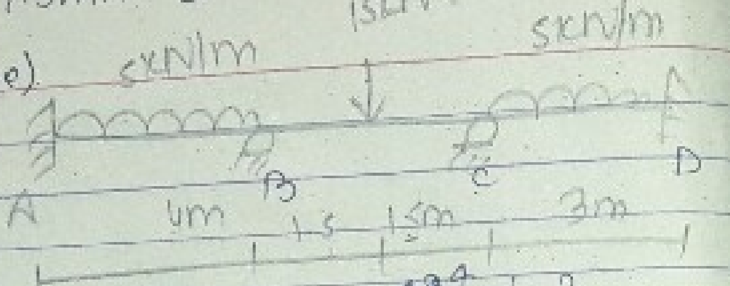
$R_B = \cancel{42.06}\ 84.12$

$R_D = 28.75$

$R_C = \cancel{107.59}\ 65.54$



Draw BMD by slope-deflection method if B & C sink by 7.5mm & A rotate by 5° anticlockwise & D rotate 5° clockwise (+ve)



Soln $EI = 5\times10^5$ MPa

(i) DKI = 2 $I = 3\times10^7\ mm^4$

(ii) Then;

(0.087)

$\therefore EI = 1.5\times10^4\ kNm^2$

$$M_{AB} = -\frac{5\times16}{12} + \frac{2EI}{4}\left(2\theta_A + \theta_B - \frac{0.0075\times3}{4}\right)$$

$= -6.67 + 1.5\times10^4\,\theta_A + 0.75\times10^4\,\theta_B - 42.1875$

$= -1353.8575 + 0.75\times10^4\,\theta_B$

$M_{BA} = 6.67 + 1.5\times10^4\,\theta_B + 0.75\times10^4\,\theta_A - 42.1875$

$= -688.0175 + 1.5\times10^4\,\theta_B$

$$M_{BC} = -\frac{15\times3}{8} + \frac{2EI}{3}(2\theta_B + \theta_C)$$

$= -5.625 + 2\times10^4\,\theta_B + \times10^4\,\theta_C$

=

$M_{CB} = 5.625 + 2\times10^4\,\theta_C + 1\times10^4\,\theta_B$

=

$M_{CD} = -3.75 + 2\times10^4\,\theta_C + \times10^4\,\theta_D + \frac{3\times0.0075}{3}$ +75

$= 2\times10^4\,\theta_C + 941.25$

$M_{DC} = 3.75 + \times10^4\,\theta_D + 1\times10^4\,\theta_C + 75$

$= 10^4\,\theta_C + 1818.75$

Then,

$M_B = 0$

or, $M_{BA} + M_{BC} = 0$

or, $-688.0175 + 1.5\times10^4\,\theta_B - 5.625 + 2\times10^4\,\theta_B + 10^4\,\theta_C = 0$

$\therefore 3.5\times10^4\,\theta_B + 10^4\,\theta_C = 693.64$

$M_C = 0$

$5.625 + 2\times10^4\,\theta_C + 10^4\,\theta_B + 941.25 + 2\times10^4\,\theta_C = 0$

or $4\times10^4\,\theta_C + 10^4\,\theta_B = -946.875$

$\therefore \theta_B = 0.0286$

$\theta_C = -0.0308$

Now,

$M_{AB} = \cancel{1568.3575}\ -1139.357$

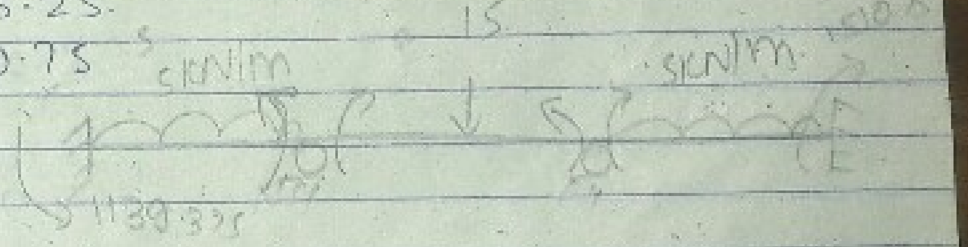
$M_{BA} = 259.0175$

$M_{BC} = \cancel{258.375}\ 258.375$

$M_{CB} = -324.375$

$M_{CD} = 325.25$

$M_{DC} = 1510.75$



Now,

$M_{BA} = 259.0175$

$R_A\times4 - 1139.375 - \frac{5\times4^2}{2} = 259.0175$

$\therefore R_A = 359.598$

$M_{CD} = 325.25$

or, $-1510.75 + R_D \times 3 - \frac{5 \times 3^2}{2} = 325.25$

$\therefore R_D = 619.5$

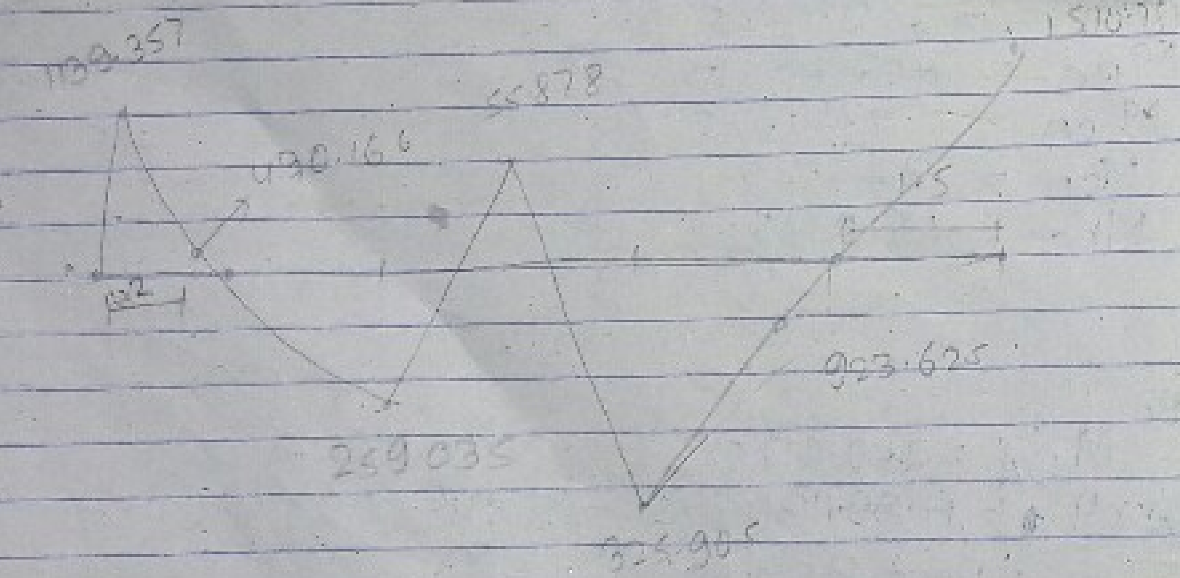
$M_{BC} = 258.375$

or, $-1510.75 + 619.5 \times 6 - 5 \times 3 \times 4.5 - 15 \times 1.5$
$+ R_{CB} = 258.375$

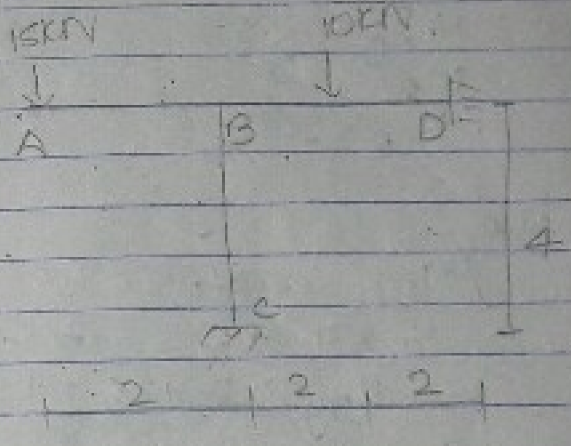
$\therefore R_C = -619.29$

$\therefore R_B = -309.808$

Now,



Q. Analyse the rigid frame shown in fig. Assume EI to be constant for all member. Draw BMD & elastic curve.



Soln

(i) DKI = 1

(ii) FEM

$FEM_{BD} = -\frac{10 \times 4}{8} = -5$

$DB = \frac{10 \times 4}{8} = 5$

$BC = 0$

$CB = 0$

(iii) Eqn

$M_{BD} = -5 + \frac{2EI}{4}(2\theta_B + \theta)$

$M_{BA} = 5 \times 2 = 10$

$M_{DB} = 5 + \frac{2EI}{4}(\theta_B)$

$M_{BC} = \frac{2EI}{4}(2\theta_B)$

$M_{CB} = \frac{2EI}{4}(\theta_B)$

Applying joint Equilibrium

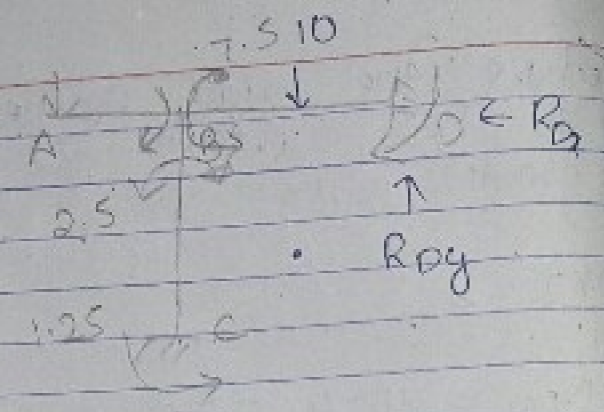
$M_B = 0$

$M_{BA} + M_{BC} + M_{BD} = 0$

or, $10 + \theta_B EI + -5 + EI\theta_B = 0$

$\therefore \theta_B = \frac{-5}{2EI}$

$\frac{wl^2}{12}$



$\therefore M_{BA} = 10$

$M_{BD} = -7.5$

$M_{DB} = 3.75$

$M_{BC} = -2.5$

$M_{CB} = -1.25$

Now,

$M_{BD} = 7.5$

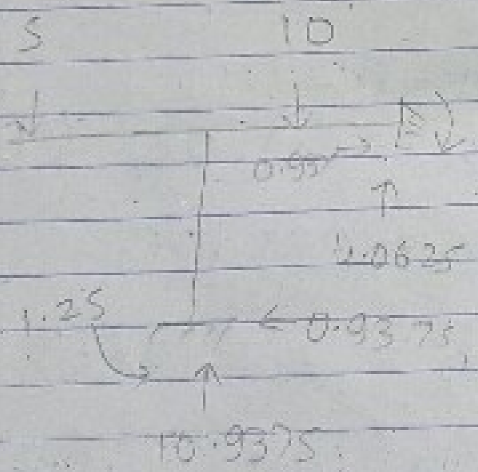
or, $R_{Dy} \times 4 + 2.5 = 7.5$

$R_{Dy} = \cancel{0.3125}\ -1.56$

$M_{BC} = 2.5$

or, $R_{Cx} \times 4 - 1.25 = 2.5$

$\therefore R_{Cx} = 0.94$



$M_{BR} = -7.5$

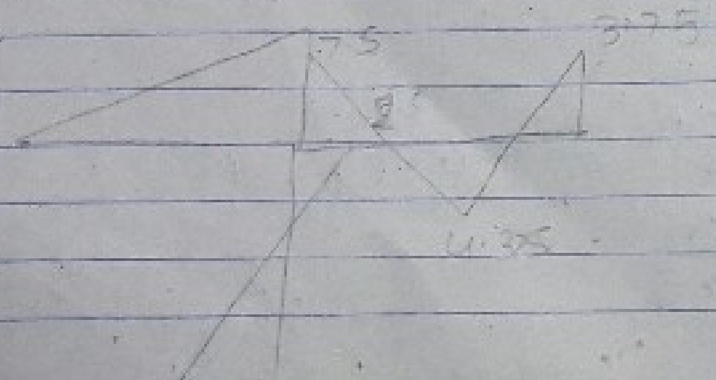
$-3.75 + R_D \times 4 - 10 \times 2 = -7.5$

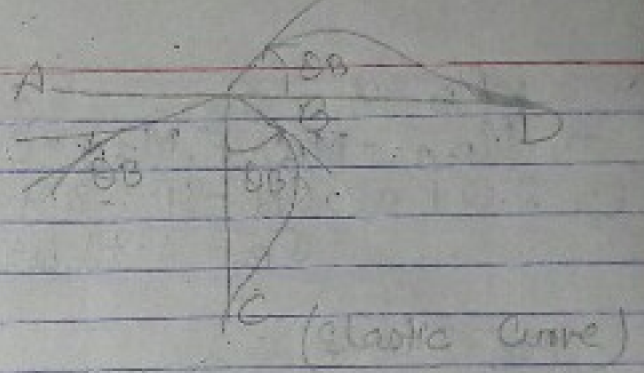
$R_D = 4.0625$

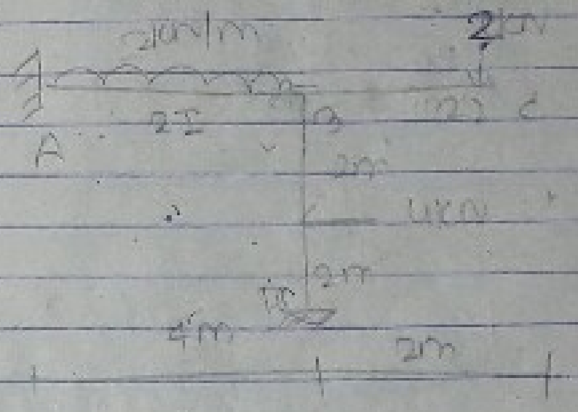
$\Sigma M_R = 2.5$

$-1.25 + H_C \times 4 = 2.5$

$\therefore H_C = 0.9375.$







(i) DKI = 1.

(ii) FEM

$AB = -\frac{2 \times 4^2}{12} = -\frac{16}{6}$

$BA = 2.67$

$BD = -\frac{4 \times 4}{8} = -2$

$DB = 2$

Eq^n

$M_{AB} = -2.67 + \frac{4EI}{4}(\theta_B) = -2.67 + \theta_B \times EI$

$M_{BA} = 2.67 + 2EI\theta_B$

$M_{BC} = -1 \times 2 \times 2 = -4$

$M_{BD} = -2 + \frac{2EI}{4}(2\theta_B) = -2 + EI\theta_B$

$M_{DB} = 2 + 0.5EI\theta_B$

Then

Joint equilibrium

$\therefore \Sigma M_B = 0$

or, $M_{BA} + M_{BC} + M_{BD} = 0$

or, $2.61 + 2EI\theta_B - 4 - 2 + EI\theta_B = 0$

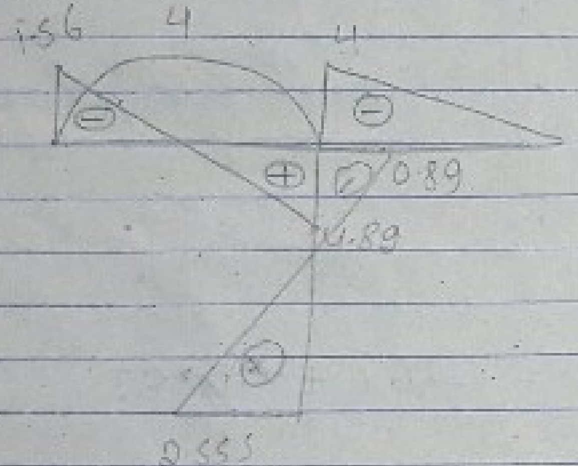
$\therefore \theta_B = 1.11/EI$

$\therefore M_{AB} = -1.56$

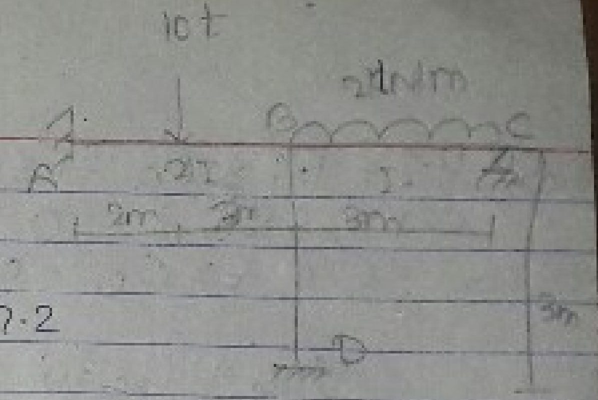
$M_{BA} = 4.89$

$M_{BD} = -0.89$

$M_{DB} = 2.555$



Soln



(i) DKI = 2 $(\theta_B \ \& \ \theta_C)$

(ii) FEM

$AB = -\dfrac{10 \times 2 \times 9}{5^2} = -7.2$

$BA = \dfrac{10 \times 3 \times 4}{5^2} = 4.8$

$BC = -\dfrac{2 \times 9}{12} = -1.5$

$CB = \dfrac{18}{12} = 1.5$

(iii) Eqn

$M_{AB} = -7.2 + \dfrac{4EI}{5}(\theta_B)$

$= -7.2 + \theta_B EI \times 0.8$

$M_{BA} = 4.8 + \dfrac{4EI}{5}(2\theta_B)$

$= 4.8 + 1.6\,\theta_B \cdot EI$

$M_{BC} = -1.5 + \dfrac{2EI}{3}(2\theta_B + \theta_C)$

$= -1.5 + 1.33\,\theta_B EI + 0.67\,\theta_C EI$

$M_{CB} = 1.5 + 0.67\,\theta_B EI + 1.33\,\theta_C \times EI$

$M_{BD} = \dfrac{2EI}{3}(2\theta_B) = 1.33\,EI\theta_B$

$M_{DB} = \dfrac{2EI}{3}(\theta_B) = 0.67\,\theta_B EI$

Joint Equillibrium

$\Sigma M_B = 0$

$M_{BA} + M_{BC} + M_{BD} = 0$

Analyse rigid frame

Soln

① DKI = 3

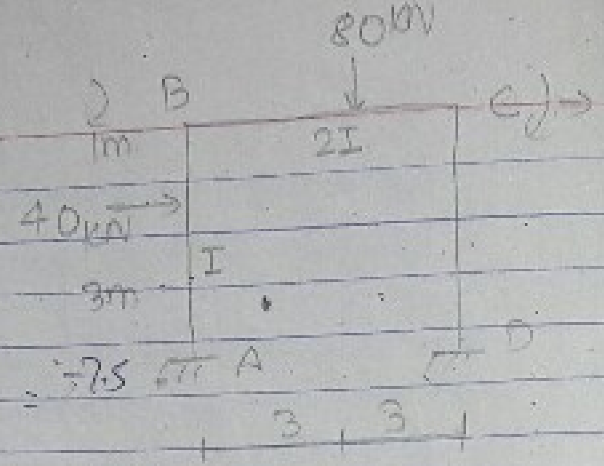
② FEM

$AB = -\frac{40\times3\times1^2}{4^2} = \frac{-30}{4} = -7.5$

$BA = \frac{40\times1\times3^2}{4^2} = \frac{90}{4} = 22.5$

$BC = \frac{-80\times6}{8} = -60$

$CB = 60$



③ Eqn.

$M_{AB} = -7.5 + \frac{2EI}{4}\left(\theta_B - \frac{3\delta}{4}\right)$

$= -7.5 + 0.5EI\theta_B - 0.375EI\delta$

$M_{BA} = 22.5 + 0.5EI\left(2\theta_B - \frac{3\delta}{4}\right)$

$= 22.5 + \theta_B EI - 0.375EI\delta$

$M_{BC} = -60 + \frac{4EI}{6}(2\theta_B + \theta_C)$

$= -60 + 1.33EI\theta_B + 0.67EI\theta_C$

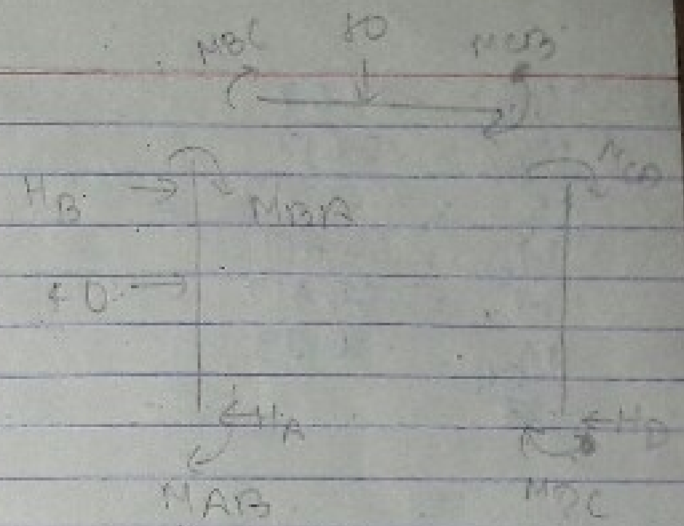
$M_{CB} = 60 + 0.67EI\theta_B + 1.33EI\theta_C$

$M_{CD} = \frac{2EI}{4}\left(2\theta_C - \frac{3\delta}{4}\right) = \theta_C EI - 0.375EI\delta$

$M_{DC} = 0.5\theta_C EI - 0.375EI\delta$

Using shear eqn.

