RCC Numericals

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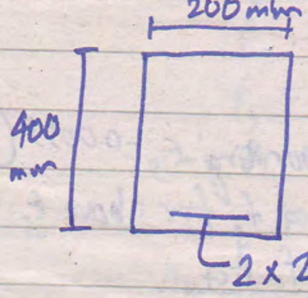
R.C.C.

Class #7

M/W:

1. Find M.R. of section : (M20, Fe500)

i)

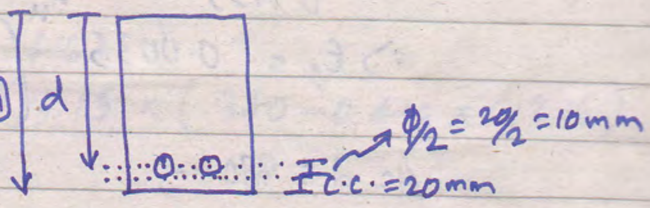


Here,

$D = 400$ mm

$\phi = 22$ mm

Let clear cover $cc = 20$mm



We know, $d = D - \frac{\phi}{2} - cc$

$= 400 - 10 - 20$

$= 370$mm

for Fe500, $x_{u,l} = 0.46d = 0.46 \times 370 = 170.2$mm

We know for SR rectangular RC section,

$x_u = \frac{f_s A_{st}}{0.36 f_{ck} b}$ & $M.R. = 0.36 f_{ck} b x_u (d - 0.416 x_u)$

For $f_s = ?$ Assuming UR section, $f_s = 0.87 f_y$

$\therefore x_u = \frac{0.87 f_y A_{st}}{0.36 f_{ck} b}$

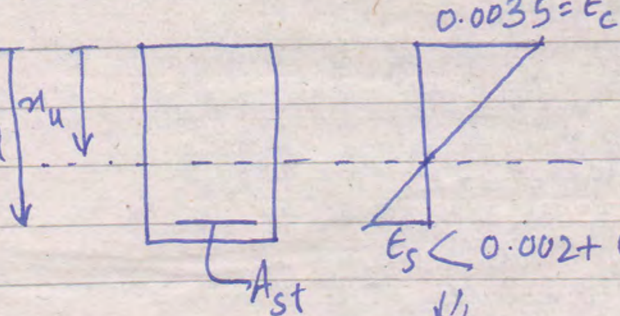
$= \frac{0.87 \times 500 \times 2 \times \pi 11^2}{0.36 \times 20 \times 200}$

$= 229.67$mm

As $x_u > x_{u,l}$, over-reinforced section.

So, assumption is incorrect.

Then, $f_s = ?$. We find f_s from ϵ_s using Table A or corresponding graph from SP16. for ϵ_s, we use strain diagram in OR case:



$\epsilon_s < 0.002 + 0.87\frac{f_y}{E_s}$

$\Downarrow$

$$\frac{\epsilon_s}{0.0035} = \frac{d - x_u}{x_u}$$

$$\Rightarrow \epsilon_s = 0.0035\left(\frac{d - x_u}{x_u}\right)$$

As d = 370 mm,

$$\epsilon_s = 0.0035\left(\frac{370 - x_u}{x_u}\right)$$

Iterative process for OR case:— (to find x_u):

1. Assume x_u
2. Calc. corresponding $\epsilon_s = 0.0035\left(\frac{d - x_u}{x_u}\right)$
3. Using SP16, get f_s for above ϵ_s
4. Calc. $x_u = \frac{f_s A_{st}}{0.36 f_{ck} b}$
5. Check if $x_{u_4} = x_{u_1}$
6. If not equal, repeat from 1. If equal, x_u is obtd.
7. Put x_u in $M_u = 0.36 f_{ck} b x_u \times (d - 0.416 x_u)$

<u>1st iteration</u>:

let $x_u = 170$ mm

$\therefore \epsilon_s = 0.0035\left(\frac{370 - 170}{170}\right) = 0.00411$

From fig. 3 SP-16, for $\epsilon_s = 0.00411$, $f_s = 431$ N/mm^2 i.e. 431 MPa

$\therefore x_u = \frac{f_s A_{st}}{0.36 f_{ck} b} = \frac{431 \times 2 \times \pi\, 11^2}{0.36 \times 20 \times 200} = 227.5$ mm

<u>2nd iteration</u>:

let $x_u = 200$ mm

Then, $\epsilon_s = 0.002975$

$f_s = 420$ MPa

$\therefore x_u = \frac{420 \times 2 \times \pi\, 11^2}{0.36 \times 20 \times 200} = 221.74$ mm

<u>3rd iteration</u>:

let $x_u = 211$ mm $\Rightarrow \epsilon_s = 0.0026 \Rightarrow f_s = 405$ MPa

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$$\therefore x_u = \frac{405 \times 2 \times \pi\, 11^2}{0.36 \times 20 \times 200} = 213.8\text{ mm}$$

<u>4th iteration</u> :

let $x_u = 212.5\text{mm}$

$\varepsilon_s = 0.00259$

$f_s = 405\text{ MPa}$

$$\therefore x_u = \frac{405 \times 2 \times \pi\, 11^2}{0.36 \times 20 \times 200} = 213.8\text{mm}$$

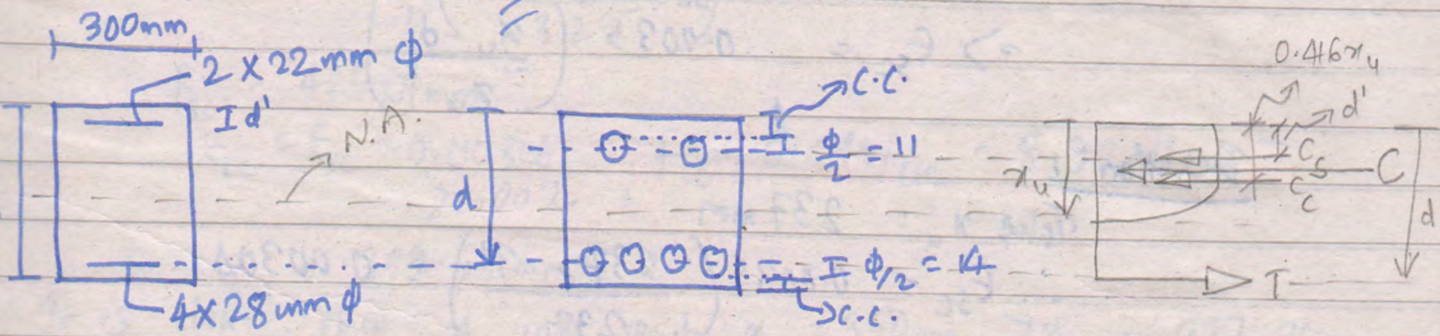
$$\therefore x_u = \frac{212.5 + 213.8}{2} = 213.15\text{mm}$$

So, $M.R. = 0.36 f_{ck}\, b x_u\,(d - 0.416 x_u)$

$= 0.36 \times 20 \times 200 \times 213.15 \times (370 - 0.416 \times 213.15)$

$= 86.35\text{ kN-m}$

ii)



Take cc = 20 mm

$\therefore d' = cc + \frac{\phi}{2} = 20 + 11 = 31\text{ mm}$

We know, $d = D - \frac{\phi}{2} - cc = 550 - 14 - 20 = 516\text{ mm}$

For DR red. section, $M.R. = 0.36 f_{ck}\, b x_u (d - 0.416 x_u) + (f_{sc} - f_{cc}) A_{sc} (d - d')$

$$x_u = \frac{f_s A_{st} - (f_{sc} - f_{cc}) A_{sc}}{0.36 f_{ck}\, b}$$

$\hookrightarrow f_s,\ f_{sc},\ f_{cc}$ are unknown.

For Fe500, $x_{u,l} = 0.46 d = 0.46 * 516 = 237.36\text{ mm}$

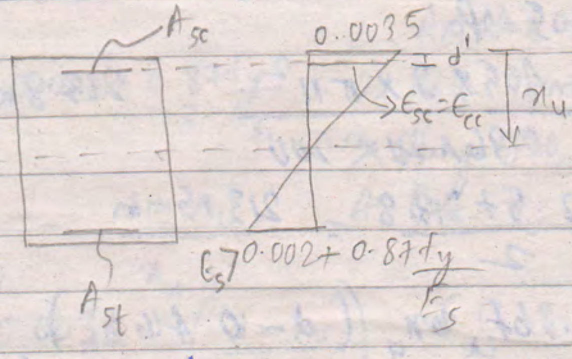
let $f_s = 0.87 f_y$ ie. UR.

Then,

$$x_u = \frac{0.87 \times 500 \times 4 \times \pi\, 14^2 - (f_{sc} - f_{cc}) \times 2 \times \pi\, 11^2}{0.36 \times 20 \times 300}$$

$$= \frac{107\,1408.76 - (f_{sc} - f_{cc}) \times 760.27}{2160}$$

for f_{sc} & f_{cc},



$$\frac{x_u}{0.0035} = \frac{x_u - d'}{\epsilon_{sc}}$$

$$\Rightarrow \epsilon_{sc} = 0.0035 \left(\frac{x_u - d'}{x_u} \right)$$

Iteration 1:

Take $x_u = 237$ mm

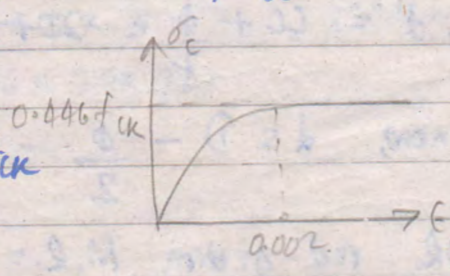
$$\therefore \epsilon_{sc} = 0.0035 \left(\frac{237 - 31}{237} \right) = 0.00304$$

From SP16, fig. 3, for $\epsilon_{sc} = 0.00304$, $f_{sc} = 422\ N/mm^2$ (or MPa)

As $\epsilon_{cc} = \epsilon_{sc} = 0.00304$,

for concrete, for $\epsilon \geq 0.002$,

f (or σ) $= 0.446 f_{ck}$



$\therefore f_{cc} = 0.446 f_{ck}$

$= 0.446 \times 20$

≈ 8.92 MPa

So, $$x_u = \frac{1071408.76 - (422 - 8.92) \times 760.27}{2160}$$

≈ 350.63 mm

Iteration 2 :

Take $x_u = 300$ mm

$\therefore \varepsilon_{sc} = 0.0035 \left(\frac{300 - 31}{300} \right) = 0.00314$

$f_{sc} = 423$ N/mm^2

$\varepsilon_{cc} = \varepsilon_{sc} = 0.00314$

for $\varepsilon_{cc} \geq 0.002$, $f_{cc} = 0.446 f_{ck} = 8.92$ N/mm^2

So,

$x_u = 350.28$ mm

Iteration 3 :

Take $x_u = 325$ mm

$\therefore \varepsilon_{sc} = 0.0035 \left(\frac{325 - 31}{325} \right) = 0.00316$

$f_{sc} = 425$ N/mm^2

$\varepsilon_{cc} = \varepsilon_{sc} = 0.00316 \therefore f_{cc} = 0.446 f_{ck} = 8.92$ N/mm^2
> 0.002

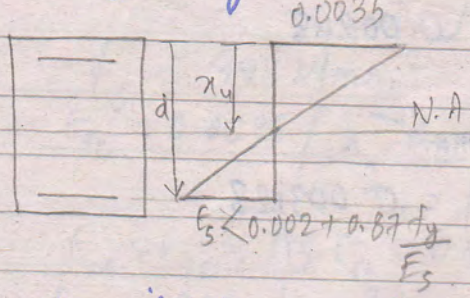
So, $x_u = 349.57$ mm

It is clear that x_u will reach $x_u = 350$ mm $> x_{u,l} = 237.36$ mm (ie. OR section)

So, initial assumption that section is UR is incorrect.

Then, design as OR section. Then, $f_s \neq 0.87 f_y$ & $f_s < 0.87 f_y$

We obtain f_s using ε_s :-



$$\frac{\varepsilon_s}{0.0035} = \frac{d - x_u}{x_u}$$

$$\Rightarrow \varepsilon_s = 0.0035 \left(\frac{516 - x_u}{x_u} \right)$$

Then, we iterate again :-

Iteration 1 :

let $x_u = 237 mm$

$$\epsilon_s = 0.0035\left(\frac{516-237}{237}\right) = 0.00412$$

From SP-16, fig. 3, for $\epsilon_s = 0.00412$,

$f_s = 432\ N/mm^2$

$$\epsilon_{sc} = 0.0035\left(\frac{x_u - d'}{x_u}\right)$$

$$= 0.0035\left(\frac{237-31}{237}\right)$$

$$= 0.00304$$

$\epsilon_{cc} = \epsilon_{sc}$

for f_{sc}, from SP-16, fig. 3 for $\epsilon_{sc} = 0.00304$

$f_{sc} = 421\ N/mm^2$

for f_{cc}, if $\epsilon_{cc} > 0.002$, $f_{cc} = 0.446 f_{ck} = 8.92\ N/mm^2$

$\Rightarrow 0.00304 > 0.002$,

$$\therefore x_u = \frac{f_s A_{st} - (f_{sc} - f_{cc}) A_{sc}}{0.36 f_{ck} b}$$

$$= \frac{432 \times 4 \times \pi \times 14^2 - (421 - 8.92) \times 2 \times \pi 11^2}{0.36 \times 20 \times 300}$$

$$= 347.56 mm$$

Iteration 2 :

let $x_u = 292 mm$

$$\epsilon_s = 0.0035\left(\frac{516-292}{292}\right) = 0.00268$$

From SP-16 fig. 3, $f_s = 410\ N/mm^2$

$$\epsilon_{sc} = 0.0035\left(\frac{292-31}{292}\right) = 0.003128$$

$\epsilon_{cc} = \epsilon_{sc}$

From SP-16, $f_{sc} = 423\ N/mm^2$. As $\epsilon_{cc} > 0.002$, $f_{cc} = 0.446 f_{ck} = 8.92\ N/$

$\therefore x_u = \dfrac{f_s A_{st} - (f_{sc} - f_{cc}) A_{sc}}{0.36 f_{ck} b}$

$= \dfrac{410 \times 4 \times \pi \times 14^2 - (423 - 8.92) \times 2 \times \pi 11^2}{0.36 \times 20 \times 300}$

$= 321.77 \text{ mm}$

Iteration 3:

let $x_u = 320 \text{mm}$

$\epsilon_s = 0.0035 \dfrac{d - x_u}{x_u} = 0.0035 \left(\dfrac{516 - 320}{320}\right) = 0.00214$

From SP-16, fig. 3, f_s for $\epsilon_s = 0.00214$, $f_s = 384 \text{ N/mm}^2$

$\epsilon_{sc} = 0.0035 \dfrac{x_u - d'}{x_u} = 0.0035 \left(\dfrac{320 - 31}{320}\right) = 0.00316$

$\epsilon_{cc} = \epsilon_{sc}$

for $\epsilon_{cc} \geq 0.002$, $f_{cc} = 0.446 f_{ck} = 8.92 \text{ N/mm}^2$

for $\epsilon_{sc} = 0.00316$, from fig. 3, $f_{sc} = 423 \text{ N/mm}^2$

$\therefore x_u = \dfrac{f_s A_{st} - (f_{sc} - f_{cc}) A_{sc}}{0.36 f_{ck} b}$

$= \dfrac{384 \times 4\pi 14^2 - (423 - 8.92) \times 2\pi 11^2}{0.36 \times 20 \times 300}$

$= 292.12 \text{mm}$

Iteration 4:

let $x_u = 306 \text{ mm}$

$\epsilon_s = 0.0035 \left(\dfrac{d - x_u}{x_u}\right) = 0.0035 \left(\dfrac{516 - 306}{306}\right) = 0.0024$

$f_s = 398 \text{ N/mm}^2$

$\epsilon_{sc} = 0.0035 \left(\dfrac{x_u - d'}{x_u}\right) = 0.0035 \left(\dfrac{306 - 31}{306}\right) = 0.003145$

$\epsilon_{cc} = \epsilon_{sc}$. As $\epsilon_{cc} > 0.002$, $f_{cc} = 0.446 f_{ck} = 8.92 \text{ N/mm}^2$

$f_{sc} = 422 \text{ N/mm}^2$

$\therefore x_u = \dfrac{398 \times 4 \times \pi \times 14^2 - (422 - 8.92) \times 2 \times \pi \times 11^2}{0.36 \times 20 \times 300} = 308.43 \text{ mm}$

<u>Iteration 5</u> :

Let $x_u = 307.21\,mm$

$$\epsilon_s = 0.0035\left(\frac{d-x_u}{x_u}\right) = 0.0035\left(\frac{\cancel{56}\,516-307.21}{307.21}\right) = 0.00238$$

$f_s = 396\ N/mm^2$

$$\epsilon_{sc} = 0.0035\left(\frac{x_u-d'}{x_u}\right) = 0.0035\left(\frac{307.21-31}{307.21}\right) = 0.003146$$

$f_{sc} = 422\ N/mm^2$

$\epsilon_{cc} = \epsilon_{sc} > 0.002$, $\therefore f_{cc} = 0.446 f_{ck} = 8.92\ N/mm^2$

$$\therefore x_u = \frac{396 \times 4 \times \pi \times 14^2 - (422-8.92) \times 2 \times \pi \times 11^2}{0.36 \times 20 \times 300}$$

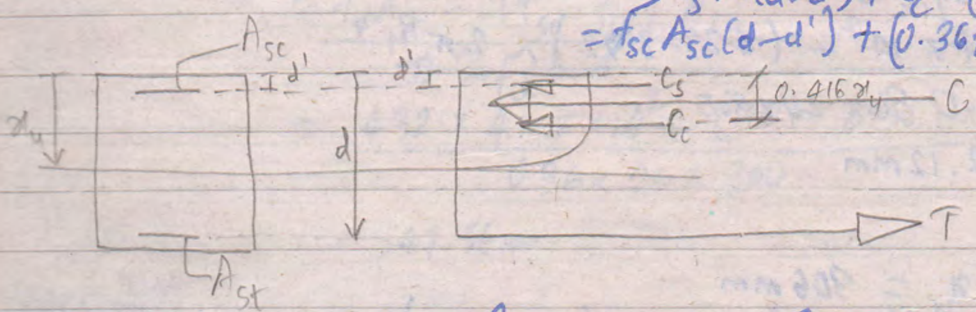
$= 306.16\,mm$

Take $x_u = \dfrac{307.21+306.16}{2} = 306.68\ mm > x_{u,l} = 237.36\,mm$. So, OR ok

$\therefore$ M.R. cap. $(M_u) = C \times L.A.$ or $T \times L.A.$

$\rightarrow C_s \times (d-d') + C_c \times (d - 0.416\,x_u)$

$= f_{sc} A_{sc}(d-d') + (0.36 f_{ck}\, b x_u - f_{cc} A_{sc})(d-0.416\,x_u)$



$A_{sc} = 2 \times \pi \times 11^2 = 760.265\ mm^2$

$A_{st} = 4 \times \pi \times 14^2 = 2463\ mm^2$

$d' = 31\,mm$, $d = 516\ mm$, $b = 300\,mm$

$f_{sc} = 422\ MPa$, $f_{cc} = 8.92\ MPa$

$\therefore$ M.R. $= 410.27\ kN\text{-}m$

If $M_u = 0.36 f_{ck}\, b x_u\,(d - 0.416\,x_u) + (f_{sc} - f_{cc})\,A_{sc}\,(d-d')$ is used instead, as provided in notes, M.R. $= 409.62\ kN\text{-}m$

2. Design beam subjected to : (Consider M20, Fe500)

(i) $M_u = 500 kN\text{-}m$

$\underline{Sol^n}$:-

x-sectional dimensions not given. So,

(a) Considering M_u is small (Design as SR, UR)

$d > d_l \Rightarrow d > \sqrt{\frac{M_u}{Qb}}$ — (1)

where, $Q = 0.36 f_{ck} \frac{x_{u,l}}{d}\left[1 - 0.416 \frac{x_{u,l}}{d}\right]$

For Fe500, $\frac{x_{u,l}}{d} = 0.46$

As M20, $f_{ck} = 20\ MPa$

$\therefore Q = 0.36 \times 20 \times 0.46\left[1 - 0.416 \times 0.46\right] = 2.67\ N/mm^2$

Take $b = \frac{d}{2}$. Then, from (1), in the limiting condition,

$d_l = \sqrt{\frac{M_u}{Qb}}$

$\Rightarrow d_l = \sqrt{\frac{500 \times 10^6 Nmm}{2.67 \times d/2\ N/mm^2}}$

$\Rightarrow d_l^2 = \frac{374.53 \times 10^6\ \cancel{N}\ mm^3}{d_l\ \cancel{N}}$

$\Rightarrow d_l = 720.82\ mm = d_l$

Adopt $d = 750 mm$ (for UR, $d > d_l$)

$b = \frac{d}{2} = 375 mm > 200mm$ OK.

For A_{st}, $M_u = T \times LA = f_s A_{st}(d - 0.416 x_u)$

$\Rightarrow A_{st} = \frac{M_u}{f_s(d - 0.416 x_u)}$

For UR, $f_s = 0.87 f_y$ $\therefore A_{st} = \frac{500 \times 10^6\ Nmm}{0.87 \times 500\ N/mm^2 (750 - 0.416 x_u) mm}$ — (2)

For $x_u \Rightarrow \quad C = T$

$\Rightarrow 0.36 f_{ck} b x_u = f_s A_{st} = 0.87 f_y A_{st}$

$\Rightarrow x_u = \dfrac{0.87 f_y}{0.36 f_{ck} b} A_{st} = \dfrac{0.87 \times 500}{0.36 \times 20 \times 375} A_{st} = 0.1611 A_{st}$

So, from (2),

$A_{st} = \dfrac{1149425.3 \text{ mm}^{2}}{(750 - 0.416 \times 0.1611 A_{st})}$

$\Rightarrow 750 A_{st} - 0.067 A_{st}^2 - 1149425.3 = 0$

$\therefore A_{st} = 9361.45, \quad 1832.58 \text{ mm}^2$

Taking lowes value,

$A_{st} = 1832.58 = n \times A$

where n = no. of reinforcement bars

A = cross-section area of a single bar

Assuming $\phi = 20$mm bars,

$1832.58 = n \times \pi \times 10^2$

$\therefore n = 5.8$

Adopt $n = 6 \times 20$mm ϕ bars

So, width (b) = 375mm

Depth (D) :-

As $D = d + \dfrac{\phi}{2} + c.c.$ & assuming $c.c = 20$mm,

$D = 750 + \dfrac{20}{2} + 20 = 780$mm

(b) Considering M_u is large, (Design as DR)

$M_{u,1} = 0.6 \times 500$kN-m $= 300$ kN-m

So, $\quad M_{u,1} = 300$ kN-m $= 0.36 f_{ck} b x_{u,1} (d - 0.416 x_{u,1})$

As Fe500, $\dfrac{x_{u,1}}{d} = 0.46 \Rightarrow x_{u,1} = 0.46d$

$\therefore 300 \times 10^6 = 0.36 \times 20 \times b \times 0.46d \, (d - 0.416 \times 0.46d)$

Take $b = \frac{d}{2}$

$\therefore 300 \times 10^6 = 0.36 \times 20 \times \frac{d}{2} \times 0.46d \left(d - 0.416 \times 0.46d\right)$

$\Rightarrow 300 \times 10^6 = 1.656 d^2 \times 0.8086 d$

$\therefore d = 607.34 mm$

Adopt $d = 610 mm$

Then, $b = \frac{d}{2} = 305mm > 200mm$ OK.

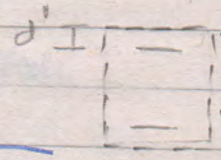
So, $M_{u,l} = 0.36 f_{ck} b x_{u,l} \left(d - 0.416 x_{u,l}\right)$

where, $x_{u,l} = 0.46d = 0.46 \times 610 = 280.6 mm$

$\therefore M_{u,l} = 0.36 \times 20 \times 305 \times 280.6 \left(610 - 0.416 \times 280.6\right)$

$= 303.95 kN\text{-}m$

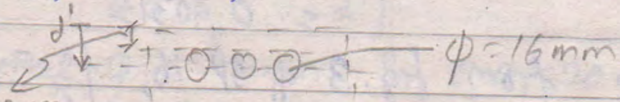
Now, $A_{st} = A_{st1} + A_{st2}$



$= \frac{M_{u,l}}{0.87 f_y \left(d - 0.416 x_{u,l}\right)} + \frac{(M_u - M_{u,l}) \text{ ie. } M_{add}}{0.87 f_y \left(d - d'\right)}$

Assuming $\phi = 16mm$ for upper / compression steel bars, and clear cover $CC = 20 mm$,

$d' = CC + \frac{\phi}{2}$



$= 20 + \frac{16}{2} = 28 mm$

(Compression bars dimension with Clear Cover)

$\therefore A_{st} = \frac{303.95 \times 10^6}{0.87 \times 500 \left(610 - 0.416 \times 280.6\right)} + \frac{\left(500 - 303.95\right) \times 10^6}{0.87 \times 500 \left(610 - 28\right)}$

$= 1416.54 + 774.38$

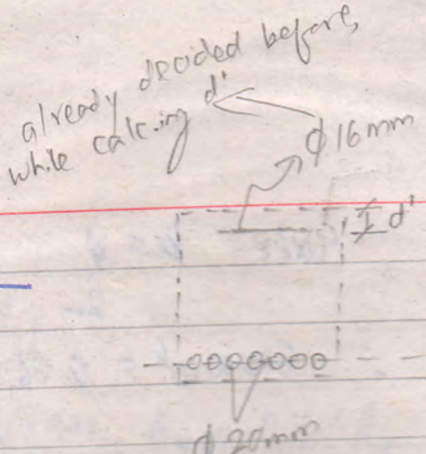
$= 2190.92 mm^2$

Taking 20mm ϕ tension bars, $n \times \pi \, 10^2 = 2190.92$

$\Rightarrow n = 6.97$

Adopting $7 \times 20 mm$ bars in tension (lower) region,

for compression bars,

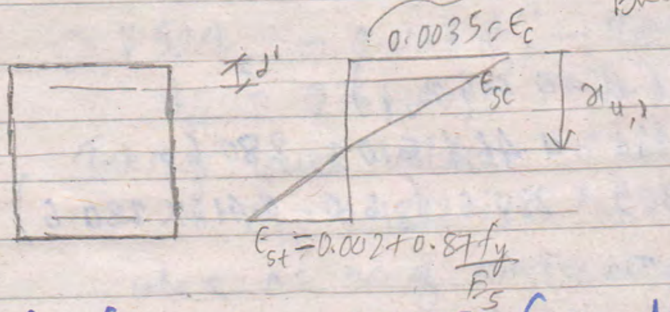


$$A_{sc} = \frac{(M_u - M_{u,l}) \text{ ie } M_{add}}{(f_{sc} - f_{cc})(d-d')}$$

$$= \frac{(500 - 303.95) \times 10^6}{(f_{sc} - f_{cc})(610 - 28)}$$

$$= \frac{196.05 \times 10^6}{582(f_{sc} - f_{cc})}$$

Strain diag. of constituent Balanced section.



$$\frac{\epsilon_{sc}}{x_{u,l} - d'} = \frac{0.0035}{x_{u,l}}$$

$$\Rightarrow \epsilon_{sc} = 0.0035\left(\frac{x_{u,l} - d'}{x_{u,l}}\right)$$

for f_{sc}, $\epsilon_{sc} = 0.0035\left(\frac{x_{u,l} - d'}{x_{u,l}}\right)$

$$= 0.0035\left(\frac{280.6 - 28}{280.6}\right)$$

$$= 0.00315$$

from fig. 3 of SP-16, $f_{sc} = 423 \text{ N/mm}^2$

And $\epsilon_{cc} = \epsilon_{sc}$. for f_{cc} with $\epsilon_{cc} = 0.00315 > 0.002$,

$$f_{cc} = 0.446 f_{ck} = 0.446 \times 20 = 8.92 \text{ N/mm}^2$$

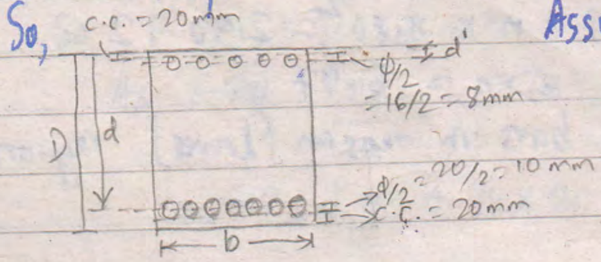
$$\therefore A_{sc} = \frac{196.05 \times 10^6 \text{ N-mm}}{582(423 - 8.92) \text{ N/mm}^2 \cdot \text{mm}} = 813.50 \text{ mm}^2$$

$$\Rightarrow n \times \pi 8^2 = 813.50$$

$$\Rightarrow n = 4.04$$

Adopt 5×16 mm ϕ steel bars for compression.

So,



Assuming c.c. = 20mm for tension bars as well,

Depth(D) = d + 10 + 20 = 610 + 30 = 640mm

Width(b) = 305mm

(ii) $M_u = 500\,kN\text{-}m$ (M20, Fe500)

$b = 300\,mm$

$D = 600\,mm$

Sol^n:—

$x_{u,l} = 0.46d$

For d, taking $c.c. = 20\,mm$ & tension bar $\phi = 20\,mm$,

$$d = D - c.c. - \frac{\phi}{2} = 600 - 20 - \frac{20}{2} = 570\,mm$$

$\therefore x_{u,l} = 0.46 \times 570 = 262.2\,mm$

$$M_{u,l} = 0.36 f_{ck}\, b x_{u,l}\,(d - 0.416 x_{u,l})$$

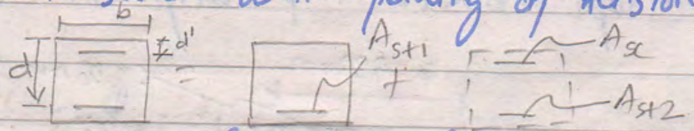
$$= 0.36 \times 20 \times 300 \times 262.2\,(570 - 0.416 \times 262.2)$$

$$= 261.05\,kN\text{-}m$$

Given $M_u = 500\,kN\text{-}m > M_{u,l} = 261.05\,kN\text{-}m$.

So, design as DR rect. section with yielding of tension steels.

Now,



$$A_{st} = A_{st1} + A_{st2}$$

$$= \frac{M_{u,l}}{0.87 f_y (d - 0.416 x_{u,l})} + \frac{(M_u - M_{u,l}) \text{ i.e. } M_{add}}{0.87 f_y (d - d')}$$

For d', let compression steel $\phi = 16\,mm$ & $c.c. = 20\,mm$

Then, $d' = cc + \phi/2 = 20 + 16/2 = 28\,mm$

$$\therefore A_{st} = \frac{261.05 \times 10^6}{0.87 \times 500 (570 - 0.416 \times 262.2)} + \frac{(500 - 261.05) \times 10^6}{0.87 \times 500 (570 - 28)} = 1301.98 + 1013.49$$

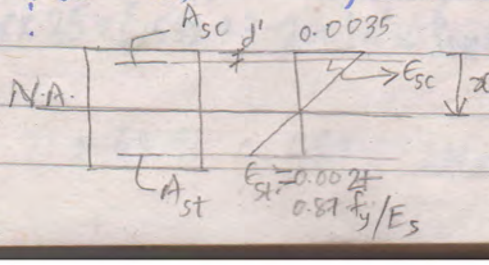
$\Rightarrow A_{st} = 2315.47\,mm^2$

$\Rightarrow n \times \pi\, 10^2 = 2315.47 \Rightarrow n = 7.37$. Adopt $8 \times 20\,mm\,\phi$ tension bars.

For compression bars,

$$A_{sc} = \frac{(M_u - M_{u,l}) \text{ i.e. } M_{add}}{(f_{sc} - f_{cc})(d - d')} = \frac{(500 - 261.05) \times 10^6}{(f_{sc} - f_{cc})(570 - 28)} = \frac{0.4408 \times 10^6}{f_{sc} - f_{cc}}$$

For f_{sc} & f_{cc}:



$$\frac{\epsilon_{sc}}{x_{u,l} - d'} = \frac{0.0035}{x_{u,l}}$$

$$\Rightarrow \epsilon_{sc} = 0.0035 \; \frac{x_{u,l} - d'}{x_{u,l}}$$

$$= 0.0035 \; \frac{262.2 - 28}{262.2}$$

$$= 0.003126$$

From Fig. 3 of SP-16, f_{sc} for $\epsilon_{sc} = 0.003126$ for Fe 500 is $423 \, N/mm^2$

$\therefore f_{sc} = 423 \, N/mm^2$

$\epsilon_{cc} = \epsilon_{sc}$ & for f_{cc}, since $\epsilon_{cc} = 0.003126 > 0.002$,

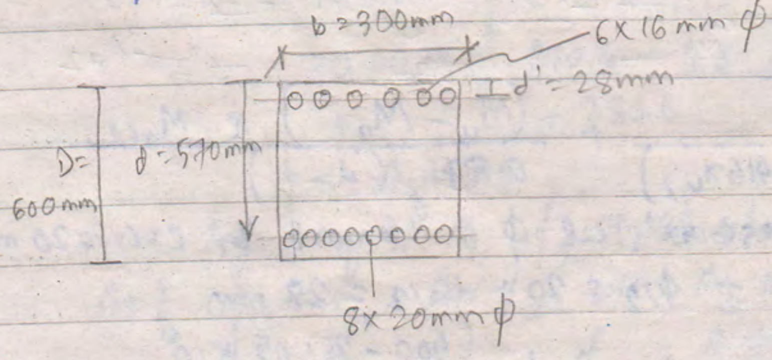
$f_{cc} = 0.446 f_{ck} = 0.446 \times 20 = 8.92 \, N/mm^2$

$$\therefore A_{sc} = \frac{0.4408 \times 10^6}{423 - 8.92} = 1064.53 \, mm^2$$

$\Rightarrow n \times \pi 8^2 = 1064.53 \, mm^2$

$\Rightarrow n = 5.29$

Adopt $6 \times 16 \, mm \, \phi$ compression (upper) steel bars.



Class #9

Design a flange beam for the following data: (intermediate beam)

$M_u = 500 \, kN\text{-}m$ ______ (i)

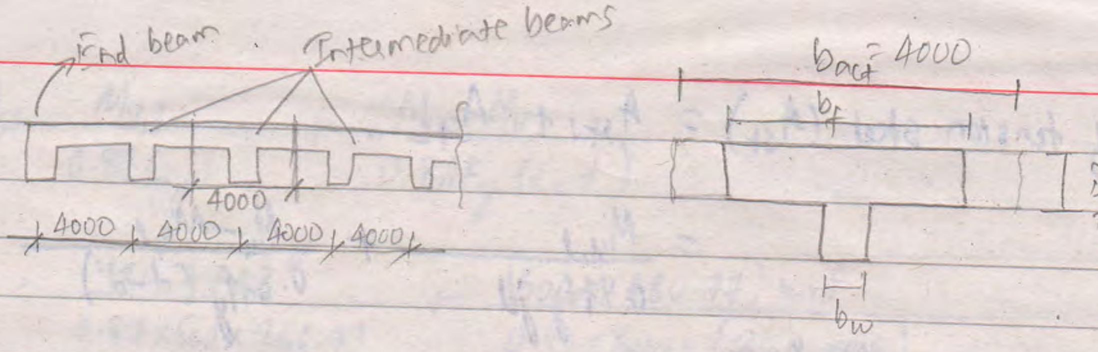
$= 1100 \, kN\text{-}m$ ______ (ii)

$b_{act} = 4000 \, mm$

$D_f = 125 \, mm$

$l = 5m$ (Continuous)

Consider M25, Fe 500



(i) $M_u = 500\,kN\text{-}m$

Soln:-

If $M_u < M_{u,l}$, design as SR URS

if $M_u > M_{u,l}$, design as DR URS (ie. yielding of tension steel)

For $M_{u,l}$:

$x_{u,l} = 0.46\,d$

For 'd',

$$d = \frac{Span}{20} \text{ to } \frac{Span}{15} = \frac{Span}{16}\,(say) = \frac{5000}{16} = 312.5\,mm$$

Adopt $D = 350\,mm$

Then, $d = D - c.c. - \frac{\phi}{2}$

Assuming tension bar $\phi = 20\,mm$ & $c.c. = 20\,mm$,

$d = \left(350 - 20 - \frac{20}{2}\right) = 320\,mm$. $b_w = \frac{d}{2} = 160 < 200$ $\therefore b_w = 200$

& $b_f = \frac{l_o}{6} + b_w + 6D_f = \frac{0.7 \times 5000}{6} + 200 + 6 \times 125 = 1533.33\,mm \ngtr b_{act} = 4000\,mm$ OK.

$\therefore x_{u,l} = 0.46 \times 320 = 147.2\,mm > D_f = 125\,mm$

So, N.A. lies in web.

$0.43\,x_{u,l} = 63.296\,mm$

$D_f = 125\,mm$

$D_f > 0.43\,x_{u,l}$. So, use respective M.R. expn:-

$\therefore M_{u,l} = 0.446 f_{ck} (b_f - b_w) y_f \left(d - \frac{y_f}{2}\right) + 0.36 f_{ck}\, b_w\, x_{u,l} (d - 0.416\,x_{u,l})$

where, $y_f = 0.15\,x_{u,l} + 0.65\,D_f = 0.15 \times 147.2 + 0.65 \times 125 = 103.33\,mm < D_f$ OK

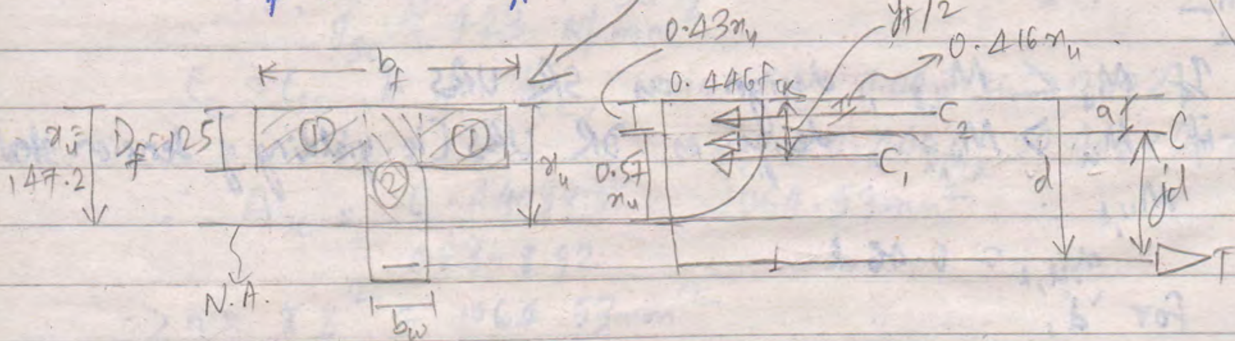
$\therefore M_{u,l} = 0.446 \times 25 \times (1533.33 - 200) \times 103.33 \times \left(320 - \frac{103.33}{2}\right) + 0.36 \times 25 \times 200 \times 147.2 \times (320 - 0.416 \times 147.2)$

$= 412.21 + 68.56 = 480.77\,kNm < M_u$ $\therefore$ Design as DR URS.

Area of tension steel (A_{st}) = $A_{st1} + A_{st2}$.

$$= \frac{M_{u,l}}{0.87 f_y\, jd} + \frac{M_u - M_{u,l}}{0.87 f_y (d-d')}$$

As $x_{u,l} = 147.2\,mm$,
$D_f > 0.43\, x_{u,l}$



(Balanced Section)

For jd :-

$$C_1 \times \frac{y_f}{2} + C_2 \times 0.416\, x_{u,l} = C \times a \quad ; \quad a = (1-j)d$$

$$\Rightarrow \underbrace{0.446 f_{ck}\, y_f (b_f - b_w)}_{C_1} \times \frac{y_f}{2} + \underbrace{0.36 f_{ck}\, b_w\, x_{u,l}}_{C_2} \times 0.416\, x_{u,l} = C \times a$$

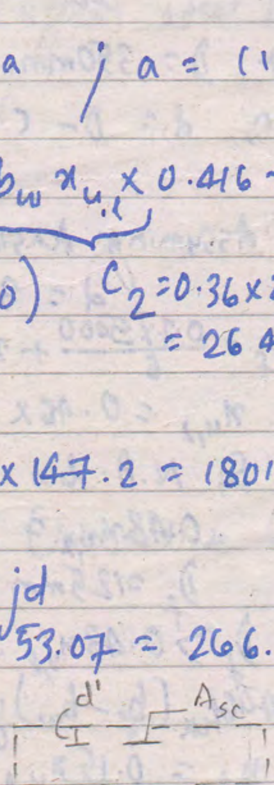
$C_1 = 0.446 \times 25 \times 103.33\,(1533.33 - 200)$ $\qquad C_2 = 0.36 \times 25 \times 200 \times 147.2$
$= 1536.17\,kN$ $\qquad = 264.96\,kN$

$\therefore C = C_1 + C_2 = 1801.13\,kN$

$$\Rightarrow 1536.17 \times \frac{103.33}{2} + 264.96 \times 0.416 \times 147.2 = 1801.13 \times a$$

$\Rightarrow a = 53.07\,mm = (1-j)d = d - jd$

$\therefore jd = d - 53.07 = 320 - 53.07 = 266.93\,mm$



For d', let compression bars be 16mm ϕ

Then, $d' = c.c. + \frac{\phi}{2}$. Assuming c.c = 20 mm,

$d' = 20 + \frac{16}{2} = 28\,mm$

$$\therefore A_{st} = \frac{M_{u,l}}{0.87 f_y jd} + \frac{M_u - M_{u,l}}{0.87 f_y (d-d')}$$

$$= \frac{480.77 \times 10^6}{0.87 \times 500 \times 266.93} + \frac{(500 - 480.77) \times 10^6}{0.87 \times 500 \times (320-28)}$$

$$= 4140.48 + 151.39$$

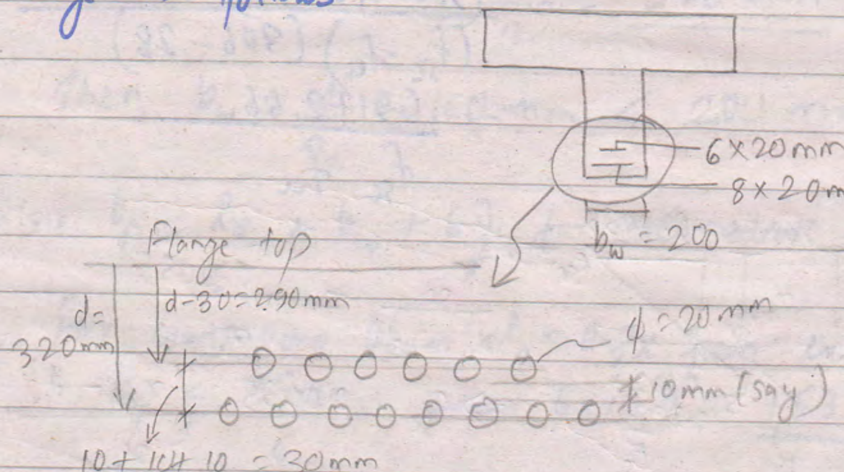
$$= 4291.87 \text{ mm}^2$$

$$\Rightarrow n \times \pi\, 10^2 = 4291.87$$

$$\Rightarrow n = 13.66$$

Adopt 14×20mm ϕ tension bars

Arrange as follows:



New d to the centroid of tension bars:

$$290 \times 6 + 320 \times 8 = d \times 14$$

$$\Rightarrow d = 307.14 \text{mm}$$

$$\therefore \text{New } A_{st} = \frac{480.77 \times 10^6}{0.87 \times 500 \times (307.14 - 53.07)} + \frac{(500 - 480.77) \times 10^6}{0.87 \times 500 \times (307.14 - 28)}$$

$$= 4350.05 + 158.37$$

$$= 4508.42 \text{mm}^2$$

$$\Rightarrow n \times \pi 10^2 = 4508.42$$

$$\Rightarrow n = 14.35$$

Adopt 15×20mm ϕ tension bars

Arrange: 8 below, 7 above. New $d = \dfrac{290 \times 7 + 320 \times 8}{15} = 306$ mm

New $A_{st} = \dfrac{480.77 \times 10^6}{0.87 \times 500 \times (306 - 53.07)} + \dfrac{(500 - 480.77) \times 10^6}{0.87 \times 500 \times (306 - 28)}$

$= 4369.66 + 159.02$

$= 4528.68 \text{ mm}^2$

$\Rightarrow n \pi 10^2 = 4528.68$

$\Rightarrow n = 14.41$

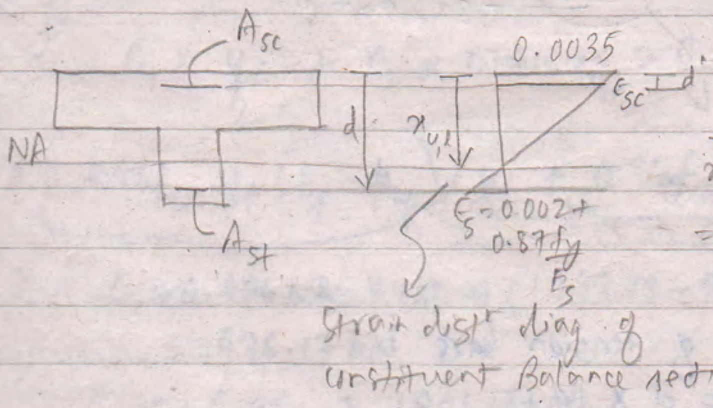
Adopt 15 × 20mm φ tension bars.

Now,

Area of compression steel $(A_{sc}) = \dfrac{M_u - M_{u,1}}{(f_{sc} - f_{cc})(d - d')}$

$= \dfrac{(500 - 480.77) \times 10^6}{(f_{sc} - f_{cc})(306 - 28)}$

$= \dfrac{69172.66}{f_{sc} - f_{cc}}$



$\dfrac{\varepsilon_{sc}}{x_{u,1} - d'} = \dfrac{0.0035}{x_{u,1}}$

$\Rightarrow \varepsilon_{sc} = 0.0035 \left(\dfrac{x_{u,1} - d'}{x_{u,1}}\right)$

$\varepsilon_{sc} = 0.0035 \left(\dfrac{x_{u,1} - d'}{x_{u,1}}\right) = 0.0035 \left(\dfrac{147.2 - 28}{147.2}\right)$

$= 0.00283$

From SP-16, fig. 3 for $\varepsilon_{sc} = 0.00283$, $f_{sc} = 415 \text{ N/mm}^2$

$\varepsilon_{cc} = \varepsilon_{sc} = 0.00283 > 0.002$

$\therefore f_{cc} = 0.446 f_{ck} = 0.446 \times 25 = 11.167 \text{ N/mm}^2$

$\therefore A_{sc} = \dfrac{69172.66}{415 - 11.167} = 171.29 \text{ mm}^2$

$\Rightarrow n \times \pi 8^2 = 171.29 \therefore n = 0.85$

Adopt 1 × 16 mm ϕ compression bar.

(ii) $M_u = 1100 kN\text{-}m$

Soln.:—

$d = \frac{\text{Span}}{20}$ to $\frac{\text{Span}}{15} = \frac{\text{Span}}{16} = \frac{5000}{16} = 312.5$ mm

Adopt $D = 350$ mm

Then, $d = D - c.c. - \frac{\phi}{2}$

let $c.c. = 20$ mm, ϕ = tension bar ϕ = 20 mm

$\therefore d = 350 - 20 - \frac{20}{2} = \underline{320 \text{ mm}}$

Then, $b_w = \frac{d}{2} = 160$ mm < 200 mm. Take $b_w = \underline{200 \text{ mm}}$

Also $b_f = \frac{l_0}{6} + b_w + 6D_f$ for intermediate beams.

For continuous beam, $l_0 = 0.7 \times$ span $= 0.7 \times 5000 = 3500$ mm

$\therefore b_f = \frac{3500}{6} + 200 + 6 \times 125 = \underline{1533.33 \text{ mm}} < b_{act} = 4000$ mm OK.

Then, $x_{u,l} = 0.46d = 0.46 \times 320 = 147.2$ mm $> D_f = 125$ mm

So, N.A. lies in web.

Also, $0.43 x_{u,l} = 0.43 \times 147.2 = 63.296$ mm

$D_f = 125$ mm $> 0.43 x_{u,l} = 63.296$ mm

So, $D_f > 0.43 x_{u,l}$

Then, M.R. eqn for $D_f > 0.43 x_{u,l}$ is:—

$M_{u,l} = 0.446 f_{ck} (b_f - b_w) y_f \left(d - \frac{y_f}{2}\right) + 0.36 f_{ck} b_w x_{u,l} (d - 0.416 x_{u,l})$

$y_f = 0.15 x_{u,l} + 0.65 D_f = 0.15 \times 147.2 + 0.65 \times 125 = 103.33$ m $< D_f = 125$ m OK.

$\therefore M_{u,l} = 0.446 \times 25 (1533.33 - 200) \times 103.33 \times \left(320 - \frac{103.33}{2}\right) + 0.36 \times 25 \times 200 \times 147.2 (320 - 0.416 \times 147.2) = 412.21 + 68.56 = 480.78$ kN-m.

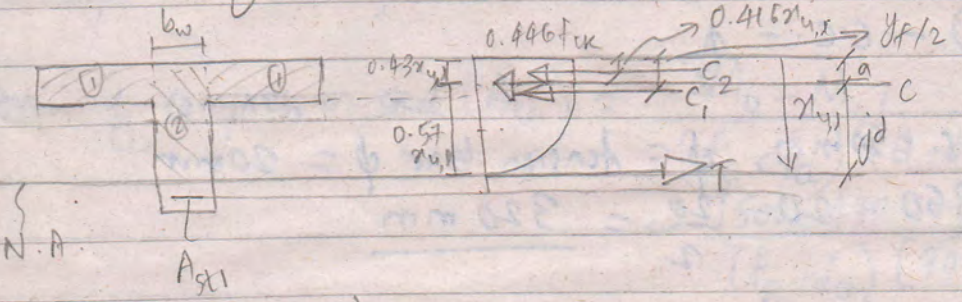
$M_u = 1100$ kN-m $> M_{u,l} = 480.78$ kN-m

So, design as DR URS (ie yielding of tension steel)

Area of tension steel

$$A_{st} = A_{st1} + A_{st2}$$

$$= \frac{M_{u,l}}{0.87 f_y \, jd} + \frac{M_u - M_{u,l}}{0.87 f_y (d - d')}$$



(Balanced section)

$$C_1 \times \frac{y_f}{2} + C_2 \times 0.416 x_{u,l} = C \times a$$

$C_1 = 0.446 f_{ck} (b_f - b_w) y_f = 0.446 \times 25 \times (1533.33 - 200) \times 103.33$
$= 1536.17$ kN

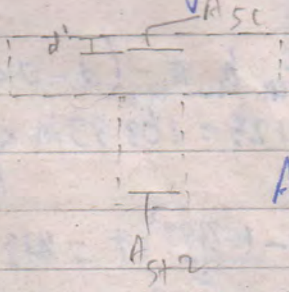
$C_2 = 0.36 f_{ck} b_w x_{u,l} = 0.36 \times 25 \times 200 \times 147.2 = 264.96$ kN

$\therefore C = C_1 + C_2 = 1801.13$ kN

So, $1536.17 \times \frac{103.33}{2} + 264.96 \times 0.416 \times 147.2 = 1801.13 \times a$

$\therefore a = 53.07$ mm

$\therefore jd = d - a = 320 - 53.07 = 266.93$ mm



Let compression steel bars be 16mm ϕ with c.c. = 20mm ϕ

Then, $d' = c.c. + \frac{\phi}{2} = 20 + \frac{16}{2} = 28$ mm

$$A_{st1} = \frac{M_{u,l}}{0.87 f_y \, jd} = \frac{480.78 \times 10^6}{0.87 \times 500 \times 266.93} = 4140.57 \text{ mm}^2$$

$$A_{st2} = \frac{M_u - M_{u,l}}{0.87 f_y (d-d')} = \frac{(1100 - 480.78) \times 10^6}{0.87 \times 500 \times (320-28)} = 4874.98 \text{ mm}^2$$

$A_{st} = A_{st1} + A_{st2} = 9015.55 \text{ mm}^2$

$\Rightarrow n \, \pi \, 10^2 = 9015.55 \text{ mm}^2$

$\Rightarrow n = 28.69$

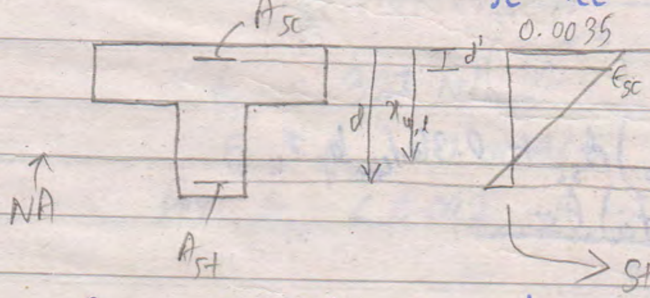
Adopt 30×20mm ϕ in tension steels to compensate for any shift in effective depth 'd'.

Now,

$$A_{sc} = \frac{M_u - M_{u,l}}{(f_{sc} - f_{cc})(d-d')}$$

$$= \frac{(1100 - 480.78) \times 10^6}{(f_{sc} - f_{cc})(320-28)}$$

$$= \frac{2120616.44}{f_{sc} - f_{cc}}$$



$$\frac{\epsilon_{sc}}{x_{u,l} - d'} = \frac{0.0035}{x_{u,l}}$$

Strain diag. of constituent Balanced section

$$\epsilon_{sc} = 0.0035 \, \frac{x_{u,l} - d'}{x_{u,l}}$$

$$= 0.0035 \, \frac{147.2 - 28}{147.2}$$

$$= 0.00283$$

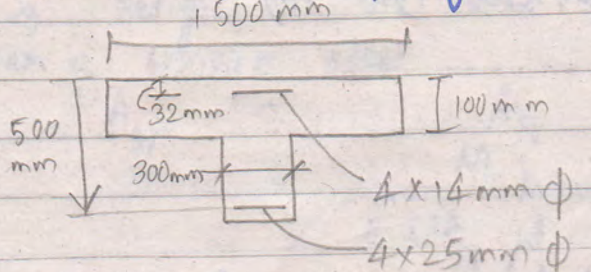
from fig 3 of SP16, f_{sc} for $\epsilon_{sc} = 0.00283$ for Fe500, $f_{sc} = 415 \text{ N/mm}^2$

$\epsilon_{cc} = \epsilon_{sc} = 0.00283$. For $\epsilon_{cc} \geq 0.002$, $f_{cc} = 0.446 f_{ck} = 0.446 \times 25 = 11.15 \text{ N/mm}^2$

$$\therefore A_{sc} = \frac{2120616.44}{415 - 11.15} = 5251 \text{ mm}^2$$

$\Rightarrow n \times \pi 8^2 = 5251 \Rightarrow n = 26.11$ Adopt 27×16mm ϕ bars in compr^n.

(*) Analyze the M.R. capacity of the given section considering M20, Fe415



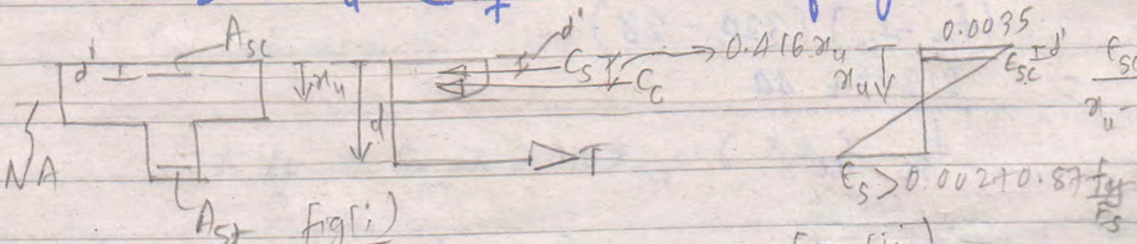
Sol^n:- $d = 500$ mm, $D_f = 100$ mm

For Fe 415, $x_{u,l} = 0.48d = 0.48 \times 500 = 240$ mm

$x_u = ?$, $MR = ?$

Assuming $x_u \leq x_{u,l}$ (UR) ($f_s = 0.87f_y$)

& $x_u \leq D_f$ (N.A. in flange)



$$\frac{\epsilon_{sc}}{x_u - d'} = \frac{0.0035}{x_u}$$

Fig (i) Fig (ii)

$T = C$

$$\Rightarrow f_s A_{st} = C_s + C_c = (f_{sc} - f_{cc})A_{sc} + 0.36 f_{ck} b_f x_u$$

$$\Rightarrow x_u = \frac{f_s A_{st} - (f_{sc} - f_{cc})A_{sc}}{0.36 f_{ck} b_f}$$

$$\therefore x_u = \frac{0.87 f_y A_{st} - (f_{sc} - f_{cc})A_{sc}}{0.36 f_{ck} b_f}$$

$$= \frac{0.87 \times 415 \times 4 \times \pi \times 12.5^2 - (f_{sc} - f_{cc}) \times 4 \times \pi \times 7^2}{0.36 \times 20 \times 1500}$$

$$= 65.64 - 0.057(f_{sc} - f_{cc})$$

For f_{sc} & f_{cc} :- from fig.(ii)

$$\epsilon_{sc} = 0.0035 \frac{x_u - d'}{x_u} = 0.0035\left(\frac{x_u - 32}{x_u}\right)$$

Trial 1: let $x_u = D_f = 100$ mm

$$\therefore \epsilon_{sc} = 0.0035\left(\frac{100 - 32}{100}\right) = 0.00238$$

from fig. 3 of SP-16, f_{sc} for $\epsilon_{sc} = 0.00238$ for Fe 415 :

$\therefore f_{sc} = 341 \ N/mm^2$

$\epsilon_{cc} = \epsilon_{sc} = 0.00238.$

for concrete, $\epsilon_{cc} \geq 0.002$ gives $f_{cc} = 0.446 f_{ck}$

$= 0.446 \times 20$

$= 8.92 \ N/mm^2$

$\therefore x_u = 65.64 - 0.057 \times (341 - 8.92) = 46.71 \ mm$

So x_u lies betn 46.71 mm & 100 mm

Thus, $x_u < D_f = 100$ & $x_u < x_{u,l} = 240$ mm

i.e., it is clear that the final x_u produced through our iteration will be within our assumptions, hence it will be our final and the actual 'x_u' of the given section.

Trial 2 : let $x_u = \frac{46.71 + 100}{2} = 73$ mm

$\epsilon_{sc} = 0.0035 \left(\frac{73 - 32}{73} \right) = 0.001965$

$f_{sc} = 327 \ N/mm^2$

$\epsilon_{cc} = \epsilon_{sc} = 0.001965$

for $\epsilon_{cc} < 0.002$, $f_{cc} = 446 f_{ck} (\epsilon_{cc} - 250 \epsilon_{cc}^2)$

$= 446 \times 20 \ (0.001965 - 250 \times 0.001965^2)$

$= 8.917 \ N/mm^2$

$\therefore x_u = 65.64 - 0.057 (327 - 8.917) = 47.51$ mm

Trial 3 : let $x_u = 50$ mm

$\epsilon_{sc} = 0.0035 \left(\frac{50 - 32}{50} \right) = 0.00126 = \epsilon_{cc}$

$f_{sc} = 248 \ N/mm^2$

$f_{cc} = 446 \times 20 \ (0.00126 - 250 \times 0.00126^2) = 7.70$

$\therefore x_u = 65.64 - 0.057 (248 - 7.70) = 51.94$ mm

Trial 4 : let $x_u = 50.97$ mm

$\epsilon_{sc} = 0.0035 \left(\frac{50.97 - 32}{50.97} \right) = 0.0013 = \epsilon_{cc} \therefore f_{sc} = 259 \ N/mm^2$

$f_{cc} = 446 \times 20 (0.0013 - 250 \times 0.0013^2) = 7.827 \, N/mm^2$

$\therefore x_u = 65.64 - 0.057 (259 - 7.827) = 51.32 \, mm$

Trial 5: let $x_u = 51.145 \, mm$

$$\epsilon_{sc} = 0.0035 \left(\frac{51.145 - 32}{51.145} \right) = 0.00131 = \epsilon_{cc}$$

$f_{sc} = 259 \, N/mm^2$

$f_{cc} = 446 \times 20 (0.00131 - 250 \times 0.00131^2) = 7.858 \, N/mm^2$

$\therefore x_u = 65.64 - 0.057 (259 - 7.858) = \underline{51.32 \, mm}$

$x_u = 51.32 \, mm < x_{u,l} = 240 \, mm$

$< D_f = 100 \, mm$

ie. assumption is valid.

$M.R. = C_s (d - d') + C_c (d - 0.416 x_u)$

$= A_{sc} (f_{sc} - f_{cc}) (d - d') + 0.36 f_{ck} b_f x_u (d - 0.416 x_u)$

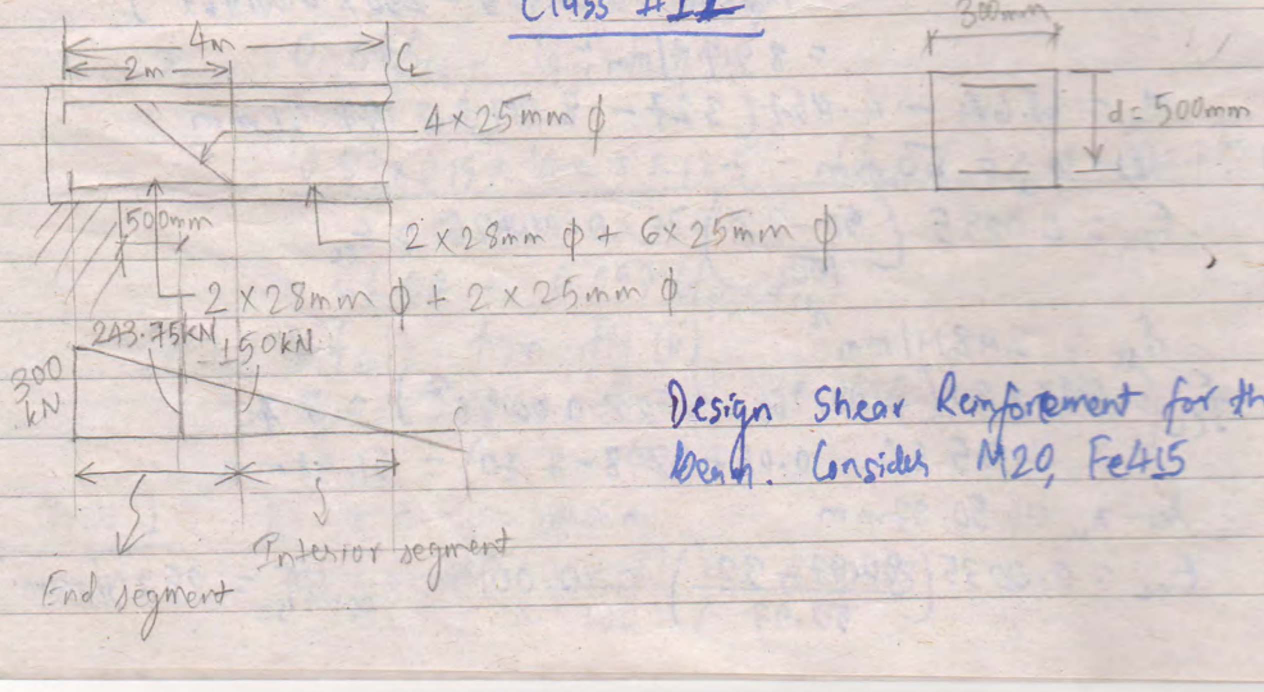
$= 4 \times \pi 7^2 (259 - 7.858)(500 - 32) + 0.36 \times 20 \times 1500 \times 51.32 (500 - 0.416 \times 51.$

$= 72.37 + 265.29$

$= 337.66 \, kN\text{-}m.$

Class #11

(*)



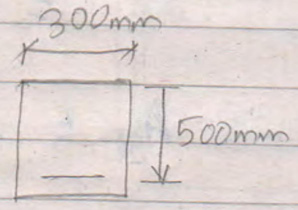
Design Shear Reinforcement for the following Beam. Consider M20, Fe415

Then, $V_{bb} = 0.87 f_y A_{st} \sin\alpha \times 4$; taking $\alpha = 45^\circ$,

$V_{bb} = 0.87 \times 415 \times$

Class #12

S.N. Sinha
Example 4.6



Design RC beam considering M20, Fe415 and:-

$M_u = 175 kN\text{-}m$

$V_u = 25 kN$

$T_u = 10 kN\text{-}m$

Sol^n:-

Firstly, $V_{ue} = V_u + \frac{1.6 T_u}{b} = 25 + \frac{1.6 \times 10 \times 10^3 kN\text{-}\cancel{mm}}{300 \cancel{mm}}$

$= 78.33$ kN

Check if $V_{ue} > V_{uc}$ or $V_{ue} \leq V_{uc}$

But $V_{uc} = \tau_{uc} \times bd$

But $\tau_{uc} = f(A_{st})$

But $A_{st} = ?$

So, assuming $V_{ue} \leq V_{uc}$, longitudinal reinf. are provided for flexural BM only (not for eqvt BM) & transverse reinf. for min. shear only.

So, design for $M_u = 175 kN\text{-}m$. But SR or DR?

If $M_u < M_{u,l}$, SR. If $M_u > M_{u,l}$, DR.

So, $M_{u,l} = 0.36 f_{ck} b x_{u,l} (d - 0.416 x_{u,l})$

$x_{u,l}$ for Fe415 $= 0.48d = 0.48 \times 500 = 240$ mm

$\therefore M_{u,l} = 0.36 \times 20 \times 300 \times 240 (500 - 0.416 \times 240) = 207.44 kN\text{-}m$

As $M_u < M_{u,l}$, design as SR URS.

For SR ORS,

$$A_{st} = \frac{M_u}{0.87 f_y (d - 0.416 x_u)}$$

where,

$$x_u = \frac{0.87 f_y A_{st}}{0.36 f_{ck} b}$$

$$= \frac{0.87 \times 415 \times A_{st}}{0.36 \times 20 \times 300} = 0.167 A_{st}$$

$$\therefore A_{st} = \frac{M_u}{0.87 \times 415 (500 - 0.416 \times 0.167 A_{st})}$$

$$= \frac{175 \times 10^6}{361.05 (500 - 0.0695 A_{st})}$$

Solving for A_{st},

$$A_{st} = 1154.74\, mm^2$$

Adopting 20mm ϕ tension bars,

$$\pi 10^2 \times n = 1154.74$$

$$\Rightarrow n = 3.67$$

Adopt $4 \times 20mm\ \phi$ tension bars at bottom and nominal reinforcement of $2 \times 12mm\ \phi$ at top (Cl. 26.5.1.7 ⇒ (b) "... shall be at least one longitudinal bar in each corner...")

Then, checking for validity of assumption ($V_{ue} \le V_{uc}$):

For $V_{uc} = \tau_{uc} \times bd$

For τ_{uc} from table 19 of IS456 with M20 &

$$100 \frac{A_{st}}{bd} = 100 \times \frac{4 \times \pi 10^2}{300 \times 500} = 0.837$$

$$\therefore \tau_{uc} = 0.56 + \frac{0.62 - 0.56}{1 - 0.75} \times (0.837 - 0.75)$$

$$= 0.581\, N/mm^2$$

$$\therefore V_{uc} = 0.581 \times 300 \times 500 = 87.15\, kN$$

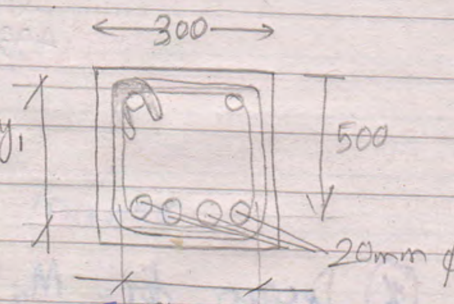
As $V_{ue} = 78.33\, kN < V_{uc} = 87.15\, kN$, initial assumption wa correct.

Now, for shear reinforcement,

As $V_{ue} \le V_{uc}$, for shear, min. shear reinforcement in the form of vertical stirrups shall be provided.

Consider 8mm ϕ two legged v. stirrups:

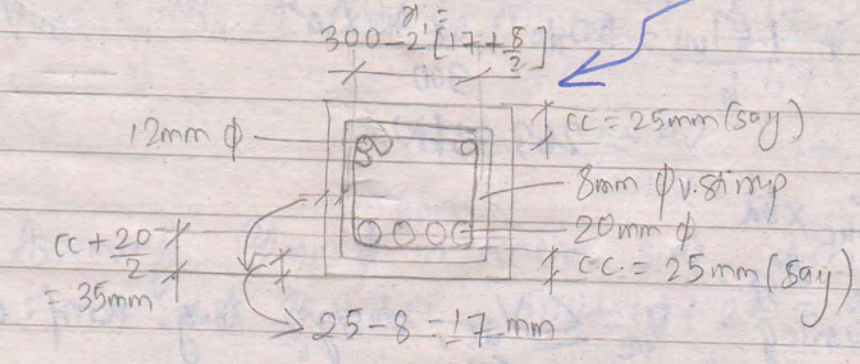
$$S_v = \frac{0.87 f_y A_{sv}}{0.4b} \quad [\text{Cl. } 26.5.1.6]$$



$$= \frac{0.87 \times 415 \times 2 \times \pi 4^2}{0.4 \times 300}$$

$$= 302.47\text{mm} \not> S_{v,max}$$

$S_{v,max}$ = min. of i) $x_1 = 300 - 2\left[\frac{8}{2} + 17\right] = 258\text{mm}$



ii) $\frac{x_1 + y_1}{4} = \frac{258 + 500 + \frac{20}{2} + \frac{8}{2} - (25 - \frac{8}{2})}{4}$

$= 187.75$ mm

iii) $0.75d = 0.75 \times 500 = 375$mm

iv) 300mm

$\therefore S_v = 187.75$mm

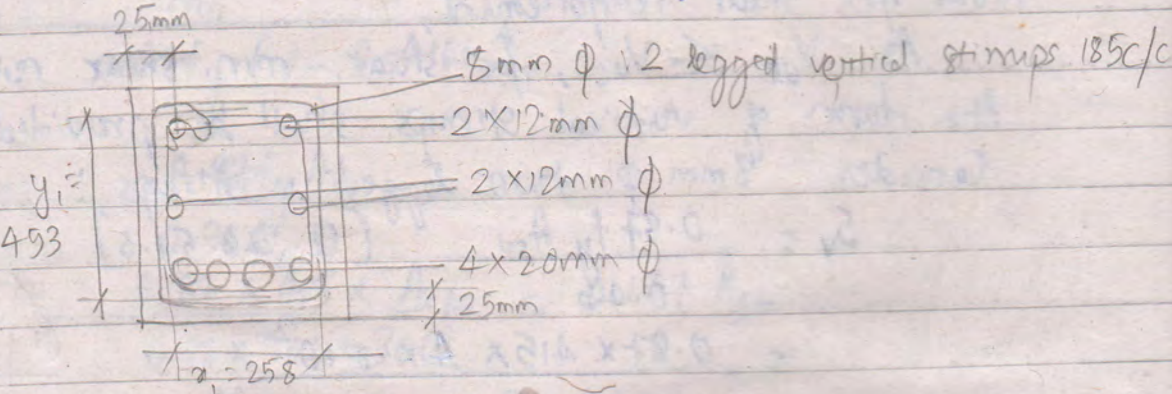
Adopt $S_v = 185$mm

Since $d = 500$ mm > 450mm, provide side reinforcements of

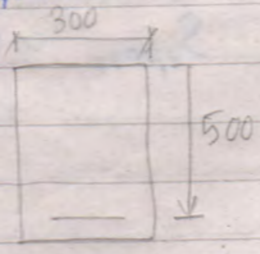
$A_s^{side} = 0.1\%$ of $bd = \frac{0.1}{100} \times 300 \times 500 = 150\text{mm}^2$

Adopting 12mm ϕ bars, $n \times \pi 6^2 = 150 \Rightarrow n = 1.32 \sim 2$

Provide 2 x 12mm ϕ bars one on each vertical face of the beam section.



(*) Design for $M_u = 50kN\text{-}m$, $V_u = 50kN$, $T_u = 40kN\text{-}m$
M20, Fe415



Soln:-

$$V_{ue} = V_u + \frac{1.6T_u}{b} = 50 + \frac{1.6 \times 40 \times 10^3}{300}$$
$$= 263.33kN$$

$V_{uc} = \tau_{uc} \times bd$

For τ_{uc}, $A_{st} = ?$

Assuming $V_{ue} \leq V_{uc}$, design long. reinf. only for flexural BM

So design for $M_u = 50kN\text{-}m$

But SR or DR?

$M_{u,l} = 0.36 f_{ck} b x_{u,l} (d - 0.416 x_{u,l})$

$x_{u,l} = \cancel{0.87 f_y A_{st}}$ $0.48d = 0.48 \times 500 = 240mm$

$\therefore M_{u,l} = 0.36 \times 20 \times 300 \times 240 \times (500 - 0.416 \times 240) = 207.44kN\text{-}m$

As $M_u < M_{u,l}$, design as SR URS.

So, for SR URS,

$$A_{st} = \frac{M_u}{0.87 f_y (d - 0.416 x_u)}$$

$$x_u = \frac{0.87 f_y A_{st}}{0.36 f_{ck} b} = \frac{0.87 \times 415 \times A_{st}}{0.36 \times 20 \times 300} = 0.167 A_{st}$$

$$\therefore A_{st} = \frac{50 \times 10^6}{0.87 \times 415 (500 - 0.416 \times 0.167 A_{st})}$$

$A_{st} = 288.54\ mm^2$

Then, $\dfrac{100 A_{st}}{bd} = \dfrac{100 \times 288.54}{300 \times 500} = 0.192$

From table 19 for M20,

$\tau_{uc} = 0.28 + \dfrac{0.36 - 0.28}{0.25 - 0.15} \times (0.192 - 0.15)$

$= 0.314\ N/mm^2$

$\therefore V_{uc} = \tau_{uc} \times bd = 47.04\ kNm$

Check: $V_{ue} > V_{uc}$. So, our assumption is incorrect.

Now assume $V_{ue} > V_{uc}$.

So, design long. reinf. for eqvt. BM (M_{ue}) & transverse reinf. for eqvt. SF (V_{ue}) resp.

$M_{ue} = M_u \pm M_{ut}$; $M_{ut} = T_u \dfrac{(1 + D/b)}{1.7}$

$= \dfrac{40}{1.7}\left(1 + \dfrac{D}{b}\right)$

Assuming 20mm ϕ bars in tension side, with c.c = 25mm,

$D = \cancel{cc}\ d + \dfrac{20}{2} + cc = 500 + 10 + 25 = 535\ mm$

$\therefore M_{ut} = \dfrac{40}{1.7}\left(1 + \dfrac{535}{300}\right) = 65.49\ kN\text{-}m$

As $M_{ut} > M_u$, long. reinforcements for both tension and compression sides are to be provided for resisting moments M_{ue1} & M_{ue2} resp. where,

$M_{ue1} = M_u + M_{ut} = 50 + 65.49 = 115.49\ kN\text{-}m$

$M_{ue2} = M_u - M_{ut} = 50 - 65.49 = -15.49\ kN\text{-}m$

M_{ue1} must be resisted by tension bars of 20mmϕ (As assumed before for calc.^{n} of D). But SR or DR?

$M_{u,l} = 0.36 f_{ck} b x_{u,l} (d - 0.416 x_{u,l}) = 207.44\ kNm$ (as b4)

$\because M_{ue1} = 115.49 < M_{u,l} = 207.44$, design as SR URS.

For SR, URS,

$$A_{st} = \frac{M_{ue1}}{0.87 f_y (d - 0.416 x_u)};$$

$$x_u = \frac{0.87 f_y A_{st}}{0.36 f_{ck} b}$$

$$= \frac{0.87 \times 415 \times A_{st}}{0.36 \times 20 \times 300} = 0.167 A_{st}$$

$$\therefore A_{st} = \frac{115.49 \times 10^6}{0.87 \times 415 (500 - 0.416 \times 0.167 A_{st})}$$

$\Rightarrow A_{st} = 709.73 \text{ mm}^2$

$\Rightarrow n \times \pi 10^2 = 709.73$

$\Rightarrow n = 2.25$

Adopt 3×20mm ϕ tension bars at bottom $[A_{st} = 942.4 \text{mm}^2]$

M_{ue2} must be resisted by compression bars at top.

Assuming CC = 25mm & compression bars of 16mm ϕ.

$$d' = 25 + \frac{16}{2} = 33\text{mm}$$

$$A_{sc} = \frac{M_{ue2}}{0.87 f_y (d - d')}$$

$$= \frac{15.49 \times 10^6}{0.87 \times 415 (500 - 33)}$$

$= 91.86 \text{ mm}^2$

$\Rightarrow n \times \pi 8^2 = 91.86$

$\Rightarrow n = 0.45$

Adopt 2×16 mm ϕ bars at top [$\because$ coz code says there must be at least 1 bar at each corner for design in torsion]

Check for assumption that $V_{ue} > V_{uc}$:

For τ_{uc}, $\frac{100 A_{st}}{bd} = 100 \times \frac{3 \times \pi 10^2}{300 \times 500} = 0.628$

From table 19, for M20,

$$\tau_{uc} = 0.48 + \frac{0.56-0.48}{0.75-0.50} \times (0.628-0.50)$$

$$= 0.521 \ N/mm^2$$

$$\therefore V_{uc} = \tau_{uc} \times bd = 78.14 kN$$

As $V_{ue} > V_{uc}$, assumption was correct.
$(263 > 78)$

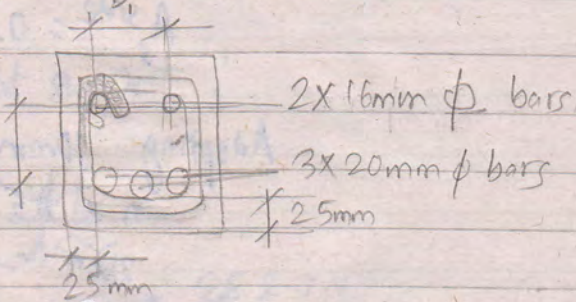
Now,

transresse reinforcements:

Since $V_{ue} > V_{uc}$, shear reinf. designed for eqvt. SF(V_{ue})

$$S_v = \frac{0.87 f_y A_{sv} b_1 d_1}{T_u + 0.4 V_u b_1}$$

Adopting 2 legged 8 mm ϕ vertical stirrups,



$$A_{sv} = 2 \times \pi 4^2 = 100.53 mm^2$$

$$b_1 = 300 - 2[25 + 16/2] = 234mm$$

$$d_1 = 500 - \quad [d'] = 467 \ mm$$

$$\therefore S_v = \frac{0.87 \times 415 \times 100.53 \times 234 \times 467}{40 \times 10^6 + 0.4 \times 50 \times 10^3 \times 234} = 88.77 mm \le S_{v,max}$$

$S_{v,max}$ = Min. of i) $\frac{0.87 f_y A_{sv}}{(\tau_{ue} - \tau_{uc}) b} = \frac{0.87 \times 415 \times 100.53}{\left[\frac{263 \times 10^3}{300 \times 500} - 0.521\right] \times 300}$

$$= 98.18 mm$$

ii) $x_1 = 300 - 2[25 - 8/2] = 258mm$

iii) $\frac{x_1 + y_1}{4} = \frac{258 + 500 + \frac{20}{2} + \frac{8}{2} - (25 - 8/2)}{4} = 187.75 mm$

iv) $0.75d = 375mm$

v) $300mm$

$$\therefore S_v = 88.77mm$$

Adopt $S_v = 85mm$. But, for convenience in constrn, a larges spacing may be used by adopting a larges dia. vertical stirrup bar Say, $10\,mm\,\phi$

Then, corresponding spacing S_v is given by:-

$$S_v = \frac{S_v \text{ for } 8mm\,\phi \text{ stirrup}}{A_{sv} \text{ for } 8mm\,\phi \text{ stirrup}} \times A_{sv} \text{ for } 10mm\,\phi \text{ stirrup}$$

$$= \frac{88.77}{2\times\pi 4^2} \times 2\times\pi 5^2$$

$$= 138.7$$

Say, $S_v = 135\,mm$ (2-legged)

So, use $10mm\,\phi$ V. stirrup @ $135mm$ c/c

Now, as $d > 450\,mm$, provide side reinforcements:

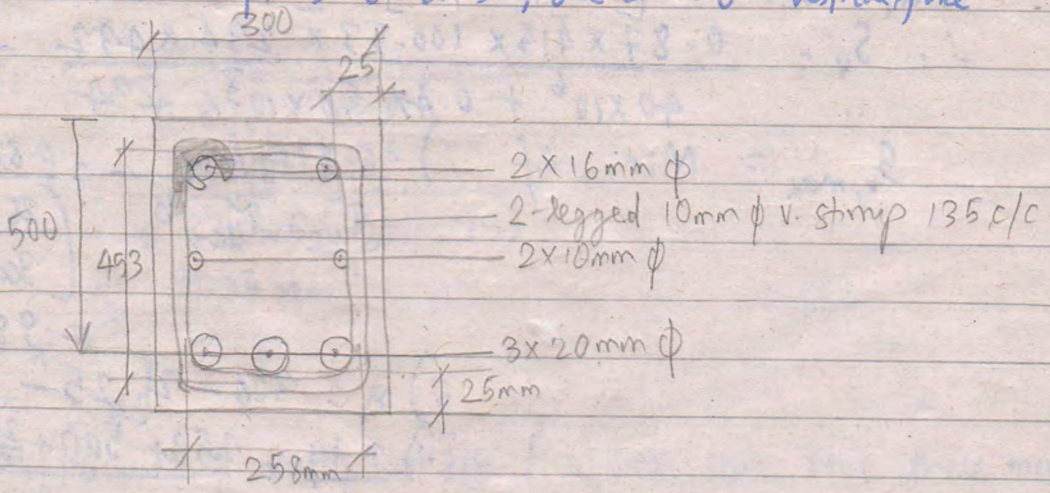
$$A_s^{side} = 0.1\% \text{ of } bd = \frac{0.1}{100} \times 300 \times 500 = 150\,mm^2$$

Adopting $10mm\,\phi$ bars,

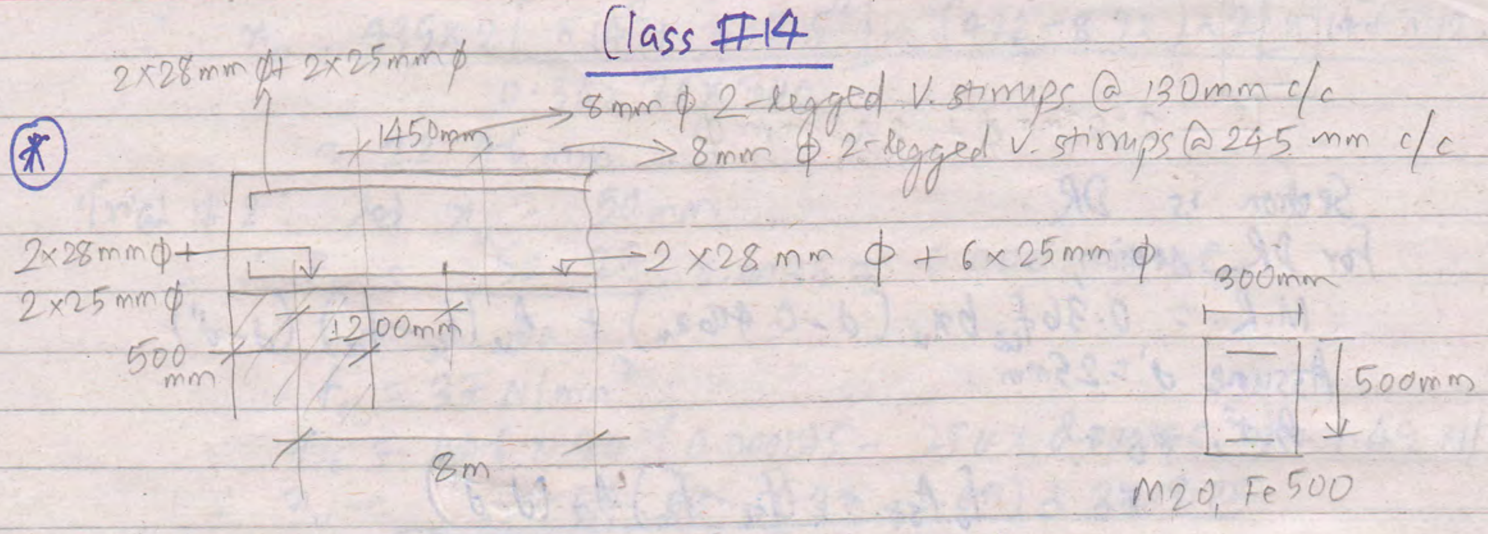
$$n \times \pi 5^2 = 150$$

$$\Rightarrow n \approx 1.9 \sim 2$$

Adopt $2 \times 10mm\,\phi$ side bars, one on each vertical face.