$\Sigma M_B = 0$



$M_{AB} + H_A\times4 - 40\times1 + M_{BA} = 0$

$\therefore 4H_A = 40 - M_{BA} - M_{BA}$ —①

$M_C = 0$

$M_{DC} + H_D\times4 + M_{CD} = 0$

$\therefore 4H_D = -(M_{CD} + M_{DC})$

Then $4(H_A + H_D) = 40 - (M_{AB} + M_{BA} + M_{CD} + M_{DC})$

$\therefore M_{AB} + M_{BA} + M_{CD} + M_{DC} = -120$

or, $-7.5 + 1.5\theta_B EI - 2\times0.375EI\delta + 22.5 + 1.5\theta_C EI = -120$

$\therefore 1.5EI\theta_B + 1.5EI\theta_C - 1.5EI\delta = -135$ —①

Then

From joint Equillibrium:

$M_B = 0$

$M_{BA} + M_{BC} = 0$

or, $22.5 + 2.33EI\theta_B + 0.67EI\theta_C - 60 - 0.375EI\delta = 0$

$\therefore 2.33EI\theta_B + 0.67EI\theta_C - 0.375EI\delta = 37.5$

$M_C = 0$

$M_{CD} + M_{CB} = 0$

or, $60 + 2.33EI\theta_C + 0.67EI\theta_B - 0.375EI\delta = 0$

$\therefore \theta_B = \frac{39.367}{EI}$, $\delta = \frac{110}{EI}$

$\theta_C = -\frac{19.367}{EI}$

$\therefore M_{AB} = -29.07$

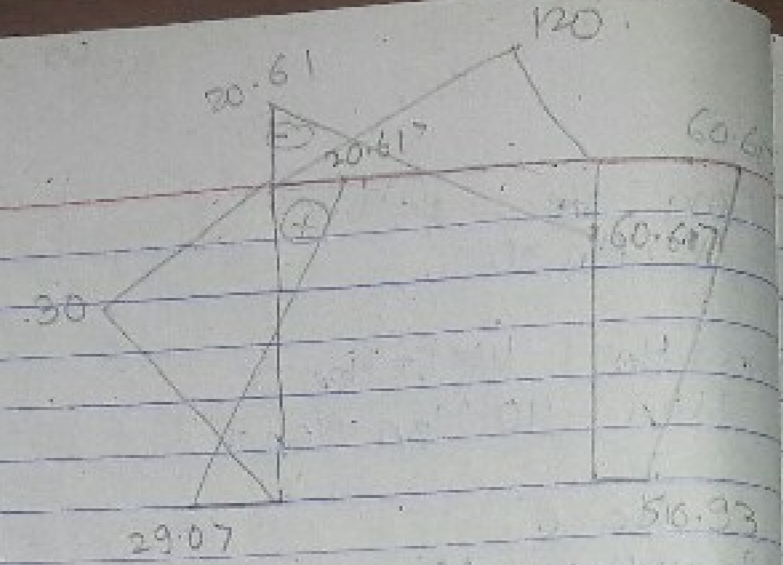
$M_{BA} = 20.617$

$M_{BC} = -20.617$

$M_{CB} = 60.617$

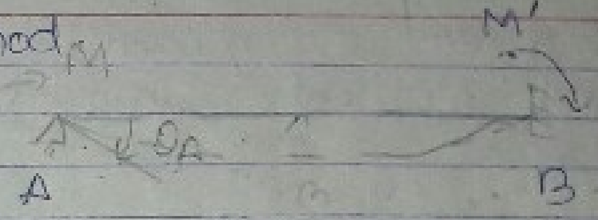
$M_{CD} = -60.617$

$M_{DC} = -50.93$



Moment Distribution Method

(clockwise +ve)



Stiffness

Carryover moment

Carryover factor

Distribution faction

when

- Stiffness : moment require to rotate one end by unit rotation when rotation is permitted at that end is called stiffness of that member.

Let θ_A be rotation at joint A due to application of moment M at joint A, and moment M' is produced at B due to effect of moment M at A, then

Stiffness of member AB

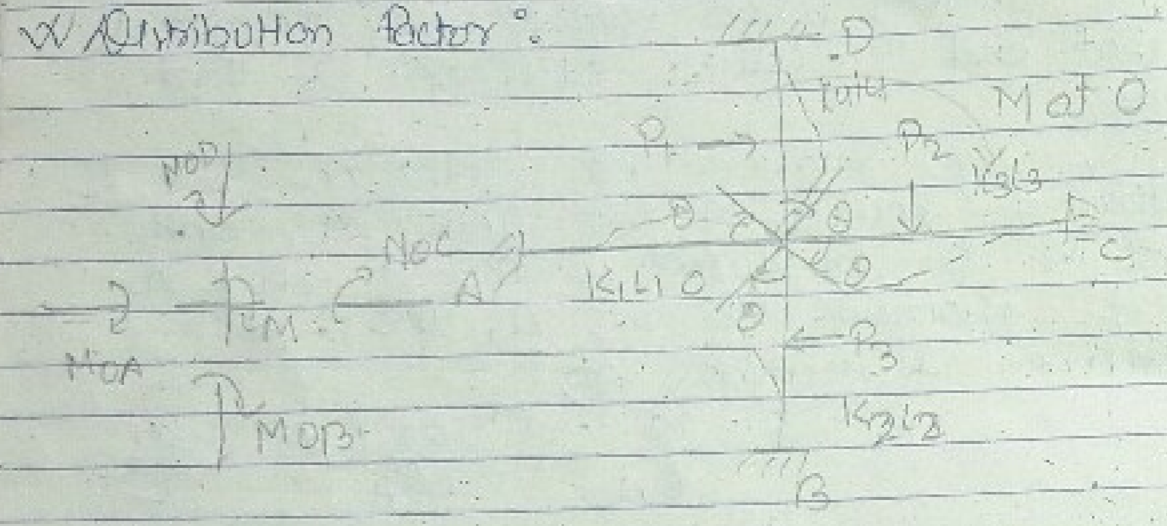
$$= K = \frac{M}{\theta_A} = \frac{4EI\,\theta_A}{L\theta_A} = \frac{4EI}{L}$$

$$M' = \frac{2EI}{L}$$

- Carryover moment : If a moment is applied at one end of member allowing rotation of that end & fixing another far end some moment develop at far end of structure due to application of moment at joint this moment at far end joint is called carryover moment. Here, M be moment applied at A when allowing rotation and M' is moment developed at far end moment. This M' is called carryover moment.

Carry over factor:
It is ratio of carry over moment to applied moment is called carryover factor $= \frac{M'}{M} = \frac{1}{2}$

Distribution factor:



$$DF \text{ for } OA = \frac{M_{OA}}{M}$$

When a moment M is applied to rigid joint where no. of member are meeting, the applied moment is shared by members meeting at joint. The ratio betn share moment by a member to applied member at joint is called D.F of that member.

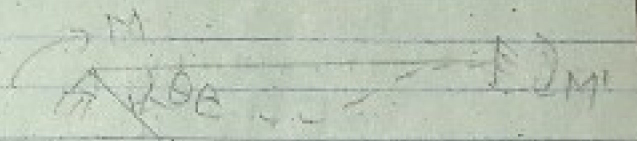
Let M_{OA} be the shared moment by mem. OA & M is the applied moment at O

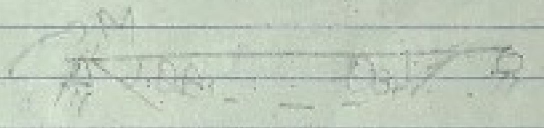
Then

$$D.F \text{ of } OA = \frac{M_{OA}}{M}$$

Derivation of stiffness, carry over moment, carry over factor & distribution factor.

Let us consider a beam AB, then by using superposition method by using M & M' in different then,

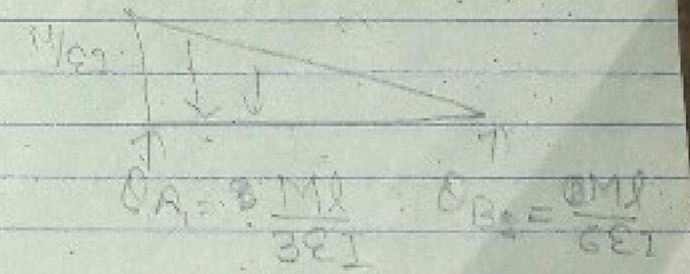




$\theta_B = \theta_{B_2} - \theta_{B_1} = 0$

or, $\frac{M'L}{3EI} - \frac{M'L}{6EI} = 0$

$\frac{M'}{M} = \frac{1}{2}$



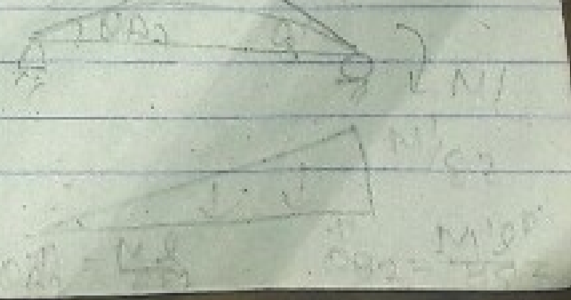
Again

$\theta_A = \theta_{A_1} - \theta_{A_2}$

$= \frac{ML}{3EI} - \frac{M'L}{6EI}$

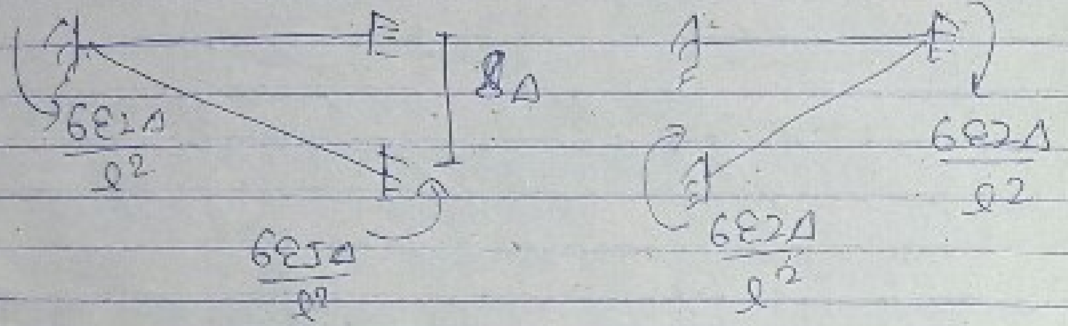
$= \frac{2M'L}{3EI} - \frac{M'L}{6EI}$

$= \frac{1}{2} \times \frac{M'L}{EI} = \frac{ML}{4EI}$

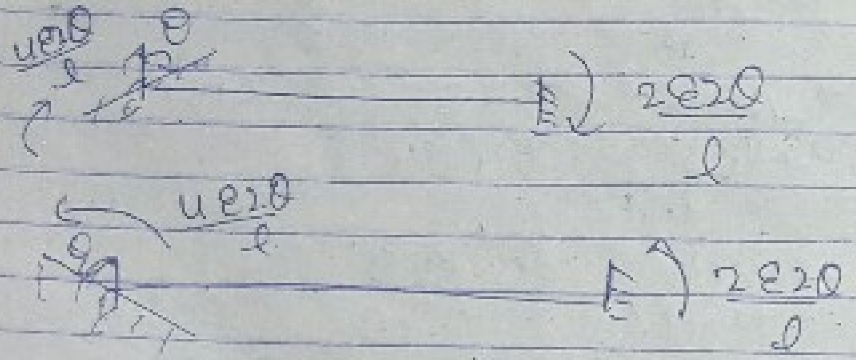


Application to a continous beam with know settlement & rotation as joint.

(i) If rightside support settlement by Δ in fixe beam. then fixed end moment's anticlock-wise $\frac{6EI\Delta}{l^2}$



(ii) If leftside support rotated by θ, in fixed beam, fixed end moments are



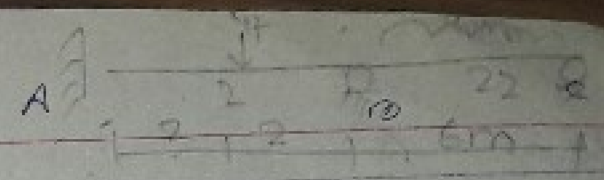
Q. If support A, rotate by clockwise 1/15° and support B settle by 3mm downward. determine support moment

$E = 2\times10^5 \ N/mm^2$

$I = 4\times10^4 cm^4$

$EI = 2\times10^{11}\times4\times10^4\times10^{-8}$

$= 8\times10^7 \ Nm^2 = \cancel{8\times10^5} \ tm^2 \approx 8000 \ tm^2$



(i) DKI = 2

(ii) FEM.

Due to lading, $AB = \frac{4\times4}{8} = -2$, $BA = 2$

$BC = -\frac{2\times6^2}{12} = -6$, $CB = 6$

Due to settement at B,

$AB = \frac{-6\times EI\Delta}{l^2} = \frac{-6\times8000\times3\times10^{-3}}{4^2} = -9$

$BA = -9$

$BC = \frac{6EI\Delta}{l^2} = \frac{6\times8000\times3\times10^{-3}}{6^2} = 4\times2 = 8$

$CB = 8$

Due to rotation

$AB = \frac{4EI\theta}{l} = \frac{4\times8000\times0.00116}{4} = 9.308$

$BA = \frac{2EI\theta}{l} = 4.65$

$BC = CB = 0$

Total $AB = -1.692$

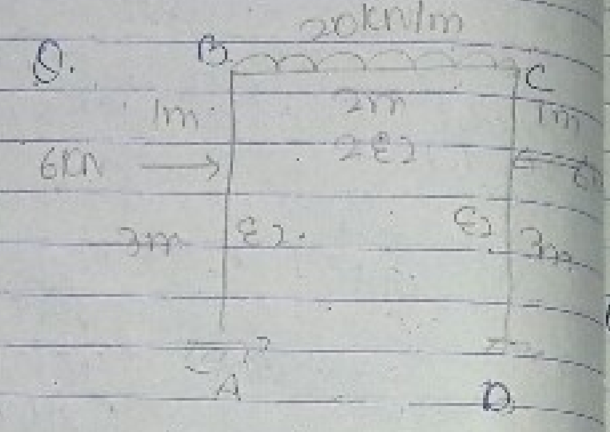
$BA = -2.35$

$BC = 2$

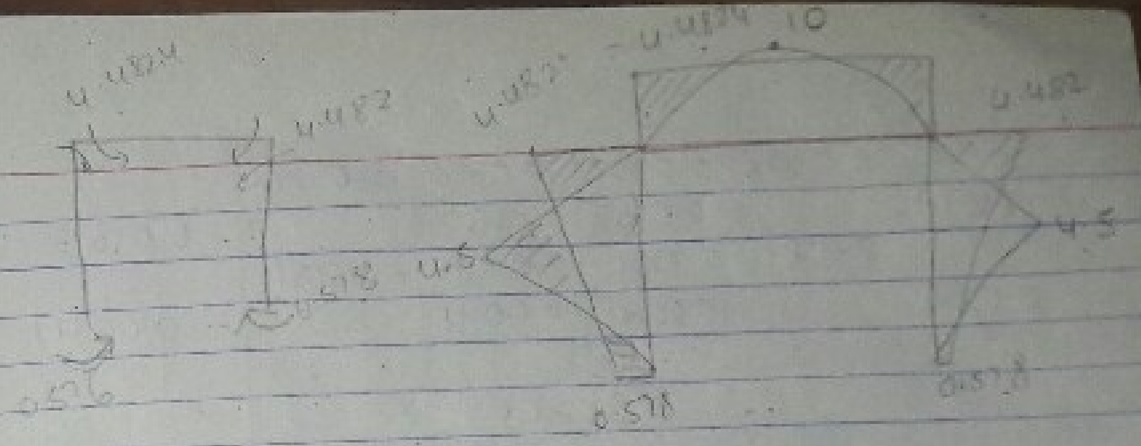
$CB = 14$

Application of Moment distribution of frame
(i) Non-sway frame
(ii) sway frame

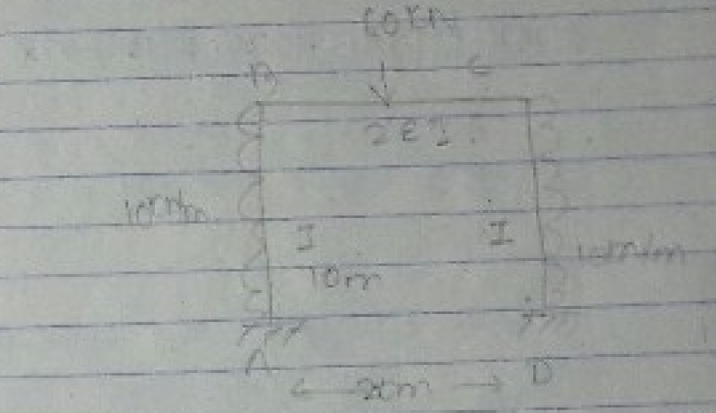
Non-sway frame. Q.



Joint	A	B		C		D
Mem	AB	BA	BC	CB	CD	DC
K	EI	EI	4EI	4EI	EI	EI
ΣK	—	5EI		5EI		—
DF	—	0.2	0.8	0.8	0.2	—
FEM	-1.125	3.375	-6.67	6.67	-3.375	1.125
BM		0.659	2.636	-2.636	-0.659	
COM	0.3295		-1.318	1.318		-0.3295
		0.2636	1.0544	-1.0544	-0.2636	
	0.1318		-0.5272	0.5272		-0.1318
		0.1055	0.42176	-0.42176	-0.1055	
	0.05275		-0.2109	0.2109		-0.0527
		0.0422	0.1687	-0.1687	-0.0422	
	0.0211		-0.0844	0.0844		-0.0211
		0.0169	0.0675	-0.0675	-0.0169	
	0.00844		-0.03376	0.03376		-0.00844
		0.00675	0.027	-0.027	-0.00675	
	0.0034		-0.0135	0.0135		-0.0034
	-0.578	4.46895	-4.4824	4.4824	-4.46895	0.578

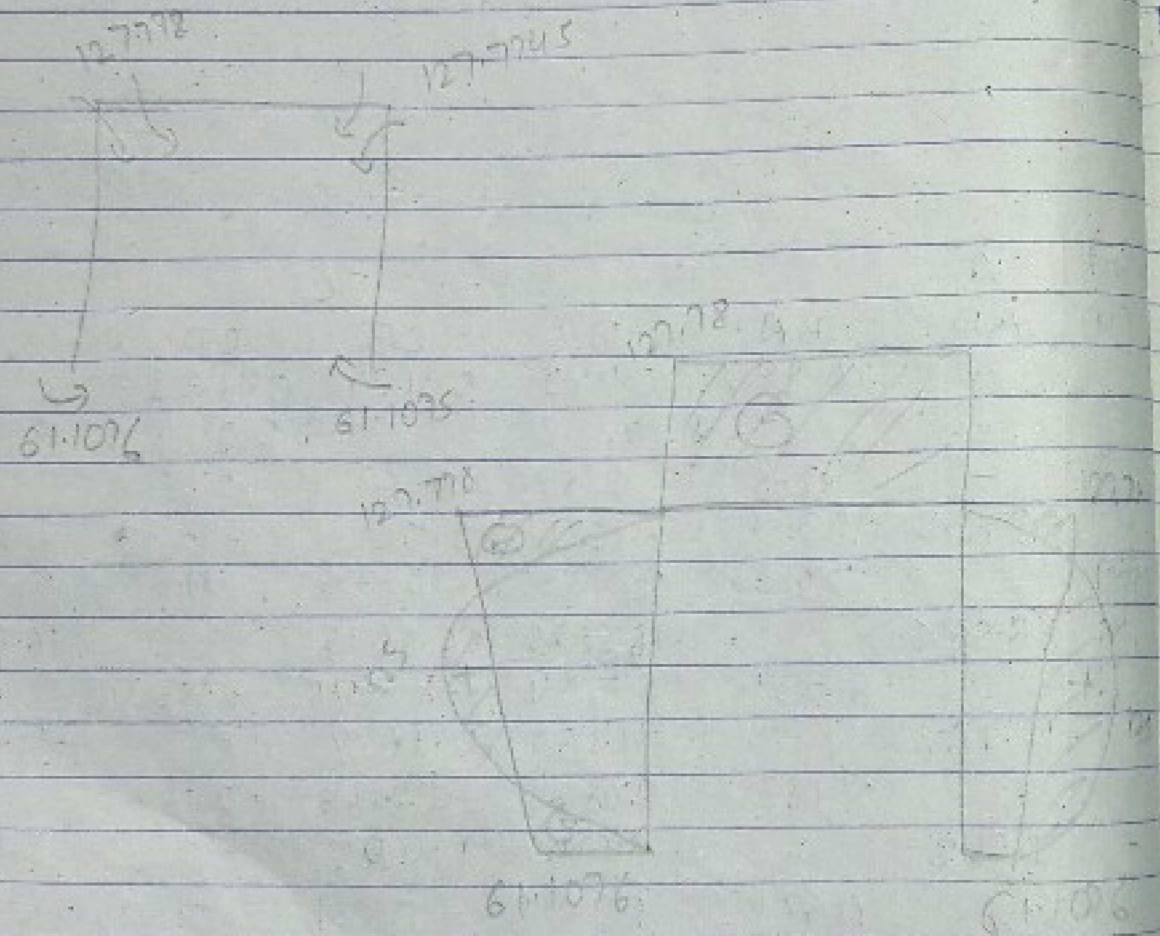


Q.