Class #14



- Check development length of bar at support
- Check shear capacity at the pt. of curtailment

Soln:—

(i) Check for development length of bar at support:

Minimum development length $(L_d) = \frac{\phi f_s}{4\tau_{bd}}$ → 26.2.1.1

For $\phi = 28mm$,

$$L_d = \frac{28 \times 0.87 \times 500}{4 \times (1.6 \times 1.2)} = 1585.94 \text{ mm}$$

For $\phi = 25mm$,

$$L_d = \frac{25 \times 0.87 \times 500}{4 \times (1.6 \times 1.2)} = 1416.02 \text{ mm}$$

L_d = Larger of the two = 1585.94 mm

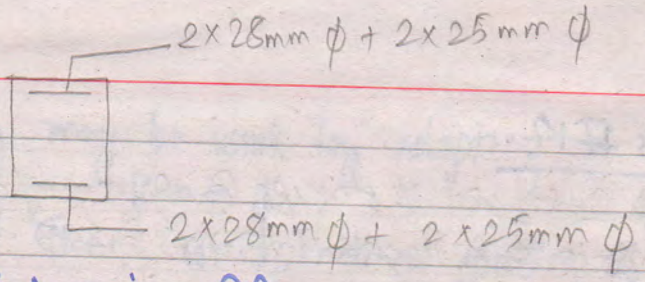
Then, provided dev. length at support $= 1.3\frac{M_1}{V_u} + l_d$

Assuming 27 kN/m UDL and the full span of beam as 16m,

$$V_u = \text{SF at support} = \frac{Wl}{2} = \frac{27 \times 16 \times 10^3}{2} = 216000 \text{ N}$$

M_1 = M.R. of beam at support.

For M_1,



Section is DR.

For DR section,

$$M.R. = 0.36 f_{ck} b x_u (d - 0.416 x_u) + A_{sc}(f_{sc} - f_{cc})(d - d')$$

Assume $d' = 25$ mm

But, $x_u = ?$

$$x_u = \frac{f_s A_{st} - (f_{sc} - f_{cc}) A_{sc}}{0.36 f_{ck} b}$$

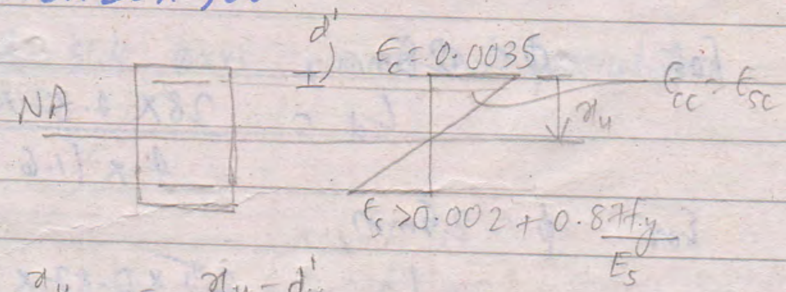
f_s, f_{sc}, f_{cc} are unknown.

For Fe500, $x_{u,l} = 0.46d = 0.46 \times 500 = 230$ mm

find x_u & compare with $x_{u,l}$ to check if UR or OR

Assume UR $\Rightarrow f_s = 0.87 f_y = 0.87 \times 500 = 435$ N/mm²

$$\text{So, } x_u = \frac{435 \times A_{st} - (f_{sc} - f_{cc}) A_{sc}}{0.36 \times 20 \times 300}$$



$$\frac{x_u}{0.0035} = \frac{x_u - d'}{\epsilon_{sc} = \epsilon_{cc}}$$

$$\Rightarrow \frac{x_u}{0.0035} = \frac{x_u - d'}{\epsilon_{sc}} \Rightarrow \epsilon_{sc} = \frac{x_u - d'}{x_u} \times 0.0035$$

Trial #1 : let $x_u = x_{u,l} = 230$ mm

$$\therefore \epsilon_{sc} = \frac{230 - 25}{230} \times 0.0035 = 0.00312$$

from fig. 3, $f_{sc} = 422$ N/mm² for $\epsilon_{sc} = 0.00312$ & Fe500

$\epsilon_{cc} = \epsilon_{sc} = 0.00312 > 0.002 \therefore f_{cc} = 0.446 f_{ck} = 8.92$ N/mm²

$$\therefore x_u = \frac{435 \times 2[\pi 14^2 + \pi 12.5^2] - (422 - 8.92) \times 2[\pi 14^2 + \pi 12.5^2]}{0.36 \times 20 \times 300}$$

$= 22.46$ mm

Trial #2: let $x_u = 50$ mm

$$\therefore \varepsilon_{sc} = \frac{50-25}{50} \times 0.0035 = 0.000175 = \varepsilon_{cc}$$

$f_{sc} = 37\ N/mm^2$

$f_{cc} = 446 \times 20\,(0.000175 - 250 \times 0.000175^2) = 1.49\ N/mm^2$

$$\therefore x_u = \frac{962764.63 - (37 - 1.49) \times 2213.25}{2160}$$

$= 409.33$ mm

Trial #3: let $x_u = 200$ mm

$\therefore \varepsilon_{sc} = 0.0030625 = \varepsilon_{cc}$

$f_{sc} = 420\ N/mm^2$

$f_{cc} = 8.92\ N/mm^2$

$\therefore x_u = 24.51$ mm

Trial #4: let $x_u = 112.26$ mm

$\therefore \varepsilon_{sc} = 0.00272 = \varepsilon_{cc}$

$f_{sc} = 410\ N/mm^2$

$f_{cc} = 446 \times 20\,(0.00272 - 250 \times 0.00272^2) = 7.76\ N/mm^2$

$\therefore x_u = 33.57$ mm

Trial #5: let $x_u = 73$ mm

$\therefore \varepsilon_{sc} = 0.0023 = \varepsilon_{cc}$

$\therefore f_{sc} = 391\ N/mm^2$

$f_{cc} = 8.92\ N/mm^2$

$\therefore x_u = 54.23$ mm

Trial #6: let $x_u = 63.6$ mm

$\therefore \varepsilon_{sc} = 0.00212$

$\therefore f_{sc} = 380\ N/mm^2$

$f_{cc} = 8.92\ N/mm^2$ $\therefore x_u = 65.49$ mm

Trial #7 : let $x_u = 64.55$mm

$\therefore E_{sc} = 0.00214 = \varepsilon_{cc}$

$\therefore f_{sc} = 382 N/mm^2$

$f_{cc} = 8.92\ N/mm^2$

$\therefore x_u = 63.45$mm

$\therefore$

$x_u = 64mm < x_{u,l} = 230mm$

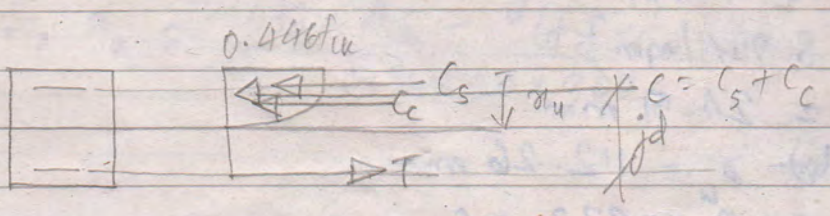
$\therefore$ Section is UR ie. assumption is correct.

$\therefore$ M.R. $= 0.36 f_{ck} b x_u (d - 0.416 x_u) + (f_{sc} - f_{cc}) A_{sc} (d - d')$

$= 0.36 \times 20 \times 300 \times 64 \times (500 - 0.416 \times 64) + (382 - 8.92) \times 2\pi[14^2 + 12.5^2] \times (500 - 25)$

$= 65.44$ kN-m $+ 392.22$ kN-m

$= 457.66$ kN-m



M.R. $= T \times jd$

$\Rightarrow 457.66 \times 10^6 = f_s A_{st} \times jd = 0.87 f_y \times 2\pi[14^2 + 12.5^2] \times jd$

$\therefore jd = 475.36$ mm

Now,

l_d = Additional anchorage dist. of bar

$= x_1$ + eqv. embedment length

for 90° bend $= 8\phi = 8 \times 28 = 224$mm

$x_1 = \frac{b_s}{2} - c.c. - 3\phi = \frac{500}{2} - 20 - 3 \times 28 = 146$mm

c.c. say 20 mm

$\therefore l_d = 370$ mm

$\therefore$ Provided development length $= 1.3\dfrac{M_1}{V} + l_d$

$= 1.3 \times \dfrac{457.66\,kNm}{216\,kN} + 0.37\,m$

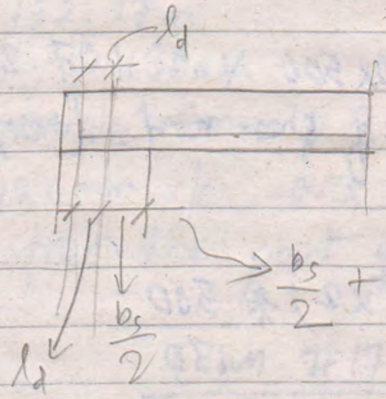
$= \cancel{27.91}\ 3.12\,m$

$= 3120\,mm$

Here,

$L_d = 1585.94\,mm < 1.3\dfrac{M_1}{V} + l_d = 3120\,mm$

OK.



$\dfrac{b_s}{2} + l_d = \dfrac{500}{2} + 370 = 620\,mm \not< \dfrac{L_d}{3} = 528.67\,mm$

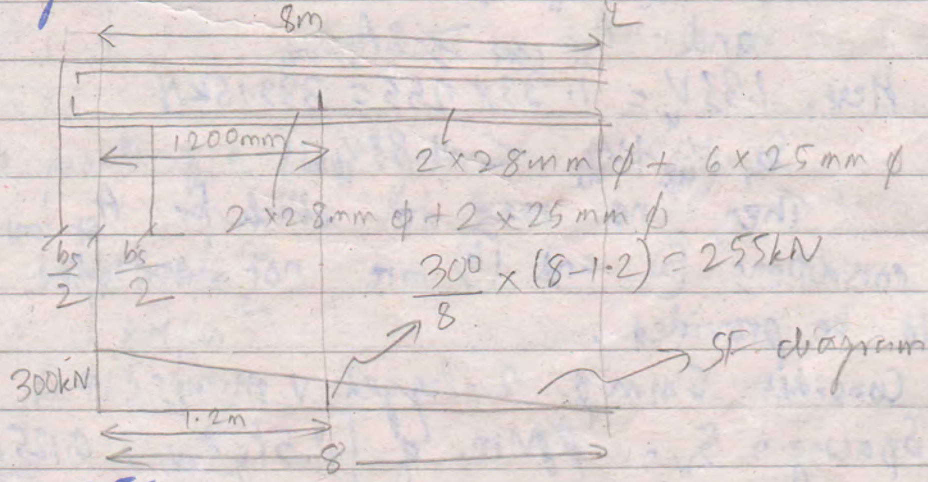
OK.

(ii) Checking shear capacity at the point of curtailment:

(a)$_1$ check if $V_{uc} + V_{us} \geq 1.5 V_u$:

V_u = SF at the point of curtailment

let SF diagram be as:



$\therefore V_u = 255\,kN$

V_{uc} = shear resisting cap. of concrete
$= \tau_{uc} \times bd$

From table 19 of IS456,

$$\frac{100 A_{st}}{bd} = 100 \times \frac{2 \times [\pi 14^2 + \pi 12.5^2]}{300 \times 500} = 1.475\%$$

$$\therefore \tau_{uc} = 0.67 + \frac{0.72-0.67}{1.50-1.25} \times (1.475-1.25)$$

$$= 0.715 \ N/mm^2$$

$$\therefore V_{uc} = 0.715 \times 300 \times 500 \ N = 107.25 kN$$

V_{us} = shear resisting cap. of shear reinf. (here, v. stirrups)

$$= 0.87 f_y A_{sv} \frac{d}{S_v}$$

$$= 0.87 * 500 * (\pi 4^2 \times 2) * \frac{500}{130}$$

$$\doteq 168.19 kN$$

$V_{uc} + V_{us} = 275.44 kN$

$1.5 V_u = 1.5 \times 255 = 382.5 kN$

As $V_{uc} + V_{us} < 1.5 V_u$, another condition is checked.

(b) Check if: $V_{uc} + V_{us} \geq 1.33 V_u$

and $A_{st\ cont} \geq 2 A_{st\ req.}$

Here, $1.33 V_u = 1.33 \times 255 = 339.15 kN$

So, $V_{uc} + V_{us} < 1.33 V_u$

Then, no need to check for $A_{st\ cont} \geq 2A_{st\ req.}$

(c) As conditions (a) and (b) are not satisfied, excess stirrups should be provided.

Consider 6mm ϕ 2-legged V. stirrups

Spacing $= S_v =$ Min. of $\left(2.5 f_y \frac{A_{sv}}{b}, \ \frac{0.125 d}{\beta_b} \right)$

$A_{sv} = 2 \times \pi 3^2 = 56.55 mm^2$
$b = 300mm$
$d = 500mm$

$$\beta_b = \frac{A_{st} \text{ curtailed}}{A_{st}} = \frac{4 \times \pi 12.5^2}{2 \times \pi 14^2 + 6 \times \pi 12.5^2} = 0.47$$

$$\therefore S_v = \text{Min. of} \left(\frac{2.5 \times 500 \times 56.55}{300}, \frac{0.125 \times 500}{0.47} \right)$$

$$= \text{Min. of} (235.625, 132.97)$$

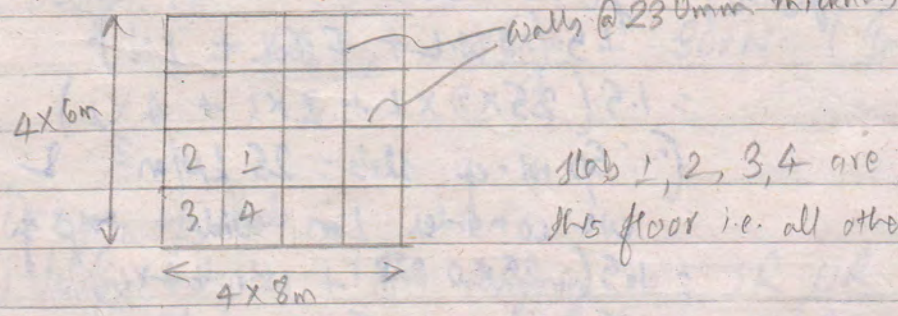
$$= 132.97 m$$

Say, $S_v = 130mm$

Therefore, provide 6mm ϕ 2-legged V. stirrups extra at spacing of 130mm c/c over a distance of $0.75d = 0.75 \times 500 = 375mm$ from the point of curtailment.

Class #17

Continuous two-way slab design :-



Slab 1, 2, 3, 4 are the representative panels of this floor i.e. all others are one of these.

Design slab panels of a floor given in the fig.
Take:-

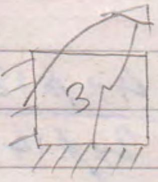
$LL = 4 \; kN/m^2$
$FF = 1 \; kN/m^2$
M20, Fe415

Soln:— Here, l_x = shorter span of panel = 6 m. l_y = longer span of panel = 8 m. $\frac{l_y}{l_x} = 1.33 < 2$

$\therefore$ Two-way slab

Design of slab panel (3) :—

discontinuous edges



(1) Preliminary Design :—

For eff. depth 'd',

As the floor consists of panels more than one, design as continuous slab.

For continuous slab panel,

$\frac{l_x}{d} = 32$; l_x = shorter span

$\Rightarrow d = \frac{l_x}{32} = \frac{6}{32} = 0.1875\,m = 187.5\,mm$

Adopt D = 200 mm

$\therefore d = D - cc - \frac{\phi}{2} = 200 - 15 - \frac{10}{2} = 180\,mm$

(2) Analysis of slab :—

(i) Design load intensity $(W_u) = \gamma_f (DL + LL)$ → F.O.S. = 1.5 for DL & LL

$= 1.5\,(S.W.L + F.F.L + LL)$

$= 1.5\,(25 \times D \times 1 + 1 \times 1 + 4 \times 1)$

[$\because$ Sp. wt. of slab = 25 kN/m³ & we consider 1 m wide strip of slab]

$= 1.5\,(25 \times 0.2 \times 1 + 1 \times 1 + 4 \times 1)$

$= 15$ kN/m span

(ii) Design BM $[M_{ux} = \alpha_x W_u l_x^2 \; ; \; M_{uy} = \alpha_y W_u l_x^2]$

Shorter span :—

BM at support (Discontinuous) $(M_1) = 0$

BM at mid-span $(M_2) = \alpha_x W_u l_x^2 = \alpha_x \times 15 \times 6^2$

for α_x, from IS456, table 26, for $l_y/l_x = 1.33$, for two adjacent edges discontinuous, positive mom. at mid-span

$$\alpha_x = 0.049 + \frac{0.053-0.049}{1.4-1.3} \times (1.33-1.3)$$

$$= 0.0502$$

$\therefore M_2 = 27.108\ kNm$

BM at support (Continuous) $(M_3) = \alpha_x W_u l_x^2$

$$\alpha_x = 0.065 + \frac{0.071 - 0.065}{1.4 - 1.3} \times (1.33-1.3)$$

$$= 0.0668$$

$\therefore M_3 = \alpha_x w_u l_x^2 = -36.072\ kNm$

Longer span :-

BM at discont. support $(M_4) = 0$

BM at mid-span $(M_5) = \alpha_y W_u l_x^2$

$\alpha_y = 0.035$

$\therefore M_5 = 0.035 \times 15 \times 6^2 = 18.9\ kNm$

BM at continuous support $(M_6) = \alpha_y W_u l_x^2$

$\alpha_y = 0.047$

$\therefore M_6 = -0.047 \times 15 \times 6^2 = -25.38\ kNm$

(iii) Design SF :

$$V_{ux} = \frac{W_u l_x}{3} = \frac{15 \times 6}{3} = 30kN$$ (In shorter span)

(3) Design of slab :-

(i) Verify depth : (for designing as SR UR section)

Check $d > d_{bal} = \sqrt{\frac{M_{ux}}{Qb}} = \sqrt{\frac{36.072 \times 10^6}{Q \times 1000}}$; $b = 1\ m = 1000\ mm$, $M_{ux} = 36.072 \times 10^6\ Nmm$

$$Q = 0.36 f_{ck} \frac{x_{u,l}}{d}\left(1 - 0.416 \frac{x_{u,l}}{d}\right)$$

$x_{u,l}$ for Fe415 $= 0.48d \Rightarrow x_{u,l}/d = 0.48$

$f_{ck} = 20\ MPa = 20\ N/mm^2$

$\therefore Q = 2.766 \quad \therefore d_{bal} = 114.198\ mm < d = 180\ mm$ OK

(ii) Reinforcement bars :-

Shorter span :-

A_{st_1} (At discont. edge) = 50% of A_{st_2}

A_{st_2} (At mid-span) = $\dfrac{M_2}{0.87f_y\,(d-0.416x_u)}$

$= \dfrac{27.108\times10^6}{0.87\times415\,(180-0.416\times x_u)}$

$x_u = \dfrac{0.87f_y A_{st2}}{0.36f_{ck}\,b}$

$= \dfrac{0.87\times415\times A_{st_2}}{0.36\times20\times1000} = 0.0501\,A_{st_2}$

$\therefore A_{st_2} = \dfrac{75081.01}{180-0.0208A_{st_2}}$

$\Rightarrow 180A_{st_2} - 0.0208A_{st_2}^2 - 75081.01 = 0$

$\Rightarrow 0.0208A_{st_2}^2 - 180A_{st_2} + 75081.01 = 0$

$A_{st_2} = 8214.416\,mm^2,\ 439.430\,mm^2$ } both $> A_{st}^{min}$ $(= 0.12\%\ bD)$ $= 216\,mm^2$

As $\phi = 10mm$,

$n = \dfrac{8214.416}{\pi5^2},\ \dfrac{439.430}{\pi5^2}$

$= 105\ ,\ 6$

Also, since c/c spacing 's' $\times$ n = b = 1000 mm

(Actually $s(n-1) = b$)

$\Rightarrow s = 1000/n$ mm

$\therefore s = \dfrac{1000}{105},\ \dfrac{1000}{\cancel{105}\ 6}$

$= \cancel{1}8mm,\ 165\,mm$ → say 150 mm

Ridiculous.

$\therefore$ let A_{st_2} = 10 mm ϕ @ 150 mm c/c in 1m width.

Then, $A_{st_1} = 50\%$ of $A_{st_2} = 10\text{mm}\,\phi$ @ 300mm c/c (in 1m width)

A_{st_3} (At continuous edge) $= \dfrac{M_3}{0.87 f_y (d - 0.416 x_u)}$

$= \dfrac{36.072 \times 10^6}{0.87 f_y (180 - 0.416 \times 0.0501 A_{st_3})}$

$\Rightarrow A_{st_3} = \dfrac{0.1 \times 10^6}{180 - 0.021 A_{st_3}}$

$\Rightarrow 0.021 A_{st_3}^2 - 180 A_{st_3} + \cancel{10^5} = 0$

~~$\therefore A_{st_3} = 8571.43,\ 5.5 \times 10^{-4}$~~ → drop.

~~$\therefore n = \dfrac{8571.43\ \text{mm}^2}{\pi 5^2\ \text{mm}^2} = 110$~~

~~$\therefore S = \dfrac{1000}{110} =$~~

$\therefore A_{st_3} = 7974.27\,\text{mm}^2,\ 597.16\,\text{mm}^2$ } both $> A_{st}^{min} = 0.12\% \cdot bD = 240\ \text{mm}^2$

$\therefore n = \dfrac{7974.27\ \text{mm}^2}{\pi 5^2\ \text{mm}^2},\ \dfrac{597.16\ \text{mm}^2}{\pi 5^2\ \text{mm}^2}$

$= 102,\ 8$

$\therefore S = \dfrac{1000\ \text{mm}}{102},\ \dfrac{1000\ \text{mm}}{8}$

$= 9\ \text{mm},\ 125\ \text{mm}$ ✓ say 110mm

Longer span :-

A_{st_4} (At discont. edge) $= 50\%$ of A_{st_5}

A_{st_5} (At mid-span) $= \dfrac{M_5}{0.87 f_y (d - 0.416 x_u)}$

$= \dfrac{18.9 \times 10^6}{0.87 \times 415 (180 - 0.416 x_u)}$

$x_u = \dfrac{0.87 f_y A_{st_5}}{0.36 f_{ck} b} = \dfrac{0.87 \times 415 \times A_{st_5}}{0.36 \times 20 \times 1000} = 0.0501 A_{st_5}$

$\therefore A_{st_5} = \dfrac{52347.32}{180 - 0.0208 A_{st_5}}$

$\Rightarrow 0.0208 A_{st_5}^2 - 180 A_{st_5} + 52347.32 = 0$

$\therefore A_{st_5} = 8352.53\ mm^2,\ 301.31\ mm^2$

$\therefore n = \dfrac{8352.53}{\pi 5^2},\ \dfrac{301.31}{\pi 5^2}$

$= 107,\ 4$

$\therefore S = \dfrac{1000\ mm}{107},\ \dfrac{1000\ mm}{\cancel{107}\ 4}$

$= 9\ mm,\ 250\ mm$ (say 220 mm)

$\therefore A_{st_5} = 10\ mm\ \phi$ @ 220 mm c/c

$\therefore A_{st_4} = 50\%$ of $A_{st_5} = 10\ mm\ \phi$ @ 440 mm c/c

But 440 mm c/c > $S_{max} = Min(3d, 300)$
$= Min(540, 300)$
$= 300\ mm$

$\therefore A_{st_4} = 10\ mm\ \phi$ @ 300 mm c/c.

A_{st_6} (At continuous edge) $= \dfrac{M_6}{0.87 f_y (d - 0.416 x_u)}$

$= \dfrac{25.38 \times 10^6}{0.87 \times 415 (180 - 0.0208 A_{st_6})}$

$\Rightarrow A_{st_6} = \dfrac{70294.97}{180 - 0.0208 A_{st_6}}$

$\Rightarrow 0.0208 A_{st_6}^2 - 180 A_{st_6} + 70294.97 = 0$

$\therefore A_{st_6} = 8243.89\ mm^2,\ 409.95\ mm^2$

$\therefore n = \dfrac{8243.89\ mm^2}{\pi 5^2\ mm^2},\ \dfrac{409.95\ mm^2}{\pi 5^2\ mm^2}$

$= 105,\ 6$

$\therefore S = \dfrac{1000\ mm}{105},\ \dfrac{1000\ mm}{6}$

$= 9\ mm,\ 165\ mm \rightsquigarrow$ Say 160 mm

$\therefore A_{st_2} = 10\text{mm}\ \phi$ @ 160 mm c/c

(iii) Check slab for shear (in shorter direcn):

$\tau_{uv} = \tau_{uv_x} = \dfrac{V_{ux}}{bd} = \dfrac{30 \times 10^3\ N}{1000\text{mm} \times 180\text{mm}} = 0.16 N/mm^2$

$k\tau_{uc}$: for k, from IS456 Cl. 40.2.1.1, for $D \geq 200$mm,
$k = 1.2$

for τ_{uc}, from Table 19 of IS 456, for M20 &
% reinforcement @ discont. edge $= 100 \times \dfrac{A_{st_1}}{bd}$

$= 100 \times \dfrac{1}{1000\text{mm} \times 180\text{mm}} \times \dfrac{1000}{300} \times \pi 5^2 \text{mm}^2$

$= 0.145\ \%$

$\therefore \tau_{uc} = 0.28\ N/mm^2 \therefore k\tau_{uc} = 0.336 N/mm^2$

As τ_{uv} $(0.16\ N/mm^2)$ $< k\tau_{uc} = 0.336\ N/mm^2$, ok.

(4) Check slab for serviceability in deflxn:-

is $\dfrac{l_x}{d} \leq \alpha\beta\gamma\delta\lambda$?

Here, $\dfrac{l_x}{d} = \dfrac{6000}{180} = 33.33$

α = Basic span-to-depth ratio (in x ie shorter span ofc.)
from Cl. 23.2.1, as one edge is simply supported (discont.)
& the other is continuous,

$\alpha = \dfrac{20+26}{2} = 23$

β = Mod. factor for $l_x > 10$m. Here, As $l_x < 10$m, $\beta = 1$

γ = Mod. factor for tension steel at mid-span (in shorter span ofc.)
from fig. 4 : $f_s = 0.58 f_y \dfrac{A_{st_2}^{req.}}{A_{st_2}^{provided}}$

$$\frac{A_{st_2}^{reqd.}}{A_{st_2}^{prov.}} = \frac{\frac{1000}{\cancel{160}165} \times \pi 5^2}{\frac{1000}{150} \times \pi 5^2} = \frac{150}{165} = 0.909$$

$\therefore f_s = 0.58 \times 415 \times 0.909$
$= 218.81 \ N/mm^2$

% reinforcement at mid-span (along shorter span afc)

$= 100 \times \frac{A_{st_2}^{prov}}{bd}$

$= 100 \times \frac{1000/150 \times \pi 5^2}{1000 \times 180}$

$= 0.291\ \%$

$\therefore$ from fig. 4, $\gamma = 1.69$

δ = Mod. factor for compression steel = 1

λ = Mod. factor for shape of beam = 1

$\therefore \alpha\beta\gamma\delta\lambda = 23 \times 1 \times 1.69 \times 1 \times 1 = 38.87$

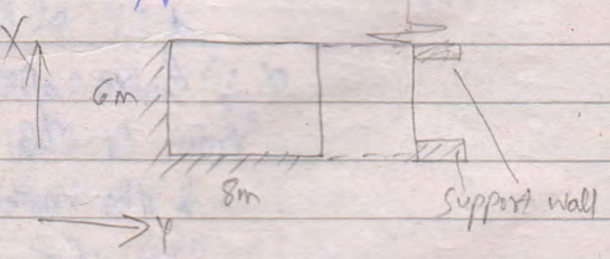
As $\frac{l_x}{d} = 33.33 < \alpha\beta\gamma\delta\lambda = 38.87$, ok.

(5) Detailing of reinforcement :

(i) Check for development length of bars at support :

Shorter span :-

is $\frac{1.3 M_l}{V_u} + l_d \geq L_d$?



M_l = M.R. cap at support
= M.R. of A_{st_1}
$\simeq$ 50% of M.R. of A_{st_2} since $A_{st_1} = 50\%$ of A_{st_2}
(infact M.R. of A_{st_1} will be greater than just 50% of M.R. of A_{st_2})
$= 0.5 \times M_2$
$= 0.5 \times 27.108$
$\simeq 14\ kNm$

V_u = S.F. at support = V_{ux} (in shorter span) = 30 kN (from step 2)
l_d = Additional anchorage length (beyond center of support)
$= \frac{230}{2} - \text{C.C.} = 115 - 20 = 95$

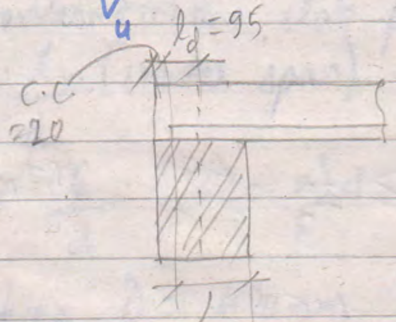
$\therefore \frac{1.3 M_l}{V_u} + l_d = \frac{1.3 \times 14 \times 10^6}{30 \times 10^3} + 95 = 701.6\text{ mm}$ = Available dev. length

$L_d = \frac{0.87 f_y \phi}{4 \tau_{bd}}$; τ_{bd} from Pg 43 of IS456 : for M20 concrete & deformed bars $\tau_{bd} = 1.6 \times 1.2 = 1.92\text{ N/mm}^2$

$= \frac{0.87 \times 415 \times 10}{4 \times 1.92}$

$= 470.12\text{ mm}$ = Reqd. dev. length

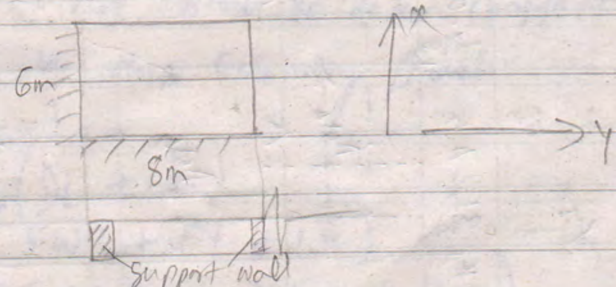
As $\frac{1.3 M_l}{V_u} + l_d = 701.6\text{ mm} > L_d = 470.12\text{ mm}$, OK.



$\frac{230}{2} + 95 - 210 > \frac{L_d}{3} = \frac{470.12}{3} = 156.7\text{ mm}$ OK.

Longer span :-

M_l = 50% of M.R. of A_{st_5}
$= 0.5 \times M_g$
$= 0.5 \times 18.9$ kNm
$= 9.45$ kNm



$V_u = V_{uy} = \frac{W_u l_x}{4}\left(2 - \frac{l_x}{l_y}\right)$

$= \frac{15\text{ kN/m} \times 6\text{ m}}{4}\left(2 - \frac{6}{8}\right)$

$= 28.125$ kN

$\therefore$ Provided dev. length $= \frac{1.3 M_1}{V_u} + l_d$

$$= 1.3 \times \frac{9.45 \times 10^6 \ N\text{-mm}}{28.125 \times 10^3 \ N} + \left(\frac{230}{2} - 20\right)$$

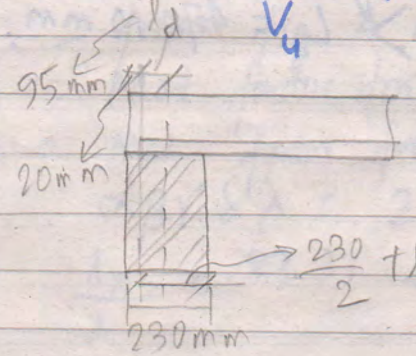
$$= 531.8 \text{ mm}$$

Reqd. min. dev. length $= L_d = \frac{\phi f_s}{4 \tau_{bd}} = \frac{\phi \, 0.87 f_y}{4 \tau_{bd}}$

$$= \frac{10 \times 0.87 \times 415}{4 \times 1.92}$$

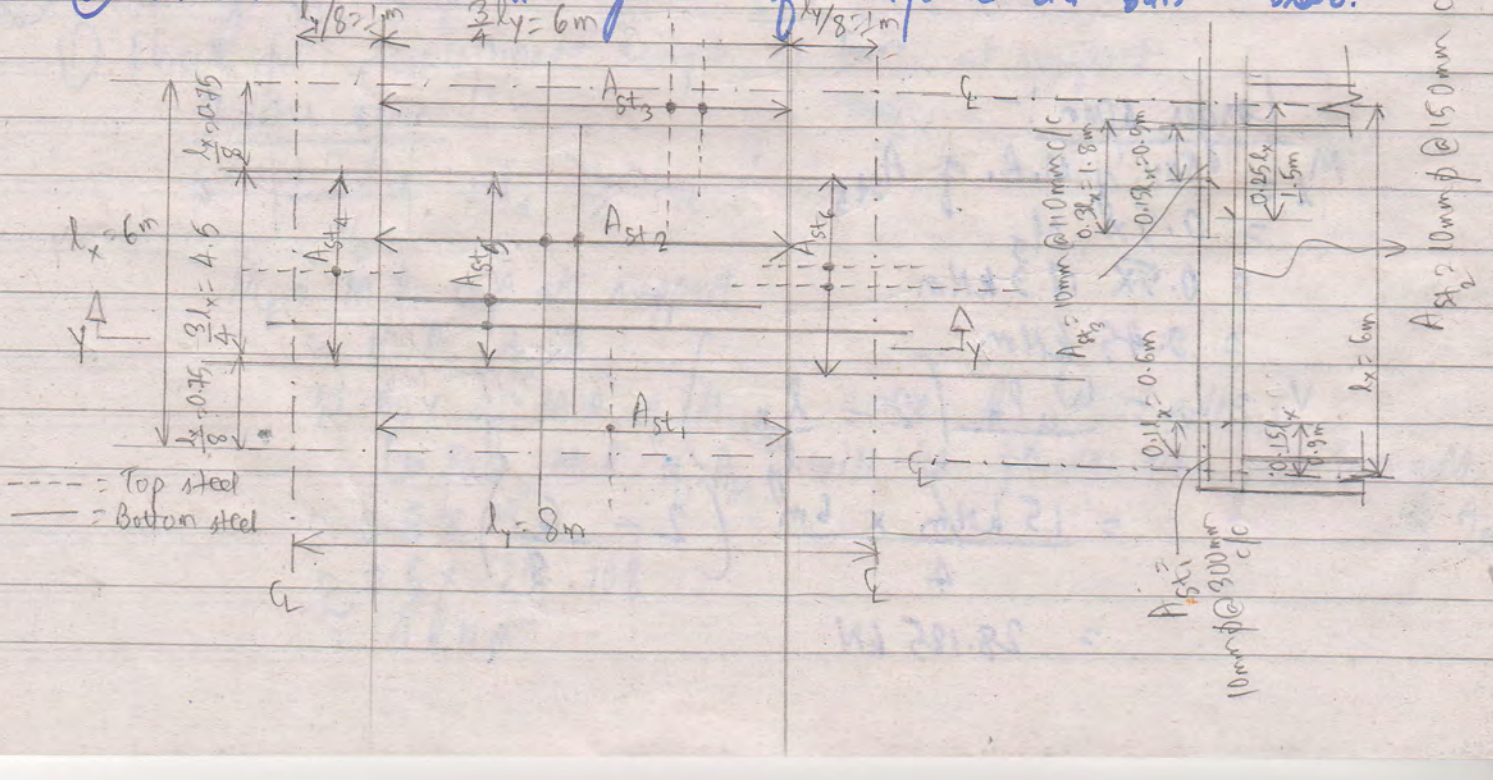
$$= 470.12 \text{ mm}$$

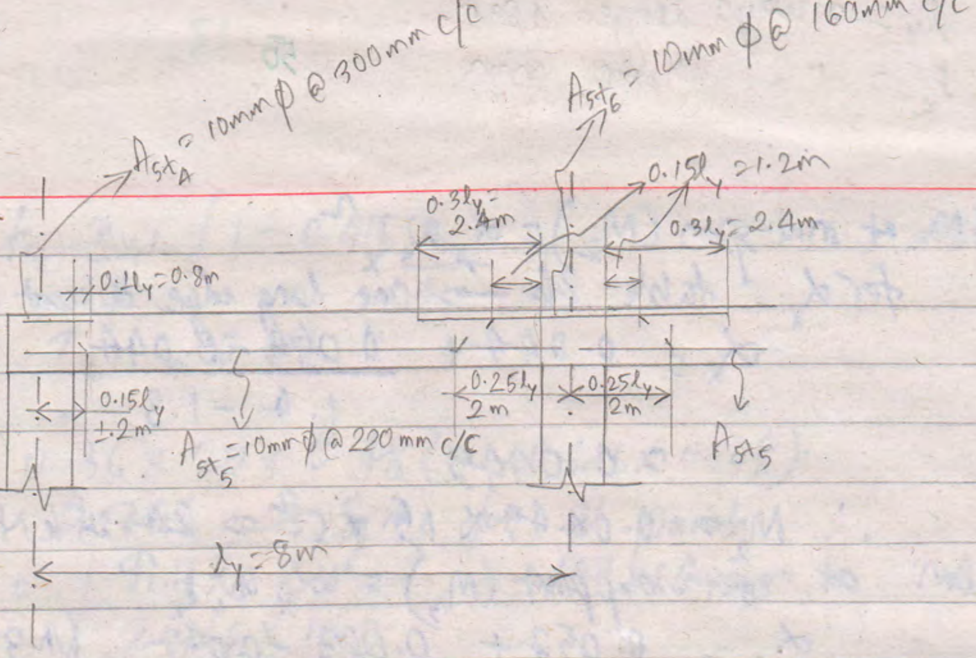
As $\frac{1.3 M_1}{V_u} + l_d = 531.8 \text{ mm} > L_d = 470.12 \text{ mm}$ OK.



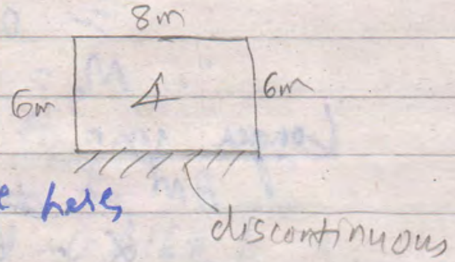
$\frac{230}{2} + l_d = 210 \text{ mm} \geq \frac{L_d}{3} = \frac{470}{3} = 157 \text{ mm}$ OK

(ii) Curtailment and arrangement of reinforcement bars in slab:





Section through X-X



Design of slab panel (4) :-

(1) Preliminary design :

For continuous slab panel as is the case here

$$\frac{l_x \text{ (shorter span)}}{d} = 32$$

$$\Rightarrow \frac{6}{d} = 32 \Rightarrow d = 0.1875m = 187.5mm$$

Taking $D = 200mm$ & c.c. = 15mm, & φ of steel = 10mm,

$$d = D - c.c. - \frac{\phi}{2} = 200 - 15 - \frac{10}{2} = 180mm$$

This shall be taken constant for all other remaining panels.

(2) Analysis of slab :

(i) Design load intensity $(W_u) = \gamma_f (DL + LL)$

$= 1.5 (S.W.L + FF + LL)$

$= 1.5 (25 \times 0.2 \times 1 + 1 \times 1 + 4 \times 1)$ for all panels

$= 15$ kN/m span → This will be the same for too

(ii) Design B.M. $[M_{ux} = \alpha_x W_u l_x^2 ; M_{uy} = \alpha_y W_u l_x^2]$

Shorter span :-

BM at discont. support $(M_1) = \alpha_x W_u l_x^2 = \alpha_x$ 0 .

l_y = always longer span
l_x = " shorter span

BM at mid-span $(M_2) = \alpha_x W_u l_x^2$
for α_x, table 26 → One long edge discont.
$$\alpha_x = 0.044 + \frac{0.047 - 0.044}{1.4 - 1.3} \times (1.33 - 1.3)$$
$= 0.0449$
$\therefore M_2 = 0.0449 \times 15 \times 6^2 = 24.25 \text{ kNm}$
BM at cont. support $(M_3) = \alpha_x W_u l_x^2$
$$\alpha_x = 0.057 + \frac{0.063 - 0.057}{1.4 - 1.3} (1.33 - 1.3)$$
$= 0.0588$
$\therefore M_3 = 0.0588 \times 15 \times 6^2 = -31.752 \text{ kNm}$

Longer span
BM at ~~discont~~ left cont. support $(M_4) = \alpha_y W_u l_x^2$
$\alpha_y = 0.037$
$\therefore M_4 = 0.037 \times 15 \times 6^2 = -19.98 \text{ kNm}$
BM at mid-span $(M_5) = \alpha_y W_u l_x^2$
$= 0.028 \times 15 \times 6^2$
$= +15.12 \text{ kNm}$
BM at right cont. support $(M_6) = \alpha_y W_u l_x^2$
$\alpha_y = 0.037$
$\therefore M_6 = 0.037 \times 15 \times 6^2 = -19.98 \text{ kNm}$

(iii) Design S.F. :-
$$V_{ux} = \frac{W_u l_x}{3} = \frac{15 \times 6}{3} = 30 \text{ kN}$$ → This is also a constant for all panels.

(3) Design of slab:
(i) Check for depth : (for designing as a SR UR section)
is $d > d_{bal}$?
$d = 180$ mm
Max B.M. in the shorter span
$$d_{bal} = \sqrt{\frac{M_{ux}}{Qb}} = \sqrt{\frac{31.752 \times 10^6}{Q \times 1000}}$$

$$Q = 0.36 f_{ck} \frac{x_{u,l}}{d}\left(1 - 0.416 \frac{x_{u,l}}{d}\right)$$

For Fe415, $\frac{x_{u,l}}{d} = 0.48$

$\therefore Q = 0.36 \times 20 \times 0.48 (1 - 0.416 \times 0.48)$

$= 2.766 \rightsquigarrow$ Same for all panels

$\therefore d_{bal} = \sqrt{\frac{31.752 \times 10^6}{2.766 \times 1000}} = 107.14 \text{ mm}$

As $d = 180 \text{ mm} > d_{bal} = 107.14 \text{ mm}$ OK.