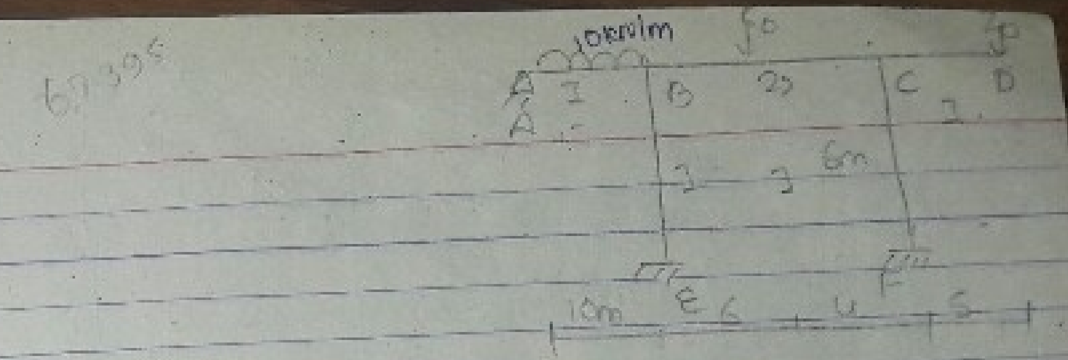
Joint	A	B		C		D
Mem	AB	BA	BC	CB	CD	DC
K	—	0.4EI	0.4EI	0.4EI	0.4EI	—
ΣK	—	0.8EI		0.8EI		—
DF	—	0.5	0.5	0.5	0.5	—
FEM	-83.33	83.33	-150	150	-83.33	83.33
BM		33.335	33.335	-33.335	-33.335	
COM	16.6675		-16.6675	16.6675		-16.6675
		8.334	8.334	-8.334	-8.334	
	4.167		-4.167	4.167		-4.167
		2.0835	2.0835	-2.0835	-2.0835	
	1.042		-1.042	1.042		-1.042
		0.521	0.521	-0.521	-0.521	
	0.2605		-0.2605	0.2605		-0.2605
		0.1303	0.1303	-0.1303	-0.1303	
	0.065		-0.0651	0.0651		-0.0651

	0.0326	0.0326	-0.0326	-0.0326	-0.0163
0.0163		-0.0163	0.0163		
	0.0081	0.0081	-0.0081	-0.0081	-0.0041
0.0041		-0.0041	0.0041		
-61.1076	127.7745	-127.778	127.778	-127.7743	61.1075

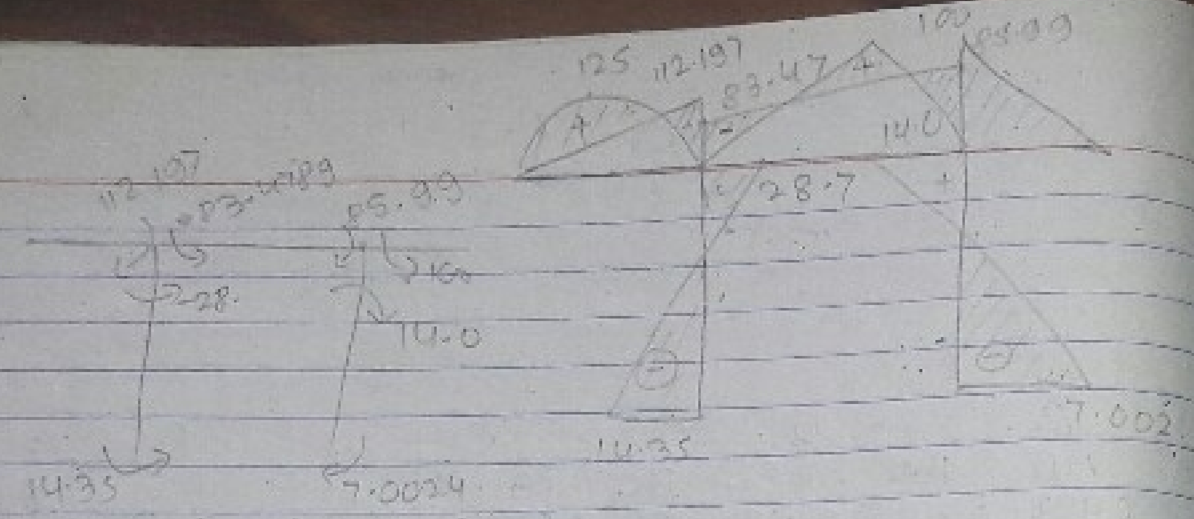


67.395

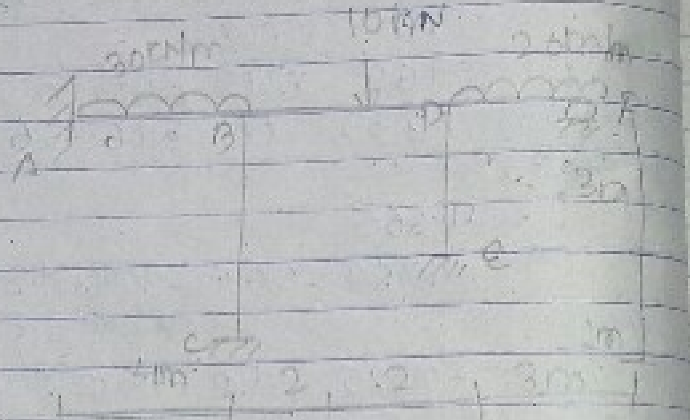
Q.



Joint	A	B			C			F	E
Mem	AB	BA	BE	BC	CB	CD	CF	FC	EB
K	0.4EI	0.3EI	0.67EI	0.8EI	0.8EI	–	0.67EI	0.67EI	0.67EI
ΣK	"		1.77EI			1.47EI			
DF	1	0.169	0.379	0.452	0.544	–	0.456	1	1
FEM	-83.33	83.33	0	-51.6	86.94	-100	0	0	0
BM	83.33								
COM		41.665							
		-11.389	-25.543	-30.463	7.398		6.202		
				3.699	-15.231			3.1	-12.7715
		-0.625	-1.402	-1.672	8.286		6.945		
				4.143	-0.836			3.473	-0.701
		-0.7	-1.57	-1.873	0.455		0.381		
				0.227	-0.936			0.191	-0.785
		-0.038	-0.086	-0.103	0.509		0.427		
				0.2950	-0.051			0.2134	-0.043
		-0.043	-0.096	-0.115	0.0277		0.0232		
				0.0139	-0.058			0.012	-0.0048
		-0.0023	-0.0053	-0.0063	0.032		0.0264		
				0.016	-0.0031			0.013	-0.0026
	0	112.1977	-28.7023	-83.4779	85.8926	-100	14.0016	7.0024	-14.35

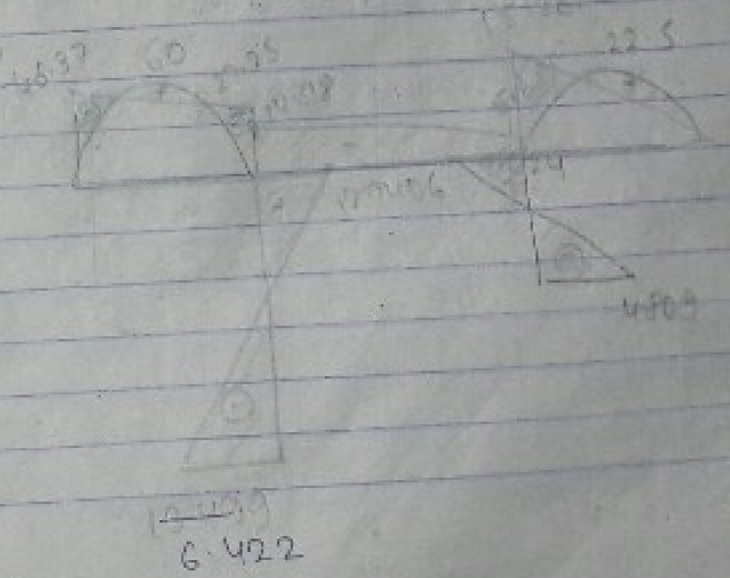


Q.



Joint	A	B		D				F	C	E
Mem	AB	BA	BC	BD	DB	DE	DF	FD	CB	ED
K	EI	EI	EI	EI	EI	1.33EI	EI	--	EI	1.33
ΣK	--		3EI			3.33		--	--	--
DF	–	0.33	0.33	0.34	0.3	0.4	0.3	–	1	1
FEM	-40	40	0	-5	5	0	-15	15	0	0
~~BM~~		~~-11.55~~	~~-11.55~~	~~-11.9~~	~~3~~	~~4~~	~~3~~	~~-15~~		
				~~1.5~~	~~-5.95~~		~~7.5~~	—	-5.775	
BM								-15		
COM							-7.5			
BM		+11.55	-11.55	-11.9	5.25	7	5.25			
	-5.775			2.625	-5.95				-5.775	3.5

A	B		D				F	C	E
AB	BA	BC	BD	DB	DE	DF	FD	CB	ED
	-0.866	-0.866	-0.893	1.785	2.38	1.785		-0.433	1.19
			0.893	-0.446					
-0.433	-0.295	-0.295	-0.304	0.134	0.1784	0.134		-0.147	0.0892
			0.067	-0.152					
-0.147	-0.022	-0.022	-0.023	0.0456	0.061	0.0456		-0.011	0.030
			0.023	-0.011					
-0.011	-0.0076	-0.0076	-0.008	0.0033	0.0044	0.0033		-0.0038	
			0.0017	-0.004					
-0.0038									
-46.3698	~~19.4654~~ 27.894	-12.7406	-14.5183	5.656	9.624	-15.282	0	-~~13.5499~~ 6.422	4.809



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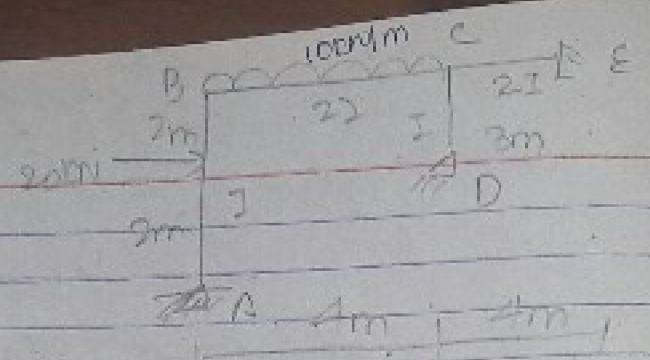
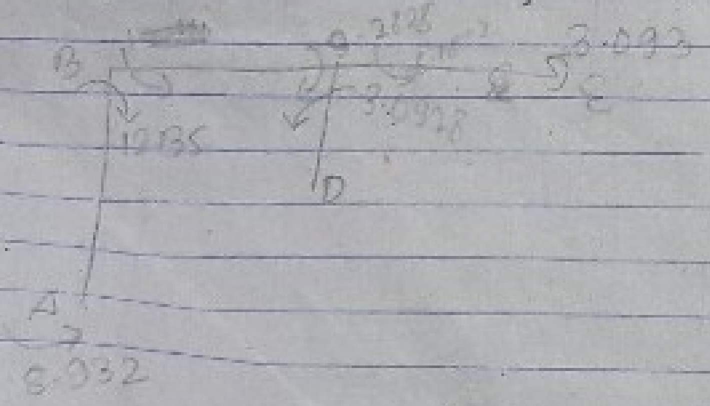
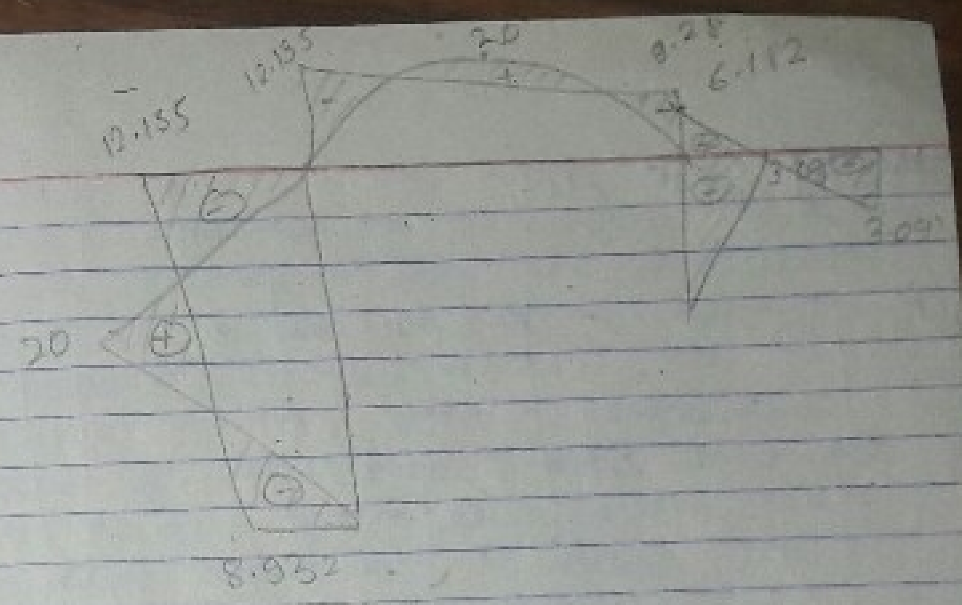
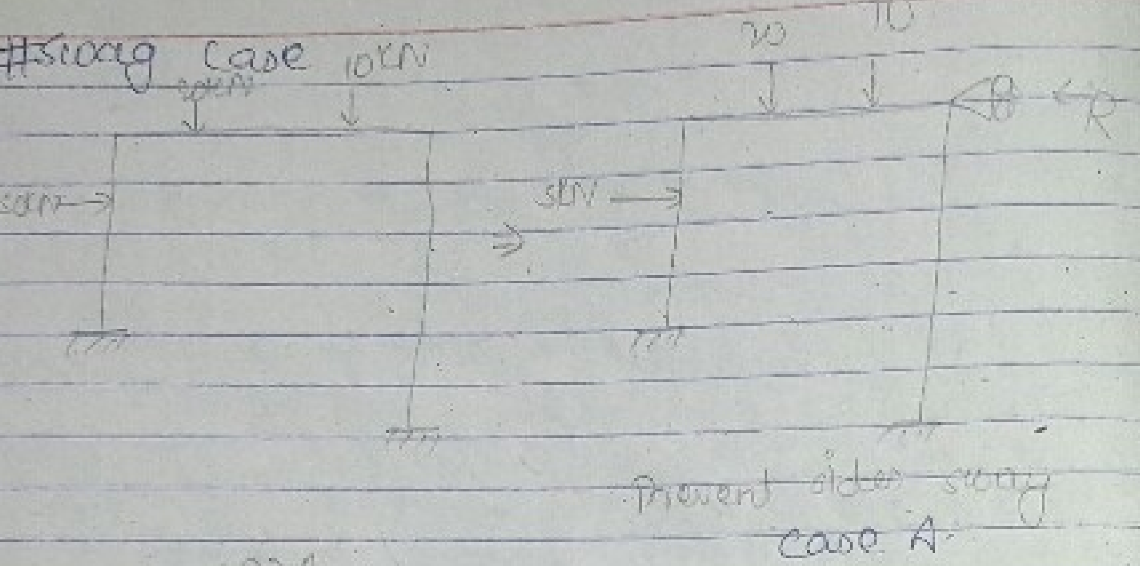


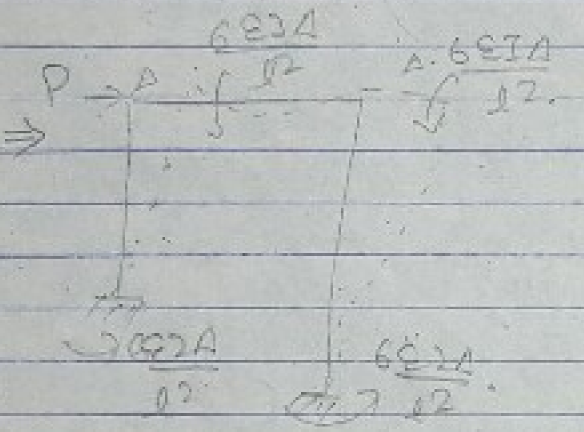
Joint	A	B		C			E	D
Mem	AB	BA	BC	CB	CD	CE	EC	DC
K	—	EI	2EI	2EI	EI	2EI	—	—
SK	—	3EI		5EI			—	—
DF	—	0.333	0.667	0.4	0.2	0.4	—	—
FEM	-10	10	-13.33	13.33	0	0	0	0
BM		1.109	2.221	-5.332	-2.666	-5.332		
COM	0.555		-2.666	1.111			-2.666	
		0.888	1.778	-0.444	-0.222	-0.444		
	0.444		-0.222	0.889			-0.222	
		0.074	0.148	-0.356	-0.178	-0.356		
	0.037		-0.178	0.074			-0.178	
		0.059	0.1187	-0.0296	-0.0148	-0.0296		
	0.03		-0.0148	0.059			-0.0148	
		0.005	0.0099	-0.0236	-0.012	-0.0236		
	0.002		-0.012	0.005			-0.012	
	-8.932	12.135	-12.1472	9.2828	-3.0928	-6.1852	-3.093	0





Sway case





Introduce Sway
Case B.

~~Sway~~ Correction

Case A :

- Apply all given loads and determine resulting end moment by MDM as before without sway.
- Having end moments, then we can determine holding force 'R' by eqn of static

Case B :

- Introduce some arbitrary horizontal displacement Δ by applying pushing force P at same pt of application & along same line of action as holding force R.
- Compute initial end moment developed in mem due to Δ. these end moment expressed in terms of $EI\Delta$. Assume $EI\Delta$ by any convinent amount & determine resulting end moments by MDM.
- Using static equil. eqn. determine pushing force 'P'

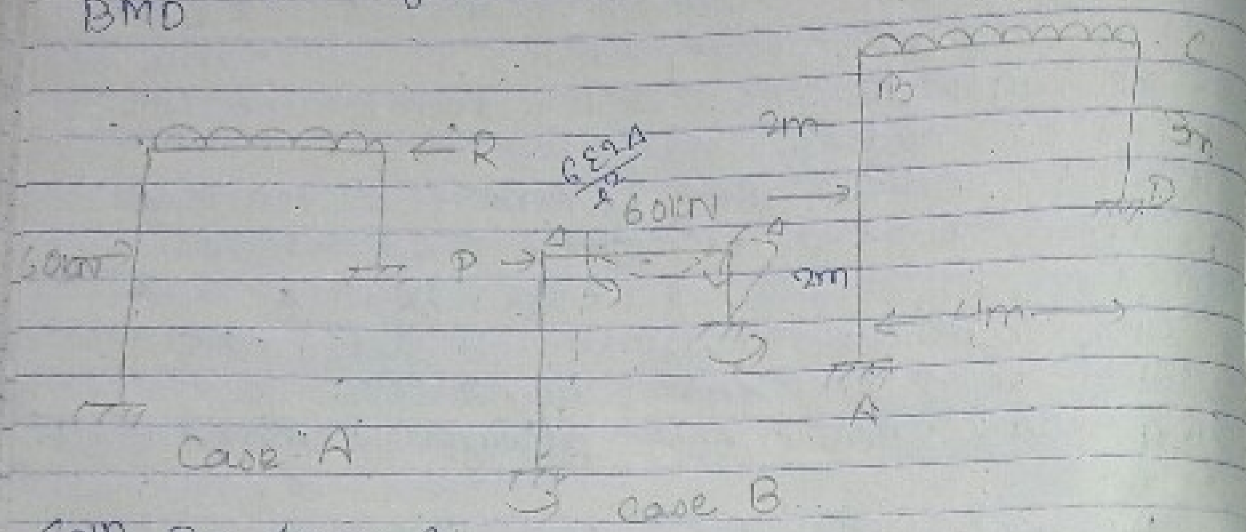
Now, to get final end moment for given frame, we superimpose case A & B to superimpose, we must take case A moment as they are since external load for this case, are same specified one's but we combine these results with any multiplied factor of case B result

i.e. Sway Co-rection factor F

$$= \frac{\text{Resultant sway force in Case A}}{\text{" \quad " \quad " \quad case B}} = \frac{R}{P}$$

$\therefore$ Final Moment = Non-Sway Moment (case A) + F x Sway Moment (case B)

Q. Analyse rigid frame by MDM & Draw BMD



Soln For Case A,

Joint	A	B		C		D
Mem	AB	BA	BC	CB	CD	DC
ΣK	EI	EI	EI	EI	1.33EI	—
K	—	2EI		2.33		—
DF		0.5	0.5	0.429	0.571	—
FEM	-30	30	-53.33	53.33	0	0
BM		11.665	11.665	-22.879	-30.45	
COM	5.8325		-11.4395	5.8325		-15.226
		5.7198	5.7198	-2.502	-3.33	
	2.860		-1.251	2.860		-1.665
		0.6255	0.6255	-1.2270	-1.633	
	0.3128		-0.613	0.3128		-0.8165
		0.3065	0.3065	-0.134	-0.179	
	0.1532		-0.067	0.1533		-0.089
		0.0335	0.0335	-0.0657	-0.087	
	0.017		-0.033	0.017		-0.0437
		0.016	0.016	-0.0073	-0.0097	
	0.008		-0.0036	0.008		-0.0048

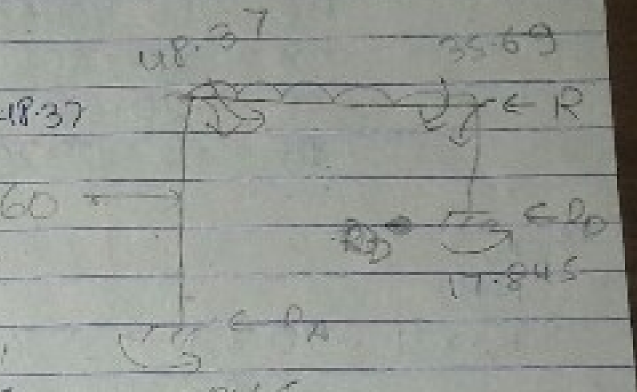
AB	BA	BC	CB	CD	DC
-20.8165	48.3663	-48.3708	35.698	-35.6897	-17.845

Now,

For AB,

$R_A \times 4 - 20.8165 - 60 \times 2 = -48.37$

$\therefore R_A = 23.1166$



Then

From CD,

$-R_D \times 3 + 17.845 = -35.69$

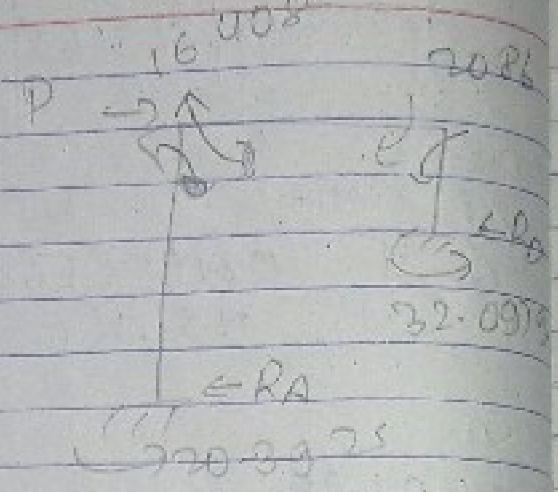
$\therefore R_D = \cancel{17.845}\ 17.845$

$\therefore R = \cancel{53.276}\ 19.038\ kN$

Case B: Let $EI\Delta = 65$

Joint	A	B		C		D
Mem	AB	BA	BC	CB	CD	DC
ΣK	EI	EI	EI	EI	1.33EI	—
K	—	2EI				—
DF	—	0.5	0.5	0.429	0.571	—
FEM	$\frac{6EI\Delta}{L^2} = -24.375$	-24.375	0	0	-43.33	-43.33
BM		12.1875	12.1875	18.589	24.741	
	6.0937		9.294	6.0937		12.3707
		-4.647	-4.647	-2.614	-3.4795	
	-2.324		-1.307	-2.324		-1.7398
		0.6535	0.6535	0.997	1.327	
	0.327		0.498	0.327		0.6635
		-0.249	-0.249	-0.14	-0.187	
	-0.125		-0.07	-0.125		-0.093
		0.035	0.035	0.0536	0.0714	
	0.0175		0.0268	0.0175		0.0357
		-0.0134	-0.0134	-0.0075	-0.0099	
	-0.0067		-0.00375	-0.0067		-0.005

AB = −20.3925
BA = −16.4084
BC = 16.40465
CB = 20.8606
CD = −20.867
DC = −32.0979



Then,
AB
$R_A \times 4 - 20.3925 = 16.408$
$\therefore R_A = \cancel{0.996}\ 9.2$

For CD
$32.0979 - R_D \times 3 = -20.86$
$\therefore R_D = 17.65$

$\therefore P = \cancel{18.6486}\ 26.85$ KN

Then,
Sway Co-factor $= \frac{R}{P} = 0.709$

Now, Final Moment
AB = (Non-Sway M)(A) + F × (Sway B)
= −20.8165 + 0.709 (−20.3925)
= −35.2757

BA = 48.3663 + 0.709 (−16.4084)
= 36.734

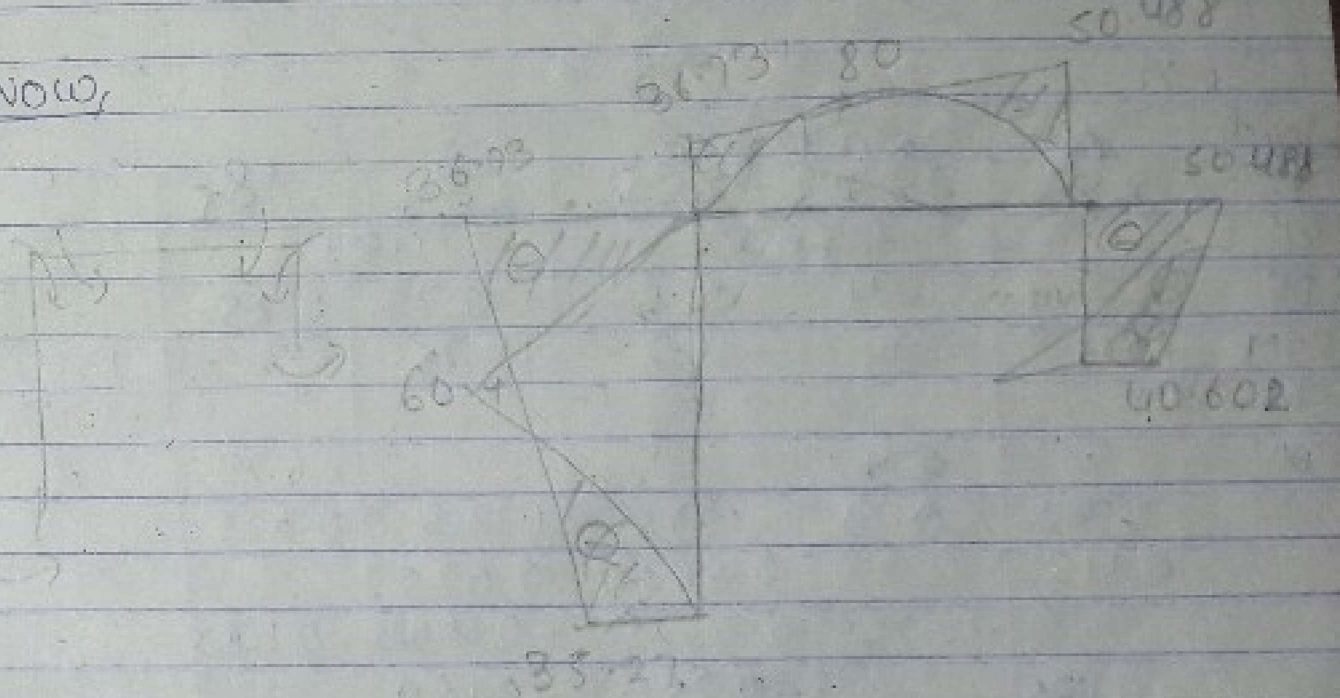
BC = −48.3663 + 0.709 (16.40465)
= −36.734

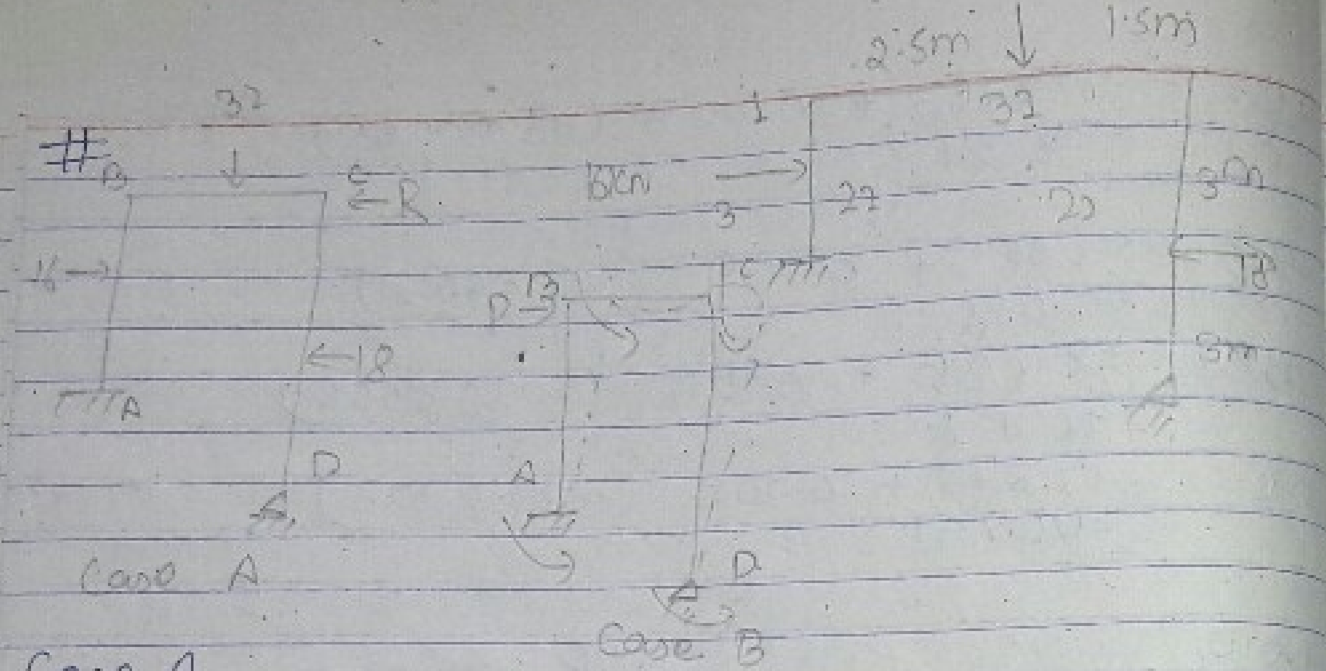
CB = 35.698 + 0.709 (20.8606)
= 50.488

CD = −35.698 + 0.709 (−20.8606)
= −50.488

DC = −17.845 + 0.709 (−32.0979)
= −40.602.

Now,





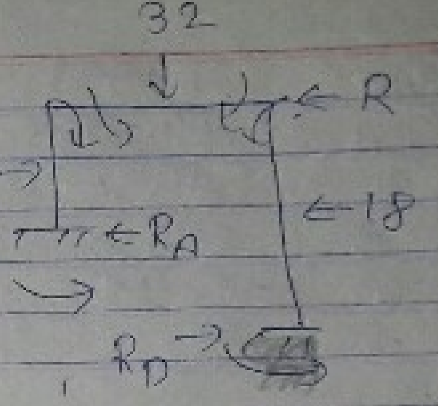
Case A

Joint	A	B		C		D
	AB	BA	BC	CB	CD	DC
K	2EI	2EI	3EI	3EI	EI	—
ΣK	—	5EI		4EI		—
DF	—	0.4	0.6	0.75	0.25	—
FEM	−3	9	−11.25	18.75	−13.5	13.5
BM						−13.5
COM		~~0.9~~			−6.75	
	~~0.45~~	0.9	1.35	1.125	0.375	
	0.45		0.5625	0.675		
		−0.225	−0.3375	−0.50625	−0.16875	
	−0.1125		−0.253	−0.169		
		0.1012	0.1518	0.1268	0.0423	
	0.0506		0.063	0.076		
		−0.0252	−0.0378	−0.057	−0.019	
	−0.0126		−0.0285	−0.0189		
		0.0114	0.017	0.014	0.0047	
	0.0057		0.0071	0.00855		
	−2.6188	9.7624	−9.7554	20.0242	−20.016	0

Now,

$R_A \times 4 - 2.6188 - 16 \times 1 = -9.7554$

$\therefore R_A = 2.215$



$R_D \times 6 - 18 \times 3 = -20.016$

$\therefore R_D = 5.664$

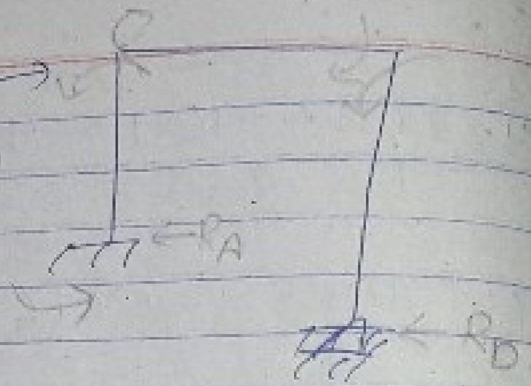
$\therefore R = 1.449$

Case B, $EI\Delta = 65$

Solution on to page no 6 ***

Joint	A	B		C		D
Mom.	AB	BA	BC	CB	CD	DC
K	=					
ΣK	—					
DF	—	0.4	0.6	0.75	0.25	
FEM	−24.375	−24.375	0	0	−10.833	−10.833
BM						10.833
COM					5.4167	
		9.75	14.625	4.06	1.353	
	4.875		2.03	7.3125		
		−0.812	−1.218	−5.484	−1.828	
	−0.406		−2.742	−0.609		
		1.0968	1.6452	0.457	0.152	
	0.5484		0.229	0.823		
		−0.0916	−0.1374	−0.6173	−0.206	
	−0.0458		−0.309	−0.0687		
		0.1236	0.1854	0.0515	0.017	
	0.0618		0.0257	0.0927		
		−0.0103	−0.01545	−0.0695	−0.023	
	−0.005		−0.0347	−0.0077		
		0.014	0.021	0.00578	0.0019	
	0.0069		0.003	0.01		

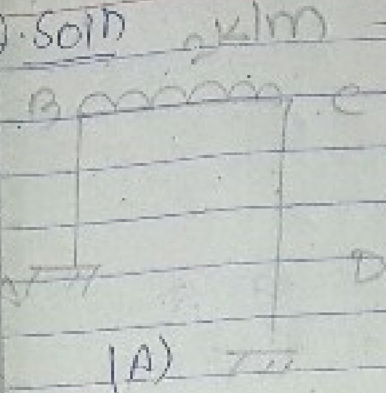
$M_{AB} = -19.339$
$BA = -14.3045$ 27.696
$BC = 14.3078$ 27.724
$CB = 5.956$ 8.024
$CD = -5.95$
$DC = 0$

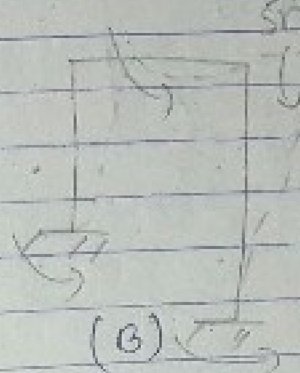


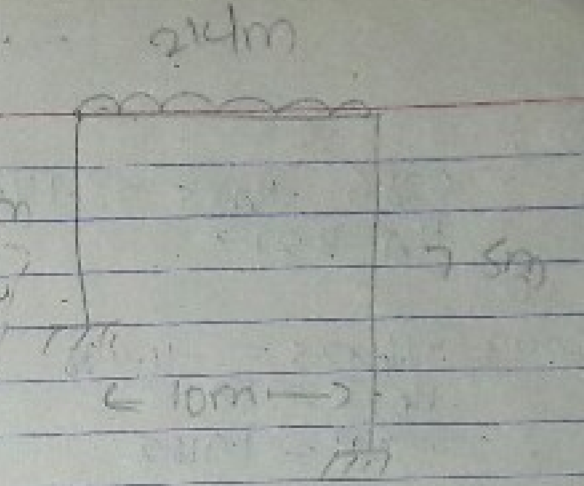
from
AB,
$R_A \times 4 - 19.339 = 14.30$
$\therefore R_A = 8.41$

CD,
$R_D \times 6 = 5.95$
$\therefore R_D = 0.99$
$\therefore P = 9.4$

Soln







Case A.

Joint	A	B		C		D
Mem	AB	BA	BC	CB	CD	DC
K	–	0.8EI	0.4EI	0.4EI	0.53EI	–
ΣK	–	1.2EI		0.933EI		–
DF	–	0.67	0.33	0.429	0.571	–
FEM	0	0	–16.67	16.67	0	0
BM		11.169	5.5011	–7.15	–9.519	
COM	5.584		–3.575	2.751		–4.759
		2.395	1.180	–1.18	–1.571	
	1.1976		–0.59	0.589		–0.785
		0.395	0.195	–0.253	–0.336	
	0.198		–0.126	0.097		–0.168
		0.084	0.0416	–0.041	–0.055	
	0.042		–0.021	0.021		–0.027
		0.014	0.0069	–0.009	–0.012	
	0.007		–0.0045	0.0034		–0.006
	7.0286	14.057	–14.062	11.498	–11.493	–5.743

AB,

$7.0286 - H_A \times 5 = -14.057$

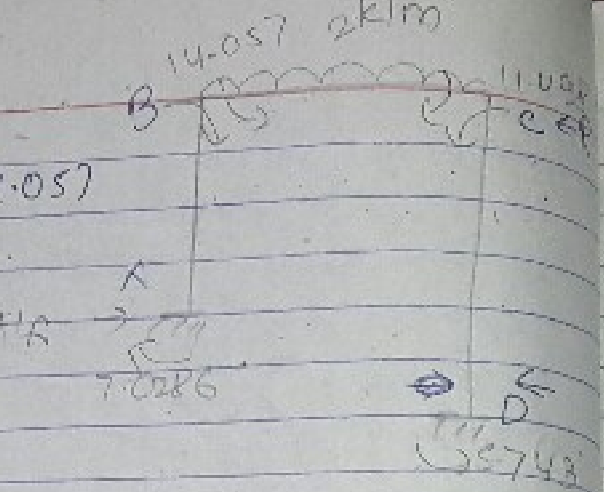
$H_A = 4.217$

For CD,

$5.743 - H_D \times 7.5 = -11.498$

$H_D = 2.2988$

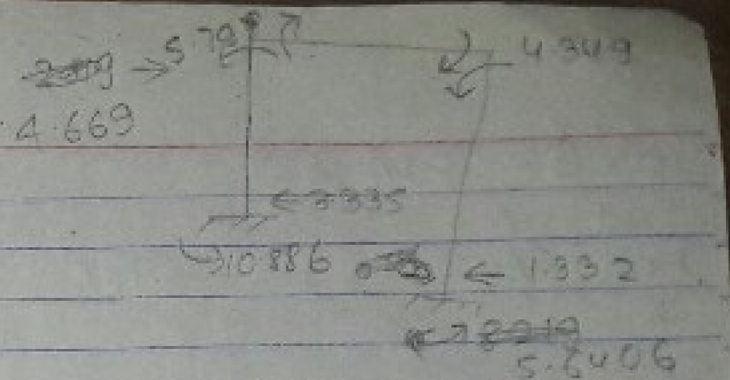
$\therefore R = 1.9182$



Case B -

Joint	A	B		C		D
Mem	AB	BA	BC	CB	CD	DC
K						
ΣK						
DF		0.67	0.33	0.429	0.571	
FEM	~~-24.375~~ -15.6	~~-24.375~~ -15.6	0	0	-6.93	-6.93
BM		10.452	5.148	2.973	3.957	
COM	5.226		1.486	2.574		1.978
		-0.996	-0.49	-1.104	-1.469	
	-0.498		-0.552	-0.245		-0.735
		0.~~122~~ 370	0.182	0.105	0.1399	
	0.061		0.053	0.091		0.07
		-0.036	-0.017	-0.039	-0.052	
	-0.018		-0.0195	-0.0085		-0.026
		0.013	0.006	0.00364	0.0048	
	0.0065		0.0018	0.003		0.0024
	-10.886	-5.797 ~~-6.167~~ ~~-6.045~~	5.7983	4.353	-4.349	~~8.219~~ 5.6406

0.68



$\therefore P = 4.669$

Cofator = 0.411

Then, $M_{AB} = 2.554$

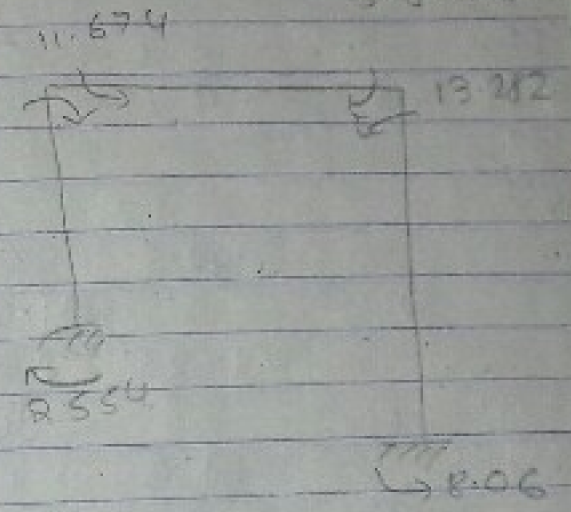
$M_{BA} = 11.674$

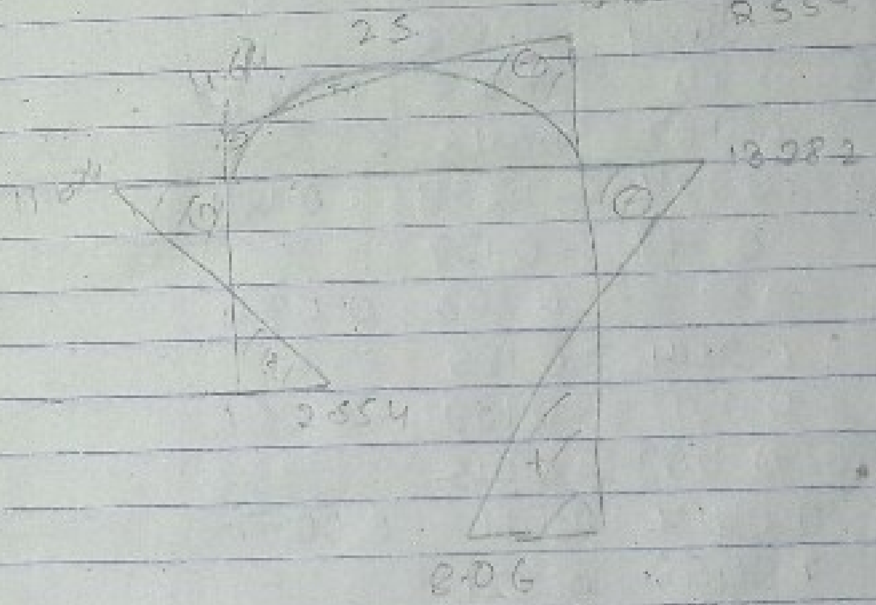
$M_{BC} = -11.674$

$M_{CB} = 13.282$

$M_{CD} = -13.282$

$M_{DC} = -8.06$



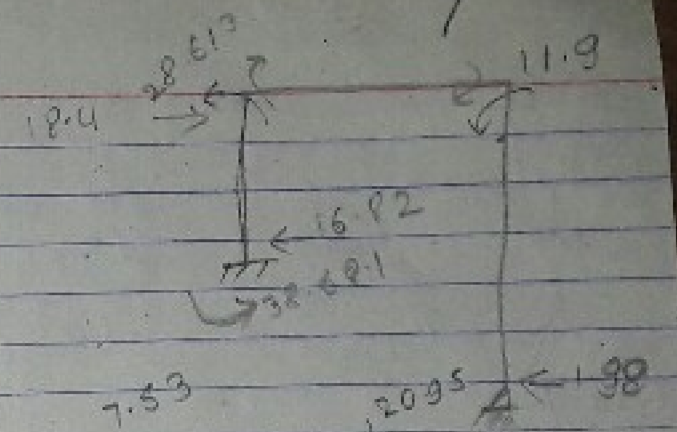


****

Case B, $8.2\Delta = 65$

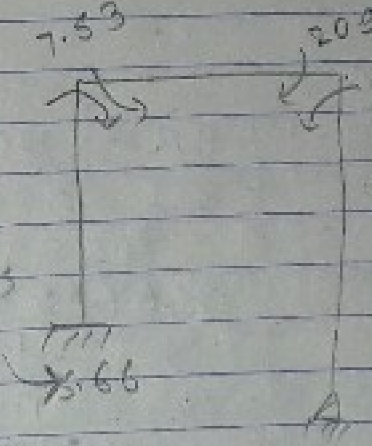
Joint	A	B		C		D
Mem	AB	BA	BC	CB	CD	DC
K						
ΣK						
DF		0.4	0.6	0.75	0.25	
FEM	-48.75	-48.75	0	0	-21.667	-21.667
BM						21.667
COM					10.833	
		19.5	29.25	8.125	2.708	
	9.75		4.0625	14.625		
		-1.625	-2.4375	-10.969	-3.656	
	-0.8125		-5.484	-1.2188		
		2.1936	3.29	0.914	0.3084	
	1.097		0.457	1.645		
		-0.183	-0.274	-1.234	-0.411	
	-0.0914		-0.617	-0.137		
		0.247	0.37	0.103	0.034	
	0.1234		0.0514	0.185		
		-0.021	-0.031	-0.139	-0.046	
	-0.01		-0.069	-0.015		
		0.0276	0.0414	0.011	0.0037	
	0.0138		0.0056	0.021		
		-0.0022	-0.0034	-0.016	-0.005	
	-0.0011		-0.008	-0.002		
	-38.681	-28.613	28.604	11.899	-11.9	0