(ii) Reinforcement bars :-

Shorter span :-

A_{st_1} (At discont. edge) $= 50\%$ of A_{st_2}

$$A_{st_2} \text{ (At mid-span)} = \frac{M_2}{0.87 f_y (d - 0.416 x_u)}$$

$$= \frac{24.425 \times 10^6}{0.87 \times 415 (180 - 0.416 x_u)}$$

$$= \frac{67165.2}{180 - 0.416 x_u}$$

$$x_u = \frac{0.87 f_y A_{st_2}}{0.36 f_{ck} b} = \frac{0.87 \times 415 A_{st_2}}{0.36 \times 20 \times 1000} = 0.0501 A_{st_2}$$

$$\therefore A_{st_2} = \frac{67165.2}{180 - 0.416 \times 0.0501 A_{st_2}} = \frac{67165.2}{180 - 0.0208 A_{st_2}}$$

$\Rightarrow 0.0208 A_{st_2}^2 - 180 A_{st_2} + 67165.2 = 0$

$\therefore A_{st_2} = 8263.06 \text{ mm}^2, \; 390.79 \text{ mm}^2$

$$\therefore n = \frac{8263.06}{\pi 5^2}, \; \frac{390.79}{\pi 5^2}$$

$= 106 \quad , \quad 5$

$$\therefore \text{Spacing } (S) = \frac{1000}{106}, \; \frac{1000}{5}$$

$= 9 \text{ mm}, \; 200 \text{ mm} \rightsquigarrow$ say 180 mm

$\therefore A_{st_2} = 10mm\ \phi$ @ $180mm$ c/c

$\therefore A_{st_1} = 10mm\ \phi$ @ $360mm$ c/c

But max. spacing $= Min.(3d, 300mm)$

$= Min(540, 300)$

$= 300\ mm$

$\therefore A_{st_1} = 10mm\ \phi$ @ $300mm$ c/c

A_{st_3} (At cont. edge) $= \dfrac{M_{ag}}{0.87f_y(d - 0.416x_u)}$

$\Rightarrow A_{st_3} = \dfrac{31.752 \times 10^6 / (0.87 \times 415)}{180 - 0.0208A_{st_3}}$

$\Rightarrow A_{st_3} = \dfrac{87943.498}{180 - 0.0208A_{st_3}}$

$\Rightarrow 0.0208A_{st_3}^2 - 180A_{st_3} + 87943.498 = 0$

$\therefore A_{st_3} = 8134.05\ mm^2,\ 519.79\ mm^2$

$\therefore n = 104,\ 7$

$\therefore S = 9\ mm,\ 142\ mm \rightarrow$ Say $130\ mm$

$\therefore A_{st_3} = 10mm\ \phi$ @ $130mm$ c/c

Longer span :-

A_{st_4} (At left cont. edge) $= \dfrac{M_4}{0.87f_y(d - 0.416x_u)}$

$= \dfrac{19.98 \times 10^6}{0.87 \times 415(180 - 0.416x_u)}$

$x_u = \dfrac{0.87f_y A_{st4}}{0.36f_{ck}b} = 0.0501A_{st4}$

$\therefore A_{st_4} = \dfrac{55338.6}{180 - 0.0208A_{st4}}$

$\Rightarrow 0.0208A_{st_4}^2 - 180A_{st4} + 55338.6 = 0$

$\therefore A_{st4} = 8334.6\ mm^2,\ 319.2 mm^2$

$\therefore n = 107,\ 5$

$\therefore S = 9$ mm, 200mm → say 180 mm

$\therefore A_{st_4} = 10$ mm ϕ @ 180 mm c/c

A_{st_6} (At right cont. edge) $= A_{st_4} = 10$ mm ϕ @ 180mm c/c $\left[\because M_4 = M_6\right]$

$A_{st_5} = \dfrac{M_5}{0.87 f_y (d - 0.416 x_u)}$

$\Rightarrow 0.0208 A_{st_5}^2 - 180 A_{st_5} + 41877.86 = 0$

$\therefore A_{st_5} = 8414.57 \text{ mm}^2, \; 239.27 \text{ mm}^2 \ngtr A_{st}^{min} = 0.12\% \, bd = 240 \text{ mm}^2$

$\therefore A_{st_5} = 240 \text{ mm}^2$

$\therefore n = \dfrac{240}{\pi 5^2} \simeq 4$

$\therefore S = \dfrac{1000}{4} \simeq 250$ mm say 230mm

$\therefore A_{st_5} = 10$mm ϕ @ 230 mm c/c

(iii) Check slab for shear (in shorter span):

$\tau_{uv} = \dfrac{V_{ux}}{bd} = \dfrac{30 \times 10^3}{1000 \times 180} = 0.16 \text{ N/mm}^2$

Same for all

$k\tau_{uc} = 1.2 \times \tau_{uc} = 1.2 \tau_{uc}$

τ_{uc}: ~~k = 1.2 for D = 200mm~~ Table 19

% reinf. @ discont. edge $= 100 \times \dfrac{A_{st_1}}{bd}$

$= 100 \times \dfrac{1000}{300} \times \dfrac{\pi 5^2}{1000 \times 180}$

$= 0.145\%$

$\therefore \tau_{uc} = 0.28 \text{ N/mm}^2$

$\tau_{uv} = 0.16 \text{ N/mm}^2 < k\tau_{uc} = 0.336 \text{ N/mm}^2$

OK.

④ Check slab for serviceability in deflexn:-

$$\frac{l_x}{d} = \frac{6000}{180} = 33.33$$

is $\frac{l_x}{d} \leq \alpha \beta \gamma \delta \lambda$

α = Basic span-eff depth ratio (in shorter span)

$$= \frac{20 + 26}{2} = 23$$

$\beta = 1$

γ = Mod. factor for tension steel (in shorter span) (mid-span)

Fig. 4 :

$$f_s = 0.58 f_y \frac{A_{st_2}\ \text{reqd.}}{A_{st_2}\ \text{prov.}} = 0.58 \times 415 \times \frac{180}{200} = 216.63$$

% reinf. $= 100 \times \frac{A_{st_2}}{bd}$

$$= 100 \times \frac{1000}{180} \times \frac{\pi 5^2}{1000 \times 180}$$

$$= 0.242\%$$

from fig. 4, $\gamma \simeq 1.75$

$\delta = 1$

$\lambda = 1$

$\therefore \alpha \beta \gamma \delta \lambda = 23 \times 1 \times 1.75 \times 1 \times 1 = 40.25$

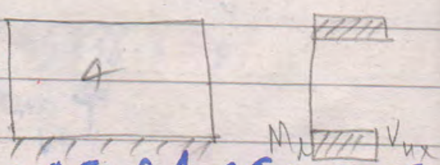
As $\frac{l_x}{d} = 33.33 \leq \alpha \beta \gamma \delta \lambda = 40.25$ OK.

⑤ Detailing of reinforcement :-

(i) Check for dev. length of bars at support :-

<u>Shorter span</u> :-

$$\frac{1.3 M_1}{V_{ux}} + l_d$$



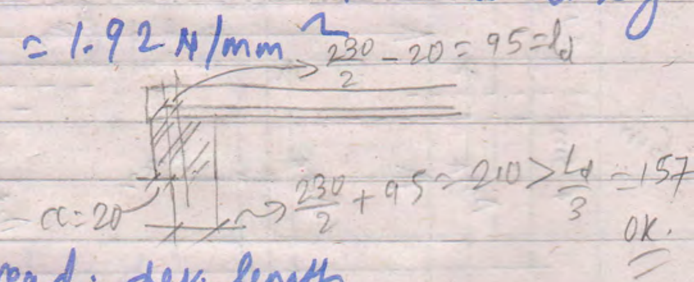
M_1 = MR at support (discont.) = 50% of $M_2 = 0.5 \times 24.25 = 12.125$ kNm

$V_u = V_{ux} = 30\,kN$

$l_d = \frac{230}{2} - c.c. = 115 - 20 = 95\ mm$

$\therefore 1.3\frac{M_1}{V_{ux}} + l_d = 1.3 \times \frac{12.125 \times 10^6}{30 \times 10^3} + 95 = 620.416\ mm$ = Provided dev. length

$L_d = \frac{0.87 f_y \phi}{4 \tau_{bd}}$; $\tau_{bd} = 1.92\ N/mm^2$



$= \frac{0.87 \times 415 \times 10}{4 \times 1.92}$

$= 470.12\ mm$ = Min. reqd. dev. length

As $1.3\frac{M_1}{V_{ux}} + l_d = 620\ mm > L_d = 470\ mm$, OK.

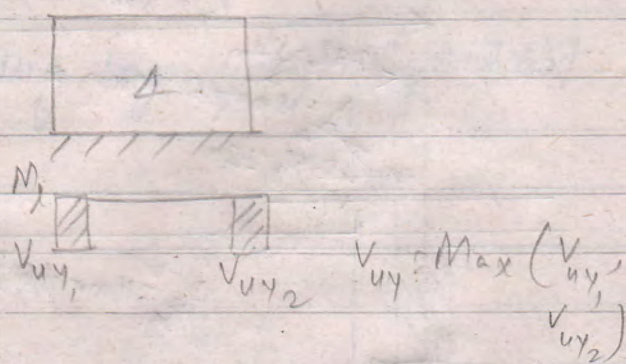
Longes span :-

$M_1 \approx M_4 = 19.98\ kNm$

$V_u = V_{uy} = \frac{W_u l_x}{4}\left(2 - \frac{l_x}{l_y}\right)$

$= \frac{15 \times 6}{4}\left(2 - \frac{6}{8}\right)$

$= 28.125\ kN$



$l_d = \frac{230}{2} - c.c. = 115 - 20 = 95\,mm$

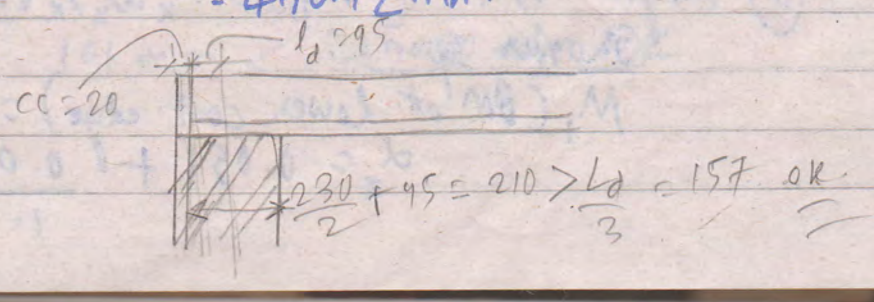
$\therefore$ Provided dev. length $= 1.3\frac{M_1}{V_u} + l_d = 1.3 \times \frac{19.98 \times 10^6}{28.125 \times 10^3} + 95$

$= 1018.52\,mm$

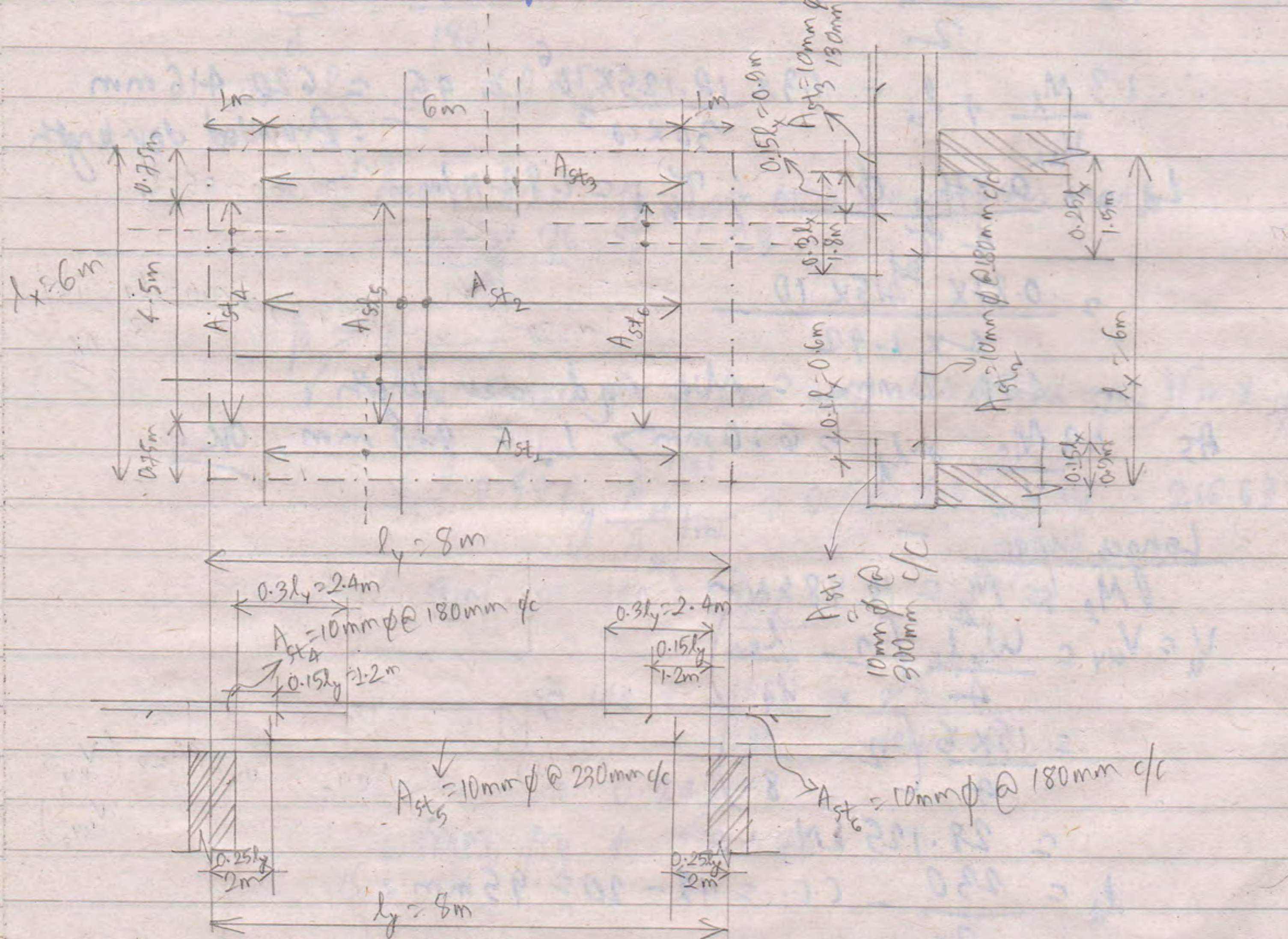
Min. reqd. dev. length $= L_d = \frac{0.87 f_y \phi}{4\tau_{bd}}$; $\tau_{bd} = 1.92\ N/mm^2$

$= 470.12\ mm$

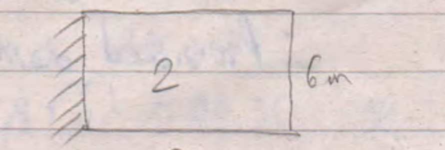
$\therefore$ OK.



(ii) Curtailment and arrangement of reinforcement bars in slab :-



Design of slab panel (2) :



(1) Preliminary design :

$D = 200mm$, $d = 180 mm$

(2) Analysis of slab :

(i) Design load intensity : $W_u = 15kN/m$ span

(ii) Design BM : $[M_{ux} = \alpha_x W_u l_x^2 ; M_{uy} = \alpha_y W_u l_x^2]$

Shorter span :

M_1 (BM at lower cont. edge) $= \alpha_x W_u l_x^2$

$$\alpha_x = 0.051 + \frac{0.055 - 0.051}{1.4 - 1.3} (1.33 - 1.3)$$

$= 0.0522$

$\therefore M_1 = -28.188$ kNm

M_3 (BM at upper cont. edge) $= M_1 = -28.188$ kNm

M_2 (BM at mid-span along shorter span):

$$\alpha_x = 0.039 + \frac{0.041 - 0.039}{1.4 - 1.3}(1.33 - 1.3)$$

$$= 0.0396$$

$\therefore M_2 = 21.384$ kNm.

<u>Longer span</u>:-

M_4 (BM at discont. edge) : $\alpha_y = 0$

$\therefore M_4 = 0$

M_5 (BM at mid-span) : $\alpha_y = 0.028$

$\therefore M_5 = 15.12$ kNm

M_6 (BM at cont. edge along longer span) : $\alpha_y = 0.037$

$\therefore M_6 = -19.98$ kNm

(iii) Design S.F. :

$$V_{ux} = \frac{W_u l_x}{3} = 30 \text{ kN}$$

(3) Design of slab :

(i) Check for depth :

is $d \geq d_{bal}$?

$d = 180$ mm

$$d_{bal} = \sqrt{\frac{M_{ux}}{Q b}} = \sqrt{\frac{28.188 \times 10^6}{Q \times 1000}}$$

$$Q = 0.36 f_{ck} \frac{x_{u,1}}{d}\left(1 - 0.416 \frac{x_{u,1}}{d}\right)$$

$$= 2.766$$

$\therefore d_{bal} = 101$ mm $< d = 180$ mm OK.

(ii) Reinforcement bars :

Shorter span:

$A_{st_1} = \frac{M_1}{0.87f_y(d-0.416x_u)}$

$= \frac{28.188 \times 10^6}{0.87 \times 415(180-0.416 \times x_u)}$

$x_u = \frac{0.87f_y A_{st_1}}{0.36 f_{ck} b} = 0.0501 A_{st_1}$

$\therefore A_{st_1} = \frac{78072.29}{180 - 0.0208 A_{st_1}}$

$\Rightarrow 0.0208 A_{st_1}^2 - 180 A_{st_1} + 78072.29 = 0$

$\Rightarrow A_{st_1} = 8195.87\ mm^2,\ 457.97 mm^2$

$\Rightarrow n = 105,\ 6$

$\Rightarrow s = 9 mm,\ 165 mm$ → say 150mm

$\therefore A_{st_1} = 10mm\ \phi$ @ 150mm c/c

$A_{st_3} = A_{st_1} = 10mm\ \phi$ @ 150 mm c/c $[M_1 = M_3]$

$A_{st_2} = \frac{M_2}{0.87f_y(d-0.416x_u)}$

$\Rightarrow 0.0208 A_{st_2}^2 - 180 A_{st_2} + 59227.25 = 0$

$\Rightarrow A_{st_2} = 8311.24\ mm^2,\ 342.61 mm^2$

$\Rightarrow n = 106,\ 5$

$\Rightarrow s = 9 mm,\ 200 mm$ → say 180mm

$\therefore A_{st_2} = 10 mm\ \phi$ @ 180 mm c/c

Longer span:

$A_{st_4} = 50\%$ of A_{st_5}

$A_{st_5} = \frac{M_5}{0.87f_y(d-0.416x_u)}$

$\Rightarrow 0.0208 A_{st_5}^2 - 180 A_{st_5} + 41877.86 = 0$

$\therefore A_{st_5} = 8414.58,\ 239.27 mm^2 < 0.12\% bD = 240\ mm^2$

59

$\therefore A_{st_5} = 240\,mm^2$

$\Rightarrow n = 4$

$\Rightarrow s = 250\,mm$ say $230\,mm$

$\therefore A_{st_4} = 10\,mm\ \phi$ @ $460\,mm$ c/c

But Max. spacing $= Min.(3d, 300)$

$= \cancel{Min(3\times 80, 300)}$

$= \cancel{180} = Min.(3\times 180, 300)$

$= 300\,mm$

$\therefore A_{st_4} = 10\,mm\ \phi$ @ $300\,mm$ c/c

& $A_{st_5} = 10\,mm\ \phi$ @ $230\,mm$ c/c

$A_{st_6} = \dfrac{M_c}{0.87 f_y (d - 0.416 x_u)}$

$\Rightarrow 0.0208 A_{st_6}^2 - 180 A_{st_6} + 55338.6 = 0$

$\therefore A_{st_6} = 8334.63\,mm^2, 319.21\,mm^2$

$\Rightarrow n = 107, 5$

$\Rightarrow s = 9\,mm, 200\,mm$

say $180\,mm$

$\therefore A_{st_6} = 10\,mm\ \phi$ @ $180\,mm$ c/c

(iii) Check slab for shear (in shorter span):

is $\tilde{\tau}_{uv} \le k\tilde{\tau}_{uc}$?

$\tilde{\tau}_{uv} = \dfrac{V_{ux}}{bd} = \dfrac{30\times 10^3}{1000\times 180} = 0.16\,N/mm^2$

$k = 1.2$ for $D = 200\,mm$ from Cl. 40.2.1.1

for $\tilde{\tau}_{uc}$ from Table 19,

% reinf at support along shorter span $= 100 \times \dfrac{A_{st_1} = A_{st_3}}{bd}$

$= 100 \times \dfrac{1}{\cancel{1000}\times 180} \dfrac{\cancel{1000}}{150} \times \pi 5^2$

$= 0.291\,\%$

$\therefore \tilde{\tau}_{uc} = 0.36 + \dfrac{0.48 - 0.36}{0.50 - 0.25} \times (0.291 - 0.25) = 0.379\,N/mm^2$

$\tau_{uv} = 0.16\ N/mm^2 < k\tau_{uc} = 0.455\ N/mm^2$

OK.

(4) Check for serviceability in deflxn: (in shorter span)

is $\frac{l_x}{d} \leq \alpha\beta\gamma\delta\lambda$?

$\frac{l_x}{d} = \frac{6000}{180} = 33.33$

$\alpha = 26$ [$\because$ in shorter span, the panel is continuous]

$\beta = 1$ [$\because l_x < 10\ m$]

γ = Mod. factor for tension steel @ mid-span in shorter span

$f_s = 0.58 f_y \frac{A_{st_2\,req}}{A_{st_2\,prov.}}$

$= 0.58 \times 415 \times \frac{180}{200}$

$= 216.63$

% reinf. at mid-span in shorter span

$= 100 \frac{A_{st_2}}{bd} = 100 \times \frac{1}{1000 \times 180} \times \frac{1000}{180} \times \frac{\pi}{4} \cdot 5^2$

$= 0.242\%$

from fig. 4, $\gamma \approx 1.76$

$\delta = 1$

$\lambda = 1$

$\therefore \alpha\beta\gamma\delta\lambda = 45.76$

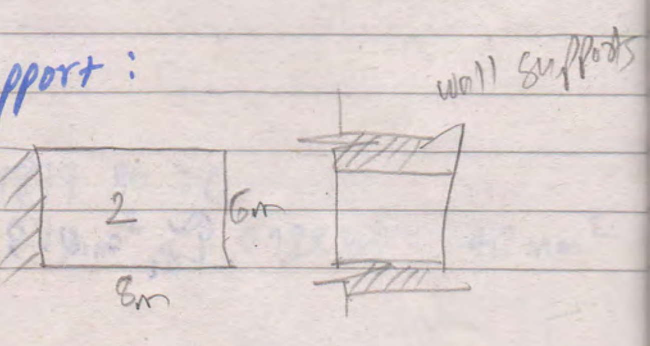
As $\frac{l_x}{d} < \alpha\beta\gamma\delta\lambda$, OK.

(5) Detailing of reinforcement:

(i) Check for dev. length of bars at support:

Shorter span:

is $1.3\frac{M_1}{V_u} + l_o \geq L_d$?

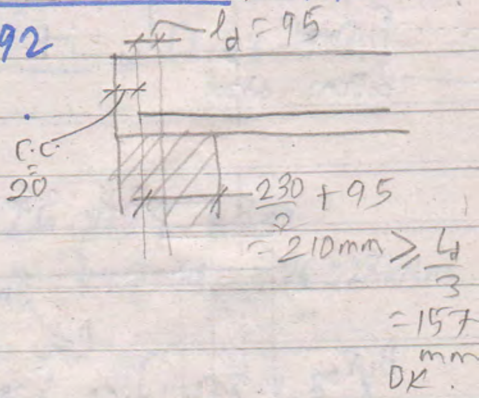


M_ℓ = M.R. at support (either one)
$\approx M_1 = 28.188$ kNm
$V_u = V_{ux} = 30$ kN
$l_d = \frac{230}{2} - c.c. = 115 - 20 = 95$ mm

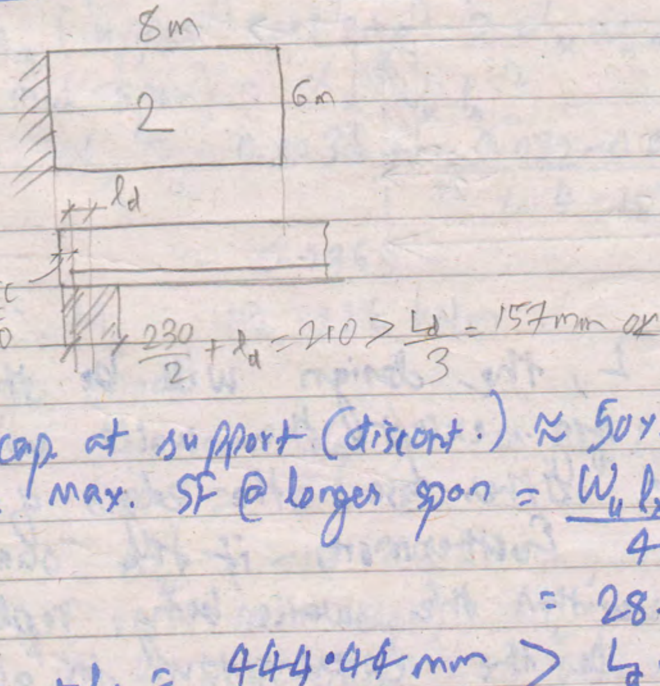
$\therefore$ Provided dev. length along shorter span $= \frac{1.3 M_\ell}{V_u} + l_d$
$= 1316.48$ mm

Min. reqd. dev. length $= L_d = \frac{0.87 f_y \phi}{4 \tau_{bd}}$
$= \frac{0.87 \times 415 \times 10}{4 \times 1.92} = 470$ mm

As $\frac{1.3 M_\ell}{V_u} + l_d > L_d$, OK.



longer span :-



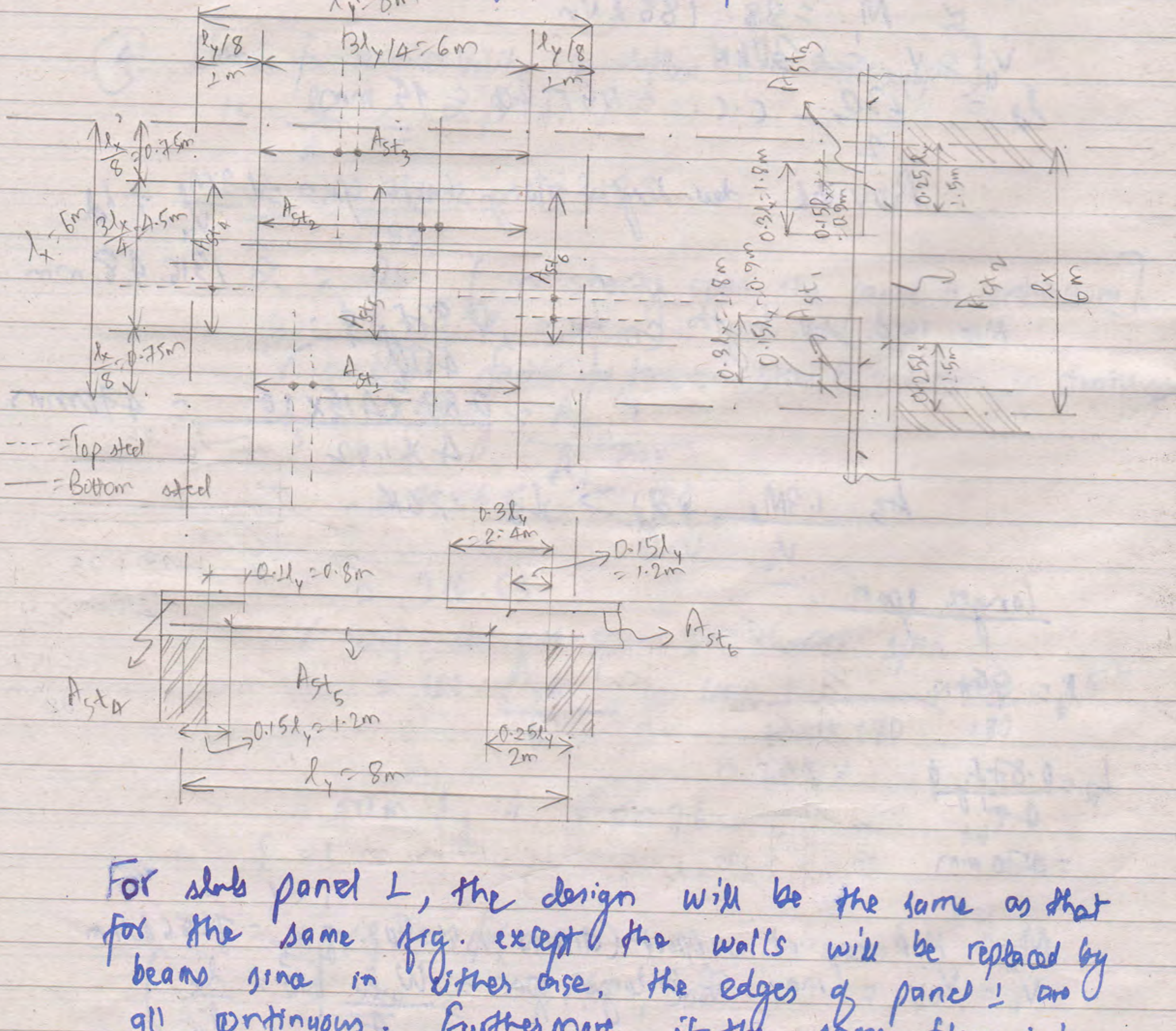
$l_d = 95$ mm

$L_d = \frac{0.87 f_y \phi}{4 \tau_{bd}}$
$= 470$ mm

M_ℓ = M.R. cap. at support (discont.) $\approx$ 50% of $M_5 = 7.56$ kN m
$V_u = V_{uy}$ = max. SF @ longer span $= \frac{W_u l_x}{4}\left(2 - \frac{l_x}{l_y}\right)$
$= 28.125$ kN

$\therefore \frac{1.3 M_\ell}{V_u} + l_d = 444.44$ mm $> L_d = 470$ mm OK.

(ii) Curtailment and arrangement of reinforcement bars in slab:-

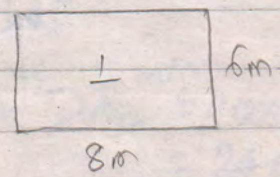


For slab panel L, the design will be the same as that for the same fig. except the walls will be replaced by beams since in either case, the edges of panel L are all continuous. Furthermore, if the same floor is to be designed with the walls being replaced by beams, one slab will be the representative for all others since then, every slab panel will be continuous in all four edges (being connected rigidly to beams at every edge).
Hence, the design for panel L is done using

wall thickness = 300mm which is eqvt. to designing any panel in a beam system with beam ~~thickness~~ width = 300mm

① Preliminary Design:

$\frac{l_x}{d} = 32 \Rightarrow d = 180mm, D = 200mm$



② Analysis of slab:

(i) Design load intensity: $W_u = 15 kN/m$ span

(ii) Design BM:

Shorter span:

M_1 (lower cont. edge) $= \alpha_x W_u l_x^2$

$$\alpha_x = 0.047 + \frac{0.051 - 0.047}{1.4 - 1.3} \times (1.33 - 1.3)$$

$$= 0.0482$$

$\therefore M_1 = 0.0482 \times 15 \times 6^2 = -26.028$ kNm

M_3 (upper cont. edge) $= \alpha_x W_u l_x^2 = M_1 = -26.028$ kNm

M_2 (mid-span) $= \alpha_x W_u l_x^2$

$$\alpha_x = 0.036 + \frac{0.039 - 0.036}{1.4 - 1.3} (1.33 - 1.3)$$

$$= 0.0369$$

$\therefore M_2 = 19.926$ kNm

Longer span:

M_4 (left cont. edge) $= \alpha_y W_u l_x^2$

$\alpha_y = 0.032$

$\therefore M_4 = -17.28$ kNm

M_6 (right cont. edge) $= M_4 = -17.28$ kNm

M_5 (mid-span) $= 0.024 \times 15 \times 6^2 = 12.96$ kNm

(iii) Design S.F:

$$V_{ux} = \frac{W_u l_x}{3} = \frac{15 \times 6}{3} = 30 kN$$

(3) Design of slab :

(i) Verify depth :

is $d > d_{bal}$?

$$d_{bal} = \sqrt{\frac{M_{ux}}{Qb}}$$

$$= \sqrt{\frac{26.028 \times 10^6}{2.766 \times 1000}} = 97 mm$$

$d = 180 mm > d_{bal} = 97 mm$. OK

(ii) Reinforcement bars :

Shorter span :

A_{st_1} (lower cont. edge) = A_{st_3} (upper cont. edge) $[M_1 = M_3]$

$$= \frac{M_1}{0.87 f_y (d - 0.416 x_u)}$$

$$= \frac{26.028 \times 10^6}{0.87 \times 415 (180 - 0.0208 A_{st_1})}$$

$\Rightarrow 0.0208 A_{st_1}^2 - 180 A_{st_1} + 72089.74 = 0$

$\Rightarrow A_{st_1} = 8232.86 mm^2, 420.98 mm^2$

$\Rightarrow n = 105, 6$

$\Rightarrow S = 9 mm, 165 mm$ → say 150 mm

$\therefore A_{st_1} = A_{st_3} = 10 mm \phi$ @ 150 mm c/c

$$A_{st_2} = \frac{M_2}{0.87 f_y (d - 0.416 x_u)}$$

$\Rightarrow 0.0208 A_{st_2}^2 - 180 A_{st_2} + 55189.03 = 0$

$\Rightarrow A_{st_2} = 8335.53 mm^2, 318.31 mm^2$

$\Rightarrow n = 107, 5$

$\Rightarrow S = 9 mm, 200 mm$ → say 180 mm

$\therefore A_{st_2} = 10 mm \phi$ @ 180 mm c/c

Longer span :

A_{st_4} (left cont. edge) = A_{st_6} (right cont. edge) $[M_4 = M_6]$

$$A_{st_4} = \frac{M_4}{0.87 f_y (d - 0.416 x_u)}$$

$\Rightarrow 0.0208 A_{st_4}^2 - 180 A_{st_4} + 47860.41 = 0$

$\Rightarrow A_{st_4} = 8379.24\ mm^2,\ 274.60\ mm^2 > A_{st}^{min} = 0.12\% \ bD$
$= 240\ mm^2$
OK.

$\Rightarrow n = 107,\ 4$

$\Rightarrow S = 9\ mm,\ 250\ mm$
$\rightarrow$ say 230 mm

$\therefore A_{st_4} = A_{st_6} = 10\ mm\ \phi$ @ 230 mm c/c

$$A_{st_5} = \frac{M_5}{0.87 f_y (d - 0.416 x_u)}$$

$\Rightarrow 0.0208 A_{st_5}^2 - 180 A_{st_5} + 35895.3 = 0$

$\Rightarrow A_{st_5} = 8449.61\ mm^2,\ 204.24\ mm^2 < A_{st}^{min} = 0.12\% \ bD$
$= 240\ mm^2$

$\therefore A_{st_5} = 240\ mm^2$

$\Rightarrow n = 4$

$\Rightarrow S = 250\ mm$ say 230 mm

$\therefore A_{st_5} = 10\ mm\ \phi$ @ 230 mm c/c

(iii) Check slab for shear : (in shorter span)

is $\tau_{uv} \le k \tau_{uc}$?

$$\tau_{uv} = \frac{V_{ux}}{bd} = \frac{30 \times 10^3}{1000 \times 180} = 0.16\ N/mm^2$$

$k = 1.2$ [40.2.1.1 ; D = 200 mm]

τ_{uc} : % reinf. at lower (or upper) edge of shorter span

$$= 100 \times \frac{A_{st_1} = A_{st_3}}{bd} = 100 \times \frac{\cancel{1000}}{150} \times \frac{\pi 5^2}{\cancel{1000} \times 180} = 0.291$$

$$\therefore \tau_{uc} \text{ (Table 19)} = 0.36 + \frac{0.48 - 0.36}{0.50 - 0.25} (0.291 - 0.25)$$

$= 0.379\ N/mm^2$

As $\tau_{uv} = 0.16\ N/mm^2 < k\tau_{uc} = 0.455\ N/mm^2$, OK.

(4) Check for serviceability in deflxn (shorter span):–

is $\frac{l_x}{d} \leq \alpha\beta\gamma\delta\lambda$?

$\frac{l_x}{d} = \frac{6000}{180} = 33.33$

$\alpha = 26$ [continuous]

$\beta = 1$ [$l_x < 10m$]

γ :

$f_s = 0.58 f_y \frac{A_{st2}\ \text{req.}}{A_{st2}\ \text{prov.}} = 0.58 \times 415 \times \frac{180}{200}$

$= 216.63$

% reinf. at midspan $= 100 \times \frac{A_{st2}}{bd} = 100 \times \frac{1000}{180} \times \frac{\pi 5^2}{1000 \times 180}$

$= 0.242\%$.

from fig. 4, $\gamma \approx 1.76$

$\delta = 1$, $\lambda = 1$

$\therefore \alpha\beta\gamma\delta\lambda = 45.76$

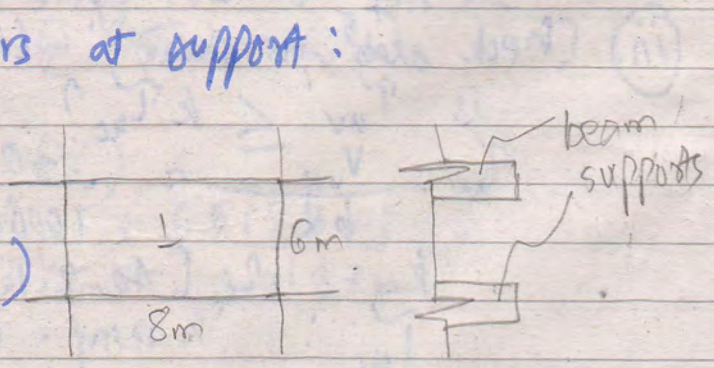
As $\frac{l_x}{d} = 33.33 < \alpha\beta\gamma\delta\lambda = 45.76$, OK.

(5) Detailing of reinforcement:

(i) Check for dev. length of bars at support:

Shorter span:

is $\frac{1.3 M_1}{V_u} + l_d \geq L_d$?



M_1 = M.R. cap at lower (or upper) edge $= M_1 = M_3$

$= 26.028$ kNm

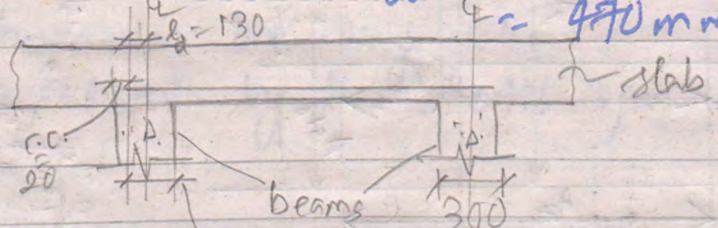
$V_u = V_{ux}$ = Max SF along shorter span = 30kN

$l_d = \frac{300}{2} - \text{c.c.} = 150 - 20 = 130$ mm

$\frac{1.3 M_1}{V_u} + l_d = 1257.88\ mm$ = Provided dev. length

Min. reqd. dev. length $(L_d) = \frac{0.87 f_y \phi}{4 \tau_{bd}} = \frac{0.87 \times 415 \times 10}{4 \times 1.92}$

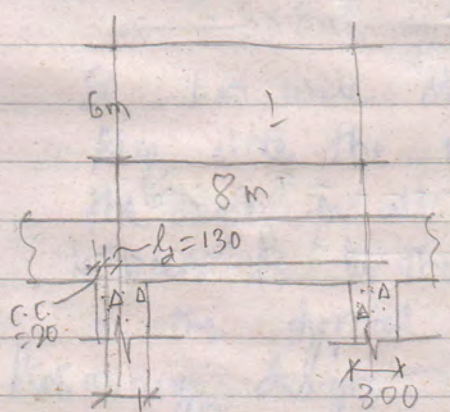
$= 470\ mm$



$\frac{1.3 M_1}{V_u} + l_d > L_d$ ∴ OK.

Longer span :

$\frac{300}{2} + 130 = 280 > \frac{L_d}{3} = 157$ OK



M_1 = M.R. cap. at left (or right) edge

$= M_4 = M_6$

$= 17.28\ kNm$

$V_u = V_{u4} = \frac{W_u l_x}{4}\left(2 - \frac{l_x}{l_y}\right)$

$= 28.125\ kN$

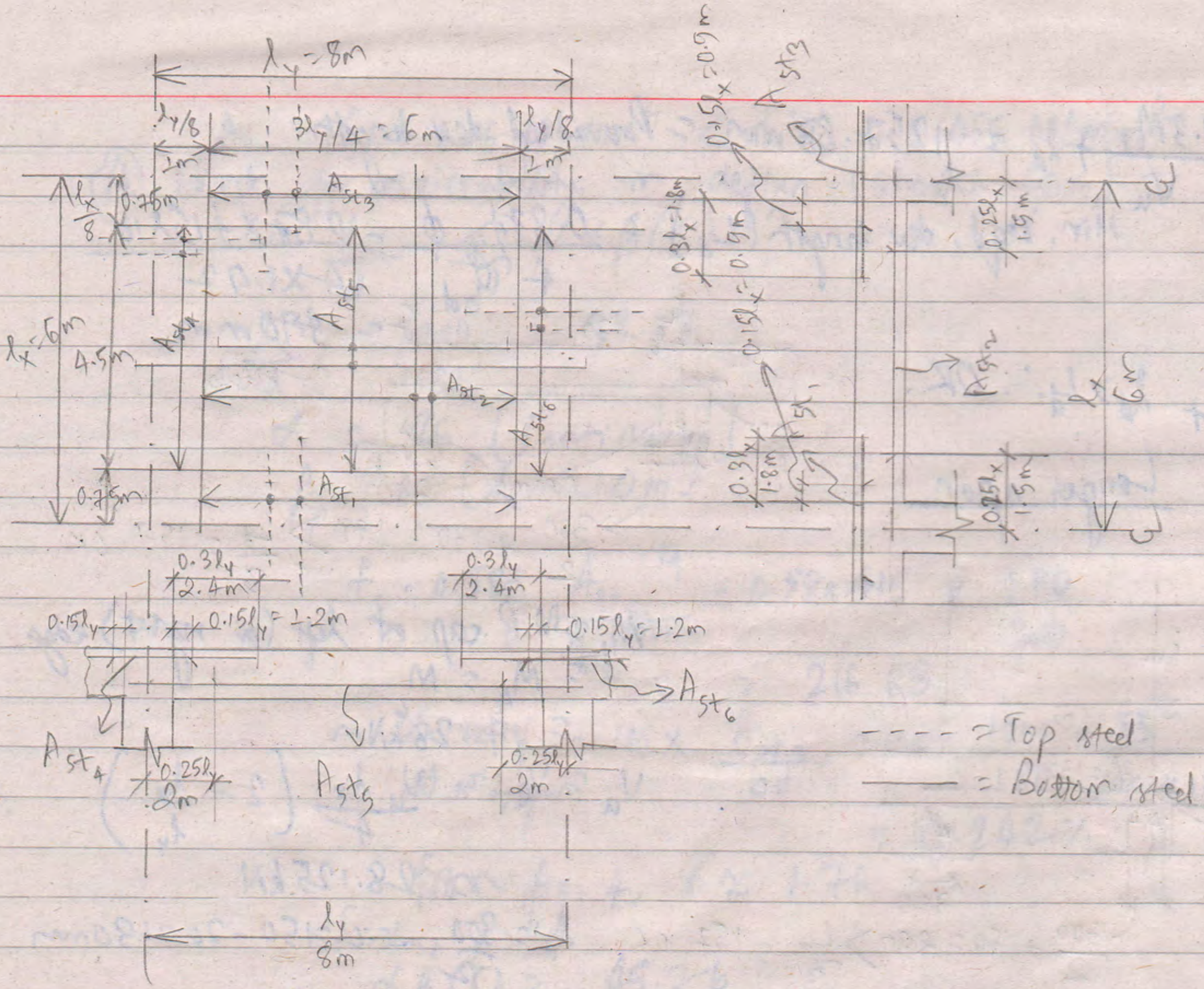
$\frac{300}{2} + 130 = 280 > \frac{L_d}{3} = 157$ OK

$l_d = \frac{300}{2} - c.c = 150 - 20 = 130\ mm$

$\frac{1.3 M_1}{V_u} + l_d = \frac{1.3 \times 17.28 \times 10^6}{28.125 \times 10^3} + 130 = 928.72\ mm$

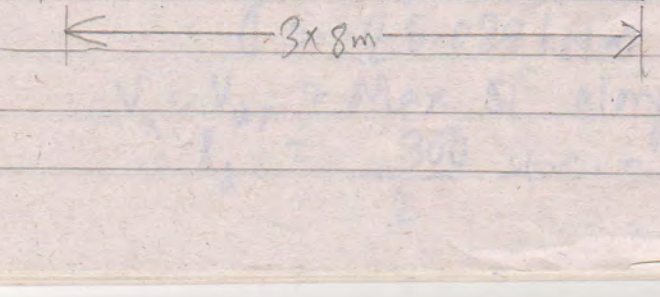
$> L_d = 470\ mm$ OK.