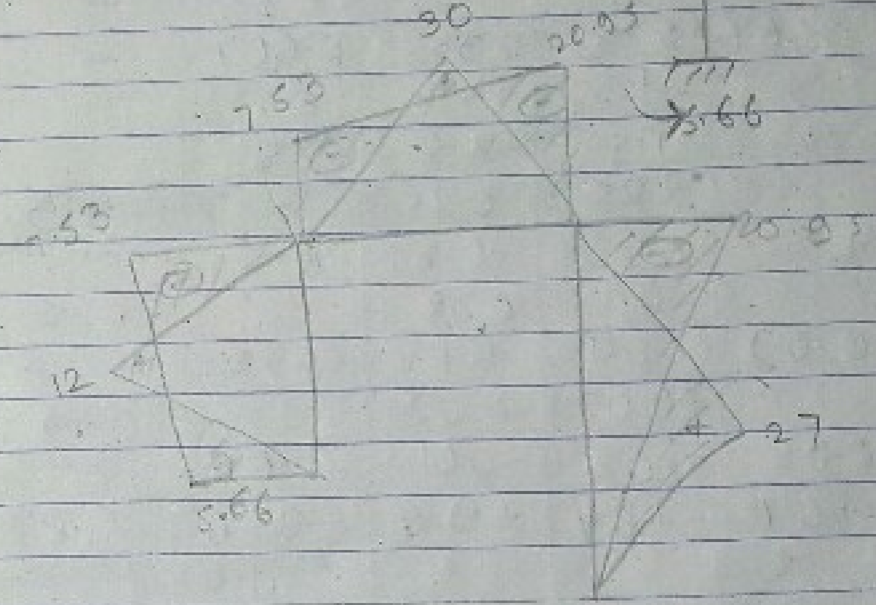
$\therefore P = 18.4$



Cofactor = 0.078

$M_{AB} = -5.66$
$M_{BA} = 7.53$
$M_{BC} = -7.53$
$M_{CD} = 20.95$
$M_{DC} = -20.95$
$M_{DC} = 0$





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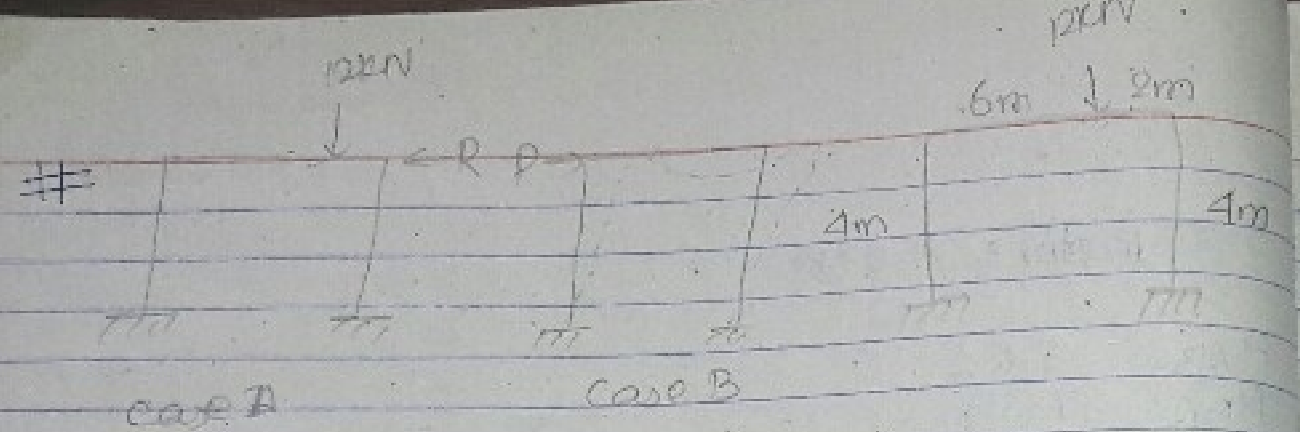


Case A.

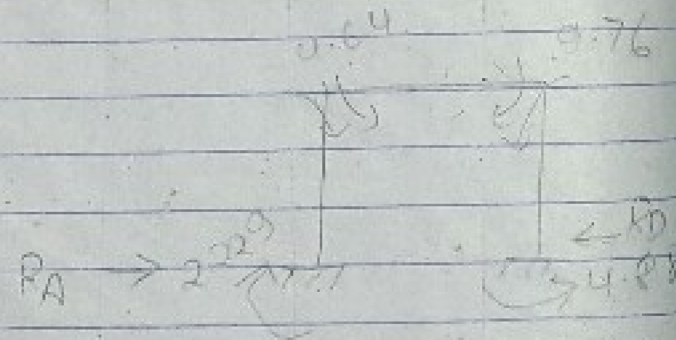
Joint	A	B		C		D
Mem	AB	BA	BC	CB	CD	DC
K	EI	EI	0.5EI	0.5EI	EI	EI
ΣK	–	1.5EI		1.5EI		–
DF	–	0.667	0.333	0.333	0.667	–
FEM	0	0	-4.5	13.5	0	0
BM		3	1.5	-4.5	-9	
COM	1.5		2.25	0.75	–	-4.5
	–	1.5	0.75	-0.25	-0.5	
	0.15		-0.13	0.38	–	-0.25
	–	0.09	0.04	-0.13	-0.25	
	0.04		-0.07	0.02		-0.13
	–	0.05	0.02	-0.01	-0.01	
	2.229	4.64	-4.64	+9.76	-9.76	-4.88

Then

$$-R_A \times 4 + 2.229 = -4.64$$

$$\therefore R_A = 1.72$$

$$R_D = 3.66$$



$$\therefore R = 1.94$$

$EI\Delta = 80,$

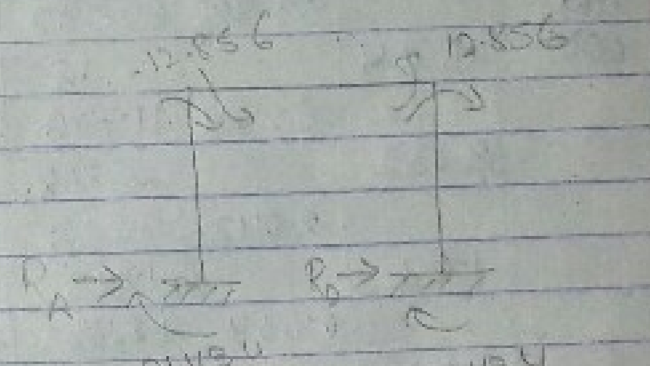
Case B.

Joint	A	B		C		D
Mem	AB	BA	BC	CB	CD	DC
DF	–	0.667	0.333	0.333	0.667	
FEM	30	30	0	0	30	30
BM		-20	-10	-10	-20	
COM	-10		-5	-5		-10
		3.333	1.667	1.667	3.333	
	1.666		0.8333	0.8333		1.666
		-0.555	-0.278	-0.278	-0.555	
	-0.278		-0.139	-0.139		-0.278
		0.093	0.046	0.046	0.093	
	0.046		0.023	0.023		0.046
		-0.015	-0.008	-0.008	-0.015	
	21.434	12.856	-12.856	-12.856	12.856	21.434

$R_A = 8.5725$ kN

$R_D = 8.5725$ kN

$P = 17.145$



Sway factor = 0.113

Then,

$M_{AB} = 2.229 + 0.113 \times 21.434 = 4.65$

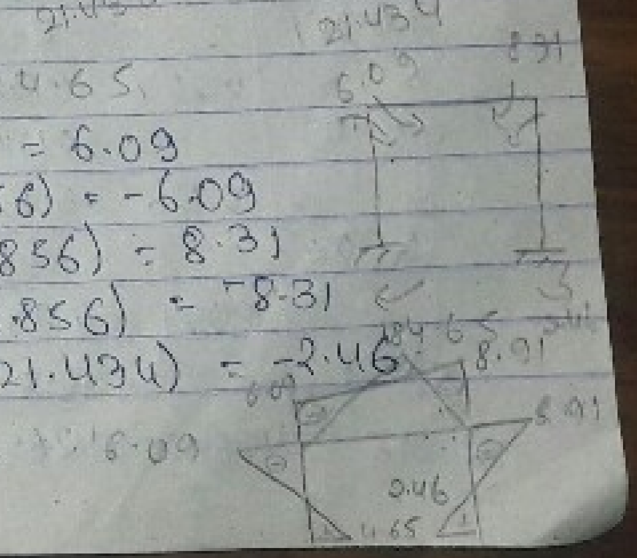
$M_{BA} = 4.64 + 0.113 \times 12.856 = 6.09$

$M_{BC} = -4.64 + 0.113 \times (-12.856) = -6.09$

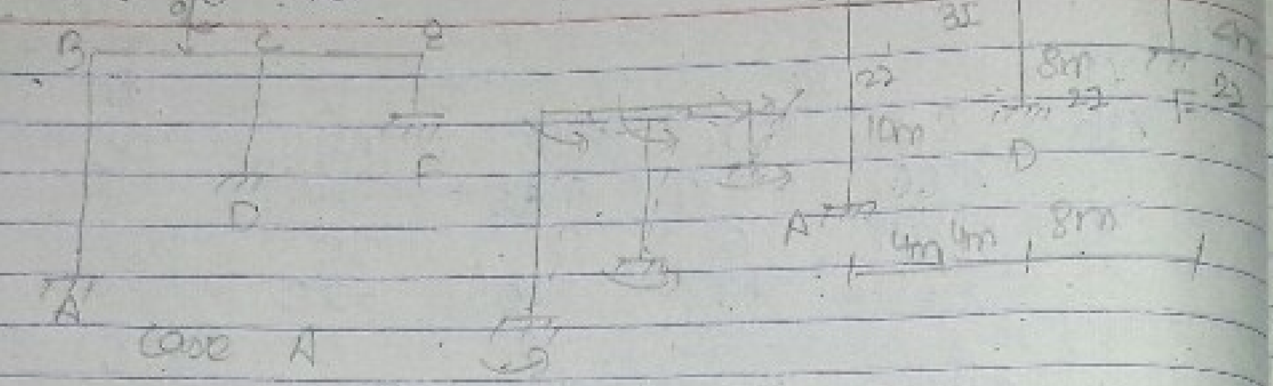
$M_{CB} = 9.76 + 0.113 \times (-12.856) = 8.31$

$M_{CD} = -9.76 + 0.113 \times (12.856) = -8.31$

$M_{DC} = -4.88 + 0.113 \times (21.434) = -2.46$



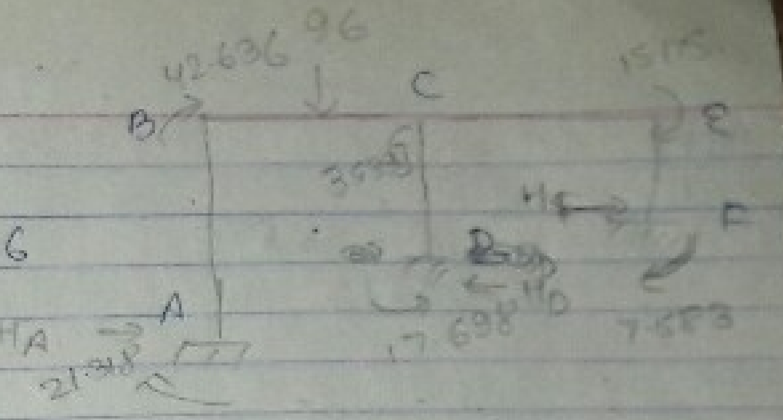
± Analyse rigid frame by MDM & draw BMD.



Case A.

Joint	A	B		C			E		F	D
Mem	AB	BA	BC	CB	CD	CE	EC	EF	FE	D
K	–	0.8EI	1.5EI	1.5EI	EI	1.5EI	1.5EI	2EI	2EI	–
ΣK	–	2.3EI		4EI			3.5EI		–	
DF	–	0.348	0.652	0.375	0.25	0.375	0.428	0.572	–	
FEM	0	0	-96	96	0	0	0	0	0	0
BM		33.408	62.592	-36	-24	-36				
COM	16.704		-18	31.296			-18			
		6.264	11.736	-11.736	-7.824	-11.736	7.704	10.296		
	3.132		-5.868	5.868		3.852	-5.868		5.148	-3
		2.042	3.826	-3.645	-2.43	-3.645	2.511	3.356		
	1.02		-1.823	1.913		1.256	-1.823		1.678	-1.21
		0.634	1.188	-1.188	-0.792	-1.188	0.780	1.043		
	0.317		-0.594	0.594		0.39	-0.594		0.521	-0.3
		0.207	0.387	-0.369	-0.246	-0.369	0.254	0.339		
	0.103		-0.1845	0.194		0.127	-0.1845		0.169	-0.12
		0.06	0.12	-0.12	-0.08	-0.12	0.08	0.106		
	0.032		-0.06	0.06		0.04	-0.06		0.05	-0.0
		0.021	0.039	-0.0375	-0.025	-0.0375	0.026	0.034		
	0.01		-0.018	0.02		0.013	-0.008		0.017	-0.0125
	21.318	42.636	-42.659	82.849	-35.397	-47.438	-15.085	15.174	7.583	-176

$\frac{6EI\Delta}{l^2}$



For AB

$21.318 - H_A \times 10 = -42.636$

$H_A = 6.3954$

For CD,

$17.698 - H_D \times 8 = -35.397$

$H_D = 6.637$

For EF

$-7.583 + H_F \times 4 = 15.175$

$H_F = 5.689$

Then,

$\therefore R = 5.447$

For $EI\Delta = 50$.

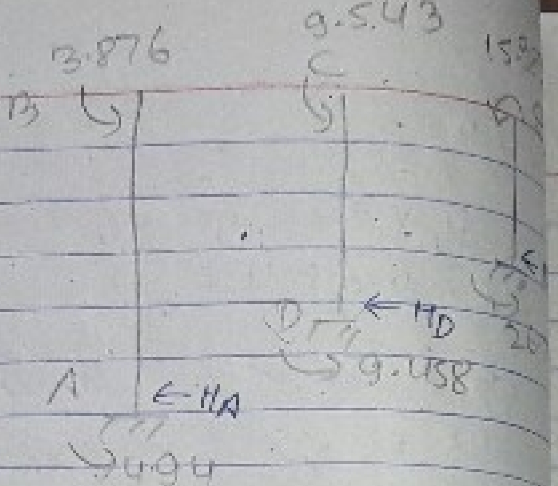
Joint	A	B		C			E		F	D
Mem.	AB	BA	BC	CB	CD	CE	EC	EF	FE	DC
DF	–	0.348	0.652	0.375	0.25	0.375	0.428	0.572	–	–
FEM	-6	-6	–	–	-9.375	–	–	-37.5	-37.5	-9.375
BM		2.088	3.912	3.516	2.344	3.516	16.05	21.45		
COM	1.044		1.758	1.956		8.025	1.758		10.725	1.172
		-0.612	-1.146	-3.743	-2.495	-3.743	-0.752	-1.005		
	-0.306		-1.871	-0.573		-0.376	-1.871		-0.503	-1.248
		0.651	1.219	0.356	0.237	0.356	0.8	1.07		
	0.326		0.178	0.6010		0.4	0.178		0.535	0.119
		-0.062	-0.116	-0.379	-0.253	-0.379	-0.076	-0.102		
	-0.031		-0.189	-0.058		-0.038	-0.189		-0.05	-0.126
		0.065	0.123	0.036	0.024	0.036	0.081	0.108		
	0.03		0.018	0.062		0.04	0.018		0.054	0.012
		-0.006	-0.0117	-0.038	-0.025	-0.038	-0.007	-0.01		
	-0.003		-0.019	-0.0058		-0.004	-0.019		-0.005	-0.0125
	-4.94	-3.876	8.856	1.739	-9.543	7.795	15.971	-15.989	-26.744	-9.458

Now,

AB,

$-4.94 + H_A \times 10 = 3.876$

$H_A = 0.8816$



CD,

$9.458 - H_D \times 8 = -9.543$

$H_D = 2.375$

EF,

$26.744 - H_F \times 4 = -15.989$

$\therefore H_F = 10.68$

$\therefore P = 13.93$

Now,

$\text{Factor} = \frac{R}{P} = \frac{5.447}{13.93} = 0.39$

Then

$M_{AB} = 21.318 + 0.39 \times (-4.94)$

$= 19.388$

$M_{BA} = 42.636 + 0.39 \times (-3.876)$

$= 41.124$

$M_{BC} = -42.659 + 0.39 \times (3.876)$

$= -41.124$

$M_{CB} = 82.849 + 0.39 \times (1.738) = 83.527$

$M_{CD} = -35.397 + 0.39 \times (-9.543) = -39.118$

$M_{CE} = -47.438 + 0.39 \times (7.795) = -44.397$

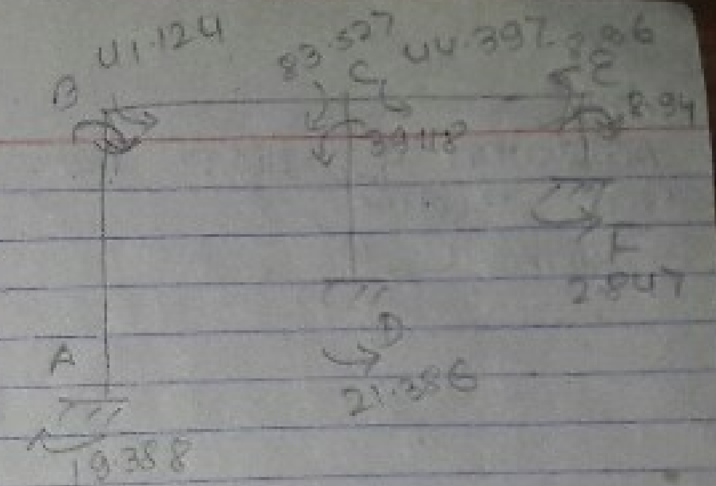
$M_{EC} = -15.192 + 0.39 \times 15.971 = -8.96$

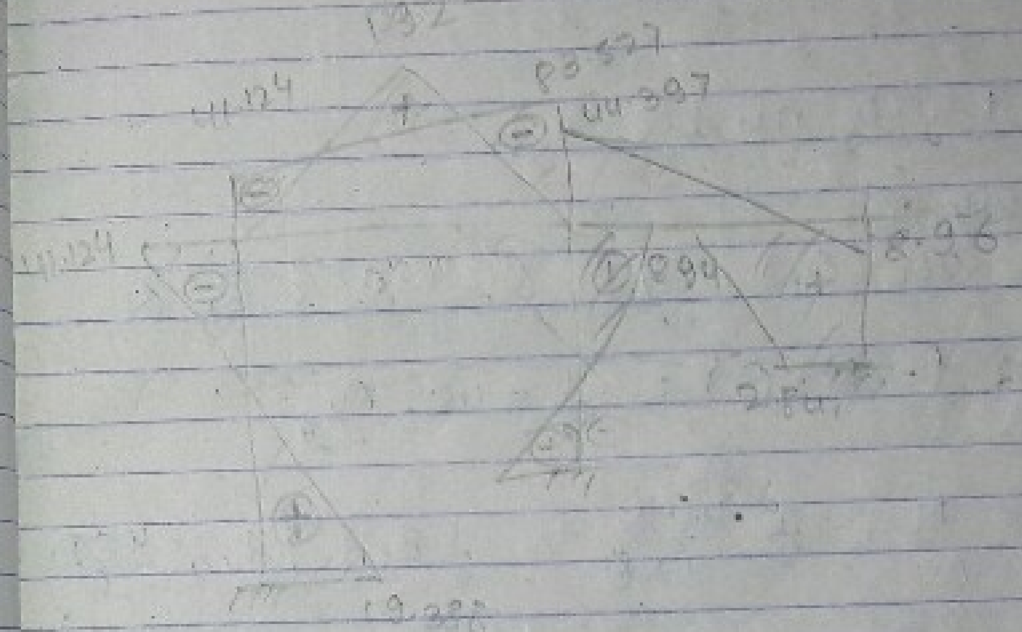
$M_{EF} = 15.174 + 0.39 \times -15.989 = 8.938$