(ii) Curtailment and arrangement of reinf. bars in slab:



Class #19

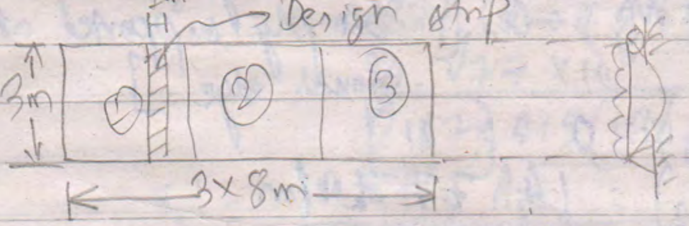
Design slab panels for the following :-
FF = 1kN/m², LL = 4kN/m², M20, Fe 415
Support : Wall of thickness 230 mm

① 

Longes span = l_y = 8m
shortes span = l_x = 3m

$$\frac{l_y}{l_x} = 2.66 > 2$$

∴ The slab behaves as one-way slab.
The moment is resisted by the shortes span only.



So, 1m wide strip along the shortes span is designed.
Also, since the support conditions for this strip will be the same for all three panels, ~~only~~ any one of them can be chosen to be the representative for designing all the others i.e. the typical panel in this slab is only one in number.

(i) Preliminary design:—

For one-way slab, for simply supported condition,

$$\frac{l_x}{d} = 25$$

$$\Rightarrow \frac{3}{d} = 25 \Rightarrow d = 120\,mm$$

Take $D = 150\,mm$ & $\phi = 10\,mm$

$$\therefore d = D - c.c. - \frac{\phi}{2} = 150 - 15 - \frac{10}{2} = 130\,mm$$

(ii) Analysis of slab:—

~~(a) Verify depth:-~~

is $d \geq \cancel{d_{bal}}$?

$d_{bal} =$

$$\frac{wl}{2}\cdot\frac{l}{2}-\frac{wl}{2}\,\frac{l}{4}$$
$$\frac{wl}{2}\left[\frac{l}{2}-\frac{1}{2}\,\frac{l}{2}\right]=\frac{wl}{2}\,\frac{l}{4}$$

(a) Design load intensity :-

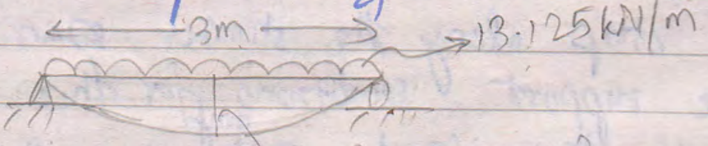
$$w_u = \gamma_f\,(DL + LL)$$
$$= 1.5\,(FF + S.W.L + LL)$$
$$= 1.5\,(1\times 1 + 25\times 0.15\times 1 + 4\times 1)$$
$$= 13.125 \text{ kN/m of span length}$$

(b) Design B.M. :- (Only shorter dirxn in one-way slab)

BM at lower edge $(M_1) = 0$ [∵ Simply supported at lower edge]

BM at upper edge $(M_3) = 0$ ["]

BM at mid-span $(M_2) = 14.77$ kN m



$$M_2 = \frac{wl^2}{8} = \frac{13.125\times 3^2}{8} = 14.77 \text{ kNm}$$

(c) Design SF :-

Max. SF (V_u) = At either support

$$= \frac{w_u l}{2} = \frac{13.125\times 3}{2} = 19.69 \text{ kN}$$

(iii) Design of slab :-

(a) Verify depth :-

is $d \geq d_{bal}$? (for SR UR design)

$$d_{bal} = \sqrt{\frac{M_{ux}}{Qb}}$$ → Max BM in shorter span = M_2

$$= \sqrt{\frac{14.77\times 10^6}{Q\times 1000}}$$ → 1m wide strip = 1000mm

$$Q = 0.36 f_{ck}\,\frac{x_{u,l}}{d}\left(1 - 0.416\,\frac{x_{u,l}}{d}\right)$$

for Fe415, $x_{u,l} = 0.48d$

$$\therefore Q = 0.36\times 20\times 0.48\,(1 - 0.416\times 0.48) = 2.766$$

$$\therefore d_{bal} = 73.1 \text{ mm}$$

As $d = 130\,mm > d_{bal} = 73.1\,mm$, OK to design as SR UR section

(b) Reinforcement bars :-

$$A_{st_1} = \frac{M_1}{0.87 f_y (d - 0.416 x_u)}$$

But $M_1 = 0$. So provide min. reinf. $= 50\%$ of A_{st_2}

$$A_{st_2} = \frac{M_2}{0.87 f_y (d - 0.416 x_u)}$$

$$= \frac{14.077 \times 10^6}{0.87 \times 415 (130 - 0.416 x_u)}$$

$$x_u = \frac{0.87 f_y A_{st_2}}{0.36 f_{ck} b} = \frac{0.87 \times 415 \times A_{st_2}}{0.36 \times 20 \times 1000} = 0.0501 A_{st_2}$$

$$\therefore A_{st_2} = \frac{40908.46}{130 - 0.0208 A_{st_2}}$$

$\Rightarrow 0.0208 A_{st_2}^2 - 130 A_{st_2} + 40908.46 = 0$

$\Rightarrow A_{st_2} = 5917.65\,mm^2,\ 332.35\,mm^2 > A_{st}^{min} = 0.12\% \cdot bD$
$= 180\,mm^2$
OK.

Taking $\phi = 10\,mm$,

$\Rightarrow n = \frac{5917.65}{\pi 5^2},\ \frac{332.35}{\pi 5^2}$

$= 76,\ 5$

$\Rightarrow S = \frac{1000\,mm}{76},\ \frac{1000\,mm}{5}$

$= 13\,mm,\ 200\,mm$ → say $180\,mm$

$\therefore A_{st_2} = 10\,mm\ \phi$ @ $180\,mm$ c/c

$A_{st_1} = 50\%$ of $A_{st_2} = 10\,mm\ \phi$ @ $360\,mm$ c/c

But Max. spacing in slab's main bars $=$ Min. $(3d, 300\,mm)$
$=$ Min. $(390, 300)$
$= 300\,mm$

$\therefore A_{st_1} = 10\,mm\ \phi$ @ $300\,mm$ c/c

$||^{ly}$, $A_{st_3} = 50\%$ of A_{st_2}
$= 10\text{mm} \phi$ @ 300 mm c/c

(c) Check slab for shear :- (in shorter span)

$$\tau_{uv} = \frac{V_u}{bd} = \frac{19.69 \times 10^3}{1000 \times 130} = 0.151 \text{ N/mm}^2$$

Shear Resisting cap. of slab $= k\tau_{uc}$

$k = 1.3$ (Cl. 40.2.1.1)

$\tau_{uc} \Rightarrow$ Table 19

$$\%\text{ reinf} = 100 \times \frac{A_{st_2}}{bd}$$

$$= 100 \times \frac{1}{1000 \times 130} \times \left(\frac{1000}{300}\right) \times \pi 5^2$$

$$= 0.201\ \%$$

$$\therefore \tau_{uc} = 0.28 + \frac{0.36 - 0.28}{0.25 - 0.15} \times (0.201 - 0.15)$$

$$= 0.3208 \text{ N/mm}^2$$

$$\therefore \underset{(0.151 \text{ N/mm}^2)}{\tau_{uv}} < \underset{(0.417 \text{ N/mm}^2)}{k\tau_{uc}} \quad \text{OK.}$$

(iv) Check for serviceability of deflan :-

is $\frac{l_x}{d} \leq \alpha\beta\gamma\delta\lambda$?

$$\frac{l_x}{d} = \frac{3000}{130} = 23.07$$

$\alpha = 20$ (Cl. 23.2.1, simply supported)

$\beta = 1$ ($l_x < 10$ m)

$\gamma \Rightarrow$ fig. 4

$$f_s = 0.58 f_y \frac{A_{st_2 \text{ reqd.}}}{A_{st_2 \text{ prov.}}}$$

$$= 0.58 \times 415 \times \frac{180}{200} = 216.63$$

$\%\ \text{reinf.} = 100 \times \dfrac{A_{st2}}{bd}$

$= 100 \times \dfrac{1}{1000 \times 130} \times \left(\dfrac{1000}{180}\right) \times \pi 5^2$

$= 0.3356\ \%$

$\therefore \gamma \approx 1.76$

$\delta = 1$

$\lambda = 1$

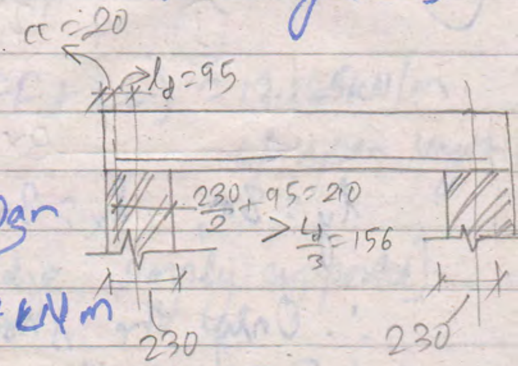
$\alpha\beta\gamma\delta\lambda = 35.2$

As $\dfrac{l_x}{d} = 23.07 < \alpha\beta\gamma\delta\lambda = 35.2$, OK.

(v) Detailing of reinforcement:

(i) Check for dev. length of bars at support :- (Only shorter span in 1-way slab)

is $\dfrac{1.3 M_1}{V_u} + l_d \geq L_d$?



M_1 = M.R. cap. at support
$\approx 50\%$ of $\underbrace{\text{MR cap at mid-span}}_{\approx M_2 = 14.77\ \text{kNm}}$

$= 7.385$ kNm

V_u = SF at support $= 19.69$ kN

$l_d = \dfrac{230}{2} - c.c = 115 - 20 = 95$ mm

$L_d = \dfrac{0.87 f_y \phi}{4\tau_{bd}}$; τ_{bd} from Cl. 26.2.1.1 $= 1.6 \times 1.2 = 1.92$

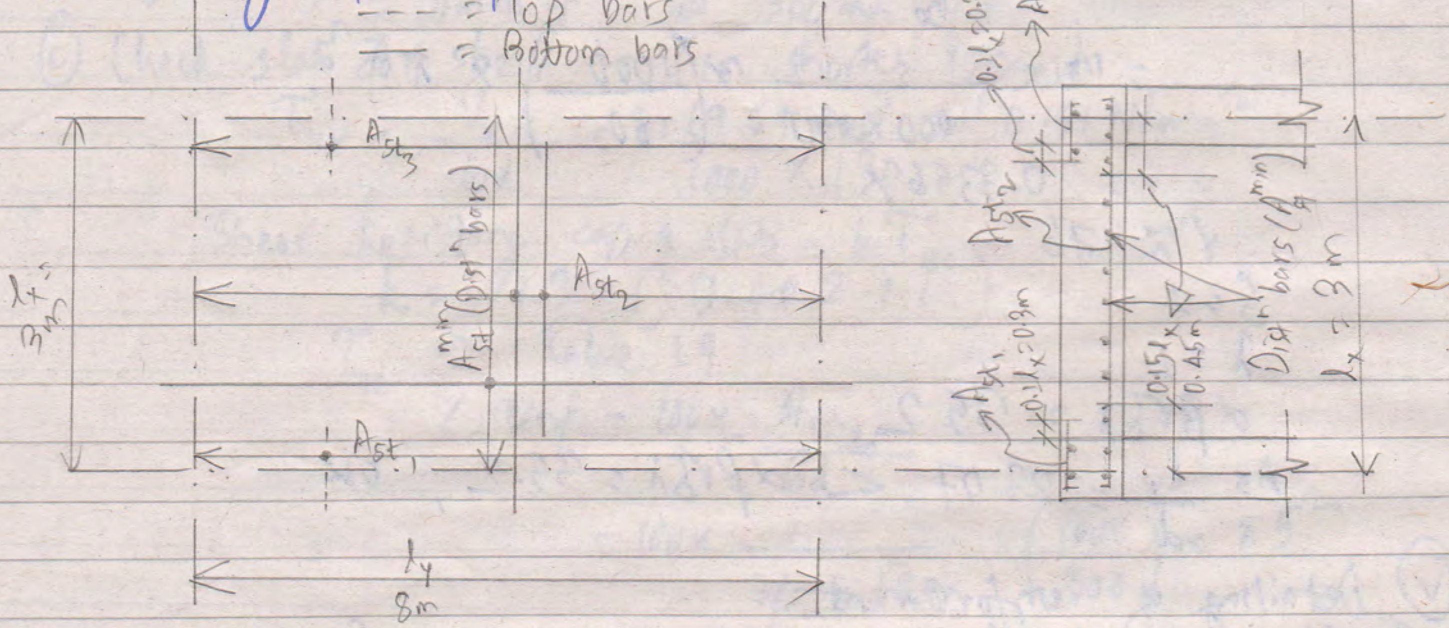
$= \dfrac{0.87 \times 415 \times 10}{4 \times 1.92} = 470.12$ mm = Min. reqd. dev. length

Provided dev. length $= \dfrac{1.3 M_1}{V_u} + l_d = 582$ mm $> L_d = 470.12$ mm

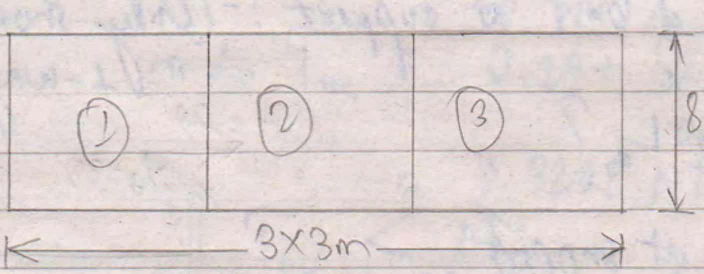
OK.

Showing distⁿ bars in plan is optional.
Should distⁿ bars be arranged in ←|+|→ format or just ←|→ ?
Torsion reinf. not reqd. in One-way slab.

(iii) Detailing of reinforcement bars in slab :-

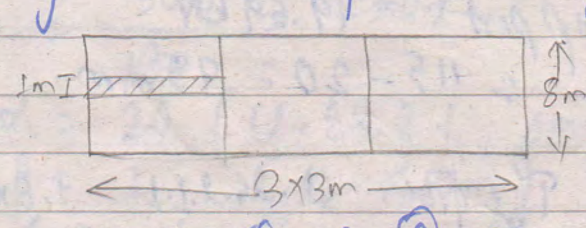


(ii)



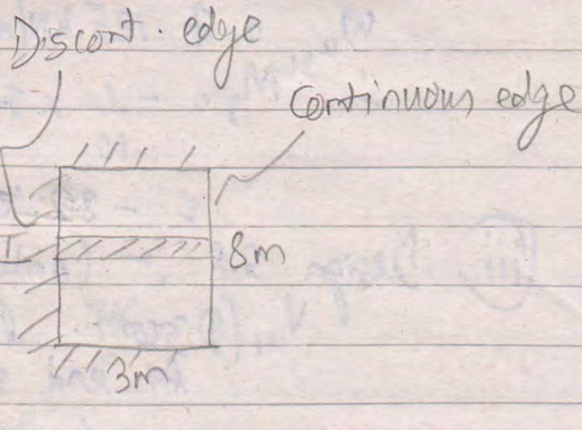
$l_y = 8m$, $l_x = 3m$. $\frac{l_y}{l_x} = 2.67 > 2$ ∴ One-way slab.

∴ Only the shorter span takes part in load sharing.
So, only the shorter span is designed



Then, panels (1) & (2) are the two typical panels of the slab. Panel (3) is the same as (1) according to support conditions.

One-way slab ≡ 1m wide beam



Design of panel ① :-

① Preliminary design :-

$\frac{l_x}{d} = 30$ (for continuous slab)

$\Rightarrow \frac{3}{d} = 30$

$\Rightarrow d = 100\,mm$

let $D = 150\,mm$

& $cc = 15\,mm$ & $\phi = 10\,mm$

$\therefore d = D - cc - \frac{\phi}{2} = 150 - 15 - \frac{10}{2} = 130\,mm$

② Analysis of slab :-

(i) Design load intensity :-

$w_u = \gamma_f(DL + LL) = 1.5(SWL + FF + LL) = 13.125\,kN/m$ of span length

(ii) Design B.M. :- (only shorter span)

M_1 (discont. edge) $= 0$ [$\because$ Discont. edge, simply supported]

M_2 (mid-span) $\Rightarrow$ Table 12

Span Moment category → Near Middle of End span

Dead load and imposed load (fixed load) = S.W. + F.F.

$= \gamma_f(25 \times 0.15 \times 1 + 1 \times 1)$

$= 1.5 \times 4.75$

$\Rightarrow w_{u_{DL}} = 7.125\,kN/m$ of span

Imposed load (not fixed load) = L.L. $= \gamma_f \times 4 \times 1 = 6\,kN/m$ of span ($w_{u_{LL}}$)

$\therefore M_2 = \underbrace{\frac{1}{12}}_{\alpha_{DL}} \times \underbrace{7.125}_{w_{u_{DL}}} \times \underbrace{3}_{l_x}^2 + \underbrace{\frac{1}{10}}_{\alpha_{LL}} \times \underbrace{6}_{w_{u_{LL}}} \times \underbrace{3}_{l_x}^2 = 10.74\,kNm$

M_3 (cont. edge) $\Rightarrow$ Table 12

Support Moment Category → At support next to the end suppor

$\alpha_{DL} = -1/10$, $\alpha_{LL} = -1/9$

$W_{u_{DL}} = 7.125$ kN/m , $W_{u_{LL}} = 6$ kN/m

$$\therefore M_3 = -\frac{1}{10} \times 7.125 \times 3^2 + -\frac{1}{9} \times 6 \times 3^2$$

$$= -12.413 \text{ kN m}$$

(iii) Design S.F. :- (only shorter span)

V_{u1} (Discont. edge) ⇒ Table 13

At end support :

$\alpha_{DL} = 0.4$, $\alpha_{LL} = 0.45$

$$\therefore V_{u1} = \alpha_{DL} \times l_x w_{DL} + \alpha_{LL} \times l_x w_{LL}$$
$$= 0.4 \times 3 \times 7.125 + 0.45 \times 3 \times 6$$
$$= 16.65 \text{ kN}$$

V_{u3} (Cont. edge) ⇒ Table 13

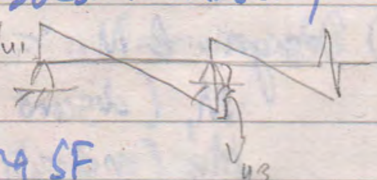
At support Next to End Support

~~In~~ Outer side :

$\alpha_{DL} = 0.6$, $\alpha_{LL} = 0.6$

$$\therefore V_{u3} = \alpha_{DL} \times w_{DL}\, l_x + \alpha_{LL} \times w_{LL}\, l_x$$
$$= 23.625 \text{ kN}$$ (Doesn't matter if -ve)

In reality, one should check S.F. at a dist = 'd' = eff. depth from face of support. But calculating SF at support (mid point of support width) & checking is more conservative.



$V_u = \text{Max.}(V_{u1}, V_{u3}) = 23.625$ kN (Note : the max. SF will always be at interior support of end span)

$= 23.625$ kN

(3) Design of slab :-

(i) Verify depth :-

is $d > d_{bal}$?

$$d_{bal} = \sqrt{\frac{M_{ux}}{Q\,b}} = \sqrt{\frac{12.413 \times 10^6}{Q \times 1000}}$$

$Q_1 = 0.36 f_{ck} \frac{x_{u,1}}{d}\left(1 - 0.416 \frac{x_{u,1}}{d}\right)$

For Fe 415, $x_{u,1} = 0.48d$

$\therefore Q_1 = 2.766$

$\therefore d_{bal} = 67$ mm

As $d = 100$ mm $> d_{bal} = 67$ mm, OK.

(ii) Reinforcement bars :-

$A_{st_1} = \frac{M_1}{0.87 f_y (d - 0.416 x_u)} = 0$

But A_{st_1} must be provided 50% A_{st_2}

$A_{st_2} = \frac{M_2}{0.87 f_y (d - 0.416 x_u)} = \frac{10.74 \times 10^6}{0.87 \times 415 (130 - 0.416 x_u)}$

$= \frac{29746.57}{130 - 0.416 x_u}$

$x_u = \frac{0.87 f_y A_{st_2}}{0.36 f_{ck} b} = 0.0501 A_{st_2}$

$\therefore A_{st_2} = \frac{29746.57}{130 - 0.0208 A_{st_2}}$

$\Rightarrow 0.0208 A_{st_2}^2 - 130 A_{st_2} + 29746.57 = 0$

$\Rightarrow A_{st_2} = 6012.127$ mm^2, 237.873 mm^2

$A_{st}^{min} = 0.12\%\ bD = \frac{0.12}{100} \times 1000 \times 150 = 180$ mm^2

$A_{st_2} > A_{st}^{min}$ OK.

$\therefore A_{st_2} = 6012.127$ mm^2, 237.873 mm^2

$\Rightarrow n = \cancel{6012}\ 77$, 4

$\Rightarrow S = 11$ mm, 250 mm

→ say 220 mm

Max. spacing = Min $(3d, 300)$ = Min $(390, 300)$

= 300 mm

OK.

$\therefore A_{st_2} = 10mm\ \phi\ @\ 220\ mm\ c/c$

$A_{st_1} = 50\%$ of $A_{st_2} = 10mm\ \phi\ @\ 440mm\ c/c$

But Max. spacing = 300mm

$\therefore A_{st_1} = 10mm\ \phi\ @\ 300mm$

$$A_{st_3} = \frac{M_3}{0.87 f_y (d - 0.416 x_u)}$$

$$= \frac{12.413 \times 10^6}{0.87 \times 415 (130 - 0.416 x_u)}$$

$\Rightarrow 0.0208 A_{st_3}^2 - 130 A_{st_3} + 34380.279 = 0$

$\Rightarrow A_{st_3} = 5973.285\ mm^2,\ 276.715\ mm^2 > A_{st}^{min}$

$\Rightarrow n = 77, 4$

$\Rightarrow S = 12mm, 250\ mm$

$\rightarrow$ say 220mm

$\therefore A_{st_3} = 10mm\ \phi\ @\ 220mm\ c/c$

(iii) Check slab for shear :-

$$\tau_{uv} = \frac{V_u}{bd} = \frac{23.625}{1000 \times 130} = 0.182\ N/mm^2$$

$k\tau_{uc}$:

for τ_{uc} ⇒ Table 19

% reinf. = $100 \frac{A_{st_3}}{bd}$ → of the section with max SF (here, the cont edge)

$$= 100 \times \frac{1}{1000 \times 130} \times \left(\frac{1000}{220}\right) \times \pi 5^2$$

$= 0.2746\ \%$

$$\therefore \tau_{uc} = 0.36 + \frac{0.48 - 0.36}{0.50 - 0.25} \times (0.2746 - 0.25)$$

$= 0.372\ N/mm^2$

$k \Rightarrow$ Cl. 40.2.1.1

for $D = 150mm$, $k = 1.3$

$k\tau_{uc} = 0.48\ N/mm^2$

As $\tau_{uv} = 0.182\ N/mm^2 < k\tau_{uc} = 0.48\ N/mm^2$ OK.

(4) Check for serviceability of deflxn :-

is $\frac{l_x}{d} \le \alpha\beta\gamma\delta\lambda$?

$\frac{l_x}{d} = \frac{3000}{130} = 23.07$

$\alpha = \frac{20+26}{2} = 23$ [One edge cont., another discont.] [Cl. 23.2.1]

$\beta = 1$ [$l_x < 10m$]

γ : fig. 4

$f_s = 0.58 f_y \frac{A_{st_2} \text{ reqd}}{A_{st_2} \text{ prov.}}$

$= 0.58 \times 415 \times \frac{220}{250}$

$= 212$

% reinf. $= 100 \frac{A_{st_2}}{bd}$

$= 100 \times \frac{1}{1000 \times 130} \times \left(\frac{1000}{220}\right) \pi 5^2$

$= 0.274\%$.

$\therefore \gamma \approx 1.78$

$\delta = 1$

$\lambda = 1$

$\therefore \frac{l_x}{d} = 23.07 < \alpha\beta\gamma\delta\lambda = 40.94$ OK.

(5) Detailing of reinforcement :-

(i) Check for dev length at supports :-

is $\frac{1.3M_1}{V_u} + l_0 \ge L_d$?

M_1 = MR cap. at discont. edge $\approx M_2 \times 50\% = 5.37$ kNm

Should dev. length be also checked for continuous support?

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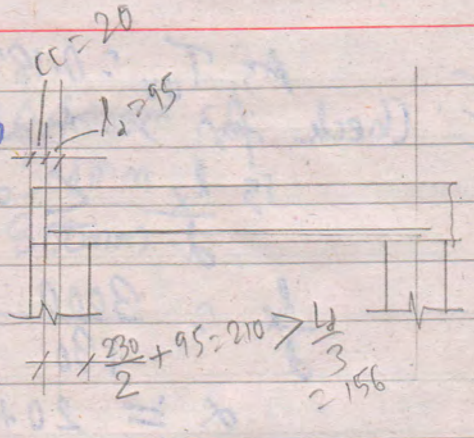
$V_u = V_{u1} = 16.65 kN$

$l_d = \frac{230}{2} - cc = 115 - 20 = 95 mm$

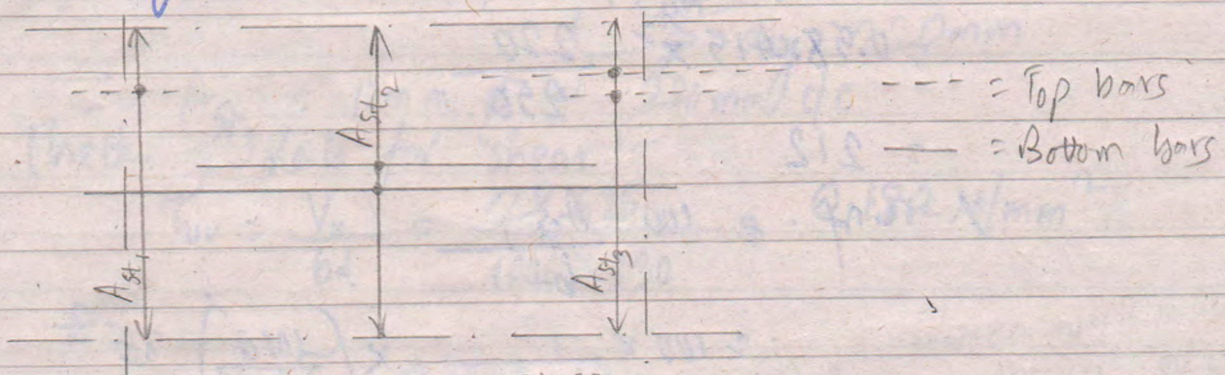
$\therefore 1.3\frac{M_1}{V_u} + l_d = 514.28 mm$

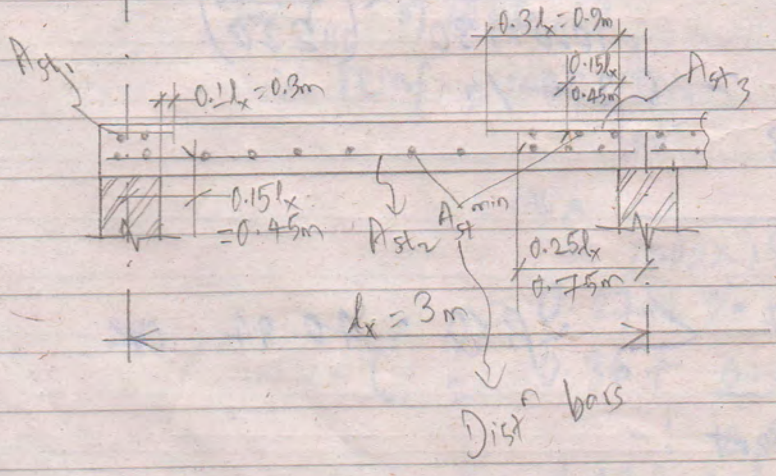
$L_d = \frac{0.87 f_y \phi}{4 \tau_{bd}} = \cancel{0.8}\ 470 mm$

$1.3\frac{M_1}{V_u} + l_d > L_d$ OK.



(ii) Detailing of reinforcement bars in slab:





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Design of panel ②:-

① Preliminary Design:-

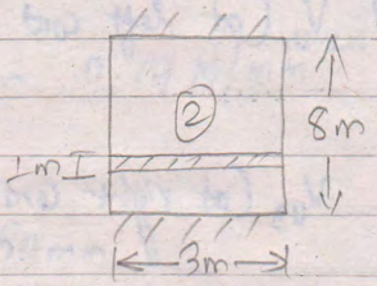
$\frac{l_x}{d} = 30$ (Contn. slab)

$\Rightarrow \frac{3}{d} = 30$

$\Rightarrow d = 100$ mm

let $D = 150$ mm

Then, $d = D - cc - \frac{\phi}{2} = 150 - 15 - \frac{10}{2} = 130$ mm.



② Analysis of slab:-

(i) Design load:-

$W_{u_{DL}}$ = Fixed load $= \gamma_f (FF + SW) = 1.5(1\times1 + 25\times0.15\times1) = 7.125$ kN

$W_{u_{LL}}$ = Non-fixed load $= \gamma_f LL = 1.5(4\times1) = 6$ kN

(ii) Design BM:- Table 12

M_1 (at left cont. edge) $= \alpha_{DL} W_{u_{DL}} l_x^2 + \alpha_{LL} W_{u_{LL}} l_x^2$

$= -\frac{1}{10} \times 7.125 \times 3^2 + -\frac{1}{9} \times 6 \times 3^2$

$= -12.413$ kNm

M_2 (at mid-span) $= \alpha_{DL} W_{u_{DL}} l_x^2 + \alpha_{LL} W_{u_{LL}} l_x^2$

$= \frac{1}{16} \times 7.125 \times 3^2 + \frac{1}{12} \times 6 \times 3^2$

$= 8.51$ kNm

M_3 (at right cont. edge) $= \alpha_{DL} W_{u_{DL}} l_x^2 + \alpha_{LL} W_{u_{LL}} l_x^2$

$= -\frac{1}{10} \times 7.125 \times 3^2 + -\frac{1}{9} \times 6 \times 3^2$

$= -12.413$ kNm

(iii) Design S.F. :- Table 13

V_{u_1} (at left cont. edge) $= \alpha_{DL} W_{u_{DL}} l_x + \alpha_{LL} W_{u_{LL}} l_x$ [Inner side]

$= 0.55 \times 7.125 \times 3 + 0.6 \times 6 \times 3$

$= 22.56\ kN$

V_{u_3} (at right cont. edge) $= \alpha_{DL} W_{u_{DL}} l_x + \alpha_{LL} W_{u_{LL}} l_x$ [Inner side]

$= 22.56\ kN$

(3) Design of slab :-

(i) Verify depth :-

is $d > d_{bal}$?

$$d_{bal} = \sqrt{\frac{M_{ux}}{Q_b}} = \sqrt{\frac{12.413 \times 10^6}{2.766 \times 1000}} = 67\ mm$$

$d = 130\ mm > d_{bal} = 67\ mm$ OK to design as SR UR section.

(ii) Reinforcement bars :-

$$A_{st_1} = \frac{M_1}{0.87 f_y (d - 0.416 x_u)} = \frac{12.413 \times 10^6}{0.87 \times 415 (130 - 0.416 x_u)}$$

$\Rightarrow 0.0208 A_{st_1}^2 - 130 A_{st_1} + 34380.28 = 0$

$\Rightarrow A_{st_1} = 5973.28\ mm^2,\ 276.72\ mm^2$

$\Rightarrow n = 77,\ 4$

$\Rightarrow S = 12\ mm,\ 250\ mm$ ✓

$\rightarrow$ say 220 mm

$\therefore A_{st_1} = 10\ mm\ \phi$ @ 220 mm c/c

$A_{st_3} = A_{st_1} = 10\ mm\ \phi$ @ 220 mm c/c

$$A_{st_2} = \frac{M_2}{0.87 f_y (d - 0.416 x_u)} = \frac{8.51 \times 10^6}{0.87 \times 415 (130 - 0.416 x_u)}$$

$\Rightarrow 0.0208 A_{st_2}^2 - 130 A_{st_2} + 23570.14 = 0$

$\Rightarrow A_{st_2} = 6063.1\ mm^2,\ 186.89\ mm^2$

$A_{st}^{min} = 0.12\%\ bD = \frac{0.12}{100} \times 1000 \times 150 = 180\ mm^2$

OK.

$\Rightarrow n = 78,\ 3 \quad \Rightarrow S = 12\ mm,\ 333\ mm$ ✓

$\rightarrow$ Say 300 mm (Max spacing)

$\therefore A_{st_2} = 10\text{mm } \phi \ @ \ 300\text{mm c/c}$

(iii) Check slab for shear :-

$$\tau_{uv} = \frac{V_u}{bd} = \frac{22.56 \times 10^3}{1000 \times 130} = 0.17 \text{ N/mm}^2$$

$k\tau_{uc}$: Cl. 40.2.1.1 :

$k = 1.3$ for $D = 150\text{mm}$

τ_{uc} : Table 19

% reinf. in max SF support $= 100 \frac{A_{st_1} \text{ or } A_{st_3}}{bd}$

$$= 100 \times \frac{1}{1000 \times 130} \times \left(\frac{1000}{220}\right) \times \pi 5^2$$

$$= 0.27\%$$

$$\therefore \tau_{uc} = 0.36 + \frac{0.48 - 0.36}{0.50 - 0.25}(0.27 - 0.25)$$

$$= 0.3696 \text{ N/mm}^2$$

$\tau_{uv} = 0.17 \text{N/mm}^2 < k\tau_{uc} = 0.48 \text{ N/mm}^2$ OK.

(4) Check for serviceability in deflxn :-

is $\frac{l_x}{d} < \alpha\beta\gamma\delta\lambda$?

$$\frac{l_x}{d} = \frac{3000}{130} = 23.07$$

$\alpha = 26$ [Continuous slab, cl. 23.2.1]

$\beta = 1$ [$l_x < 10$ m]

γ : $f_s = 0.58 f_y \frac{A_{st_2 \, req.}}{A_{st_2 \, prov}} = 0.58 \times 415 \times \frac{300}{333} = 216.85$

% tension reinf. $= 100 \frac{A_{st_2}}{bd} = 100 \times \frac{1}{1000 \times 130} \times \left(\frac{1000}{300}\right) \pi 5^2$

$= 0.201\%$

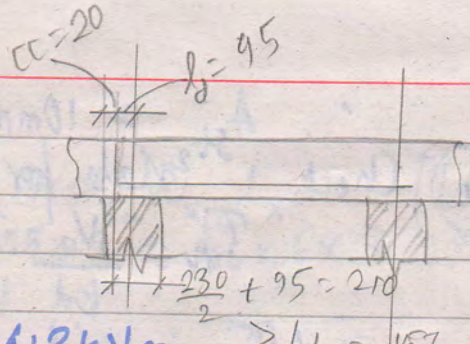
$\gamma \simeq 1.85$ from fig. 4

~~df~~ $\delta = 1$, $\lambda = 1$ As $\frac{l_x}{d} = 23.07 < \alpha\beta\gamma\delta\lambda = 48.1$, OK.

(5) Detailing of reinforcement :-

(i) Check for dev. length at supports :-

is $\dfrac{1.3M_1}{V_u} + l_d \geq L_d$?



M_1 = M.R. at cont. edge (either) = M_1 = 12.413 kNm

V_u = SF at cont. edge (either) = 22.56 kN

$l_d = \dfrac{230}{2} - cc = 115 - 20 = 95$ mm

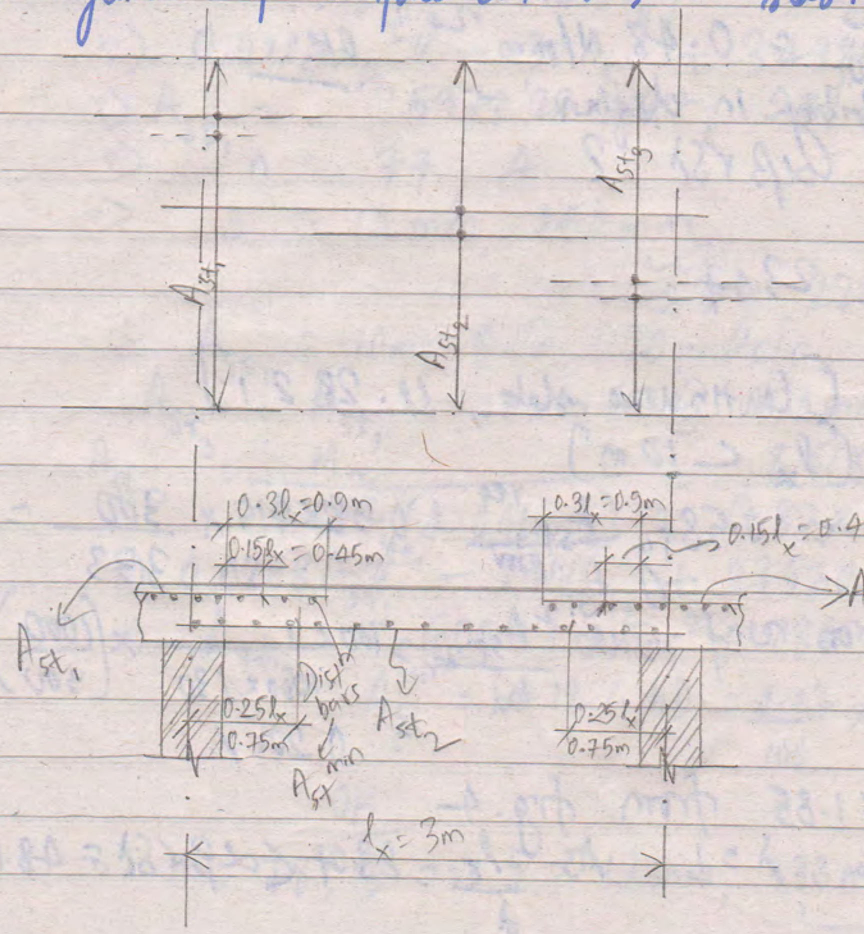
$L_d = \dfrac{0.87 f_y \phi}{4 \tau_{bd}} = 470$ mm

As $\dfrac{1.3 M_1}{V_u} + l_d$ = 810 mm > L_d = 470 mm, OK.

(ii) Arrangement of reinforcement bars in slab :-

- - - - - = Top bars

———— = Bottom bars



Class #21

Example 1 :- Design rectangular, square and circular section of column for the following data:

$P_u = 3000$ kN → Eff. length about X-axis

$l = 3.25$ m, $l_{ex} = 3$ m, $l_{ey} = 2.75$ m

↳ Unsupported length → Eff. length about Y-axis

M20, Fe415

Solⁿ:- **Rectangular section:-**

(1) Find approx. size of column:

For approximation, the eqⁿ eqʳ for axially loaded short column may be used :-

$$P_u = 0.4 f_{ck} A_c + 0.67 f_y A_s$$

$$= 0.4 f_{ck} (A_g - A_s) + 0.67 f_y A_s$$

Taking % reinf $p = 2\%$ [0.8% - 4%]

$$\Rightarrow P_u = 0.4 f_{ck} \left(A_g - \frac{2}{100} \times A_g\right) + 0.67 f_y \frac{2}{100} \times A_g$$

$$= 0.4 \times 20 \times 0.98 A_g + 0.67 \times 415 \times 0.02 A_g$$

$$\Rightarrow 3000 \times 10^3 = 13.401 A_g$$

$$\Rightarrow A_g = 223863.891 \text{ mm}^2$$

For rectangular section, let $D = 1.5b$;

D = Larger cr-sectional dimension

b = Smaller " "

[∵ $D \not> 4b$ for qualifying as non-wall ∴ $D < 4b$

And for two-way action such as in two-way slab, $D \not> 2b$

∴ $D < 2b$ is preferred]

Then, $Db = A_g$

$$\Rightarrow 1.5b^2 = 223863.891 \text{ mm}^2$$

$$\Rightarrow b = 386.32 \text{ mm}$$

$$\therefore D = 1.5b = 579.48 \text{ mm}$$

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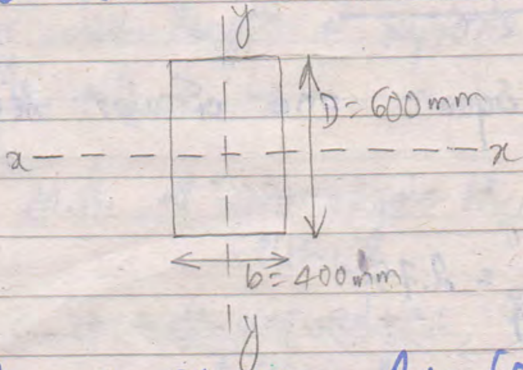
Provide 600mm X 400mm column size.



(2) Check 'e' and Slenderness Ratio (S.R.) of column:

Since load's eccentricity is not provided by the question, we only check if accidental (minimum) eccentricity is significant.

e_{min} in longer side $= \frac{l}{500} + \frac{D}{30} \nless 20mm$; $l = 3.25m$, $D = 0.6m$

$= 26.5 \text{ mm} < 0.05D = 30mm$

e_{min} in shorter side $= \frac{l}{500} + \frac{b}{30} \nless 20mm$; $l = 3.25m$, $b = 0.4m$

$= 19.83 \text{ mm} < 0.05b = 20mm$

Check for slenderness ratio about the two axes:

S.R. about X-X axis $= \frac{l_{ex}}{D} = \frac{3}{0.6} = 5 < 12$

S.R. about Y-Y axis $= \frac{l_{ey}}{b} = \frac{2.75}{0.4} = 6.875 < 12$

Since eccentricities are insignificant (ie. $< 0.05 \times$ corresp. side) and S.R. are less than 12 about both axes, the column is designed as Axially loaded column and Short column respectively.

(3) Find 'A_s' using eq^n [For Axially loaded short column]:

$P_u = 0.4 f_{ck} A_c + 0.67 f_y A_s$

$= 0.4 \times 20 \times (A_g - A_s) + 0.67 \times 415 A_s$

$= 0.4 \times 20 \times (600 \times 400 - A_s) + 0.67 \times 415 A_s$

$\Rightarrow 3000 \times 10^3 = 1920000 - 8A_s + 278.05 A_s$

$\Rightarrow A_s = 3999.26 mm^2$

C.C for columns = 40mm

Additional tie = tie for additional long. bars (ie. non-corner long. bars)

Provide 4 × 28mm ϕ and 4 × 25mm ϕ [A_s = 4426.5 mm^2]

④ Find transverse reinforcements :

$\phi_{tr} \nless 6mm$

$\nless \frac{\phi_{long}}{4} = \frac{Max.(28mm, 25mm)}{4} = \frac{28}{4} = 7mm$

∴ Provide ϕ_{tr} = 8mm

Spacing $S_v \ngtr 300mm$

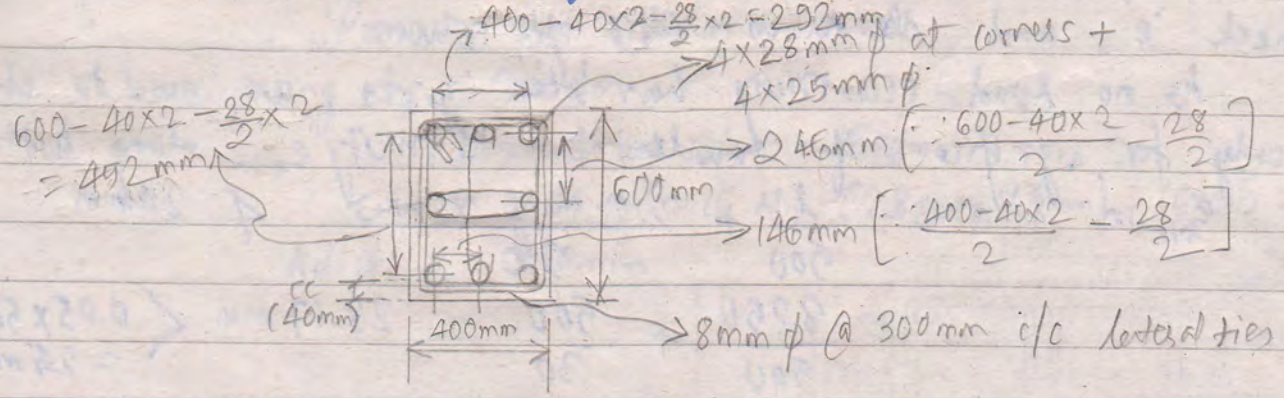
$\ngtr$ least lateral dim. of column = 400 mm

$\ngtr 16 \times \phi_{long} = 16 \times Min.(28mm, 25mm)$

$= 16 \times 25 mm$

$= 400 mm$

∴ Provide S_v = 300 mm



Spacing betn consecutive long. bars on shorter face = 146 mm > 75mm

∴ Additional tie is reqd. in shorter dirxn

Spacing betn corner long. bars on longer face = 492mm > 48ϕ_{tr} = 384 mm; provide close tie to the additional long. bars on the longer faces

Spacing betn consecutive long. bars on longer face = 246mm > 75mm

∴ Additional tie is reqd. in longer dirxn

Spacing betn corner long. bars on shorter face = 292 < 48ϕ_{tr} = 384mm. provide open tie to the additional long. bars on the shorter faces

Square section:

(1) Find out approx. size of column:

$P_u = 0.4 f_{ck} A_c + 0.67 f_y A_s$

$= 0.4 \times 20 \times (A_g - pA_g) + 0.67 \times 415 \times pA_g$

Taking $p = 2\%$,

$3000 \times 10^3 = 7.84 A_g + 5.561 A_g$

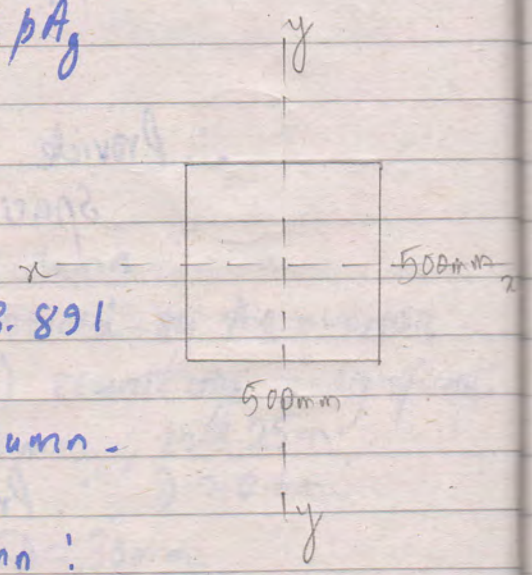
$\Rightarrow A_g = 223863.891 \, mm^2$

For square section, $b = D$

$\therefore A_g = bD = b^2 = D^2 = 223863.891$

$\therefore b = D = 473.14 mm$

Adopt 500 mm X 500mm size of column.



(2) Check 'e' and slenderness ratio of column:

As no load eccentricity has been given, we need to check only for significance of accidental eccentricity e_{min} along both axes.

e_{min} in X-axis $= \frac{l}{500} + \frac{\text{Dim. along. X-axis}}{30} \nless 20mm$

$= \frac{3250}{500} + \frac{500}{30} = 23.17 mm < 0.05 \times 500 = 25 mm$

e_{min} in Y-axis $= \frac{l}{500} + \frac{\text{Dim. along Y-axis}}{30} \nless 20 mm$

$= 23.17 mm < 0.05 \times 500 = 25 mm$

Check for slenderness ratio about two axes:

SR about x-x axis $= \frac{l_{ex}}{500} = \frac{3000}{500} = 6 < 12$

SR about y-y axis $= \frac{l_{ey}}{500} = \frac{2750}{500} = 5.5 < 12$

$\therefore$ Eccentricity is insignificant in both axes $\Rightarrow$ Axially loaded

S.R. about both axes $< 12 \Rightarrow$ Short column

③ Find 'A_s' using eq^m eq^n :- (Since column is axially loaded short column)

$$P_u = 0.4 f_{ck} A_c + 0.67 f_y A_s$$
$$= 0.4 f_{ck}(A_g - A_s) + 0.67 f_y A_s$$
$$= 0.4 f_{ck} A_g + (0.67 f_y - 0.4 f_{ck}) A_s$$
$$\Rightarrow 3000 \times 10^3 = 0.4 \times 20 \times 500 \times 500 + (0.67 \times 415 - 0.4 \times 20) \times A_s$$
$$\Rightarrow A_s = 3703.018 \text{ mm}^2$$

Provide 4×28mmϕ long. bars at corners + 4×25mmϕ long. bars.

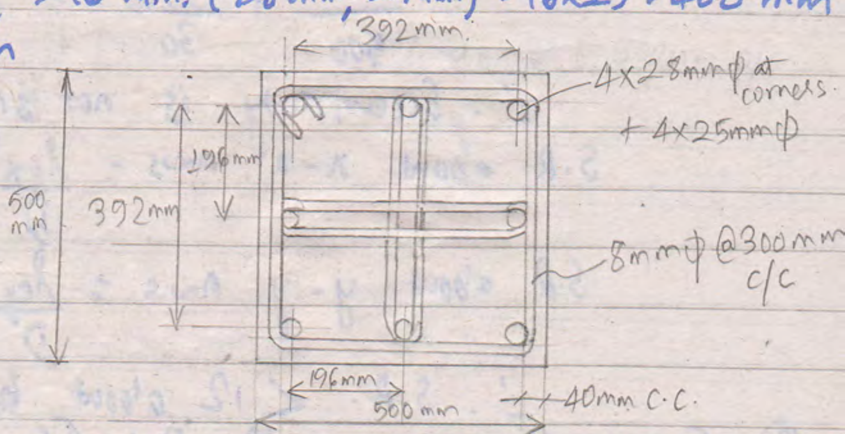
④ Find transverse reinforcements :

$$\phi_{tr} \nless 6\text{mm}$$
$$\nless \frac{\phi_{long}}{4} = \frac{\text{Max.}(28\text{mm}, 25\text{mm})}{4} = 7\text{mm}$$

$\therefore$ Provide $\phi_{tr} = 8$ mm

Spacing $S_v \ngtr 300$mm
$\ngtr$ least lat. dim. = 500mm
$\ngtr 16\phi_{long} = 16$ Min.(28mm, 25mm) = $16 \times 25 = 400$ mm

$\therefore$ Adopt $S_v = 300$mm



Spacing bet^n consecutive long. bars in either dirxn = 196mm > 75mm
So, additional ties are reqd in both dirxns for non-corner bars.
Spacing bet^n corner bars in either dirxn = 392mm > $48\phi_{tr} = 48 \times 8 = 384$mm
So provide close ties to the non-corner (additional) bars in both dirxns.

Circular section :

1. Find out approx. size of column :

$P_u = 0.4 f_{ck} A_c + 0.67 f_y A_s$

$= 0.4 f_{ck} (A_g - A_s) + 0.67 f_y A_s$

Taking $p = 2\%$;

$P_u = 0.4 f_{ck} (A_g - pA_g) + 0.67 f_y A_g p$

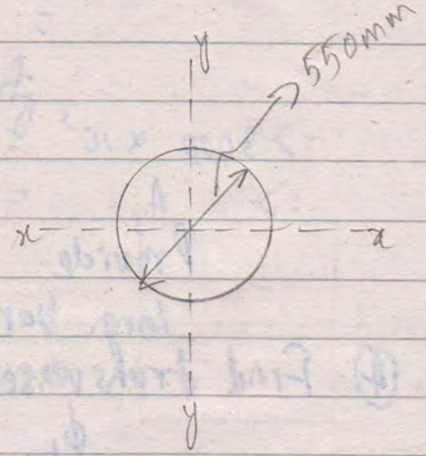
$\Rightarrow 3 \times 10^6 = 0.4 f_{ck} (1-p) A_g + 0.67 f_y p A_g$

$\Rightarrow A_g = 223863.891 \text{ mm}^2$

$\Rightarrow \frac{\pi D^2}{4} = 223863.891$

$\Rightarrow D = 533.88 \text{ mm} = $ Dia. of column.

Adopt $D = 550$ mm



2. Check 'e' and S.R. of column :-

$e_{min} = \frac{l}{500} + \frac{D}{30} \nless 20 \text{ mm}$

$= \frac{3250}{500} + \frac{550}{30} = 24.83 \text{ mm} < 0.05D = 0.05 \times 550 = 27.5 \text{ mm}$

∴ Eccentricity is not significant ⇒ Axially loaded column.

S.R. about x-x axis $= \frac{l_{ex}}{D} = \frac{3000}{550} = 5.45 < 12$

S.R. about y-y axis $= \frac{l_{ey}}{D} = \frac{2750}{550} = 5 < 12$

∴ S.R. < 12 about both axes ⇒ Short column

3. Find A_s using eq^m eq^n : [Since it is Axially loaded short column]

$P_u = 0.4 f_{ck} A_c + 0.67 f_y A_s$

$= 0.4 f_{ck} (A_g - A_s) + 0.67 f_y A_s$

$\Rightarrow 3 \times 10^6 = 0.4 \times 20 \left(\frac{\pi 550^2}{4} - A_s\right) + 0.67 \times 415 \times A_s$

$\Rightarrow A_s = 4070.863 \text{ mm}^2$

∴ Provide 7 × 28 mm ϕ long. bars (4310.265 mm^2)

④ Find transverse reinforcements :- [Lateral ties are used here; spiral ties could also be used]

$\phi_{tr} \not< 6mm$

$\not< \frac{\phi_{long}}{4} = \frac{Max.(28, 25)}{4} = 7\,mm$

$\therefore$ Adopt $\phi_{tr} = 8\,mm$

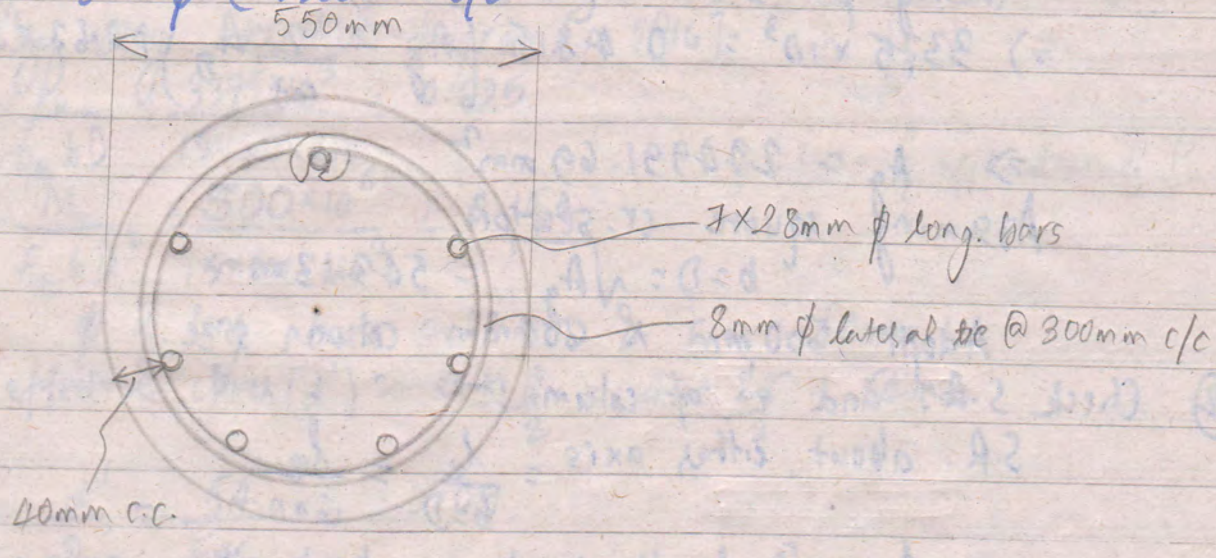
Spacing $S_v \not> 300\,mm$

$\not>$ Dia. of column = 550 mm

$\not> 16\,\phi_{long} = 16\,Min.(25, 28) = 16 \times 25 = 400\,mm$

Provide spacing = 300mm.

$\therefore$ 8mm ϕ @ 300mm c/c



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Class #22

2. Design a 4m long column for the following data:
Support condition: Both ends fixed and effectively held in position.
$P_u = 3375 kN$, $M_u = 500 kN\text{-}m$, M15, Fe415

Solution:

(1) Find approx. size of column:–

$P_u = 0.4 f_{ck} A_c + 0.67 f_y A_s$

$= 0.4 f_{ck}\left(A_g - \frac{pA_g}{100}\right) + 0.67 f_y \frac{pA_g}{100}$

Taking $p = 2\%$. $(0.8 - 4\%)$

$\Rightarrow 3375 \times 10^3 = 0.4 \times 15\left(A_g - \frac{2}{100} A_g\right) + 0.67 \times 415 \times \frac{2}{100} A_g$

$\Rightarrow A_g = 294991.69 \ mm^2$

Assuming square cr. section,

$b = D = \sqrt{A_g} = 543.13 \ mm$

Adopt 600mm x 600mm column size.

(2). Check S.R. and 'e' of column:–

S.R. about either axis $= \frac{l_e}{b=D} = \frac{l_e}{600}$

l_e = eff. length of column about either axis $= l_{ex} = l_{ey}$ [$\because$ Square]

From Table 28 of IS 456:

Both ends fixed ≡ Restrained against rotation in both ends and, effectively held in position

$\Rightarrow$ Recommended value of effective length (l_e) = $0.65 l$

$\therefore l_e = 0.65 l = 0.65 \times 4m = 2.6m$

$\therefore SR = \frac{l_e}{600} = \frac{2600}{600} = 4.33 < 12 \Rightarrow$ Short column

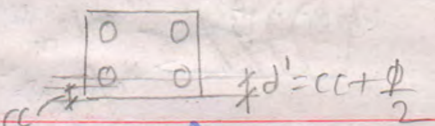
Since ~~as~~ load eccentricity has been given in the form of B.M.,

Actual eccentricity (e_{act}) $= \frac{M_u}{P_u} = \frac{500}{3375} = 148.15 \ mm > 0.05D = 0.05 \times 600 = 30 \ mm$

$\therefore$ Significant e_{act}.

$\frac{M_u}{f_{ck}\,a b^2}$ for $\frac{d'}{b}$, b = side along plane of bending

d' = effective cover to longitudinal bars = $cc + \frac{\phi}{2}$



Also, accidental eccentricity about either axis (coz square)

$e_{min} = \frac{l}{500} + \frac{D-b}{30} \not< 20\,mm$

$= \frac{4000}{500} + \frac{600}{30}$

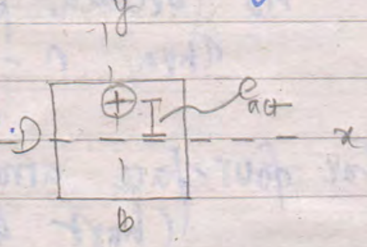
$= 28\,mm < 0.05D = 0.05 \times 600 = 30\,mm$

$\therefore$ Not significant e_{min} along either axis.

$\therefore$ The only significant eccentricity is the actual eccentricity e_{act} about the axis of $M_u = 500\,kN\text{-}m$

Either way, it is uniaxially loaded.

So, design as a uniaxially loaded short column.



③ Find 'A_s' by using interaction diagram [in SP16]

Here, $\frac{P_u}{f_{ck}bD} = \frac{3375 \times 10^3}{15 \times 600 \times 600} = 0.625$

⊕ = Point of application of P_u

$\frac{M_u}{f_{ck}bD^2} = \frac{500 \times 10^6}{15 \times 600 \times 600^2} = 0.154$

Using ϕ of long. bars = 28 mm & c.c. = 40 mm,

effective cover $(d') = cc + \frac{\phi}{2} = 40 + \frac{28}{2} = 54\,mm$

$\therefore \frac{d'}{D} = \frac{54}{600} = 0.09$

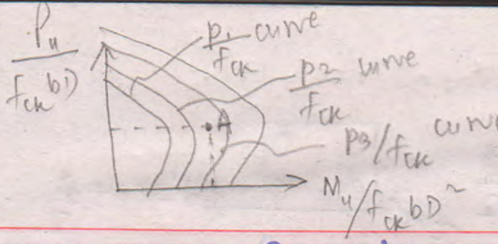
For two-face arrangement of reinforcement:

Given, $f_y = 415\,N/mm^2$

As the chart for $d'/D = 0.09$ is not provided, we find out p/f_{ck} for $d'/D = 0.05$ & $d'/D = 0.1$ from chart 31 & 32 of SP16,

$\frac{p}{f_{ck}}$ ~~for~~ $= 0.15$ for $\frac{d'}{D} = 0.05$

$= 0.16$ for $\frac{d'}{D} = 0.10$



if point A is located someplace betn p_2 & p_3 curves take higher curve ie. $\frac{p_3}{f_{ck}}$

$\therefore$ For $\frac{d'}{D} = 0.09$,

$$\frac{p}{f_{ck}} = 0.15 + \frac{0.16-0.15}{0.10-0.05}(0.09-0.05) = 0.008 + 0.15 = 0.158$$

$\cancel{\therefore p = 0.008 \times f_{ck} = 0.008 \times 15 =}$

$\therefore p = 0.158 \times f_{ck} = 0.158 \times 15 = 2.37\ \%$

$\therefore A_s = \frac{pA_g}{100} = \frac{2.37}{100} \times 600 \times 600 = 8532\ mm^2$

As decided previously, $\phi = 28\ mm\ \phi$ for long. bars

Then, $n = \frac{8532}{\pi 14^2} = 14$

For four-face arrangement of reinforcements :

(Chart 43 & 44

$\frac{p}{f_{ck}} = 0.18$ for $\frac{d'}{D} = 0.05$

$= 0.20$ for $\frac{d'}{D} = 0.10$

$$\therefore \frac{p}{f_{ck}} = 0.18 + \frac{0.20-0.18}{0.10-0.05}(0.09-0.05) = 0.196$$

$\therefore p = 0.196 \times f_{ck} = 0.196 \times 15 = 2.94\ \%$

$\therefore A_s = \frac{p}{100} A_g = 10584\ mm^2$

$\therefore$ No. of long. reinf. bars $= \frac{10584}{\pi 14^2} = 18$

(4) Find transverse reinforcements :-

$\phi_{tr} \nless 6\ mm$

$\nless \frac{\phi_{long}}{4} = \frac{28}{4} = 7\ mm$

$\therefore$ Provide $\phi_{tr} = 8\ mm$

Spacing $S_v \ngtr 300\ mm$

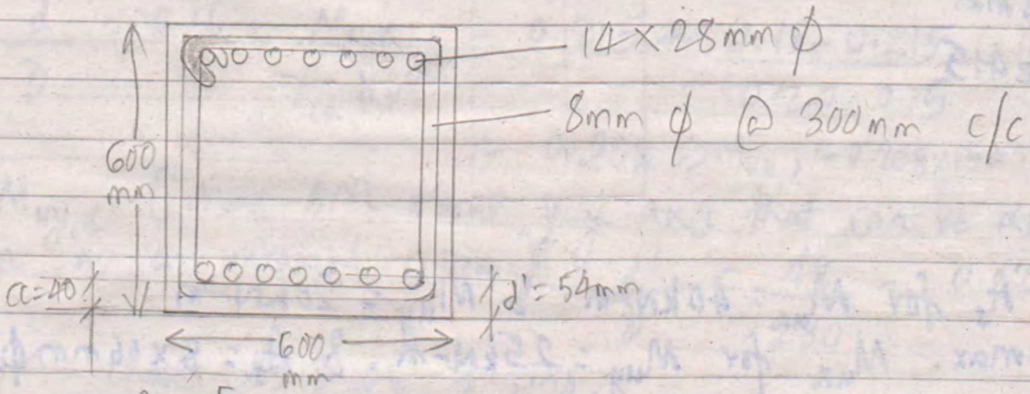
if 2 face arrangement, provide long. bars on the sides that develop tension & compression due to the eccentric load's B.M.

$\not>$ least dim. of column = 600 mm

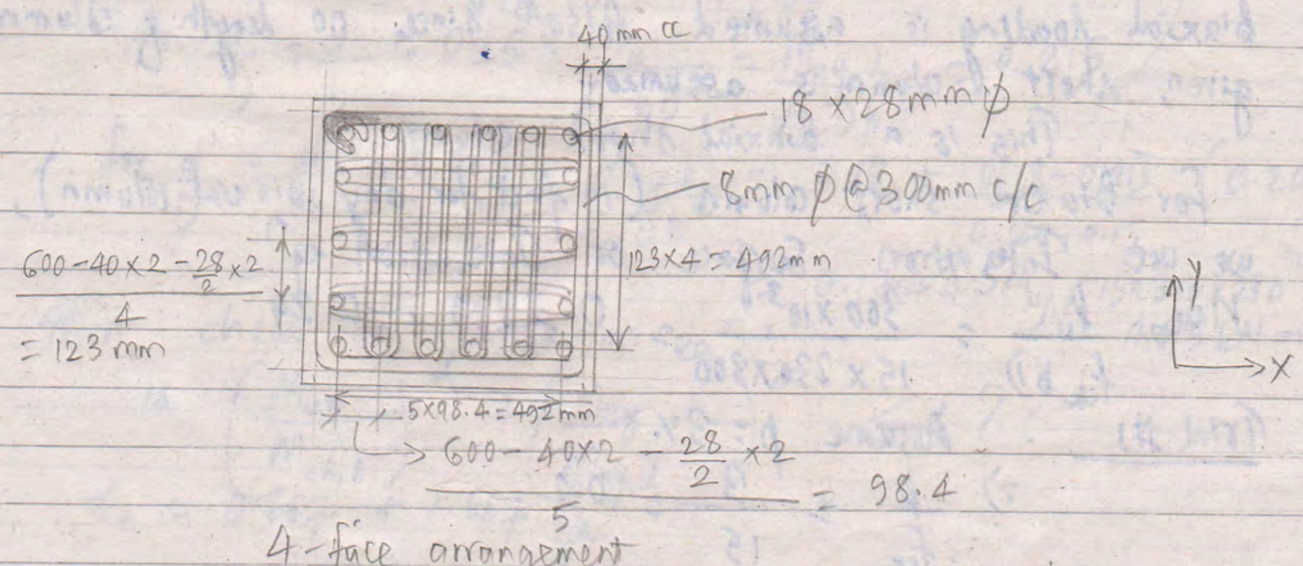
$\not>$ $16\phi_{long}$ = 16 × 28 = 448 mm

∴ Provide spacing of 300 mm

∴ 8 mm ϕ @ 300 mm c/c



2 - face arrangement



4 - face arrangement

Since spacing betn consecutive long. bars in X dirxn = 98.4 mm > 75 mm, additional tie is to be provided for bars connecting the Y-dirxn faces ie. ties reqd. in X-dirxn. Spacing betn corner bars of Y dirxn faces = 492 mm > $48\phi_{t}$ = 384 mm

So, provide close tie to bars on Y dirxn faces

Since spacing betn consecutive long. bars in Y dirxn = 123 mm > 75 mm additional four-face arrangement of reinf. bars is used in earthquake-resistant design normally.

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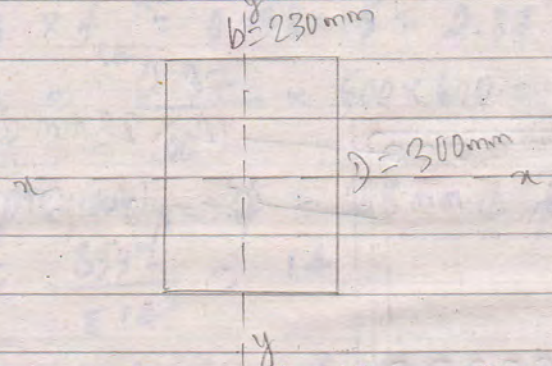
tie is to be provided in Y-dirxn. Spacing betⁿ corner bars in X-dirxn faces = 492mm > $48\phi_{tr}$ = 384mm. So, provide close tie to bars on X-dirxn faces. []]

3. Column size = 230 mm x 300 mm
P_u = 300 kN
d' = 48 mm
M15, Fe415



(i) Find 'A_s' for M_{ux} = 40 kN-m & M_{uy} = 20 kN-m
(ii) Find max. M_{ux} for M_{uy} = 25 kN-m & A_s = 8 x 16 mm ϕ

Solⁿ:—

(i) Since BM about both x-x & y-y axes have been provided, biaxial loading is assumed. Also, since no length of column is given, short column is assumed.

∴ This is a Biaxial short column.

For Biaxial short column (in fact for any biaxial column), we use Interaction Surface or Interaction eqⁿ.

Here, $\dfrac{P_u}{f_{ck} bD} = \dfrac{300 \times 10^3}{15 \times 230 \times 300} = 0.2899 \sim 0.29$

Trial #1 : Assume $p = 3\%$

$\Rightarrow \dfrac{p}{f_{ck}} = \dfrac{3}{15} = 0.2$

Assuming 4-face arrangement of long. reinf. bars,

For $M_{ux,1}$ (max. BM about x-x axis that can be resisted by the column as a uniaxial column), $\dfrac{d'}{D} = \dfrac{48}{300} = 0.16$

From charts ~~45~~ & 46 of SP16,

$l_{ex} \rightarrow$ Eff. lgth about $x-x$ axis. $\quad \frac{l_{ey}}{b}$

$\frac{l_{ex}}{D}$ = SR about $x-x$ axis.

$D \rightarrow$ lateral dim. $\perp$ $x-x$ axis

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for $\frac{d'}{D} = 0.15$: $\frac{M_u}{f_{ck} bD^2} = \frac{M_{ux,1}}{f_{ck} bD^2} = 0.215$

for $\frac{d'}{D} = 0.20$; $\frac{M_u}{f_{ck} bD^2} = \frac{M_{ux,1}}{f_{ck} bD^2} = 0.18$

interpolation

$\therefore$ for $\frac{d'}{D} = 0.16$, $\frac{M_{ux,1}}{f_{ck} bD^2} = 0.215 + \frac{0.18 - 0.215}{0.20 - 0.15}(0.16 - 0.15)$

$= 0.208 \Rightarrow M_{ux,1} = 0.208 \times 15 \times 230 \times 300^2 = 64.58 \text{ kNm}$

For $M_{uy,1}$ (max. BM about y-y axis that can be resisted by the column as a uniaxial column), $\frac{d'}{b} = \frac{48}{230} = 0.208$

From charts 45 & 46 of SP16,

for $\frac{d'}{D} = \frac{d'}{b} = 0.20$: $\frac{M_u}{f_{ck} bD^2} = \frac{M_{uy,1}}{f_{ck} Db^2} = 0.18$ extrapolation

for $\frac{d'}{D} = \frac{d'}{b} = 0.15$: $\frac{M_u}{f_{ck} bD^2} = \frac{M_{uy,1}}{f_{ck} Db^2} = 0.215$

$\therefore$ for $\frac{d'}{D} = \frac{d'}{b} = 0.208$, $\frac{M_{uy,1}}{f_{ck} Db^2} = 0.18 + \frac{0.18 - 0.215}{0.20 - 0.15}(0.208 - 0.20)$

$= 0.1744 \Rightarrow M_{uy,1} = 15 \times 300 \times 230^2 \times 0.1744$
$= 41.515 \text{ kNm}$

Then, check in Interaction eq^n:

is $\left(\frac{M_{ux}}{M_{ux,1}}\right)^{\alpha_n} + \left(\frac{M_{uy}}{M_{uy,1}}\right)^{\alpha_n} \leq 1$?

$\alpha_n = 0.667 + 1.667 \frac{P_u}{P_{uz}}$

$P_{uz} = 0.446 f_{ck} A_c + 0.75 f_y A_s$

$= 0.446 \times 15 \times \left(230 \times 300 - \frac{3}{100} \times 230 \times 300\right) + 0.75 \times 415 \times \frac{3}{100} \times 230 \times 300$

$= 1092.049 \text{ kN}$

$\therefore \alpha_n = 1.12$

$$\left(\frac{M_{ux}}{M_{ux,1}}\right)^{\alpha_n} + \left(\frac{M_{uy}}{M_{uy,1}}\right)^{\alpha_n}$$

$$= \left(\frac{40}{64.58}\right)^{1.12} + \left(\frac{20}{41.515}\right)^{1.12} = 1.026 > 1$$

Meaning $M_{ux,1}$ & $M_{uy,1}$ are too low

$\Rightarrow$ M.R cap. is too low.

Thus, increase % reinf. p to $p = 3.25\%$.

Trial #2 : $\frac{p}{f_{ck}} = \frac{3.25}{15} = 0.2167$ for d'/D

for the same $\frac{P_u}{f_{ck}bD}$, get $\frac{M_{ux,1}}{f_{ck}bD^2}$ & $\frac{M_{uy,1}}{f_{ck}Db^2}$ for $\frac{d'}{b}$

Calculate $M_{ux,1}$ & $M_{uy,1}$.

Calculate $\alpha_n = 0.667 + 1.667\frac{P_u}{P_{uz}}$; $P_u = 300kN$

$P_{uz} = 0.446f_{ck}A_c + 0.75f_yA_s$

Check if:-

$$\left(\frac{M_{ux}}{M_{ux,1}}\right)^{\alpha_n} + \left(\frac{M_{uy}}{M_{uy,1}}\right)^{\alpha_n} \leq 1$$

It should be this time. That is alright.

$\therefore$ Final $p = 3.25\%$

$\therefore A_s = \frac{p}{100}A_g = \frac{3.25}{100} \times 230 \times 300 = 2242.5\ mm^2$

(ii) $\frac{P_u}{f_{ck}bD} = 0.29$

Since $A_s = 8 \times 16mm\phi = 8 \times \pi 8^2 = 1608.495\ mm^2$

$p = \frac{A_s}{A_g} \times 100 = \frac{1608.495}{230 \times 300} \times 100 = 2.33\%$

$\therefore p/f_{ck} = \frac{2.33}{15} = 0.155$

Using Interaction surface = More than one interaction diagram used for interpolation.

Interaction eq^n:

$$\left(\frac{M_{ux}}{M_{ux,l}}\right)^{\alpha_n} + \left(\frac{M_{uy}}{M_{uy,l}}\right)^{\alpha_n} = 1$$

For $M_{ux,l}$:-

Given, $\frac{d'}{D} = 0.16$

$\frac{P_u}{f_{ck} bD} = 0.29$

$p/f_{ck} = 0.155$

For $p/f_{ck} = \cancel{0.15}\ 0.14$:

for $\frac{d'}{D} = 0.15$:

$\frac{M_{ux,l}}{f_{ck} bD^2} = 0.162$

for $\frac{d'}{D} = 0.20$:

$\frac{M_{ux,l}}{f_{ck} bD^2} = 0.14$

~~For p/f_{ck} = 0.14 0.14 :~~

~~for d'/D = 0.15 :~~

$\therefore$ for $\frac{d'}{D} = 0.16$, $\frac{M_{ux,l}}{f_{ck} bD^2} = 0.162 + \frac{0.14 - 0.162}{0.20 - 0.15}(0.16 - 0.15)$

$= 0.1576$

For $p/f_{ck} = 0.16$:

for $d'/D = 0.15$:

$\frac{M_{ux,l}}{f_{ck} bD^2} = 0.18$

for $d'/D = 0.20$:

$\frac{M_{ux,l}}{f_{ck} bD^2} = 0.155$

$\therefore$ for $d'/D = 0.16$, $\frac{M_{ux,l}}{f_{ck} bD^2} = 0.18 + \frac{0.155 - 0.18}{0.20 - 0.15}(0.16 - 0.15)$

$= 0.175$

$\therefore$ for $p/f_{ck} = 0.155$, $\frac{M_{ux,l}}{f_{ck} bD^2} = 0.1576 + \frac{0.175 - 0.1576}{0.16 - 0.14}(0.155 - 0.14)$

$= 0.17065$

$\therefore M_{ux,l} = 15 \times 230 \times 300^2 \times 0.17065 = \underline{\underline{52.99 \, kN\text{-}m}}$

for $M_{uy,l}$:

Given, $\frac{P_u}{f_{ck} bD} = 0.29$

$\frac{d'}{b} = 0.208$

$p/f_{ck} = 0.155$

For $p/f_{ck} = 0.14$:

for $d'/b = 0.20$:

$\frac{M_{uy,l}}{f_{ck} D b^2} = 0.14$

for $d'/b = 0.15$:

$\frac{M_{uy,l}}{f_{ck} D b^2} = 0.163$

$\therefore$ for $d'/b = 0.208$,

$\frac{M_{uy,l}}{f_{ck} D b^2} = 0.14 + \frac{0.14 - 0.163}{0.20 - 0.15}(0.208 - 0.20)$

$= \underline{\underline{0.1363}}$

For $p/f_{ck} = 0.16$:

for $d'/b = 0.20$:

$\frac{M_{uy,l}}{f_{ck} D b^2} = 0.154$

for $d'/b = 0.15$:

$\frac{M_{uy,l}}{f_{ck} D b^2} = 0.18$

$\therefore$ for $d'/b = 0.208$,

$\frac{M_{uy,l}}{f_{ck} D b^2} = 0.154 + \frac{0.154 - 0.18}{0.20 - 0.15}(0.208 - 0.20)$

$= \underline{\underline{0.1498}}$

$\therefore$ for $p/f_{ck} = 0.155$:

$\frac{M_{uy,l}}{f_{ck} D b^2} = 0.1363 + \frac{0.1498 - 0.1363}{0.16 - 0.14}(0.155 - 0.14)$

$= 0.1464$

$\therefore M_{uy,l} = 0.1464 \times 15 \times 300 \times 230^2 = \underline{\underline{34.86 \, kN\text{-}m}}$

$$\alpha_n = 0.667 + 1.667 \frac{P_u}{P_{uz}}$$

$$P_{uz} = 0.446 f_{ck} A_c + 0.75 f_y A_s$$
$$= 0.446 \times 15 (A_g - A_s) + 0.75 f_y A_s$$
$$= 0.446 \times 15 (230 \times 300 - 1608.495) + 0.75 \times 415 \times 1608.495$$
$$= 951.493 \text{kN}$$

$\therefore$ ~~Enter~~ $\alpha_n = 1.193$

Finally:

Interaction eq^n:—

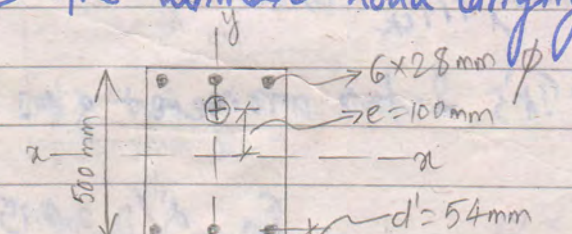
$$\left(\frac{M_{ux}}{M_{ux,1}}\right)^{\alpha_n} + \left(\frac{M_{uy}}{M_{uy,1}}\right)^{\alpha_n} = 1 \quad \text{for max. } M_{ux}$$

$$\Rightarrow \left(\frac{M_{ux}}{52.99}\right)^{1.193} + \left(\frac{25}{34.86}\right)^{1.193} = 1$$

$\Rightarrow$ $M_{ux} = 20.78$ kN-m

= Max. BM about x-x axis the column can resist for the provided dimension, steel, concrete, reinforcement, axial load & M_{uy}.

4. Determine the ultimate load carrying cap. of column:



M25, Fe 415

Sol^n:—

$A_s = 6 \times \pi\, 14^2 = 3694.51\,\text{mm}^2 \Rightarrow p = A_s/A_g \times 100 = 2.46\,\%$

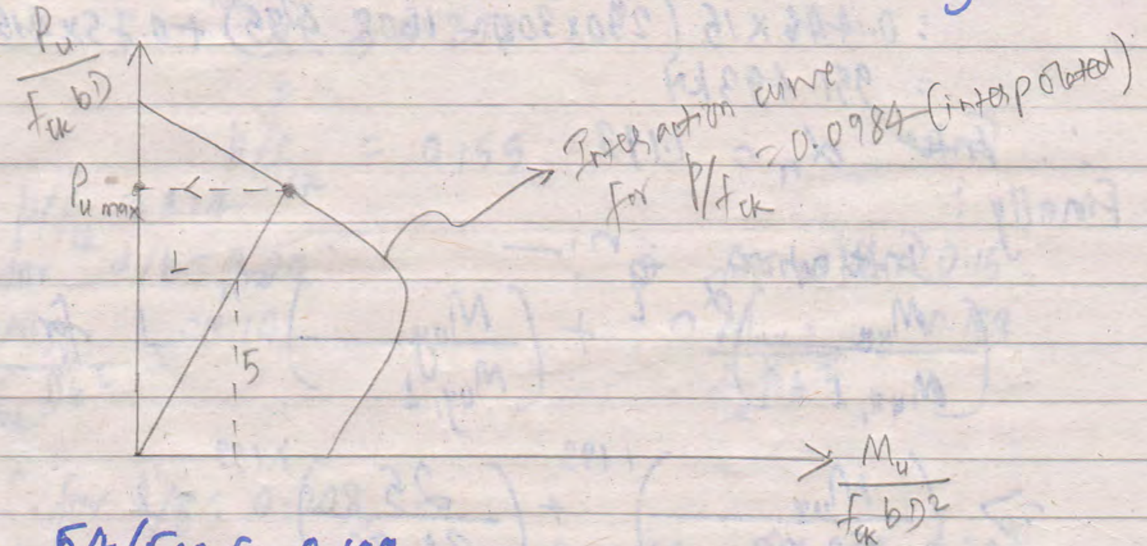
$D = 500$ mm $\quad \therefore p/f_{ck} = \frac{2.46}{25} = 0.0984$

$b = 300$ mm

Use interaction diag. to solve this problem:

$$\frac{M_{ux}/\cancel{f_{ck} b D^2}}{P_u/\cancel{f_{ck} b D}} = \frac{M_{ux}}{P_u D} = \frac{\cancel{P_u} e}{\cancel{P_u} D} = \frac{100}{500} = \frac{1}{5}$$

So, the point in the interaction curve for $P/f_{ck} = 0.0984$ where $\dfrac{M_{ux}/f_{ck}bD^2}{P_u/f_{ck}bD} = \dfrac{1}{5}$ gives the max. axial load P_u & max. uniaxial BM M_{ux} that be resisted by the column.



$d'/D = 54/500 = 0.108$

Similar to prev. q^n, since we don't have the exact curve for $P/f_{ck} = 0.0984$ & $d'/D = 0.108$, first, we operate in the premise of $P/f_{ck} = 0.08$ and then again in $P/f_{ck} = 0.10$. For each, we operate under $d'/D = 0.10$ and $d'/D = 0.15$ and interpolate for $d'/D = 0.108$. Finally, we interpolate bet^n the two std. P/f_{ck} values for our actual requirement of $P/f_{ck} = 0.0984$:

{It's 2-face arrangement of bars btw, as given}

In $P/f_{ck} = 0.08$:

In $d'/D = 0.1$:
$$\frac{P_u}{f_{ck}bD} = 0.468$$

In $d'/D = 0.15$:
$$\frac{P_u}{f_{ck}bD} = 0.455$$

$\therefore$ For $d'/D = 0.108$,
$$\frac{P_u}{f_{ck}bD} = 0.468 + \frac{0.455 - 0.468}{0.15 - 0.1}(0.108 - 0.1)$$
$$= \underline{\underline{0.466}}$$

In $P/f_{ck} = 0.10$:

$$\text{In } d'/D = 0.1: \quad \frac{P_u}{f_{ck} bD} = 0.52 \qquad\qquad \text{In } d'/D = 0.15: \quad \frac{P_u}{f_{ck} bD} = 0.5$$

$$\therefore \text{For } d'/D = 0.108, \ \frac{P_u}{f_{ck} bD} = 0.52 + \frac{0.5 - 0.52}{0.15 - 0.1}(0.108 - 0.1)$$

$$= \underline{\underline{0.5168}}$$

$\therefore$ For $p/f_{ck} = 0.0984$:

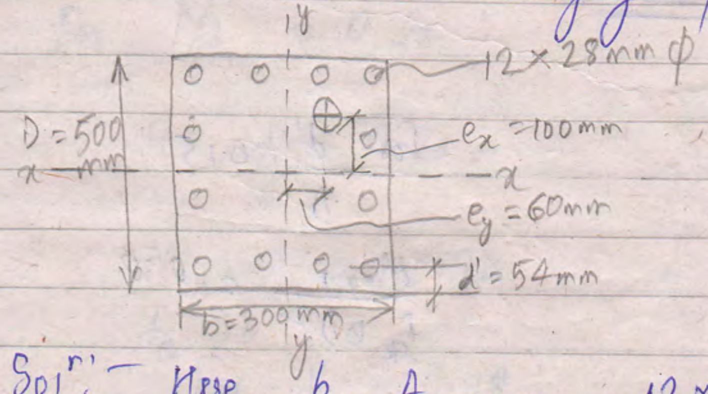
$$\frac{P_u}{f_{ck} bD} = 0.466 + \frac{0.5168 - 0.466}{0.10 - 0.08}(0.0984 - 0.08)$$

$$= 0.5127$$

$$\Rightarrow \frac{P_u}{25 \times 300 \times 500} = 0.5127$$

$$\Rightarrow P_{u_{max}} = 1922.625 \, kN$$

5. Find the ultimate load carrying capacity of column :-



Consider M25, Fe415

Sol^n:- Here, $p = \frac{A_s}{A_g} \times 100\% = \frac{12 \times \pi 14^2}{300 \times 500} \times 100\% = 4.93\%$

$\therefore p/f_{ck} = 4.93/25 = 0.197$

Since both axes have eccentricity, Biaxial Column.