$M_{FE} = \cancel{8.938}\ 7.583 + 0.39 \times (-26.744)$

$= -2.847$

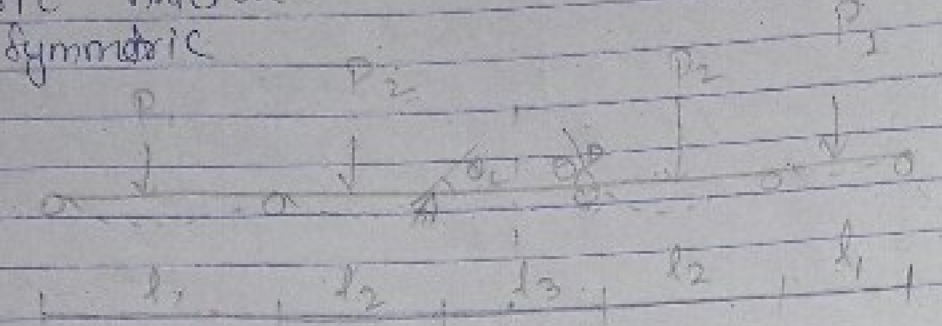
$M_{DC} = -21.386$





Application of MDM for symmetric & anti-symmetric structure.

a) Symmetric



Symmetric abt CD member Here,

$\theta_C = \theta_D$

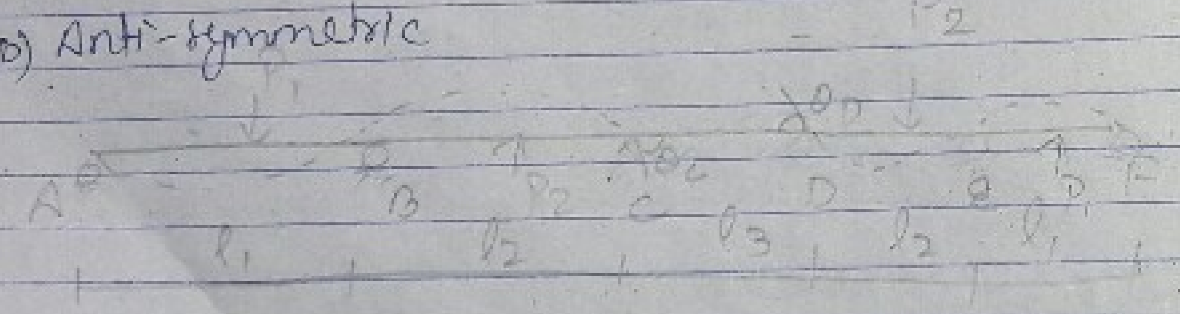
From slope deflection,

$$M_{CD} = \frac{2EI}{l_3}(2\theta_C + \theta_D) \quad \text{where } \theta_C = -\theta \ \& \ \theta_D = \theta$$

$$\therefore M_{CD} = \frac{2EI}{l_3}(-2\theta + \theta) = -\frac{1}{2} \times 4EI\,\theta/l$$

$$K_{modified} = \frac{M}{\theta} = \frac{1}{2}\frac{4EI\theta}{l \times \theta} = \frac{1}{2}K_{CD} \quad \left[K_{CD} = \frac{4EI}{l}\right]$$

b) Anti-symmetric

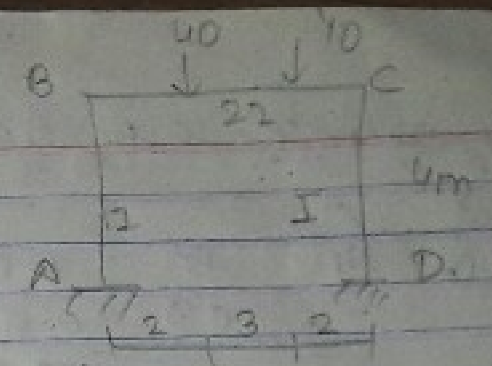


$$M_{CD} = \frac{2EI}{l}[2\theta + \theta] \qquad \theta_C = \theta_D = \theta$$

$$= \frac{6EI\theta}{l} = \frac{3}{2} \times \frac{4EI\theta}{l} = \frac{3}{2} \times K_{CD}$$

$$\therefore K_{modif} = \frac{3}{2}K_{CD}$$

Q.



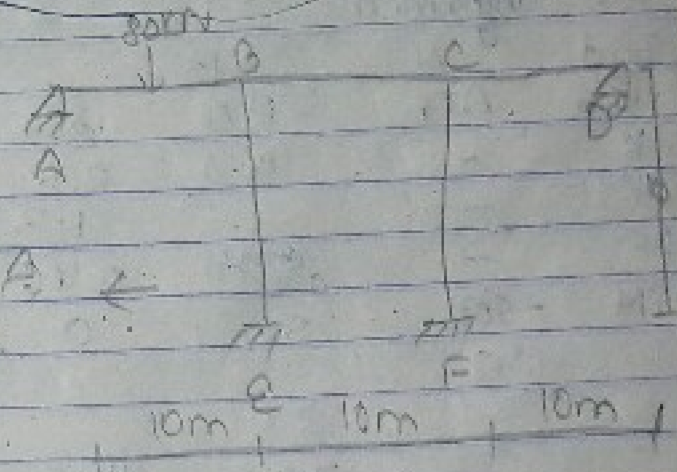
Joint	A	B	
Mem	AB	BA	BC
K	–	EI	$\frac{1}{2} \times \frac{4 \times 2EI}{L} = 0.571EI$
ΣK	–	1.571 EI	
DF	–	0.637	0.363
FEM	0	0	–57.143
		36.4	20.743
	18.2		
Total	18.2	36.4	–36.4

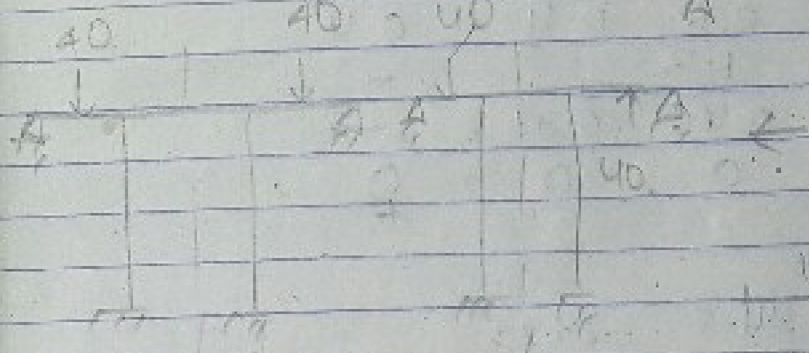
$\therefore M_{CB} = 36.4$

$M_{CD} = -36.4$

$M_{DC} = -18.2$

Q.





Symmetric (A)

antisymmetric (B)

For symmetric case than.

Joint	A	B			E
Mom	AB	BA	BC	BE	EB
K	–	0.3	0.2	0.4	0.4
ΣK	–		0.9	0.45	–
DF	–	0.33	0.22		–
FEM	-50	50	0	0	0
BM	50				
COM		25			
		-24.75	-16.5	-33.75	
					-16.875
Total:	0	50.25	-16.5	-33.75	-16.875

For antisymmetric

Joint	A	B			E
Mom	AB	BA	BC	BE	EB
K	–	0.3	0.6	0.4	0.4
ΣK	–		1.3		–
DF	–	0.23	0.46	0.31	–
FEM	-50	50	0	0	0
	50				
		25			
		-17.25	-34.5	-23.25	
					-11.625
Total	0	57.75	-34.5	-23.25	-11.625

∴ Total Moment

$M_{AB} = 0$

$M_{BA} = 108$

$M_{BC} = -51$

$M_{BE} = -57$

$M_{EB} = -28.5$

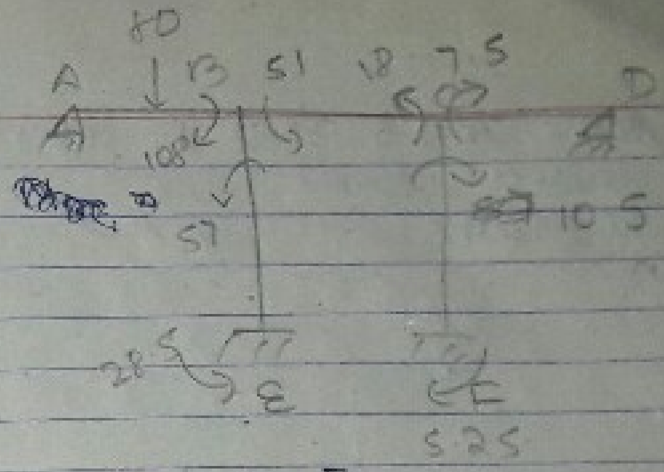
$M_{DC} = 0$

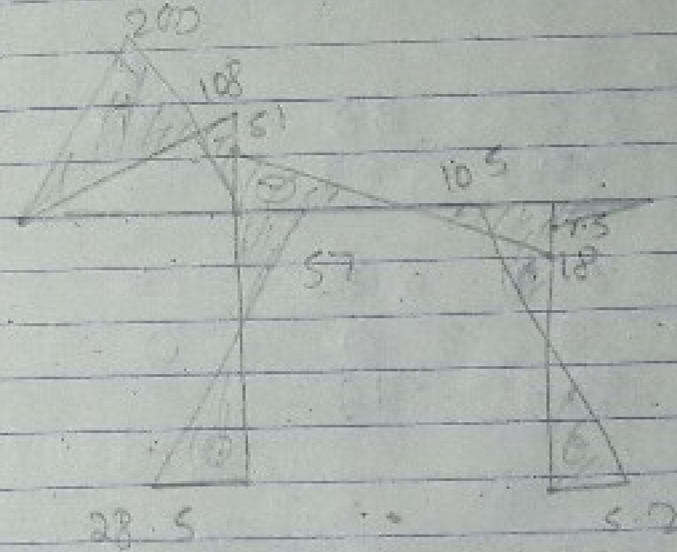
$M_{CD} = -50.25 + 57.75 = 7.5$

$M_{CF} = 33.75 + 23.25 = 10.5$

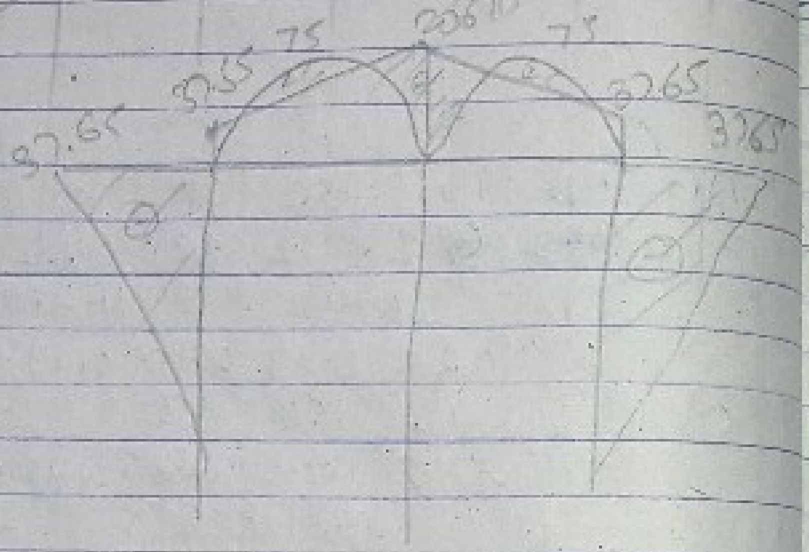
$M_{FC} = 16.875 + 11.625 = 5.25$

$M_{CB} = 16.5 - 34.5 = -18$



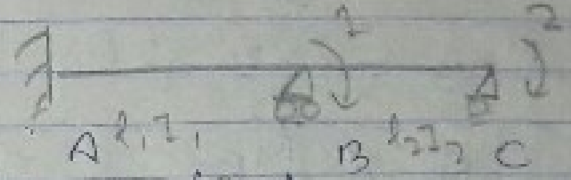


AB	BA	BC	CB	CD	CE	EC	EF	FC	DC
	0.048	0.014	0.082	0.027	0.082	0.143	0.047		
		0.041	0.00716		0.071	0.041			
0	-5.671	5.582	3.544	-7.074	3.609	5.711	-5.671		



Stiffness matrix method

Let ABC be a indeter. continous beam as shown whose Degree of freedom is 2. (fig 1)



(fig 1)

$\therefore$ Stiffness matrix

$$[K] = \begin{bmatrix} K_{11} & K_{12} \\ K_{21} & K_{22} \end{bmatrix}$$

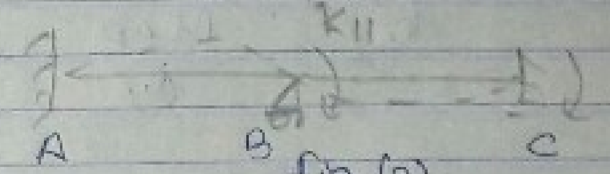


fig (2)

To find value of K_{11}, 1st stiffness coefficient, we 1st assign co-ordinate 1 at joint B & lock join C. In this case assign $\theta_B = 1$. i.e. unit displacement at B. (fig 2)

Then

$$M_{BA} = \frac{2EI_1}{l_1}(2\theta_B + \theta_A) = \frac{4EI_1\theta_B}{l_1}$$

$$M_{BC} = \frac{2EI_2}{l_2} \times 2\theta_B = \frac{4EI_2}{l_2}\theta_B$$

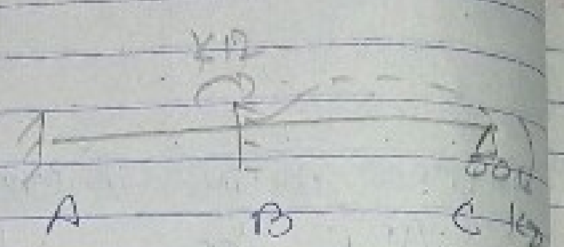
Then $K_{11} = \dfrac{M_{BA} + M_{BC}}{\theta_B} = \left(\dfrac{4EI_1}{l_1} + \dfrac{4EI_2}{l_2}\right)$

Again,

$$M_{CB} = \frac{2EI_2}{l_2} \times \theta_B$$

$$\therefore K_{21} = \frac{M_{CB}}{\theta_B} = \frac{2EI_2}{l_2}$$

Again assign co-ordinate 2 at C & lock join B. i.e. $\theta_c = 1$.



then

$$M_{BC} = \frac{2EI_2}{l_2}\theta_c$$

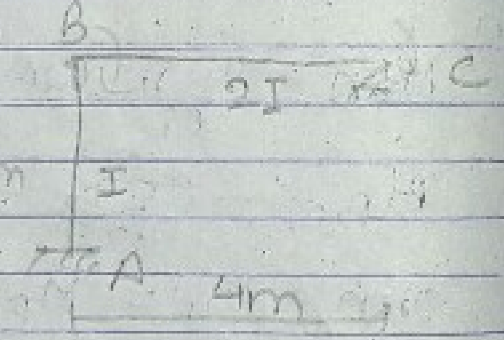
$$M_{CB} = \frac{2EI_2 \times 2\theta_c}{l_2}$$

$$\therefore K_{12} = \frac{M_{BC}}{\theta_c} = \frac{2EI_2}{l_2}$$

$$K_{22} = \frac{M_{CB}}{\theta_c} = \frac{4EI_2}{l_2}$$

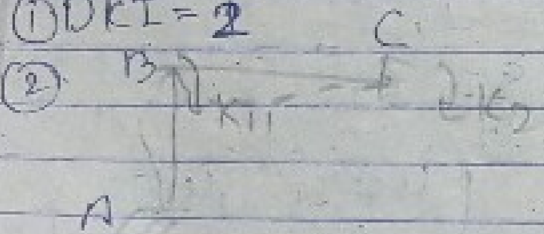
$$\therefore K = \begin{bmatrix} \frac{4EI_1}{l_1} + \frac{4EI_2}{l_2} & \frac{2EI_2}{l_2} \\ \frac{2EI_2}{l_2} & \frac{4EI_2}{l_2} \end{bmatrix}$$

Q. Generate stiffness matrix.



(1) DKI = 2

(2)

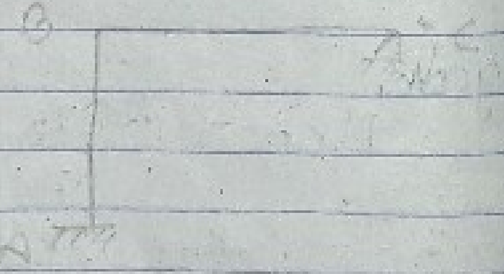


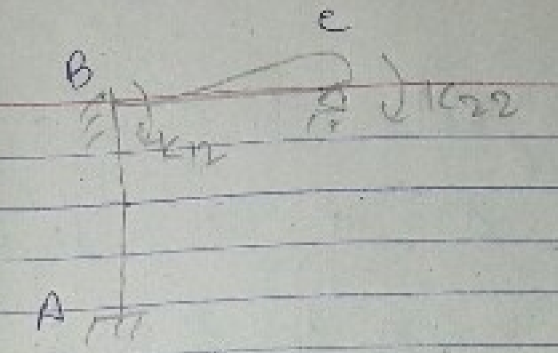
$$K_{11} = M_{BA} + M_{BC}$$

$$= \frac{4EI}{3} + \frac{8EI}{4}$$

$$= \frac{16+24}{12}EI = \frac{40}{12}EI$$

$$K_{21} = \frac{4EI}{4} = EI$$





$$K_{12} = M_{BC}$$

$$= \frac{2EI}{l} \times \theta_c$$

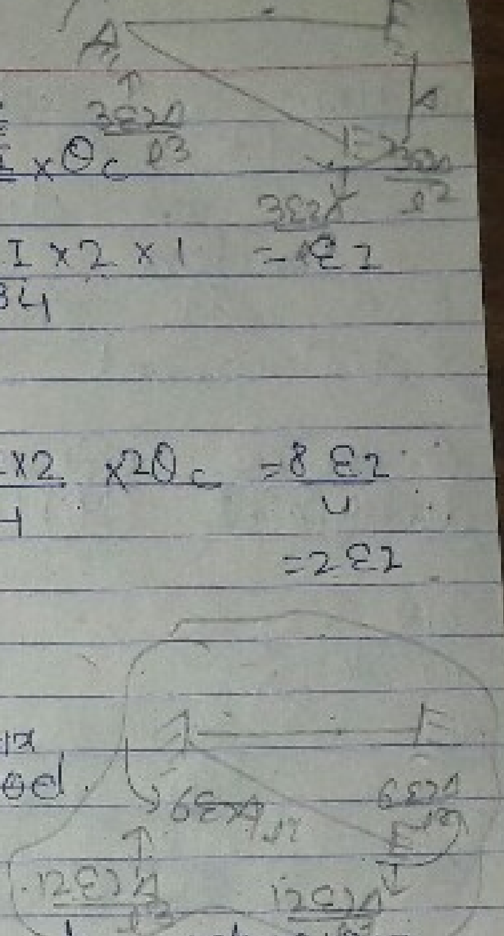
$$= \frac{2EI \times 2 \times 1}{4} = EI$$

$$K_{22} = M_{CB}$$

$$= \frac{2EI \times 2}{4} \times 2\theta_c = \frac{8EI}{4}$$

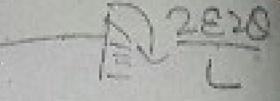
$$= 2EI$$

$$\therefore \begin{bmatrix} \frac{10}{3}EI & EI \\ EI & 2EI \end{bmatrix}$$



Procedure of stiffness matrix method.

1. Determine DKI.
2. Assign co-ordinate direction to unknown displacement
3. Impose restrain in all co-ordinate direction to get a fully restrain structure.
4. Determine the forces developed in each of co-ordinate direction of fully restrain structure, it is called P_L. [FEM calculation]
5. Determine stiffness matrix K by giving unit displacement to restrain structure in each of co-ordinate direction
6. Observing final forces in various co-ordinate direction, note down final force P.



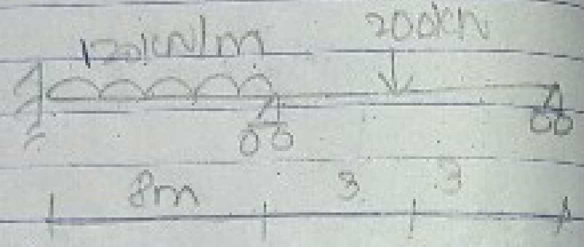
7. Form and solve stiffness eq^n

$[K][\Delta] = [P - P_L]$

to get displacement Δ in co-ordinate direction

8. Calculate mem forces using these joint displacement.

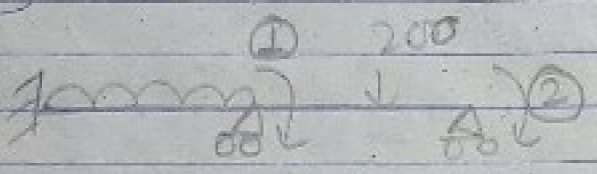
Q. Analyse continuous beam shown in fig by displacement (stiffness method) Take EI = constant



Sol^n

(i) DKI = 2

(ii)



(iii) Calculate FEM

$M_{AB} = \cancel{-106.67}\ -640$

$M_{BA} = \cancel{+106.67}\ 640$

$M_{BC} = -150$

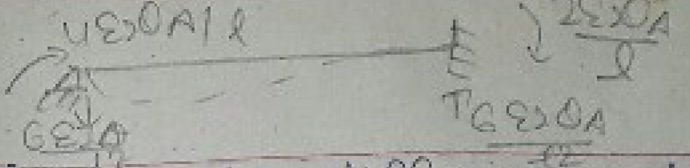
$M_{CB} = 150$

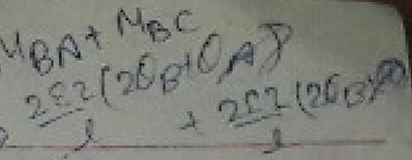
$\therefore$ The moment developed in co-ordinate dir

$P_L = \begin{bmatrix} P_{1L} \\ P_{2L} \end{bmatrix}$

$\therefore P_{1L} = 640 - 150 = 490$

$P_{2L} = 150$





(iv) Generate stiffness matrix.