For Biaxial column, Interaction Surface ie multiple Interaction Diagrams are used alongside Interaction eq^n :-

$$\left(\frac{M_{ux}}{M_{ux,1}}\right)^{\alpha_n} + \left(\frac{M_{uy}}{M_{uy,1}}\right)^{\alpha_n} = 1$$

Use interaction eq^n for Biaxial Column problems along side multiple interaction diagrams.

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$$\Rightarrow \left(\frac{P_u e_x}{M_{ux,1}}\right)^{\alpha_n} + \left(\frac{P_u e_y}{M_{uy,1}}\right)^{\alpha_n} = 1$$

Trial #1 : Assume $P_u = 3000 kN$ $\therefore \frac{P_u}{f_{ck} bD} = \frac{3000 \times 10^3}{25 \times 300 \times 500} = 0.8$

For $M_{ux,1}$:-

$$\frac{d'}{D} = \frac{54}{500} = 0.108$$

For $p/f_{ck} = 0.18$:

For $\frac{d'}{D} = 0.10$:

$$\frac{M_{ux,1}}{f_{ck} bD^2} = 0.093$$ (Chart 44 & 45 of SP16)

For $\frac{d'}{D} = 0.15$:

$$\frac{M_{ux,1}}{f_{ck} bD^2} = 0.084$$

$\therefore$ For $\frac{d'}{D} = 0.108$:

$$\frac{M_{ux,1}}{f_{ck} bD^2} = 0.093 + \frac{0.084 - 0.093}{0.15 - 0.10}(0.108 - 0.10)$$

$$= \underline{0.09156}$$

For $p/f_{ck} = 0.20$:

For $\frac{d'}{D} = 0.10$:

$$\frac{M_{ux,1}}{f_{ck} bD^2} = 0.12$$

For $\frac{d'}{D} = 0.15$:

$$\frac{M_{ux,1}}{f_{ck} bD^2} = 0.10$$

$\therefore$ For $\frac{d'}{D} = 0.108$:

$$\frac{M_{ux,1}}{f_{ck} bD^2} = 0.12 + \frac{0.10 - 0.12}{0.15 - 0.10}(0.108 - 0.10)$$

$$= \underline{0.1166}$$

$\therefore$ For $p/f_{ck} = 0.197$

$$\frac{M_{ux,1}}{f_{ck} bD^2} = 0.09156 + \frac{0.1166 - 0.09156}{0.20 - 0.18}(0.197 - 0.18)$$

ii) $\frac{M_{ux,l}}{f_{ck} b D^2} = 0.1128$

$\therefore M_{ux,l} = 0.1128 \times 25 \times 300 \times 500^2 = 211.58 \text{ kNm}$

For $M_{uy,l}$:-

$\frac{d'}{b} = \frac{54}{300} = 0.18$

For $P/f_{ck} = 0.18$:

For $\frac{d'}{b} = 0.20$: (Charts 45 & 46 of SP 16) For $\frac{d'}{b} = 0.15$:

$\frac{M_{uy,l}}{f_{ck} D b^2} = \cancel{0.095}\ 0.078$ $\qquad \frac{M_{uy,l}}{f_{ck} D b^2} = 0.085$

$\therefore$ For $\frac{d'}{b} = 0.18$:

$$\frac{M_{uy,l}}{f_{ck} D b^2} = 0.085 + \frac{0.078 - 0.085}{0.20 - 0.15} \times (0.18 - 0.15)$$

$$= \underline{0.0808}$$

For $P/f_{ck} = 0.20$:

For $\frac{d'}{b} = 0.20$: For $\frac{d'}{b} = 0.15$:

$\frac{M_{uy,l}}{f_{ck} D b^2} = 0.092$ $\qquad \frac{M_{uy,l}}{f_{ck} D b^2} = 0.101$

$\therefore$ For $\frac{d'}{b} = 0.18$:

$$\frac{M_{uy,l}}{f_{ck} D b^2} = 0.101 + \frac{0.092 - 0.101}{0.20 - 0.15} (0.18 - 0.15)$$

$$= \underline{0.0956}$$

$\therefore$ For $P/f_{ck} = 0.197$:

$$\frac{M_{uy,l}}{f_{ck} D b^2} = 0.0808 + \frac{0.0956 - 0.0808}{0.20 - 0.18} (0.197 - 0.18)$$

$= 0.0934$

$\therefore M_{uy,1} = 0.0934 \times 25 \times 500 \times 300^2 = 105.053\ kN\text{-}m$

$\alpha_n = 0.667 + 1.667 \dfrac{P_u}{P_{uz}}$

$P_{uz} = 0.446 f_{ck} A_c + 0.75 f_y A_s$
$= 0.446 \times 25 \times (A_g - pA_g) + 0.75 \times 415 \times A_s$
$= 0.446 \times 25 \times 300 \times 500 \times (1 - 0.0493) + 0.75 \times 415 \times 0.0493 \times 300 \times 500$
$= 3891.74\ kN$

$P_u = 3000\ kN$

$\therefore \alpha_n = 1.95$

Check :

$\left(\dfrac{P_u e_x}{M_{ux,1}}\right)^{\alpha_n} + \left(\dfrac{P_u e_y}{M_{uy,1}}\right)^{\alpha_n}$

$= \left(\dfrac{3000 \times 0.1}{211.58}\right)^{1.95} + \left(\dfrac{3000 \times 0.06}{105.053}\right)^{1.95}$

$= 1.975 + 2.858$

$= 4.833 \neq 1$

So, decrease $P_u \Rightarrow$ meaning ult. load cap. is less than 3000 kN

Trial# 2 : Assume $P_u = 0.4 f_{ck} A_c + 0.67 f_y A_s$ [$\because$ Exp^n for Axially loaded short column. Just as an approx.]

$= 0.4 \times 25 \times 300 \times 500 (1 - 0.0493) + 0.67 \times 415 \times 0.0493 \times 300 \times 500$

$= 3482\ kN$ (!) (Not good)

Assume $P_u = 1000\ kN \Rightarrow P_u / f_{ck} bD = 0.267$

For $M_{ux,1}$:- $d'/D = 0.108$

For P/f_{ck} = 0.18 :

For $d'/D = 0.10$:

$\dfrac{M_{ux,1}}{f_{ck} bD^2} = 0.225$

~~For P/f_{ck}~~

for $d'/D = 0.15$:

$\dfrac{M_{ux,1}}{f_{ck} bD^2} = 0.2$

$\therefore$ For $d'/D = 0.108$, $\dfrac{M_{ux,1}}{f_{ck} bD^2} = 0.225 + \dfrac{0.2 - 0.225}{0.15 - 0.10}(0.108 - 0.10)$

$= \underline{\underline{0.221}}$

For $P/f_{ck} = 0.20$:

For $d'/D = 0.10$:

$\frac{M_{ux,1}}{f_{ck} bD^2} = 0.245$

for $d'/D = 0.15$:

$\frac{M_{ux,1}}{f_{ck} bD^2} = 0.217$

$\therefore$ For $d'/D = 0.108$, $\frac{M_{ux,1}}{f_{ck} bD^2} = 0.245 + \frac{0.217 - 0.245}{0.15 - 0.10}(0.108 - 0.10)$

$= \underline{\underline{0.2405}}$

$\therefore$ For $P/f_{ck} = 0.197$,

$\frac{M_{ux,1}}{f_{ck} bD^2} = 0.221 + \frac{0.2405 - 0.221}{0.20 - 0.18}(0.197 - 0.18)$

$= 0.2376$

$\Rightarrow M_{ux,1} = 25 \times 300 \times 500^2 \times 0.2376 = 445.5\ kNm$

For $M_{uy,1}$:- $d'/b = 0.18$

For $P/f_{ck} = 0.18$:

For $d'/b = .15$:

$\frac{M_{uy,1}}{f_{ck} Db^2} = 0.2$

For $d'/b = 0.20$:

$\frac{M_{uy,1}}{f_{ck} Db^2} = 0.17$

$\therefore$ For $d'/b = 0.18$, $\frac{M_{uy,1}}{f_{ck} Db^2} = 0.2 + \frac{0.17 - 0.2}{0.20 - 0.15}(0.18 - 0.15)$

$= \underline{\underline{0.182}}$

For $P/f_{ck} = 0.20$:

For $d'/b = 0.15$:

$\frac{M_{uy,1}}{f_{ck} Db^2} = 0.216$

for $d'/b = 0.20$:

$\frac{M_{uy,1}}{f_{ck} Db^2} = 0.184$

$\therefore$ For $d'/b = 0.18$, $\frac{M_{uy,1}}{f_{ck} Db^2} = 0.216 + \frac{0.184 - 0.216}{0.20 - 0.15}(0.18 - 0.15)$

$= \underline{\underline{0.1968}}$

$\therefore$ For $P/f_{ck} = 0.197$,

$\frac{M_{uy,1}}{f_{ck} Db^2} = 0.182 + \frac{0.1968 - 0.182}{0.20 - 0.18}(0.197 - 0.18) = 0.19458$

$\Rightarrow M_{uy,1} = 0.19458 \times 25 \times 500 \times 300^2 = 218.903$ kNm

$\alpha_n = 0.667 + 1.667 \dfrac{P_u}{P_{uz}}$

$P_{uz} = 0.446 f_{ck} A_c + 0.75 f_y A_s$

$= 0.446 \times 25 \times 300 \times 500 (1 - 0.0493) + 0.75 \times 415 \times 0.0493 \times 300 \times 500$

$= 3891.74$ kN

$P_u = 1000$ kN

$\therefore \alpha_n = 1.095$

Check:

$\left(\dfrac{P_u e_x}{M_{ux,1}}\right)^{\alpha_n} + \left(\dfrac{P_u e_y}{M_{uy,1}}\right)^{\alpha_n}$

$= \left(\dfrac{1000 \times 0.1}{445.5}\right)^{1.095} + \left(\dfrac{1000 \times 0.06}{218.903}\right)^{1.095}$

$= 0.1948 + 0.242$

$= 0.437 < 1$

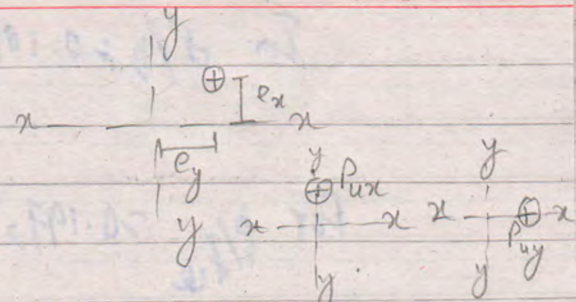
$\therefore$ The column can handle more than $P_u = 1000$ kN

Now the iteration could be continued by slightly inc. P_u and checking again if the interaction eq^n yields ≈ 1. But I'm going to interpolate P_u bet^n 1000 kN & 3000 kN with the val. of Interaction eq^n being 0.421 & 4.66 respectively, for the val. of " " at 1:

$P_u = 1000 + \dfrac{3000 - 1000}{4.833 - 0.437} (1 - 0.437)$

$\therefore P_{u_{max}} = 1256.14$ kN.

<u>Using Bressler's Method :–</u>



$$\frac{1}{P_u} = \frac{1}{P_{ux}} + \frac{1}{P_{uy}} - \frac{1}{P_{uz}} ;$$

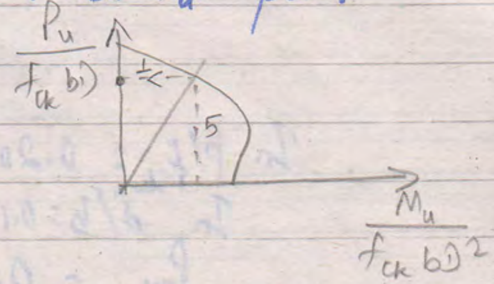
P_u = Ult. load capacity under biaxial eccentricities e_x & e_y
P_{ux}, P_{uy} = Ult. load " " uniaxial " e_x & e_y resp
P_{uz} : Ult. axial load cap.

Then,

$$\frac{M_{ux,1} / \cancel{f_{ck} bD^2}}{P_u / \cancel{f_{ck} bD}} = \frac{M_{ux,1}}{P_u D} = \frac{P_u \times e_x}{P_u D} = \frac{e_x}{D} = \frac{100}{500} = \frac{1}{5}$$

$$\frac{M_{uy,1} / \cancel{f_{ck} b D^2}}{P_u / \cancel{f_{ck} bD}} = \frac{M_{uy,1}}{b P_u} = \frac{P_u e_y}{P_u b} = \frac{e_y}{b} = \frac{60}{300} = \frac{1}{5}$$

So, for both P_{ux} & P_{uy} (corresponding to $M_{ux,1}$ & $M_{uy,1}$ resp.), we look at resp. interaction diagrams and seek a point on the curve that satisfies $\frac{M_u / f_{ck} bD^2}{P_u / f_{ck} bD} = \frac{1}{5}$. That P_u point gives P_{ux} (or P_{uy}).



For P_{ux} :– (Uniaxial about x–x)

$d'/D = 54/500 = 0.108$

In $p/f_{ck} = 0.18$:

~~for~~ In $d'/D = 0.10$: $\frac{P_{ux}}{f_{ck} bD} = 0.67$

In $d'/D = 0.15$: $\frac{P_{ux}}{f_{ck} bD} = 0.63$

$\therefore$ In $d'/D = 0.108$, $\frac{P_{ux}}{f_{ck} bD} = 0.67 + \frac{0.63 - 0.67}{0.15 - 0.10}(0.108 - 0.10)$

$= 0.6636$

In $p/f_{ck} = 0.20$:

In $d'/D = 0.10$: $\frac{P_{ux}}{f_{ck} bD} = 0.717$

In $d'/D = 0.15$: $\frac{P_{ux}}{f_{ck} bD} = 0.675$

In $d'/D = 0.108$, $\frac{P_{ux}}{f_{ck}bD} = 0.717 + \frac{0.675 - 0.717}{0.15 - 0.10}(0.108 - 0.10)$

$= 0.7103$

For $p/f_{ck} = 0.197$,

$$\frac{P_{ux}}{f_{ck}bD} = 0.6636 + \frac{0.7103 - 0.6636}{0.20 - 0.18}(0.197 - 0.18)$$

$= 0.7033$

$\Rightarrow P_{ux} = 0.7033 \times 25 \times 300 \times 500 = 2637.36\,kN$.

For P_{uy} :— (Uniaxial about y-y)

$d'/b = 54/300 = 0.18$

In $p/f_{ck} = 0.18$:

In $d'/b = 0.15$: $\frac{P_{uy}}{f_{ck}bD} = 0.637$

In $d'/b = 0.20$: $\frac{P_{uy}}{f_{ck}bD} = 0.6$

$\therefore$ In $d'/b = 0.18$, $\frac{P_{uy}}{f_{ck}bD} = 0.637 + \frac{0.6 - 0.637}{0.20 - 0.15}(0.18 - 0.15)$

$= 0.6148$

In $p/f_{ck} = 0.20$:

In $d'/b = 0.15$: $\frac{P_{uy}}{f_{ck}bD} = 0.672$

In $d'/b = 0.20$: $\frac{P_{uy}}{f_{ck}bD} = 0.635$

$\therefore$ In $d'/b = 0.18$, $\frac{P_{uy}}{f_{ck}bD} = 0.672 + \frac{0.635 - 0.672}{0.20 - 0.15}(0.18 - 0.15)$

$= 0.6498$

$\therefore$ For $p/f_{ck} = 0.197$, $\frac{P_{uy}}{f_{ck}bD} = 0.6148 + \frac{0.6498 - 0.6148}{0.20 - 0.18}(0.197 - 0.18)$

$= 0.6446$

$\Rightarrow P_{uy} = 0.6446 \times 25 \times 300 \times 500 = 2417.25\,kN$.

Then, $P_{uz} = 0.446 f_{ck} A_c + 0.75 f_y A_s$
$= 3891.74 \text{ kN}$

Finally,

$$\frac{1}{P_u} = \frac{1}{P_{ux}} + \frac{1}{P_{uy}} - \frac{1}{P_{uz}}$$

$$= \frac{1}{2637.36} + \frac{1}{2417.25} - \frac{1}{3891.74}$$

$\Rightarrow P_u = 1866 \text{ kN} =$ Max. (ultimate) load with given eccentricities that the column can resist

6. (*) Design RC column for:

$P_u = 1000 \text{ kN}$

$M_{ux, bottom} = 110 \text{ kNm}$

$M_{ux, top} = 80 \text{ kNm}$

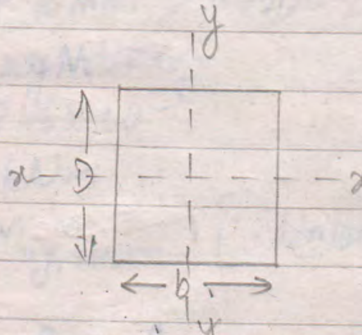
$M_{uy, bottom} = 40 \text{ kNm}$

$M_{uy, top} = 30 \text{ kNm}$

Size of column $= 300\text{mm} \times 480\text{mm}$

$l = 5.8\text{m}$, $l_{ex} = 5.4\text{m}$, $l_{ey} = 4.2\text{m}$

Consider M20, Fe415, $d' = 60\text{mm}$



$D = 480\text{mm}$
$b = 300\text{mm}$

Sol^n:—

For any axis:

In unbraced column, use larger of the two (bottom or top) moment

In braced column, use 60% of larger + 40% of smaller moment for design

Since size of column is given, start with checking S.R. & e

(1) Check S.R. & eccentricity:—

S.R. about x–x axis $= \frac{l_{ex}}{D} = \frac{5.4 \times 1000}{480} = 11.25 < 12$

$\therefore$ Short about x–x axis.

S.R. about y–y axis $= \frac{l_{ey}}{b} = \frac{4200}{300} = 14 > 12 \Rightarrow$ Long ~~along~~ about y–y axis

And unsupp. length $(l) = 5800\text{mm} < 60b = 60 \times 300 = 18000\text{mm}$

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Considering unbraced column,

$M_{ux,tot} = \text{Max.}(M_{ux,bottom}, M_{ux,top})$

[$\because$ Short about x-x axis]

$= \text{Max.}(80, 110)$

$= 110 \text{ kNm}$

$M_{uy,tot} = \text{Max.}(M_{uy,bottom}, M_{uy,top}) + M_{ay}$

Additional (secondary) BM due to P-Δ effect in long column

[$\because$ Long about y-y axis]

$= \text{Max}(40, 30) + \dfrac{P_u b}{2000}\left(\dfrac{l_{ey}}{b}\right)^2 \times K$

b = dim. of column in the plane of M_{ay} bending

K = Correction factor

$K = \dfrac{P_{uz} - P_u}{P_{uz} - P_b} \leq 1$

$P_{uz} = 0.446 f_{ck} A_c + 0.75 f_y A_s$

Since moments about both axes have been provided, it obviously indicates Biaxial loading. Now, checking for dominant moments:-

Accidental ecc. e_{min} in x-x dirxn $= \dfrac{l}{500} + \dfrac{b}{30} \nless 20\text{mm}$

$= \dfrac{5800}{500} + \dfrac{300}{30}$

$= 21.6\text{mm} > 0.05b = 15$

$\therefore$ Significant eccentricity in x-x dirxn.

BM due to this ecc. ~~in x-x dirxn~~ $= M_{ey} = P_u \times 21.6 \times 10^{-3}$

$= 1000 \times 21.6 \times 10^{-3}$

$= 21.6$ kNm

$\therefore M_{uy} = \text{Max.}(M_{ey}, M_{uy,top}, M_{uy,bottom})$

$= \text{Max.}(21.6, 30, 40)$

$= 40$ kNm

Accidental ecc. e_{min} in y-y dirxn $= \dfrac{l}{500} + \dfrac{D}{30} \nless 20$mm

$= \dfrac{5800}{500} + \dfrac{480}{30}$

$= 27.6 \text{mm} > 0.05D = 24$mm

$\therefore$ Significant eccentricity in y-y dirxn

BM due to this ecc. $= M_{ex} = P_u \times 27.6 \times 10^{-3}$

$= 27.6$ kNm

$\therefore M_{ux} = \text{Max.}(M_{ex}, M_{ux,top}, M_{uy,bottom})$ [$\because$ Unbraced column]

$= \text{Max.}(27.6, 80, 110)$

$= 110$ kNm

Since the column is long about y-y axis, additional moment is generated due to significant lateral deflxn (P-Δ effect)

Additional moment about y-y axis (M_{ay}) $= \dfrac{P_u b}{2000}\left(\dfrac{l_{ey}}{b}\right)^2 \times K$

$= \dfrac{1000 \times 300 \times 10^{-3}}{2000}\left(\dfrac{4200}{300 \cancel{\times 10^{-3}}}\right)^2 \times K$

$= 29.4 \times K$

K = Corrxn factor $= \dfrac{P_{uz} - P_u}{P_{uz} - P_b} \leq 1$

$P_{uz} = 0.446 f_{ck} A_c + 0.75 f_y A_s$

Assuming $\% p = 2\%$, $A_c = A_g - A_s = A_g - pA_g = A_g(1-p) = 300 \times 480 \times (1-0.02)$

$= 141120 \text{ mm}^2$

& $A_s = pA_g = 0.02 \times 300 \times 480 = 2880 \text{ mm}^2$

$\therefore P_{uz} = 2155.19$ kN

For P_b:- Table 60 of SP16 :-

This is for extra mom. about y-y axis.

So, $d'/b = 60/300 = 0.2$

Rectangular section:

$P_b = k_1 f_{ck} bD + k_2 pbD$

$= 0.184 \times 20 \times 300 \times 480 + 0.028 \times 2 \times 300 \times 480$

[∵ 4-face arrangement assumed]

$= 537.984 kN$

$\therefore K = \dfrac{P_{uz} - P_u}{P_{uz} - P_b} \le 1$

$= \dfrac{2155.19 - 1000}{2155.19 - 537.984}$

$= 0.714$

$\therefore M_{ay} = 29.4 * K = 21 kNm$

$M_{uy} = Max.(M_{uy,top}, M_{uy,bottom}, M_{ey}) + M_{ay}$

$= Max(30, 40, 21.6) + 21$

$= 40 + 21$

$= 61 kNm$

Due to long column about y-y axis, significant lateral defl xn. Thus, extra BM due to P-Δ effect

$\therefore M_{ux} = 110 kNm, \; M_{uy} = 61 kNm$

Then, check if :-

$$\left(\frac{M_{ux}}{M_{ux,1}}\right)^{\alpha_n} + \left(\frac{M_{uy}}{M_{uy,1}}\right)^{\alpha_n} \le 1$$

$P_u / f_{ck} bD = 1000 \times 10^3 / 20 \times 300 \times 480 = 0.3472 \sim 0.35$

For $M_{ux,1}$:-

$d'/D = 60/480 = 0.125$

$p/f_{ck} = 2/20 = 0.10$

For $d'/D = 0.10$:

For $d'/D = 0.15$:

$\frac{M_{ux,l}}{f_{ck} b D^2} = 0.132$ $\frac{M_{ux,l}}{f_{ck} b D^2} = 0.12$

$\therefore$ For $d'/D = 0.125$, $\frac{M_{ux,l}}{f_{ck} b D^2} = 0.132 + \frac{0.12 - 0.132}{0.15 - 0.10}(0.125 - 0.10)$

$= 0.126$

$\Rightarrow M_{ux,l} = 20 \times 300 \times 480^2 \times 0.126$

$= 174.02$ kN-m

For $M_{uy,l}$:-

$d'/b = 60/300 = 0.2$

$p/f_{ck} = 2/20 = 0.10$

For $d'/b = 0.2$, $\frac{M_{uy,l}}{f_{ck} D b^2} = 0.105$

$\Rightarrow M_{uy,l} = 20 \times 480 \times 300^2 \times 0.105$

$= 90.72$ kNm

$\alpha_n = 0.667 + 1.667 \frac{P_u}{P_{uz}}$

$= 0.667 + 1.667 \times \frac{1500}{2155.19}$

$= 1.44$

$\therefore \left(\frac{110}{174.2}\right)^{1.44} + \left(\frac{61}{90.72}\right)^{1.44} = 0.516 + 0.56 = 1.08 > 1$

Meaning $M_{ux,l}$ & $M_{uy,l}$ are a lil too small.

Inc. % p to 2.25 %. Calculating $M_{ux,l}$ & $M_{uy,l}$, M_{ux} & M_{uy} again with this value of $p = 2.25\%$, the above condition will be satisfied.

$\therefore A_s = \frac{2.25}{100} \times 300 \times 480 = 3240 \text{ mm}^2$

Adopt 4×20mm ϕ + 4×28mm ϕ long. bars.

Transverse reinforcements:-

$\phi_{tr} \not< 6$mm

$\not< \frac{\phi_{long}}{4} = \frac{Max.(28, 20)}{4} = \frac{28}{4} = 7mm$

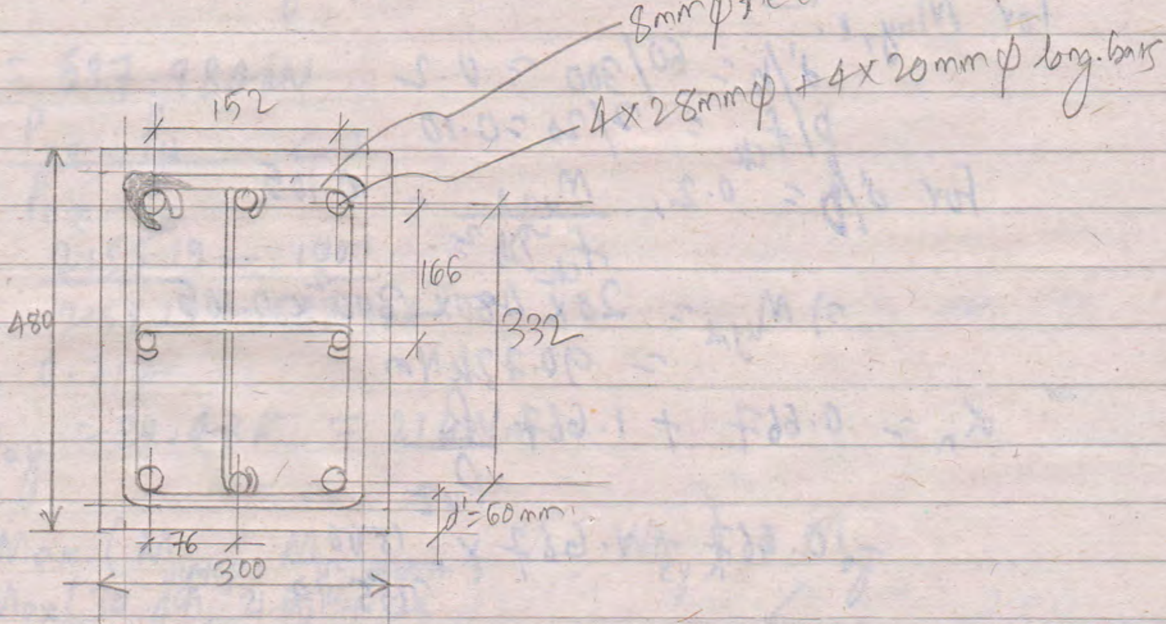
Provide $\phi_{tr} = 8mm$

Spacing (S_v) $\not> 300mm$

$\not>$ least lateral dim. of column $= 300mm$

$\not> 16\phi_{long} = 16 \times Min.(28, 20) = 16 \times 20 = 320$ mm

Provide 300mm c/c



Spacing betn consecutive long. bars in the shorter dirxn = 76mm > 75mm. So, additional tie is needed in the shorter dirxn. Spacing betn corner long. bars of longer face = 332 < $48\phi_{tr}$ = 48×8 = 384 mm. So, provide open tie to non-corner bars of longer faces [o——o]

Spacing betn consecutive long. bars in the longer dirxn = 166mm > 75mm. So additional tie is needed in the longer dirxn. Spacing betn corner long. bars of shorter face = 152 < $48\phi_{tr}$ = 384mm. So, provide open tie to non-corner bars of shorter faces

{ Practice one or two more of this kind of problem.
Eg. 9.14, 9.15, 9.16 from S.N. Sinha's book }

Eg. 9.15 Design a biaxially eccentrically loaded braced rectangular column deforming in single curvature for the following data.

Ultimate axial load $P_u = 2000\,kN$

Ultimate biaxial moments at bottom : $M_{ux1} = 225\,kNm$ & $M_{uy1} = 125\,kNm$

At top : $M_{ux2} = 175\,kNm$ & $M_{uy2} = 75\,kNm$

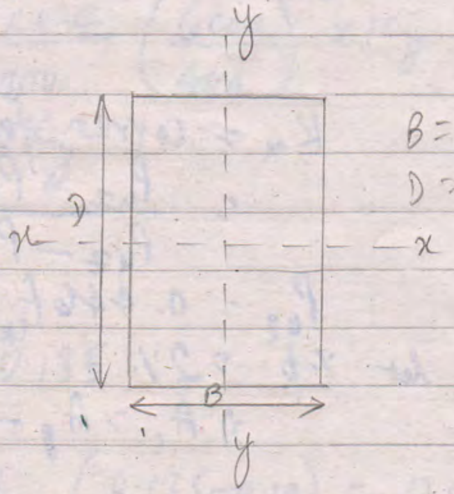
Unsupported length $(l) = 9\,m$

Effective lengths $l_{ex} = 8\,m$ & $l_{ey} = 6\,m$

Column section : B (in x-dirxn) $= 400\,mm$ and $D = 600\,mm$

Grade of concrete : M25 and Grade of steel = Fe 415

Sol^n :-



B = 400mm
D = 600mm

S.R. about x-x axis $= \frac{l_{ex}}{D} = \frac{8000}{600}$

$= 13.33 > 12$

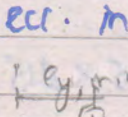
∴ Long about x-x axis

S.R. about y-y axis $= \frac{l_{ey}}{B} = \frac{6000}{400}$

$= 15 > 12$

∴ Long about y-y axis

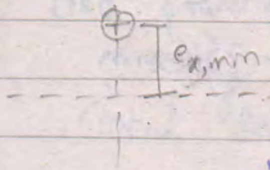
Accidental ecc. in x-x axis $(e_{y,min}) = \frac{l}{500} + \frac{B}{30} \not< 20\,mm$



$= \frac{9000}{500} + \frac{400}{30} = 31.33\,mm > 0.05B = 20\,mm$

∴ Significant ecc. in x-x axis

Acc. ecc. in y-y axis $(e_{x,min}) = \frac{l}{500} + \frac{D}{30} \not< 20\,mm$



$= \frac{9000}{500} + \frac{600}{30} = 38\,mm > 0.05D = 30\,mm$

∴ Significant ecc. in y-y axis

Accidental BM about x-x axis $(M_{ex}) = P_u e_{x,min} = 2000 \times 38 \times 10^{-3} = 76\,kN\,m$

" " " y-y axis $(M_{ey}) = P_u e_{y,min} = 2000 \times 31.33 \times 10^{-3} = 62.66\,kNm$

Top BM about x-x axis $(M_{ux,top}) = Max.(175, 76) = 175 kNm$
Top BM about y-y axis $(M_{uy,top}) = Max.(75, 62.66) = 75 kNm$
Bottom BM about x-x axis $(M_{ux,bottom}) = Max.(225, 76) = 225 kNm$
Bottom BM about y-y axis $(M_{uy,bottom}) = Max.(125, 62.66) = 125 kNm$
Since the column is long about both axes, additional moment is generated about both axes.

Extra BM about x-x axis $(M_{ax}) = \frac{P_u D}{2000}\left(\frac{l_{ex}}{D}\right)^2 \times K_x$

$$= \frac{2000 \times 0.6}{2000}\left(\frac{8000}{600}\right)^2 \times K_x$$

$$= 106.67 K_x$$

K_x = corrxn factor

$$= \frac{P_{uz} - P_u}{P_{uz} - P_{bx}} \leq 1$$

$P_{uz} = 0.446 f_{ck} A_c + 0.75 f_y A_s$

let $\%p = 2\%$

$\Rightarrow A_c = A_g - pA_g = 400 \times 600 - \frac{2}{100} \times 400 \times 600 = 235200 \text{ mm}^2$

$A_s = pA_g = 4800 \text{ mm}^2$

$\therefore P_{uz} = 0.446 \times 25 \times 235200 + 0.75 \times 415 \times 4800$
$= 4116.48 kN$

For P_{bx}: Table 60 SP16

Considering 4-face reinf.

~~$d'/D = 40/600 = 0.067$ [$\because$ Eff. cover d' taken as 40mm]~~

Considering 40mm c.c. and 28mm ϕ long. bars

Eff. cover $(d') = cc + \phi/2 = 40 + 14 = 54$ mm

$\therefore d'/D = 54/600 = 0.09$

$P_{bx} = k_1 f_{ck} bD + k_2 pbD$

$k_1 = 0.219 + \frac{0.207 - 0.219}{0.10 - 0.05}(0.09 - 0.05) = 0.2094$

$k_2 = 0.424 + \frac{0.328 - 0.424}{0.10 - 0.05}(0.09 - 0.05) = 0.3472$

$\therefore P_{bx} = 0.2094 \times 25 \times 400 \times 600 + 0.3472 \times 2 \times 400 \times 600$
$= 1423.056 kN$

$\therefore K_x = \frac{4116.48 - 2000}{4116.48 - 1423.056} = 0.786$

$\therefore M_{ax} = 106.67 \times 0.786 = 83.8 kN\text{-}m$

Extra BM about y-y axis $(M_{ay}) = \frac{P_u b}{2000}\left(\frac{l_{ey}}{b}\right)^2 \times K_y$

$= \frac{2000 \times 0.4}{2000}\left(\frac{6000}{400}\right)^2 \times K_y$

$= 90 K_y$

$K_y = \frac{P_{uz} - P_u}{P_{uz} - P_{by}} \leq 1$

For P_{by} :

$d'/b = 54/400 = \cancel{0.09}\ 0.135$

$P_{by} = k_1 f_{ck} bD + k_2 p bD$

$k_1 = 0.207 + \frac{0.196 - 0.207}{0.15 - 0.10}(0.135 - 0.10) = 0.1993$

$k_2 = 0.328 + \frac{0.203 - 0.328}{0.15 - 0.10}(0.135 - 0.10) = 0.2405$

$\therefore P_{by} = 0.1993 \times 25 \times 400 \times 600 + 0.2405 \times 2 \times 400 \times 600$
$= 1311.24 kN$

$\therefore K_y = \frac{4116.48 - 2000}{4116.48 - 1311.24} = 0.754$

$\therefore M_{ay} = 90 \times 0.754 = 67.86 kN\text{-}m$

$\therefore$ Total BM about x-x axis $(M'_{ux}) = 0.6 \times Max.(175, 225) + 0.4 \times Min(175, 225)$
$+ M_{ax}$

$= 0.6 \times 225 + 0.4 \times 175 + 83.8$

$= 288.8 kNm$

$\therefore$ Total BM about y-y axis $(M_{uy}') = 0.6 \times Max(75,125) + 0.4 \times Min(75,125) + M_{ay}$

$= 0.6 \times 125 + 0.4 \times 75 + 67.86$

$= 172.86$ kN-m

Check if :-

$$\left(\frac{M_{ux}'}{M_{ux,l}}\right)^{\alpha_n} + \left(\frac{M_{uy}'}{M_{uy,l}}\right)^{\alpha_n} \le 1$$

For $M_{ux,l}$:-

$d'/D = 54/600 = 0.09$

$p/f_{ck} = 2/25 = 0.08$

$P_u/f_{ck}bD = 2000 \times 10^3 / 25 \times 400 \times 600 = 0.33$

In $d'/D = 0.05$:

$\frac{M_{ux,l}}{f_{ck}bD^2} = 0.15$

In $d'/D = 0.10$:

$\frac{M_{ux,1}}{f_{ck}bD^2} = 0.135$

$\therefore$ for $d'/D = 0.09$, $\frac{M_{ux,l}}{f_{ck}bD^2} = 0.15 + \frac{0.135 - 0.15}{0.10 - 0.05}(0.09 - 0.05)$

$= 0.138$

$\Rightarrow M_{ux,l} = 25 \times 400 \times 600^2 \times 0.138 = 496.8$ kN-m

For $M_{uy,l}$:-

$d'/b = 54/400 = 0.135$

$p/f_{ck} = 2/25 = 0.08$

In $d'/b = 0.10$:

$\frac{M_{uy,l}}{f_{ck}Db^2} = 0.135$

In $d'/b = 0.15$:

$\frac{M_{uy,1}}{f_{ck}Db^2} = 0.123$

$\therefore$ for $d'/b = 0.135$, $\frac{M_{uy,l}}{f_{ck}Db^2} = 0.135 + \frac{0.123 - 0.135}{0.15 - 0.10}(0.135 - 0.10)$

$= 0.1266$

$\Rightarrow M_{uy,l} = 25 \times 600 \times 400^2 \times 0.1266 = 303.84$ kN-m

$\alpha_n = 0.667 + 1.667 \dfrac{P_u}{P_{uz}}$

$P_{uz} = 0.446 f_{ck} A_c + 0.75 f_y A_s = 4116.48$ kN

$P_u = 2000$ kN

$\therefore \alpha_n = 1.477$

Then,

$$\left(\frac{M_{ux}'}{M_{ux,l}}\right)^{\alpha_n} + \left(\frac{M_{uy}'}{M_{uy,l}}\right)^{\alpha_n}$$

$$= \left(\frac{288.8}{496.8}\right)^{1.477} + \left(\frac{172.86}{303.84}\right)^{1.477}$$

$= 0.4498 + 0.4347$

$= 0.884 < 1 \Rightarrow$ Safe

So, $A_s = 2\%$ of $A_g = 0.02 \times 400 \times 600 = 4800$ mm^2

Provide 8×28 mm ϕ long. bars.

Transverse reinf :-

$\phi_{tr} \nless 6$ mm

$\nless \dfrac{\phi_{long}}{4} = \dfrac{28}{4} = 7$ mm

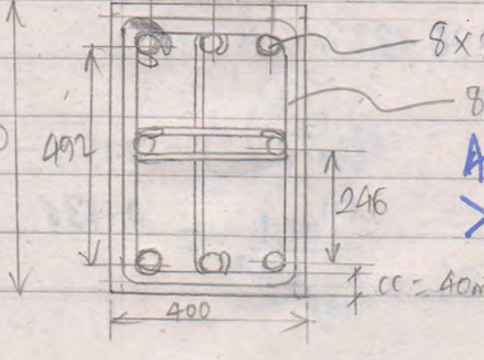
$\therefore$ Provide 8mm ϕ transverse ties reinf.

Spacing $S_v \ngtr 300$ mm

$\ngtr$ least lateral dim $= 400$ mm

$\ngtr 16\phi_{long} = 16 \times 28 = 448$ mm

$\therefore$ Provide 300mm spacing c/c.



Since spacing bet^n consecutive bars in shorter face $= 146$mm > 75 mm, additional tie is necessary in the shorter direction.

As spacing bet^n long. bars at corners of longer face $= 492$mm $> 48\phi_{tr} = 384$mm, provide closed tie bet^n non-corner bars of longer faces

Since spacing betn consecutive long. bars in longer face = 246 mm > 75 mm, additional tie is reqd. in longer dirxn. As spacing betn corner bars in shorter face = 292 mm < $48\phi_{tr}$ = 384 mm, provide open tie betn non-corner bars of shorter faces. [6]

Redo of example (8) 5 using another method (may or may not work, just an idea) :-

$$\left(\frac{P_u e_x}{M_{ux,1}}\right)^{\alpha_n} + \left(\frac{P_u e_y}{M_{uy,1}}\right)^{\alpha_n} = 1 \text{ ; find } P_{u\,max}.$$

Given: $P/f_{ck} = 0.197$, $e_x = 100$mm, $e_y = 60$mm, $d'/D = 0.108$, $d'/b = 0.18$

Now,

$$\frac{M_{ux,1}/(f_{ck}bD^2)}{P_u/(f_{ck}bD)} = \frac{M_{ux,1}}{DP_u} = \frac{P_u e_x}{DP_u} = \frac{100}{500} = \frac{1}{5}$$

$$\frac{M_{uy,1}/f_{ck}Db^2}{P_u/(f_{ck}bD)} = \frac{M_{uy,1}}{bP_u} = \frac{P_u e_y}{bP_u} = \frac{60}{300} = \frac{1}{5}$$

The idea is to use these slopes to obtain $M_{ux,1}$ & $M_{uy,1}$ and solve the interaction eqn for P_u.