$K_{11} = \frac{4EI_1}{l_1} + \frac{4EI_2}{l_2} = \frac{4EI}{8} + \frac{4EI}{6} = \frac{7}{6}EI$

$K_{12} = K_{21} = \frac{2EI_2}{l_2} = \frac{2EI}{6} = \frac{EI}{3}$

$K_{22} = \frac{4EI_2}{l_2} = \frac{4EI}{6}$

$\therefore \begin{bmatrix} 7/6\,EI & 1/3\,EI \\ 1/3\,EI & 2/3\,EI \end{bmatrix} = [K]$

(5) Final forces acting in co-ordinat direction (1) & (2) are zero.

$[P] = 0$

(6) Using stiffness Eq^n.

$[K][\Delta] = [P - P_L]$

$\Rightarrow \begin{bmatrix} 7/6\,EI & 1/3\,EI \\ 1/3\,EI & 2/3\,EI \end{bmatrix} \begin{bmatrix} \theta_B \\ \theta_C \end{bmatrix} = \begin{bmatrix} -490 \\ -150 \end{bmatrix}$

$\therefore \begin{bmatrix} \theta_B \\ \theta_C \end{bmatrix} = \begin{matrix} -415/EI \\ \frac{-17.5}{EI} \end{matrix}$

Now, Using slope deflection Eq^n

$M_{AB} = -640 + \frac{2EI}{8}\left(\frac{-415}{EI}\right) = -743.75$

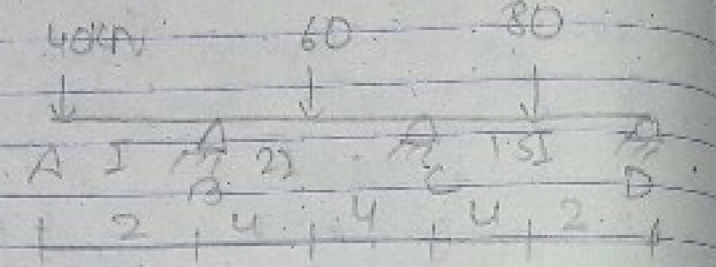
$M_{BA} = 640 + \frac{2EI}{8}\left[2 \times \left(\frac{-415}{EI}\right)\right] = 432.5$

$$M_{BC} = -150 + \frac{2EI}{6}\left(2\left(\frac{-415}{EI}\right) - \frac{17.5}{EI}\right)$$
$= -432.5$

$$M_{CB} = 150 + \frac{2EI}{6}\left[\frac{2\times -17.5}{EI} - \frac{415}{EI}\right]$$
$= 0$ ✗

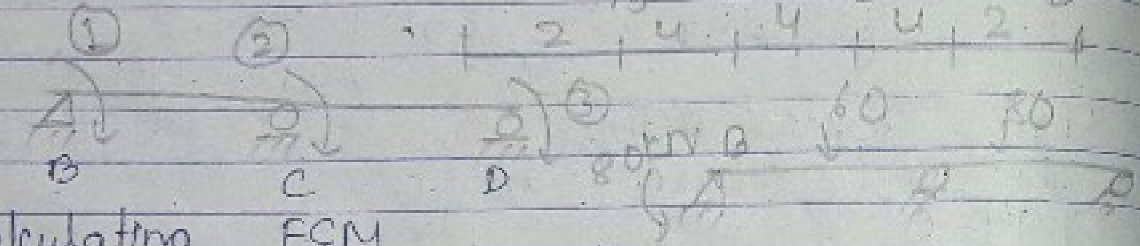
Q. (Draw BMD & SFD)

Soln



(i) DKI = 3

(ii)



(iii) Calculating FEM

$M_{AB} = M_{BA} = 0$

$M_{BC} = -60$

$M_{CB} = 60$

$M_{CD} = -35.55$

$M_{DC} = 71.111$

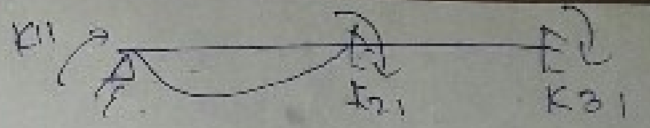
Then,

(iv) $P_L = \begin{bmatrix} P_{1L} \\ P_{2L} \\ P_{3L} \end{bmatrix} = \begin{bmatrix} -60 \\ 24.45 \\ 71.11 \end{bmatrix}$

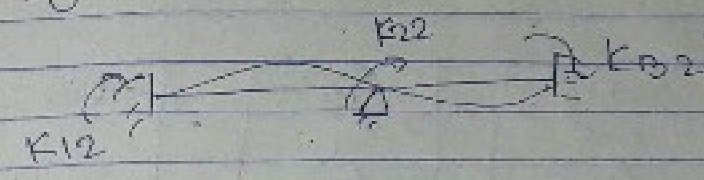
(iv) Generate stiffness matrix.

$K_{11} = M_{BC} = \frac{2EI \times 2}{8} \times \theta_B = \frac{EI}{4}$

$$K_{21} = \frac{2EI}{l_1} + \frac{2EI}{l_2}$$
$$= \frac{2\times 2EI}{8} + \frac{2\times 1.5EI}{6} = 0.5EI$$



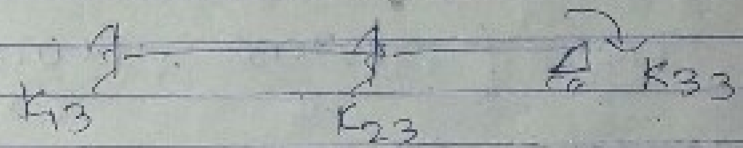
$K_{31} = 0$



$\therefore K_{12} = K_{21} = 0.5EI$

$$K_{22} = \frac{4\times 2EI}{8} + \frac{4\times 1.5EI}{6}$$
$= 2EI$

$$K_{32} = \frac{2\times 1.5EI}{6} = 0.5EI$$



$$\therefore K_{33} = \frac{4\times 1.5EI}{6} = EI$$

$$\therefore [K] = \begin{bmatrix} EI & 0.5EI & 0 \\ 0.5EI & 2EI & 0.5EI \\ 0 & 0.5EI & EI \end{bmatrix}$$

Then,

(v) Find force acting in co-ordinate direction at B, C, D are -80, 0, 0

(vi) Now, Eqn

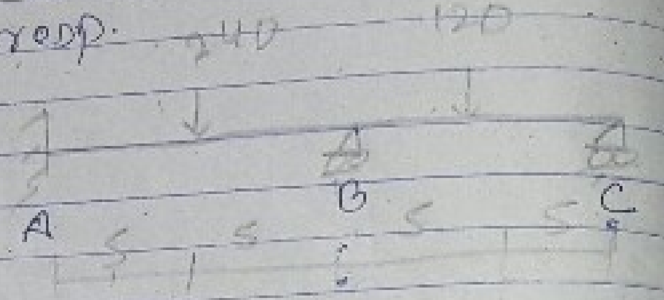
$$\begin{bmatrix} 8EI & 0.5 & 0 \\ 0.5 & 2 & 0.5 \\ 0 & 0.5EI & EI \end{bmatrix} \begin{bmatrix} \theta_B \\ \theta_C \\ \theta_D \end{bmatrix} = \begin{bmatrix} -20 \\ -24 \\ -71.11 \end{bmatrix}$$

$\therefore \theta_B = -27.0381/EI$
$\theta_C = 14.076/EI$
$\theta_D = -78.14/EI$

Now
(1) By slope deflection.

$$M_{BC} = -60 + \frac{4EI}{8}\left(-\frac{2\times27.038}{EI} + \frac{14.076}{EI}\right)$$
$$= -80$$
$$M_{CB} = 60 + \frac{4EI}{8}\left(\frac{2\times14.076}{EI} - \frac{27.038}{EI}\right)$$
$$= 60.557$$
$$M_{CD} = -35.555 + \frac{3EI}{6}\left(\frac{2\times14.076}{EI} - \frac{78.14}{EI}\right)$$
$$= -60.549$$
$$M_{BA} = \frac{2EI}{2}\left(\frac{2\times-27.038}{EI}\right) =$$

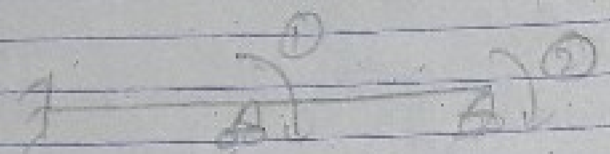
Analyse Cont. beam by stiffness if downward settlement of B & C in kN/m are $20000/EI$ & $\frac{1000}{EI}$ resp.



Soln

① $D \times 2 = 2$

②



③ Calculation of FEM

By loading + settlement

$M_{AB} = -300 - 120 = -420$

$M_{BA} = 300 - 120 = 180$

$M_{BC} = -150 + \frac{6 \times 1000}{10^2} = -90$

$M_{CB} = 150 + 60 = 210$

Then

$$P_L = \begin{bmatrix} P_{L1} \\ P_{L2} \end{bmatrix} = \begin{bmatrix} 90 \\ 210 \end{bmatrix}$$

④ Stiffness

$K_{11} = \frac{4EI}{10} + \frac{4EI}{10} = 0.8EI$

$K_{12} = K_{21} = \frac{2 \times EI}{10} = 0.2EI$

$K_{22} = \frac{4EI}{10} = 0.4EI$

$$\therefore [K] = \begin{bmatrix} 0.8EI & 0.2EI \\ 0.2EI & 0.4EI \end{bmatrix}$$

⑤ Now, P forces is 0

⑥ $[K][\Delta] = [P - P_L]$

$$\begin{bmatrix} 0.8EI & 0.2EI \\ 0.2EI & 0.4EI \end{bmatrix} \begin{bmatrix} \theta_B \\ \theta_C \end{bmatrix} = \begin{bmatrix} -90 \\ -210 \end{bmatrix}$$

$\theta_B = 21.428/EI$

$\theta_C = -535.7/EI$

∴ By slope deflection

$M_{AB} = -420 + \frac{2EI}{10}\left(\frac{21.428}{EI}\right) + 0$

$= -415.7144$

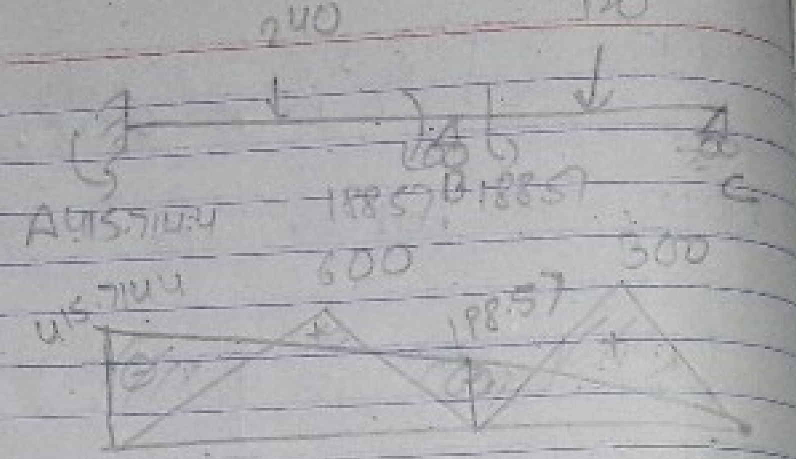
$M_{BA} = 180 + \frac{2EI}{10}\left(\frac{2 \times 21.428}{EI}\right) = 188.57$

$M_{BC} = -90 + \frac{2EI}{10}\left[\frac{2 \times 21.428}{EI} - \frac{535.7}{EI}\right]$

$= -188.568$

$M_{CB} = 210 + \frac{EI}{5}\left[-\frac{535.7 \times 2}{EI} + \frac{21.428}{EI}\right]$

$= 0$



BMD

Now, For SFD.

For AB

$-415.714 + R_A \times 10 - 240 \times 5 = -188.57$

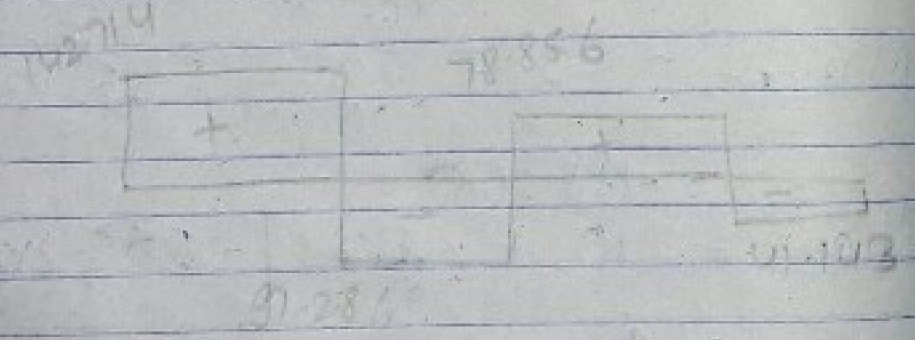
$\therefore R_A = 142.714$

For BC

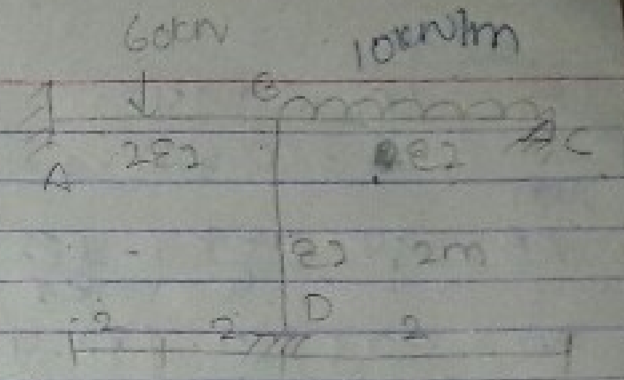
$R_C \times 10 - 120 \times 5 = -188.57$

$\therefore R_C = 41.143$

$\therefore R_B = 176.143$



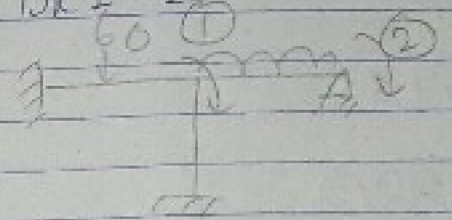
$\frac{PL}{8}$ $\frac{wl^2}{12}$



Q. Soln

(i) DK I = 2

(ii)



(iii) FEM

$M_{AB} = -30$

$M_{BA} = 30$

$M_{BD} = M_{DB} = 0$

$M_{BC} = -3.333$

$M_{CB} = 3.333$

Now, $P_L = \begin{bmatrix} 26.67 \\ 3.33 \end{bmatrix}$

(iv) Stiffness matrix

$K_{11} = \frac{4EI}{l_1} + \frac{4EI}{l_2} + \frac{4EI}{l_3} = \frac{8EI}{4} + \frac{4EI}{2} + \frac{4EI}{2}$

$= 6EI$

$K_{12} = K_{21} = \frac{2EI \times 2}{2} = EI$

$K_{22} = \frac{4EI \times 2}{2} = 2EI$

$\therefore [K] = \begin{bmatrix} 6EI & EI \\ EI & 2EI \end{bmatrix}$

(v) ~~The force~~ is 0

(vi) $[K].[\Delta] = [P - P_L]$

or, $\begin{bmatrix} 6EI & EI \\ EI & 2EI \end{bmatrix} \begin{bmatrix} \theta_B \\ \theta_C \end{bmatrix} = \begin{bmatrix} -26.67 \\ 3.33 \end{bmatrix}$

$\therefore \theta_B = -4.551/EI$
$\theta_C = 0.6081/EI$

Then

$AB = -30 + \frac{4EI}{4}\left(-\frac{4.55}{EI}\right) = -34.55$

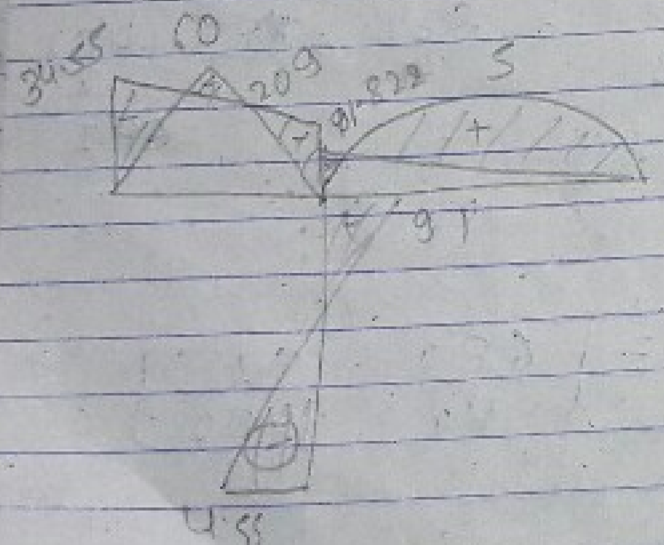
$BA = 30 + \frac{4EI}{4}\left(2\times -\frac{4.55}{EI}\right) = 20.9$

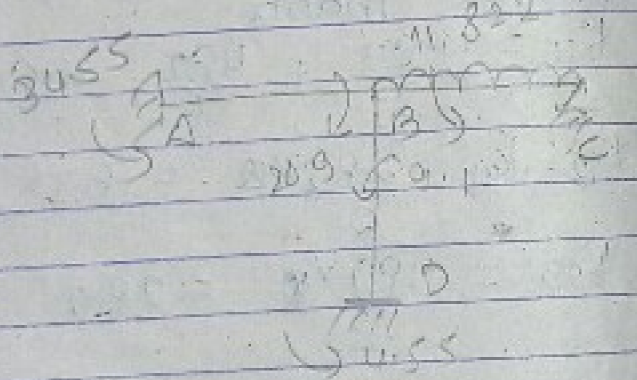
$BC = -3.33 + \frac{2EI}{2}\left(2\times -\frac{4.55}{EI} + \frac{0.608}{EI}\right) = -11.822$

$CB = 3.33 + \frac{2EI}{2}\left(2\times\frac{0.608}{EI} + \frac{-4.55}{EI}\right) = 0$

$BD = \frac{2EI}{2}\left(2\times -\frac{4.55}{EI}\right) = -9.1$

$DB = \frac{2EI}{2}\left(\frac{-4.55}{EI}\right) = -4.55$

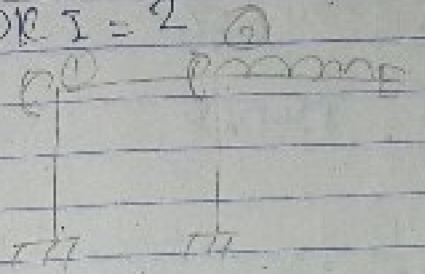


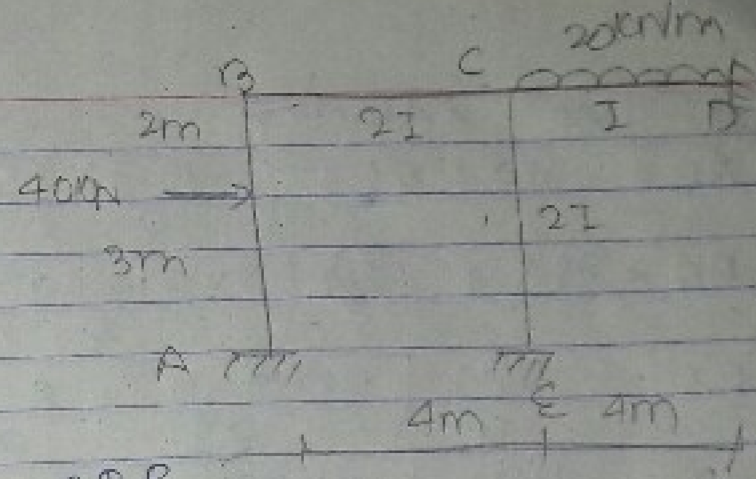


Soln

(i) DKI = 2

(ii)





(iii) $AB = -19.2$, $BA = 28.8$
$BC = CB = CE = EC = 0$
$CD = -26.67$
$DC = 26.67$
$P_L = \begin{bmatrix} 28.8 \\ -26.67 \end{bmatrix}$

(iv) Stiffness
$K_{11} = \frac{4EI}{5} + \frac{8EI}{4} = 2.8EI$

$K_{12} = K_{21} = \frac{2EI\times 2}{4} = EI$

$K_{22} = \frac{8EI}{4} + \frac{8EI}{5} + \frac{4EI}{4} = 4.6EI$

$\therefore [K] = \begin{bmatrix} 2.8EI & EI \\ EI & 4.6EI \end{bmatrix}$

(v) Force acting in co-ordinate dirn is 0.

(vi) $\begin{bmatrix} 2.8EI & EI \\ EI & 4.6EI \end{bmatrix}\begin{bmatrix} \theta_B \\ \theta_C \end{bmatrix} = \begin{bmatrix} -28.8 \\ 26.67 \end{bmatrix}$

$\therefore \theta_B = -13.39/EI$
$\theta_C = 8.71/EI$

By slope deflection

$AB = -19.2 + \frac{2EI}{5}\left(\frac{-13.39}{EI}\right) = -24.556$

$BA = 288 + \frac{2EI}{5}\left(2\times\frac{-13.39}{EI}\right) = 18.088$

$BC = \frac{4EI}{4}\left[2\times\frac{-13.39}{EI} + \frac{8.71}{EI}\right] = -18.07$

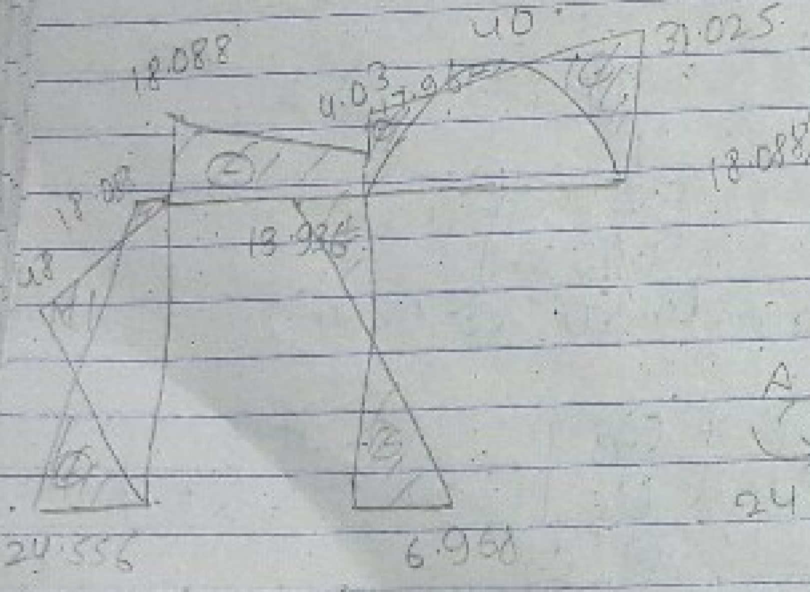
$CB = \frac{4EI}{4}\left[\frac{2\times 8.71}{EI} - \frac{13.39}{EI}\right] = 4.03$

$CE = \frac{4EI}{5}\left[\frac{2\times 8.71}{EI}\right] = 13.936$

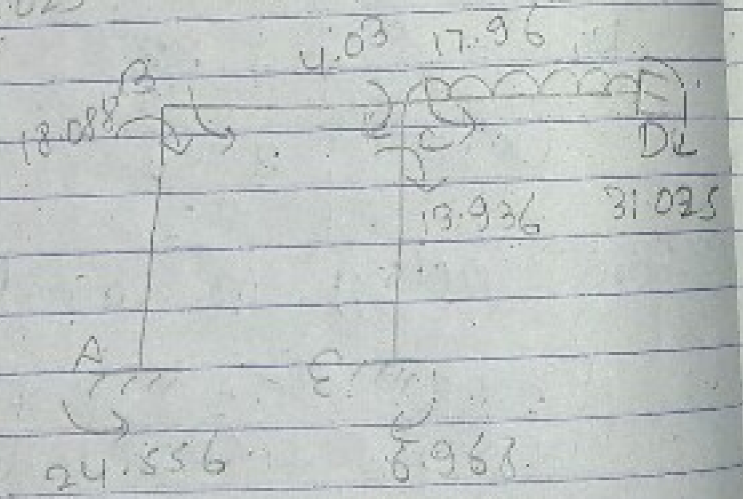
$EC = \frac{4EI}{5}\left[\frac{8.71}{EI}\right] = 6.968$

$CD = -26.67 + \frac{2EI}{4}\left[\frac{2\times 8.71}{EI}\right] = -17.96$

$DC = 26.67 + \frac{2EI}{4}\left[\frac{8.71}{EI}\right] = 31.025$

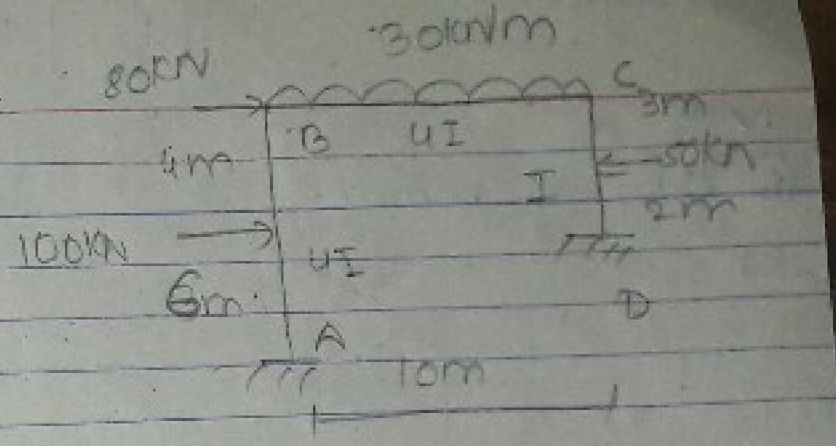


(BMD)

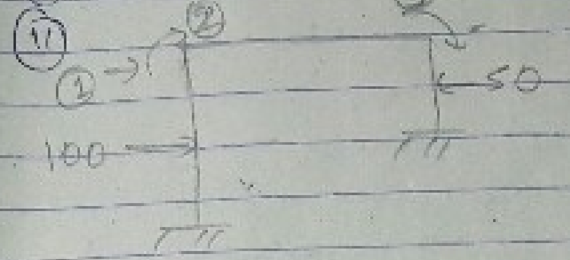


ans.
Soln

(i) DKI = 3



(ii)



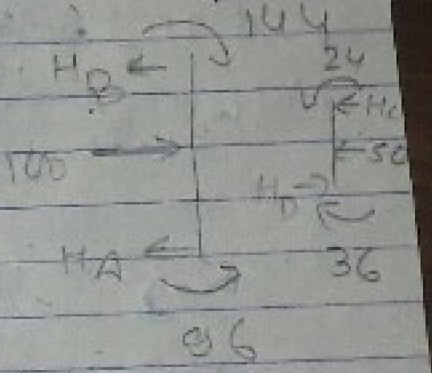
(iii) FEM

$AB = -97.959$ 96, $BC = -250$, $CD = -24$

$BA = 73.469$ 144, $CB = 250$, $DC = 24$ 36

Then

$$P_L = \begin{bmatrix} P_{1L} \\ P_{2L} \\ P_{3L} \end{bmatrix} = \begin{bmatrix} -47.2 \\ -106 \\ 226 \end{bmatrix}$$



$-96 - 100\times 4 + H_A \times 10 = -144$

$H_A = 35.2$ (←)

$-24 - H_C \times 5 - 50\times 2 = -36$

$H_C = 17.6$ (→)

$-144 + H_B \times 10 - 100\times 6 = -96$

$H_B = 64.8$ (←)

$H_D \times 5 - 36 - 50\times 3 = -24$

$H_D = 32.4$ (→)

$\therefore P_{1L} = H_B + H_C$

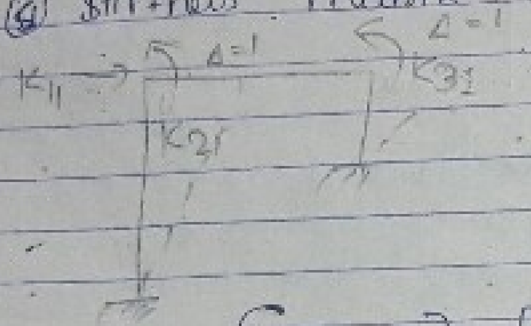
$= -64.8 + 17.6$

$= 47.2$ (←)

① Final force $P = \begin{bmatrix} P_1 \\ P_2 \\ P_3 \end{bmatrix} = \begin{bmatrix} 80 \\ 0 \\ 0 \end{bmatrix}$

Now,

② Stiffness matrix.

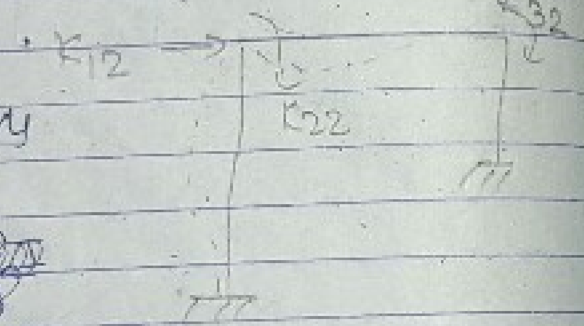


$\therefore K_{11} = \left[\frac{12 \times 4EI}{10^3}\right]_{AB} + \left[\frac{12 \times EI}{5^3}\right]_{CD}$

$= 0.144\ EI$

$K_{21} = \frac{6 \times 4EI}{10^2} = -0.24EI$

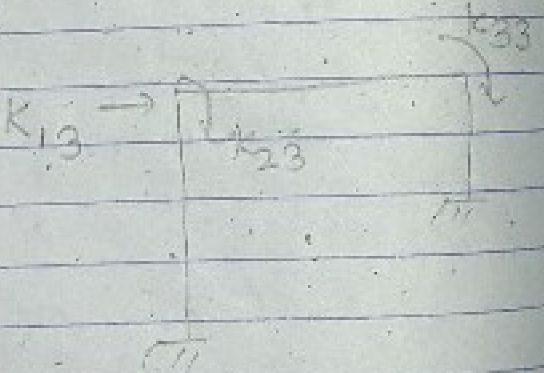
$K_{31} = -\frac{6 \times EI}{5^2} = -0.24EI$



$\therefore K_{22} = \frac{4EI \times 4}{10} + \frac{4EI \times 4}{10} = 3.2EI$

$K_{32} = \frac{2EI \times 4}{10} + \frac{2EI}{5}$

$= 0.8EI.$



$\therefore K_{33} = \frac{4 \times 4EI}{10} + \frac{4EI}{5}$

$= 2.4EI.$

$\therefore [K] = \begin{bmatrix} 0.144EI & -0.24EI & -0.24EI \\ -0.24EI & 3.2EI & 0.8EI \\ -0.24EI & 0.8EI & 2.4EI \end{bmatrix}$

Then

$[K][\Delta] = \begin{bmatrix} 127.2 \\ 106 \\ -226 \end{bmatrix}$

$\therefore$ ② $\Delta_B = 1028.92/EI$

$\theta_B = 117.94/EI$

$\theta_C = -30.58/EI$

Then,

$AB = -96 + \frac{8EI}{10}\left[\frac{117.94}{EI} - \frac{3 \times 1028.92}{10EI}\right] = -248.584$

$BA = 144 + \frac{8EI}{10}\left[\frac{2 \times 117.94}{EI} - \frac{3 \times 1028.92}{10EI}\right]$

$= 85.76$

$BC = -250 + \frac{8EI}{10}\left[\frac{2 \times 117.94}{EI} - \frac{30.58}{EI}\right]$

$= -85.76$

$CB = 250 + \frac{8EI}{10}\left[\frac{2 \times -30.58}{EI} + \frac{117.94}{EI}\right]$

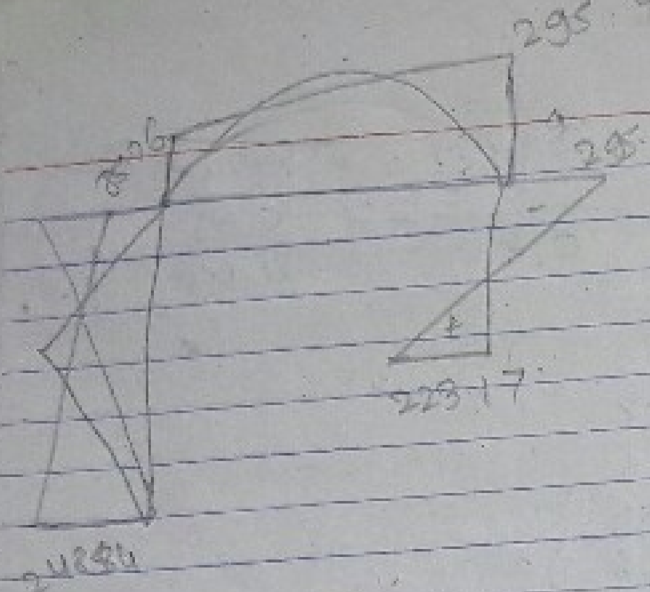
$= 295.4$

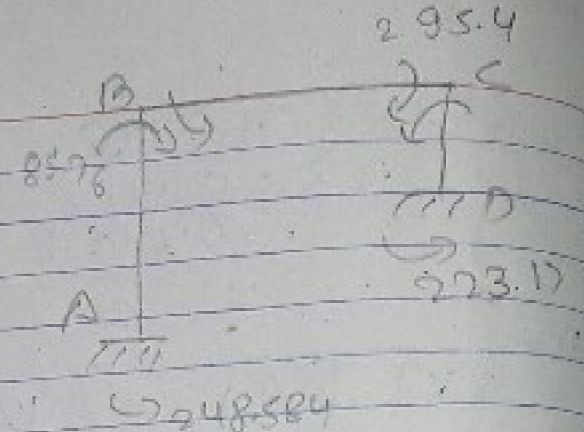
$CD = -24 + \frac{2EI}{5}\left[2 \times \left(\frac{-30.58}{EI}\right) - \frac{3 \times 1028.92}{5EI}\right]$

$= -295.4$

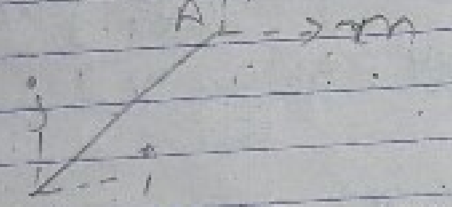
$DC = 36 + \frac{2EI}{5}\left[\frac{-30.58}{EI} - \frac{3 \times 1028.92}{5EI}\right]$

$= \cancel{23.768}\ -223.1723$





Analysis of pin jointed frame by direct stiffness method.



- Pin joint offers resistance to elongates & contraction of member meeting at that pt.

Case 1. Displacement along co-ordinate i

the displacement along i-direction = $OO' = 1$

the displacement along member = OO''

$= OO' \cos\theta$

$= \cos\theta$

The compressive force required to produce this displacement is given as

force (P) $= \frac{\Delta \times AE}{L} = \frac{AE \cos\theta}{L}$

force along i-direct $= P\cos\theta$

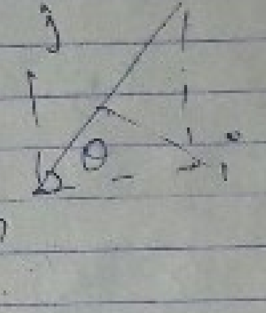
$= \frac{AE\cos^2\theta}{L} = k_{ii}$

force along j-direction $= P\sin\theta$

$= \frac{AE}{L}\sin\theta\cos\theta = k_{ji}$

$\therefore k_{mi} = -\frac{AE\cos^2\theta}{L}$

$k_{ni} = -\frac{\Sigma AE}{L}\sin\theta\cos\theta$

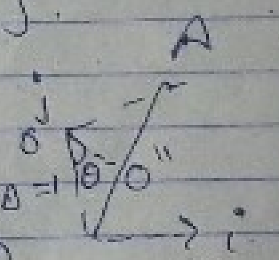


$\therefore$ Joint stiffness

$k_{ii} = \Sigma AE\cos^2\theta / L$

$k_{ji} = \Sigma AE \frac{\sin\theta\cos\theta}{L}$

Case II Displacement along co-ordinate j.



The displacement along j direction.

$OO' = 1$

The displacement along mem: OO''

$= OO'\sin\theta$

$= \sin\theta$

The compressive force required to produce this displacement is given as force (P) $= \frac{AE\sin\theta}{L}$

force along i-direction $= P\cos\theta$

$= \frac{AE}{L}\cos\theta\sin\theta = k_{ij}$

Force along j-direction = $P\sin\theta = \frac{AE\sin^2\theta}{L} = k_{jj}$

$\therefore k_{mj} = -\frac{AE}{L}\cos\theta\sin\theta$

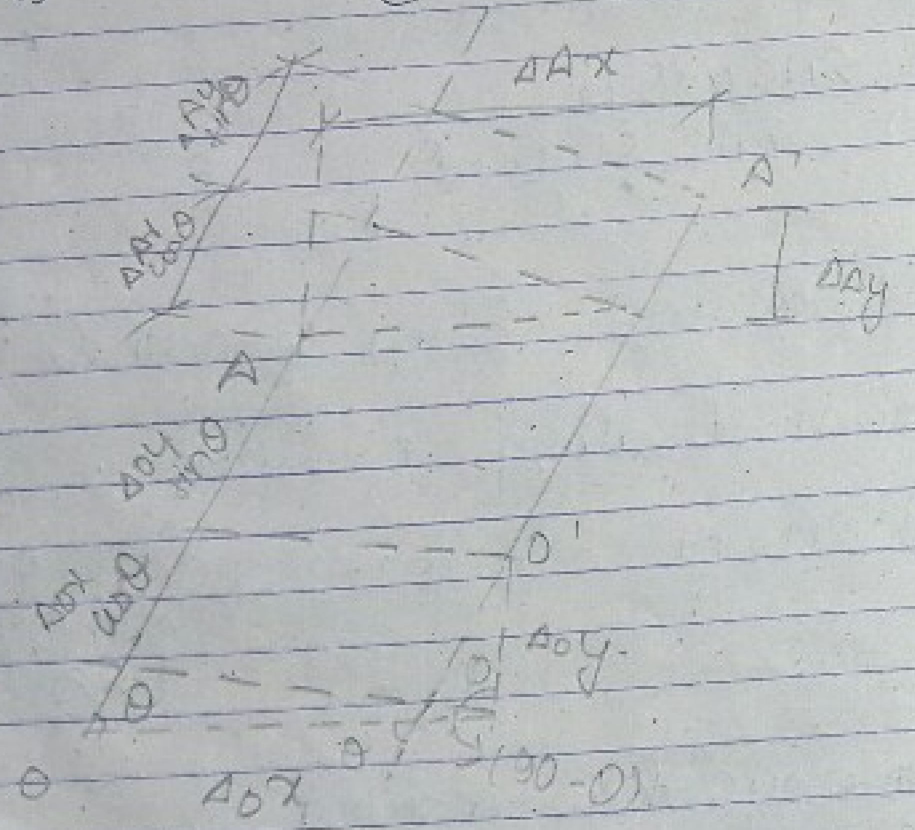
$k_{nj} = -\frac{AE}{L}\sin^2\theta$

$\therefore$ joint stiffness

$k_{ij} = \Sigma\frac{AE\cos\theta\sin\theta}{L}$

$k_{jj} = \Sigma AE\frac{\sin^2\theta}{L}$

#



Let AO be truss member joining a pin joint A & O. Let A' & O' are displaced position of joint A & O resp.

Δ_{Ox} & Δ_{Oy} be displacement along x & y dirn resp at joint O & similarly displ. at A are Δ_{Ax} and Δ_{Ay}.

Shortening of mem. due to displacement of O
$= \Delta_{Ox}\cos\theta + \Delta_{Oy}\sin\theta$

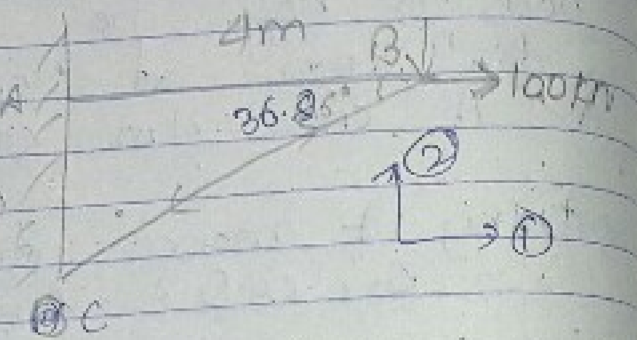
Extension of member due to displacement A
$= \Delta_{Ax}\cos\theta + \Delta_{Ay}\sin\theta$

$\therefore$ Total elongation / displacement =
$(\Delta_{Ax} - \Delta_{Ox})\cos\theta + (\Delta_{Ay} - \Delta_{Oy})\sin\theta$

$\therefore$ Mem. force $= S_{OA} = \frac{AE}{L}\left[(\Delta_{Ax} - \Delta_{Ox})\cos\theta + (\Delta_{Ay} - \Delta_{Oy})\sin\theta\right]$

where θ is always measured in anticlockwise direction

Use stiffness matrix to find mem force of given truss. Assume axial stiffness of each member $= 400\ kN/cm$.



Soln

Axial stiffness $= \frac{AE}{L}$

$= 400\ kN/cm$

$= 40000\ kN/m$

Mem	θ	AE/L	$\frac{AE}{L}\cos^2\theta$	$\frac{AE}{L}\sin^2\theta$	$\frac{AE}{L}\sin\theta\cos\theta$
BA	180	40000	40000	0	0
BC	216.83	40000	25599.93	14400.0	19200

$K_{11} = 65600 \quad K_{22} = 1440 \quad K_{21} = K_{12} = 19200$

$\therefore P = [K][\Delta]$

$$\begin{bmatrix} 100 \\ -50 \end{bmatrix} = \begin{bmatrix} 65600 & 19200 \\ 19200 & 14400 \end{bmatrix} \begin{bmatrix} \Delta_{Bx} \\ \Delta_{By} \end{bmatrix}$$

$\therefore \Delta_{Bx} = 4.16\times10^{-3}$

$\Delta_{By} = -9.03\times10^{-3}$

Now, mem. forces

$S_{BA} = \frac{AE}{L}\left((\Delta_{Ax} - \Delta_{Bx})\cos180 + (\Delta_{Ay} - \Delta_{By})\sin180\right)$

$= 166.4\ kN$

$S_{BC} = \frac{AE}{L}\left[(\overset{0}{\Delta_{Cx}} - \Delta_{Bx})\cos 216.86 + (\overset{0}{\Delta_{Cy}} - \Delta_{By})\sin 216.86\right]$

$= 83.533\ kN$.

Q. Analyse pin joined truss shown in fig by stiffness matrix method. Take area of cross-section for all $100\ mm^2$ as $E = 200\ kN/mm^2$.

$K_{11} = 75.6$

$K_{12} = K_{21} = -15.6$

$K_{22} = 170.24$

$\Delta_{ox} = 0.465$

$\Delta_{oy} = -0.309$