~~In~~ For $M_{ux,1}$:-

In $P/f_{ck} = 0.18$:-

In $d'/D = 0.10$:-

$$\frac{M_{ux,1}}{f_{ck}bD^2} = 0.133$$

In $d'/D = 0.15$:-

$$\frac{M_{ux,1}}{f_{ck}bD^2} = 0.128$$

$\therefore$ For $d'/D = 0.108$,

$$\frac{M_{ux,1}}{f_{ck}bD^2} = 0.133 + \frac{0.128 - 0.133}{0.15 - 0.10}(0.108 - 0.10)$$
$$= 0.1322$$

In $P/f_{ck} = 0.20$:-

In $d'/D = 0.10$:-

$$\frac{M_{ux,1}}{f_{ck}bD^2} = 0.142$$

In $d'/D = 0.15$:-

$$\frac{M_{ux,1}}{f_{ck}bD^2} = 0.136$$

$\therefore$ For $d'/D = 0.108$, $\frac{M_{ux,l}}{f_{ck} b D^2} = 0.142 + \frac{0.136 - 0.142}{0.15 - 0.10}(0.108 - 0.10)$

$= 0.141$

$\therefore$ For $P/f_{ck} = 0.197$,

$\frac{M_{ux,l}}{f_{ck} b D^2} = 0.1322 + \frac{0.141 - 0.1322}{0.20 - 0.18}(0.197 - 0.18)$

$= 0.1397$

$\Rightarrow M_{ux,l} = 25 \times 300 \times 500^2 \times 0.1397 = \underline{261.9 kN\text{-}m}$

For $M_{uy,l}$:-

In $P/f_{ck} = 0.18$:-

In $d'/b = 0.15$:-

$\frac{M_{uy,l}}{f_{ck} D b^2} = 0.129$

In $d'/b = 0.20$:-

$\frac{M_{uy,l}}{f_{ck} D b^2} = 0.119$

$\therefore$ For $d'/b = 0.18$,

$\frac{M_{uy,l}}{f_{ck} D b^2} = 0.129 + \frac{0.119 - 0.129}{0.20 - 0.15}(0.18 - 0.15)$

$= 0.123$

In $P/f_{ck} = 0.20$:-

In $d'/b = 0.15$:-

$\frac{M_{uy,l}}{f_{ck} D b^2} = 0.137$

In $d'/b = 0.20$:-

$\frac{M_{uy,l}}{f_{ck} D b^2} = 0.127$

$\therefore$ for $d'/b = 0.18$,

$\frac{M_{uy,l}}{f_{ck} D b^2} = 0.137 + \frac{0.127 - 0.137}{0.20 - 0.15}(0.18 - 0.15)$

$= 0.131$

$\therefore$ for $P/f_{ck} = 0.197$,

$\frac{M_{uy,l}}{f_{ck} D b^2} = 0.123 + \frac{0.131 - 0.123}{0.20 - 0.18}(0.197 - 0.18)$

$= 0.1298$

$\Rightarrow M_{uy,l} = 25 \times 500 \times 300^2 \times 0.1298 = \underline{146.025 kN\text{-}m}$

Then, $\alpha_n = 0.667 + 1.667 \dfrac{P_u}{P_{uz}}$

$$P_{uz} = 0.446 f_{ck} A_c + 0.75 f_y A_s$$
$$= 0.446 \times 25 \times 500 \times 300 (1 - 0.0493) + 0.75 \times 415 \times 0.0493 \times 500 \times 300$$
$$= 3891.74 \text{ kN}$$

$\therefore \alpha_n = 0.667 + 4.2834 \times 10^{-4} P_u$; P_u in kN

So,
$$\left(\frac{P_u \times 0.1}{261.9}\right)^{0.667 + 4.2834 \times 10^{-4} P_u} + \left(\frac{P_u \times 0.06}{146.025}\right)^{0.667 + 4.2834 \times 10^{-4} P_u} = 1$$

Iteration table :-

P_u (kN)	α_n	$f(P_u)$
1000	1.09534	0.7258
1500	1.30951	1.0126
1400	1.26667	0.9487
1450	1.28809	0.9802
1475	1.298802	0.9962
1480	1.30094	0.99949
1485	1.30308	1.00275
1482	1.3018	1.00079
1481	1.30137	1.00014
1480.5	1.301157	0.999815 ≈ 1 & < 1

Could this be correct? What about the result from Bressler's mtd?

Class # 24

① Design a footing for the following data:

Column size = 400mm x 400mm

DL = 1000 kN

LL = 400 kN } Acting on column (Axial loading)

q_{allow} = 200 kN/m²

M20, Fe 415

→ exp. only applicable for axially loaded footing.

Solⁿ:-

(i) Area of footing (A) = $\frac{1.1 \times \text{Service load}}{\text{Allowed bearing cap. of soil}} = \frac{1.1(DL+LL)}{q_{allow}}$

$\therefore A = \frac{1.1(1000+400)\,kN}{200\,kN/m^2} = 7.7\,m^2$

Adopt square footing of size 2.8m x 2.8m

$\therefore A_{provided} = 2.8^2 = 7.84\,m^2$

$\therefore L = B = 2.8\,m$

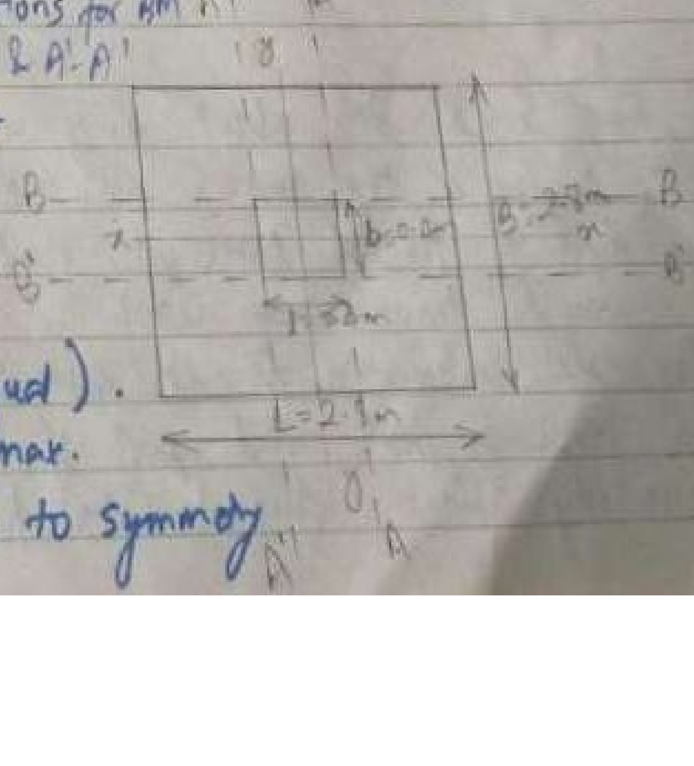
And given column dimensions : $l = b = 0.4m$

(ii) Upward soil pressure $(p_u) = \frac{P_u}{A} = \frac{1.5(DL+LL)}{7.84} = \frac{1.5 \times 1400}{7.84} = 267.85\,kN/m^2$

Adopt $p_u = 270\,kN/m^2$

(iii) Determination of max. BM, max. SF at critical sections of footing :-

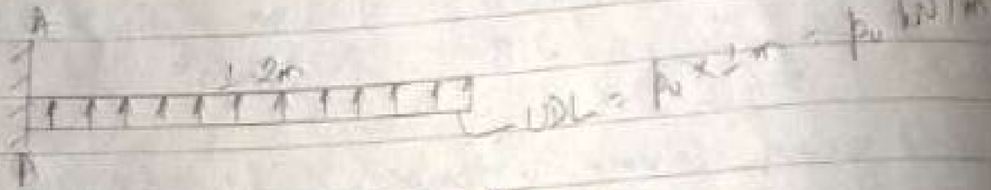
For column attached footing, crit. sections for BM are at the faces of the column : A-A & A'-A' and B-B & B'-B'. For BM about x-x axis, the max. BM occurs at B-B & B'-B' (due to symmetry about x-x axis, BM at B-B & BM at B'-B' will be equal).



IIly, for BM about y-y axis, the max. BM occurs at A-A & A'-A' (due to symmetry

about y-y axis, BM at A-A & BM at A'-A' will also be equal)
For BM about y-y at section A-A, we idealize the section A-A as a fixed support and the part of footing right of it as a cantilever beam as follows:-

(Here we ~~have~~ ~~design~~ are designing for 1m width of footing, so we have taken 1m wide strip of footing for BM calculation)
In 2-D:-



$$M_{uy} \text{ at section A-A} = (p_u \times 1.2) \times \frac{1.2}{2}$$

$$= 270 \times 1.2 \times \frac{1.2}{2}$$

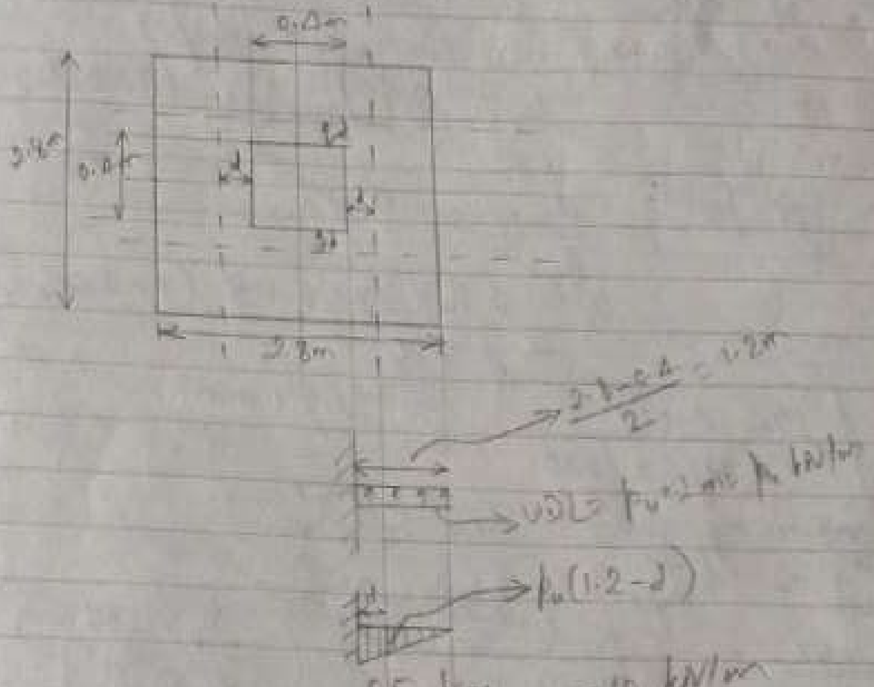
$$= 194.4 \text{ kNm}$$

$$= M_{uy} \text{ at section A'-A'}$$

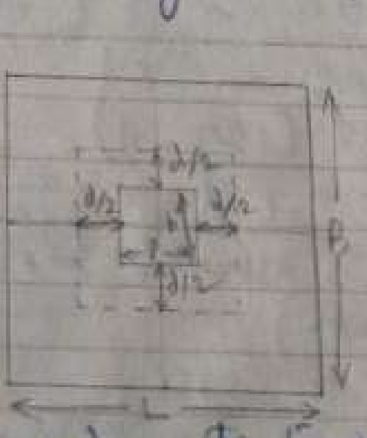
~~For~~ Since BM about y-y axis at both the critical sections A-A & A'-A' is the same, we use any one of them for calculation of depth of footing later on. Had any one been greater, the greater one would have been used.

For BM about x-x at section B-B, we idealize the section B-B as a fixed support and part of footing above it as a cantilever beam similarly. Here, due to the footing being square and the column

Being square too, M_{ux} at B-B = 194.4 kNm
And also, M_{ux} at B'-B' = 194.4 kNm
For crit. sections for one-way shear, at dist. 'd': eff depth of footing, from face of column :



∴ Crit. shear (one-way) = $p_u(1.2-d)$ on all sides/direns.
For crit. sections for two-way shear, at dist. 'd/2' from face of column:-



Crit. shear (two-way) = $p_u\left[LB-(l+d)(b+d)\right]$
in kN/m^2

Footing is very similar to slab.

(iv) Thickness of footing :-

(a) Based on moment : $d \geq d_{bal}$

Eff. depth ~~of~~, for balanced SR section :

$d = \sqrt{\dfrac{M_{u,max}}{Q\,b}}$; $M_{u,max} = 194.4\ kN\text{-}m$

b = Width of footing = 1000 mm (designed for 1m width strip)

$Q = 0.36 f_{ck} \dfrac{x_{u,l}}{d}\left(1 - 0.416\dfrac{x_{u,l}}{d}\right)$

For Fe415, $\dfrac{x_{u,l}}{d} = 0.48$

$\therefore Q = 0.36 \times 20 \times 0.48\,(1 - 0.416 \times 0.48)$
$= 2.766$

$\Rightarrow d = \sqrt{\dfrac{194.4 \times 10^6}{2.766 \times 1000}} = 265.1\ mm$

(b) Based on one-way shear : $\tau_{uv} \leq k\tau_{uc}$

$\tau_{uv} = k\tau_{uc}$

$\tau_{uv} = \dfrac{V_{u,max}}{b_0 d} = \dfrac{p_u\left(\frac{B-b}{2} - d\right)}{1000 \times d}$ [$\because b_0 = 1000\ mm$ i.e. 1m width of strip]

$= \dfrac{270\ kN/m\left(\frac{2.8-0.4}{2} - d\right)m}{1000\ mm \times d}$

$= \dfrac{270\ N/\cancel{mm} \times (1200 - d)\ \cancel{mm}}{1000\ mm \times d}$

Cl. 6.0.2.1.1

$= 0.27\,\dfrac{(1200 - d)}{d}$

$k = 1$ (assumed; since it is assumed the total depth of footing D will be $\geq 300\ mm$)

[$\because$ for slab, $A_{st}^{min} = 0.12\%$ & footing is also a type of slab]

$\tau_{uc} \Rightarrow$ Using Table 19 of IS456,

Assuming A_{st}^{min}, $\%p = \dfrac{A_{st}^{min}}{bd} = \dfrac{0.12\%\ \text{of}\ \cancel{bD}}{\cancel{bd}} \approx 0.12\%$
1000 mm

$\therefore$ For M20, $\tau_{uc} = 0.28\ N/mm^2$

ϕ for footing should be ≥ 12mm; 20mm gen. used
c.c ≥ 50mm; 75mm preferred

$$\Rightarrow 0.27 \frac{(1200-d)}{d} = 1 \times 0.28$$

$$\Rightarrow d = 589.1 \text{ mm}$$

(c) Based on two-way shear : $\tau_{uv} \leq K_s \tau_{uc}$

$$\tau_{uv} = K_s \tau_{uc}$$

$$\tau_{uv} = \frac{V_{u,max}}{b_0 d} = \frac{p_u \text{ kN/m}^2 [LB-(l+d)(b+d)]}{2(l+d+b+d) \times d}$$

perimeter of crit. sections

$$= \frac{270 \text{ kN/m}^2 \times [2.8^2-(0.4+d)^2]}{2(0.4 \times 2 + 2d) \times d}$$

$$= \frac{0.27 \text{ N/mm}^2 \times [2800^2-(400+d)^2]}{2(400 \times 2 + 2d) \times d}$$

$$K_s = \beta_c + 0.5 = \frac{B}{L} + 0.5 = 1 + 0.5 = 1.5 > 1 \therefore K_s = 1$$

$$\tau_{uc} = 0.25\sqrt{f_{ck}} = 0.25\sqrt{20} = 1.118 \text{ N/mm}^2$$

$$\therefore \frac{0.27 [2800^2 - (400+d)^2]}{2(800+2d) \times d} = 1.118$$

$$\Rightarrow d = 483 \text{ mm}$$

As d is max. for one-way shear, d = 590 mm

Assuming ϕ of reinf. bars = 20 mm & c.c. = 75 mm,

$$D = d + \phi/2 + cc = 590 + 10 + 75 = 675 \text{ mm}$$

(v) Reinforcement :-

$$A_{st} = \frac{M_u}{0.87 f_y (d - 0.416 x_u)} = \frac{194.4 \times 10^6}{0.87 \times 415 (590 - 0.416 x_u)}$$

$$x_u = \frac{0.87 f_y A_{st}}{0.36 f_{ck} b} = \frac{0.87 \times 415 A_{st}}{0.36 \times 20 \times 1000} = 0.0501 A_{st}$$

$$\therefore A_{st} = \frac{194.4 \times 10^6}{0.87 \times 415 (590 - 0.416 \times 0.0501 A_{st})} = \frac{538429.58}{590 - 0.0208 A_{st}}$$

$$\Rightarrow 0.0208 A_{st}^2 - 590 A_{st} + 538429.58 = 0$$

$$\therefore A_{st} = 27421.38 \text{ mm}^2, 944.01 \text{ mm}^2$$

Max. spacing betⁿ bars in footing = Min (3d, 450mm)

Adopting 20mm ϕ bars, for 1m width of footing,

$n = \frac{27421.38\ mm^2}{\pi 10^2\ mm^2},\ \frac{984.01\ mm^2}{\pi 10^2\ mm^2}$

$= 88\ ,\ 4$

$\therefore$ Spacing betⁿ bars :

$S = \frac{1000\ mm}{88},\ \frac{1000\ mm}{4}$

$= 11.36\ mm\ ,\ 250\ mm < Min\,(3d,\ 450\ mm)$

$= Min\,(3 \times 590,\ 450)$

$= 450\ mm$ OK.

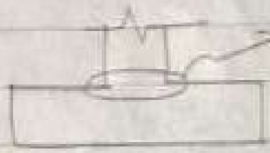
$\therefore$ 20mm ϕ @ 250 mm c/c on both dirxns since $M_{ux} = M_{uy}$ in this case.

(vi) Check bearing stress :-

Bearing stress $= \frac{1.5\,(DL + LL)}{400 \times 400}$

$= \frac{1.5\,(1000 + 400) \times 10^3\ N}{400 \times 400\ mm^2}$

$= 13.125\ N/mm^2$



bearing stress at this area i.e. area of contact of column & footing/area of column

Allowable bearing stress $= 0.45 f_{ck} \sqrt{\frac{A_1}{A_2}}$

$A_1 = (l + 4d)(b + 4d) = (400 + 4 \times 590)^2 = 7617600\ mm^2$

$A_2 = bl = 400^2 = 160000\ mm^2$

$\therefore \sqrt{\frac{A_1}{A_2}} = 6.9 \not> 2 \quad \therefore \sqrt{\frac{A_1}{A_2}} = 2$

$\therefore$ Allowable bearing stress $= 0.45 \times 20 \times 2 = 18\ N/mm^2$

Since bearing stress < Allowable bearing stress, OK.

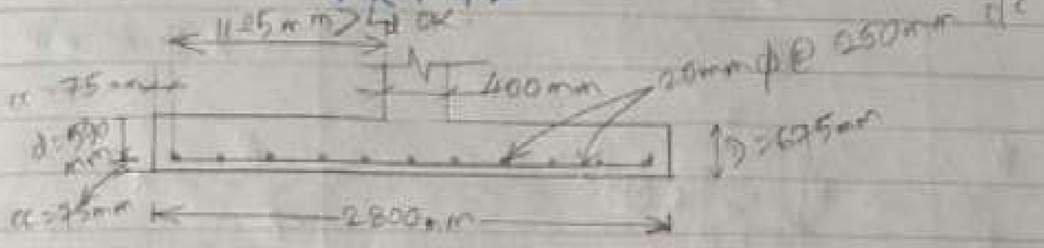
(vii) Check dev. length :-

Reqd. dev. length $(L_d) = \frac{0.87 f_y \phi}{4 \tau_{bd}}$

For τ_{bd}, (I. 26.2.1.1 :) $1.6 \times 1.2 = 1.92\ N/mm^2$

Design load = service load ×1.5
(P_u) (DL+LL)

$\therefore L_d = \dfrac{0.87 \times 415 \times 20}{4 \times 1.92} \approx 940.23\,mm$



(2) Design a rectangular footing for the following data:-
Column size : 400mm × 400mm
P_u = Design load = 1500 kN
M_u = 300 kN-m } Acting on column (Uniaxial loading)
q_{allow} = 150 kN/m^2
M25, Fe415

Soln:-

(i) Footing dimensions :- Assuming an axially loaded footing design.

Area of footing (A) $= \dfrac{1.1(DL+LL)}{q_{allow}} = \dfrac{1.1 \times P_u}{1.5 \times q_{allow}}$

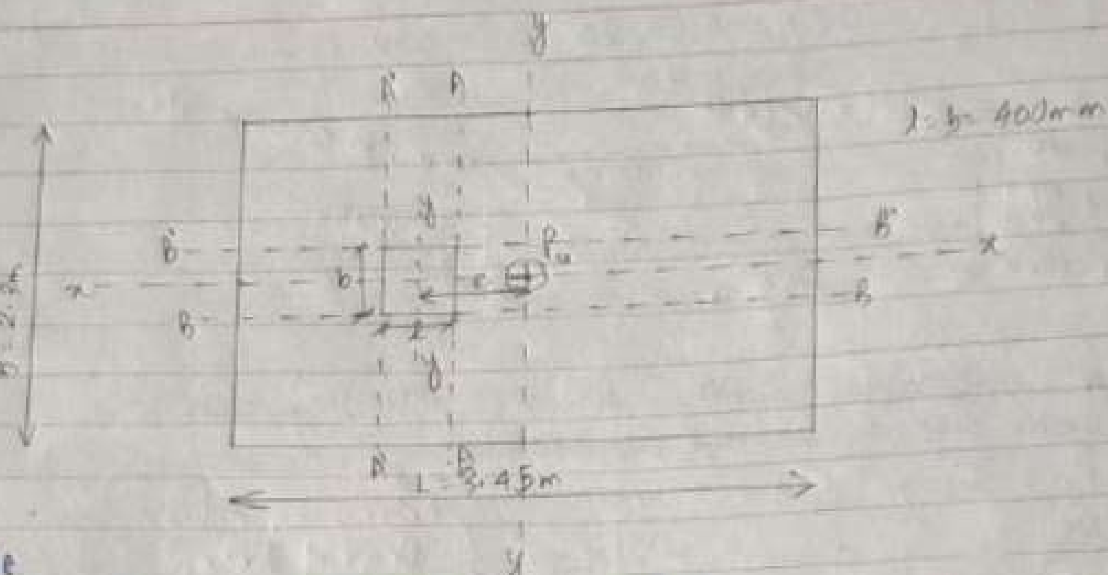
$= \dfrac{1.1}{1.5} \times \dfrac{1500\,\cancel{kN}}{150\,\cancel{kN}/m^2}$

$= 7.33\,m^2$

Adopting $L = 1.5B$, $LB = 7.33$
$\Rightarrow 1.5B^2 = 7.33$
$\Rightarrow B = 2.21\,m$

Provide $B = 2.3\,m$, $L = 3.45\,m \Rightarrow A_{prov} = 7.935\,m^2$

(ii) Upwards soil pressure :- Since axial loading condition for footing is assumed above in using the expn for 'A', we have to arrange the column such that the uniaxial loading condition of column results in coincidence of centroid of footing plan area and point of application of P_u. The arrangement, assuming the uniaxial loading of column is about its y-y axis, to its right by 'e' ie , is:
(See 11.2.6 @ SN Sinha)



where,

$$e = \frac{M_u}{P_u} = \frac{300\ \text{kN-m}}{1500\ \text{kN}} = 0.2\ \text{m}$$

In positioning the column so, P_u acts right at the centroid of footing. Thus the footing is loaded axially. Therefore, upwards soil pressure p_u can simply be obtd. as

$$p_u = \frac{P_u}{A} = \frac{1500\ \text{kN}}{7.935\ \text{m}^2} = 189.036\ \text{kN/m}^2$$

and the distribution of p_u is uniform throughout the footing's plan area.

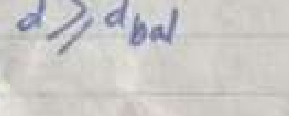
(iii) Thickness of footing:-

(a) Based on moment:- $d \geq d_{bal}$

$$d = \sqrt{\frac{M_{u,max}}{Q\,b}}$$

$M_{u,max}$ = Max. (BM about face A'A', BM about face AA, BM about face BB = BM about face B'B')

BM about face A'A':-

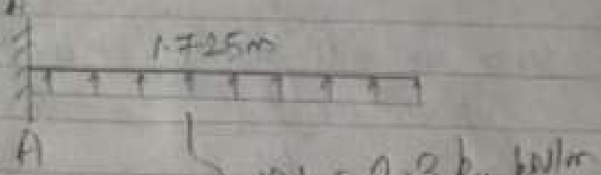


[Here, we are considering the entire width of footing for calc UDL & thus BMs. But it could've been done for 1m width too.]

UDL = p_u kN/m^2 × 2.3m = 2.3 p_u kN/m

$M_{A'A'} = 2.3 p_u * 1.325 \times \frac{1.325}{2} = 381.66\ kNm$

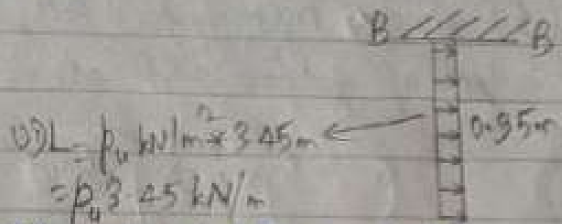
BM about face AA :–



$M_{AA} = 2.3 p_u * 1.725 \times \frac{1.725}{2}$

$= 646.88\ kNm$

BM about face BB :–



$M_{BB} = 3.45 p_u * 0.95 \times \frac{0.95}{2} = 294.29\ kN\text{-}m$

$M_{B'B'} = M_{BB}$ [$\because$ Symmetry]

$\therefore M_{u,max} = 646.88\ kNm$ about face AA

So, $d = \sqrt{\frac{M_{u,max}}{Q_1 b}}$; $b = 2.3m$ since $M_{u,max}$ is about

$Q_1 = 0.36 f_{ck} \frac{x_{u,l}}{d} \left(1 - 0.416 \frac{x_{u,l}}{d}\right)$

For Fe415, $\frac{x_{u,l}}{d} = 0.48$

$\therefore Q = 0.36 \times 25 \times 0.48 \left(1 - 0.416 \times 0.48\right) = 3.457$

$\therefore d = 285.23mm$

(b) Based on one-way shear :– $\tau_{uv} \le k\tau_{uc}$

$\tau_{uv} = \frac{V_{u,max}}{b_0 d} = k\tau_{uc} \Rightarrow d = \frac{V_{u,max}}{b_0 k \tau_{uc}}$

For k, Cl. 40.2.1.1, assuming total depth D will be > 300 mm,
$k = 1$.

For τ_{uc}, assuming % reinf. (%p) $= 100 \dfrac{A_{st}}{bd}$

Since we will provide at least $A_{st}^{min} = 0.12\%$ of bD for slabs,

$$\%p = 100 \frac{A_{st}^{min}}{bd} = 100 \times \frac{0.12\%\ bD}{bd} \approx 0.12$$

From Table 19 of IS456, for M25,

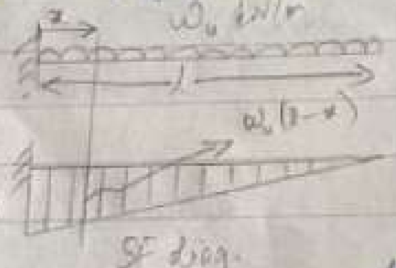
$$\tau_{uc} = 0.29\ N/mm^2$$

$$\therefore d = \frac{V_{u,max}}{b_0 k \tau_{uc}} = \frac{V_{u,max}}{b_0 * 0.29}$$

$V_{u,max}$ = Max. (S.F. at d right of face AA, SF at d left of face A'A', $\underbrace{\text{SF at d up of face A'B'} = \text{SF at d down of face BB}}_{}$)

[∵ Symmetry about x-x axis of footing]

Since span of footing right of AA is visibly longer than the span of footing left of A'A' and "SF at 'x' from support in a 'w_u' UDL cantilever $= w_u(l - x)$":



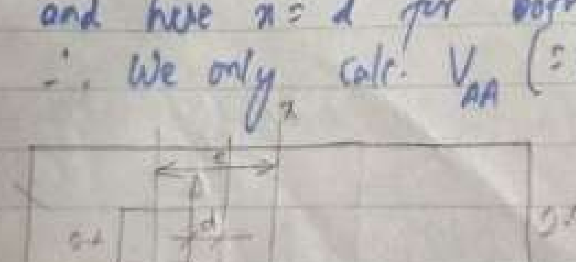
"

and here $x = d$ for both cases & $w_u = p_u$, SF $\propto$ span (l)

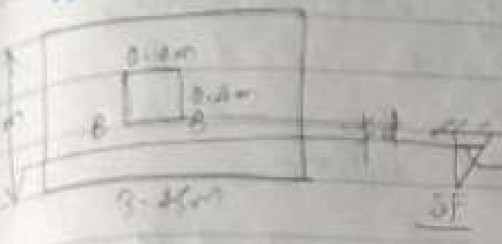
∴ We only calc. V_{AA} (= SF at 'd' right of AA) $= (p_u\ kN/m^2 * 2.3\ m) * \left(\dfrac{3.45}{2} + e - \dfrac{0.4}{2} - d\right)$

$$= 189.036 \times 2.3\ kN/m * (1.725 - d)$$

$$= 434.78\ N/mm * (1725 - d)$$



V_{BB} (= SF at 'd' below BB) $= (p_u \times 3.45) \times \left(\dfrac{2.3-0.4}{2} - d\right)$



$= 652.17\ kN/m \times (0.95m - d)$

$= 652.17\ N/mm \times (950 - d)$

$V_{u,max}$ is not easily discernable. So find 'd' using both.

$d = \dfrac{V_{AA}}{b_0 \times 0.29}$. For V_{AA}, $b_0 = 2.3m$

$\Rightarrow d = \dfrac{434.78 \times (1725 - d)}{2300 \times 0.29}$

$\Rightarrow d = 680.71\ mm$

$d = \dfrac{V_{BB}}{b_0 \times 0.29}$. For V_{BB}, $b_0 = 3.45m$

$\Rightarrow d = \dfrac{652.17 \times (950 - d)}{3450 \times 0.29}$

$\Rightarrow d = 374.89\ mm$

$\therefore d = 680.71\ mm$

(c) Based on two-way shear :- $\tau_{uv} \leq k_s \tau_{uc}$

$\tau_{uv} = \dfrac{p_u [LB - (l+d)(b+d)]}{b_0 d} = \dfrac{189.036\ kN/m^2 [3.45 \times 2.3\ m^2 - (0.4+d)^2]}{2[(0.4+d) + (0.4+d)]\ d}$

$k_s = 0.5 + \beta_c = 0.5 + \dfrac{B}{L} = 0.5 + \dfrac{2.3}{3.45} = 1.167 \not> 1 \quad \therefore k_s = 1$

$\tau_{uc} = 0.25\sqrt{f_{ck}} = 0.25\sqrt{25} = 1.25\ N/mm^2$

Then, $\tau_{uv} = k_s \tau_{uc}$

$\Rightarrow \dfrac{189.036 \times 10^{-3}\ N/mm^2 [3450 \times 2300 - (400+d)^2]}{2[800 + 2d]\ d} = 1.25$

$\Rightarrow d = 363.86\ mm$

$\therefore d =$ Largest of the three $= 680.71\ mm$

Taking $\phi = 20\,mm$ & $c.c = 75\,mm$,
Tot. depth (D) $= d + \frac{\phi}{2} + c.c. = 681 + 10 + 75 = 766\,mm$

Adopt $D = 770\,mm$
$\therefore d = D - cc - \frac{\phi}{2} = 770 - 75 - 10 = 685\,mm$

(iv) Reinforcement bars :-

In x-x dirxn :- max BM at crit. sections in y-y dirxn

$$A_{st_1} = \frac{M_{uy,max} = Max.(M_{AA}, M_{A'A'})}{0.87 f_y (d - 0.416 x_u)}$$

$$= \frac{646.88 \times 10^6}{0.87 \times 415 (d - 0.416 x_u)}$$

As $M_{uy,max} = 646.88\,kNm > M_{ux,max} = 294.29\,kNm$,
provide A_{st_1} at the bottom.

Then, $d = D - cc - \phi/2 = 770 - 75 - 10 = 685\,mm$

$$x_u = \frac{0.87 f_y A_{st_1}}{0.36 f_{ck} b} = \frac{0.87 \times 415 A_{st_1}}{0.36 \times 25 \times 2300} = 0.0174 A_{st_1}$$

$$\Rightarrow A_{st_1} = \frac{1791663.21}{d - 7.238 \times 10^{-3} A_{st_1}}$$

$\Rightarrow 7.238 \times 10^{-3} A_{st_1}^2 - 685 A_{st_1} + 1791663.21 = 0$

$\Rightarrow A_{st_1} = 91947.25\,mm^2$, $2692.15\,mm^2 > A_{st}^{min} = 0.12\%.bD = 0.12\%\times 2\ldots = 2125.\ldots$ OK.

$\Rightarrow n = \frac{91947.25}{\pi 10^2}$, $\frac{2692.15}{\pi 10^2}$

$= 293$, 9

$\Rightarrow$ Spacing (s) $= \frac{2300}{293}\,mm$, $\frac{2300}{9}\,mm$

$= 7.85\,mm$, $255.56\,mm$

→ Say 250 mm

This exp^n is valid if A_{st_2} is area of steel considering the actual width, not unit width (i.e. 'b' used in calc^n of A_{st_2} must be the entire dimension along 'b')
for further clarification, see Sinha Q 11.49

In y-y direcn :–

$$A_{st_2} = \frac{M_{xz,max} = M_{BB'} = M_{AA'} = 294.29 \times 10^6}{0.87 f_y (d - 0.416 x_u)}$$

Since A_{st_2} is to be provided above A_{st_1},

$$d = D - cc - \phi - \frac{\phi}{2} = 770 - 75 - 20 - 10 = 665 \text{ mm}$$

$$x_u = \frac{0.87 f_y A_{st_2}}{0.36 f_{ck} b} = \frac{0.87 \times 415 \times A_{st_2}}{0.36 \times 25 \times 3450} = 0.01163 A_{st_2}$$

$$\therefore A_{st_2} = \frac{294.29 \times 10^6}{0.87 \times 415 (665 - 0.416 \times 0.01163 A_{st_2})}$$

$$\Rightarrow A_{st_2} = \frac{815094.86}{665 - 0.00484 A_{st_2}}$$

$$\Rightarrow A_{st_2} = 136159.85 \text{mm}^2,\ 1236.84 \text{mm}^2 < A_{st}^{min} = 0.12\% \text{ of } bD = 0.12\% * 3450 * 770 = 3187.8 \text{mm}^2$$

$$\therefore A_{st} = 3187.8 \text{ mm}^2$$

For rectangular footing, in arranging reinf. of y-y direcns (ie shorter direcn)

$$\boxed{A_t = \frac{2A_{st_2}}{\frac{L}{B} + 1}} = \frac{2 \times 3187.8}{1.5 + 1} = 2550.24 \text{ mm}^2 > A_{st}^{min} = 0.12\% (bD) = 2125.2 \text{mm}^2 \text{ OK.}$$

are to be provided in the central zone of width $B = 2.3$m.

$\therefore$ The remaining $A_{st_2} - A_t = 637.56 \text{ mm}^2$ is to be provided in each of the end zones of tot. width $L - B = 3.45 - 2.3 = 1.15$ m.

$\therefore$ For central zone : $A_t = 2550.24 \text{ mm}^2$

$$\Rightarrow n = \frac{2550.24}{\pi 10^2} = 9$$

$$\Rightarrow S = \frac{2300}{9} = 256 \text{ mm say } 250\text{mm} < Min(3d, 450) = Min(2055, 450) = 450 \text{mm OK}$$

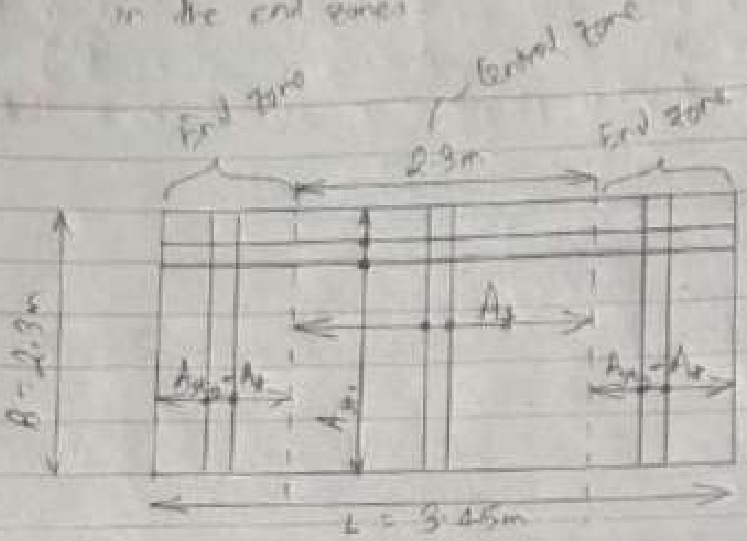
For end zones: $A_{st_2} - A_t = 637.56 \text{mm}^2 < A_{st}^{min} = 0.12\% bD) = 0.12\% * \frac{575}{*2} * 770 = 1062.6 \text{mm}^2$

$\Rightarrow n = 1062.6/\pi 10^2 = 3.4 \sim 4$ $\therefore$ Take $A_{st_2} - A_t = 1062.6 \text{mm}^2$.

$\Rightarrow S = 1150/4 = 287.5$ mm say 250 mm $< Min(3d, 450)$ OK.

Important! Not 1150/2!

(ii) Reinf. bars in the central zone are generally densely packed than in the end zones.



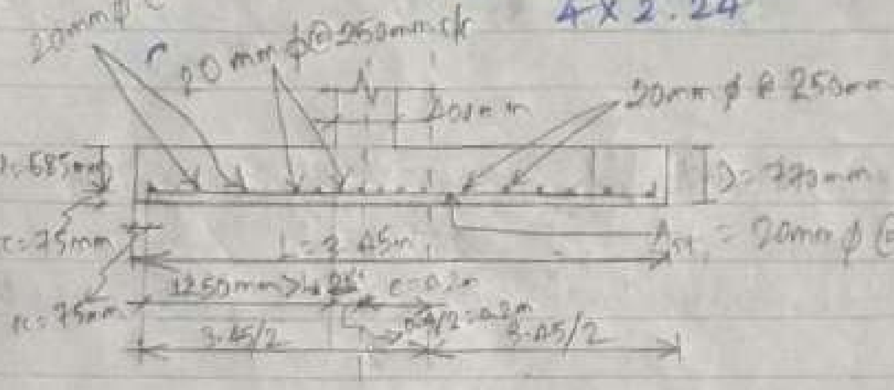
$A_{st_1} = 20mm\,\phi$ @ 250mm c/c
$A_y = 20mm\,\phi$ @ 250mm c/c
$A_{st_2} - A_y = 20mm\,\phi$ @ 250mm c/c

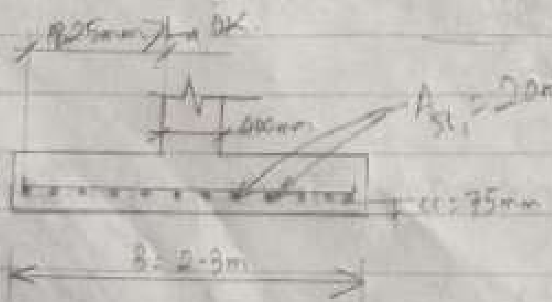
(v) Check development length of reinf. bars in both dirxns of footing:-

Reqd. dev. length in any dirxn $(L_d) = \dfrac{0.87 f_y \phi}{4\tau_{bd}}$

$\tau_{bd} = 1.6 \times 1.4 = 2.24\,N/mm^2$ [Cl. 26.2.1.1]

$\therefore L_d = \dfrac{0.87 \times 415 \times 20}{4 \times 2.24} = 805.91 \approx 806\,mm$





(vi) Check bearing stress:

Bearing stress at contact area of column & footing

$= \dfrac{1.5(DL+LL)}{\text{Area of column}} = \dfrac{P_u}{400 \times 400} = \dfrac{1500 \times 10^3\,N}{160000\,mm^2} = 9.375\,N/mm^2$

Allowable bearing stress $= 0.45 f_{ck} \sqrt{\frac{A_1}{A_2}}$

$A_1 = (b+4d)(l+4d) = (400+4\times685)^2 = 9859600$

$A_2 = bl = 400^2 = 160000$

$\therefore \sqrt{\frac{A_1}{A_2}} = 7.85 \not> 2 \quad \therefore \sqrt{\frac{A_1}{A_2}} = 2$

$\therefore$ Allow. b. stress $= 0.45\times25\times2 = 22.5\,N/mm^2$

Since bearing stress $(9.375\,N/mm^2)$ < Allowable bearing stress $(22.5\,N/mm^2)$, safe.

Redo of (2) by placing column in the centroid of footing :-

(i) Approximate area of footing :-

$$A = \frac{1.1(DL+LL)}{q_{allow}} = 1.1\times\frac{P_u}{1.5}\times\frac{1}{q_{allow}} = 7.33\,m^2$$

Say $8\,m^2$

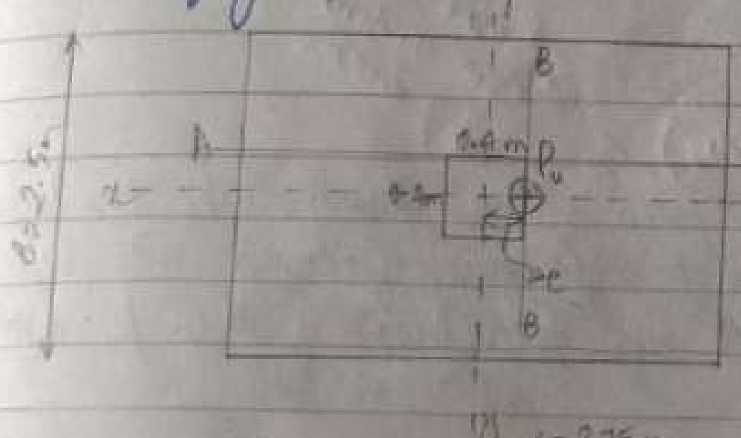
Let $L = 1.5B \therefore LB = 8 \Rightarrow 1.5B^2 = 8 \Rightarrow B = 2.31\,m$

Say $B = 2.5\,m \Rightarrow L = 3.75\,m$

$\Rightarrow A_{prov.} = LB = 2.5\times3.75 = 9.375\,m^2$

(ii) Soil pressure :-

Arranging the column at the center of footing :



$e = M_u/P_u = 0.2\,m$

In the shown loading conditions, the uniaxial loading of column produce a BM $= M_u$ about y-y axis on the footing plan. This in turn causes larger load in the right part of y-y of footing & lesser in the left of y-y given by :-

$$p_u = \frac{P_u}{A} \pm \frac{M_u}{I_{yy}} x$$

Where,

p_u = Stress on footing = Upward soil pressure on footing.

M_u = BM about y-y axis of footing

I_{yy} = MOI about y-y " " " $= \frac{BL^3}{12} = \frac{2.5 \times 3.75^3}{12} = 10.98\,m^4$

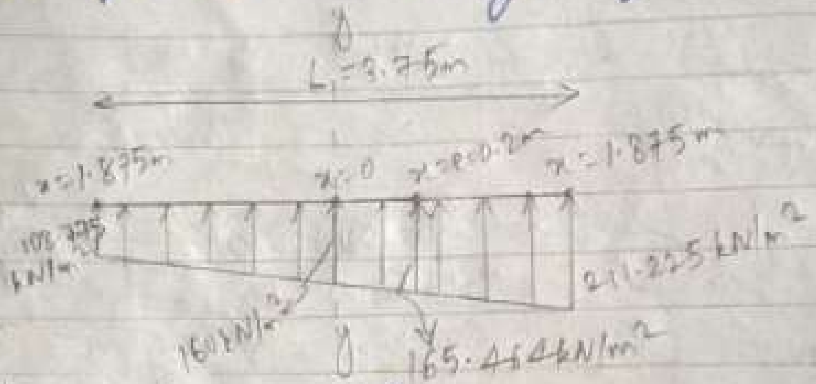
x = Dist. from y-y axis

$\pm$ = +ve for section towards right of y-y axis

$$\therefore\ p_u = \frac{1500}{9.375} \pm \frac{300}{10.98} x \qquad [\text{in kN, m}]$$

$$= 160 \pm 27.32x \quad kN/m^2$$

Thus, soil pressure dist^n along length looks like:



throughout the width B.

(iii) Thickness of footing :-

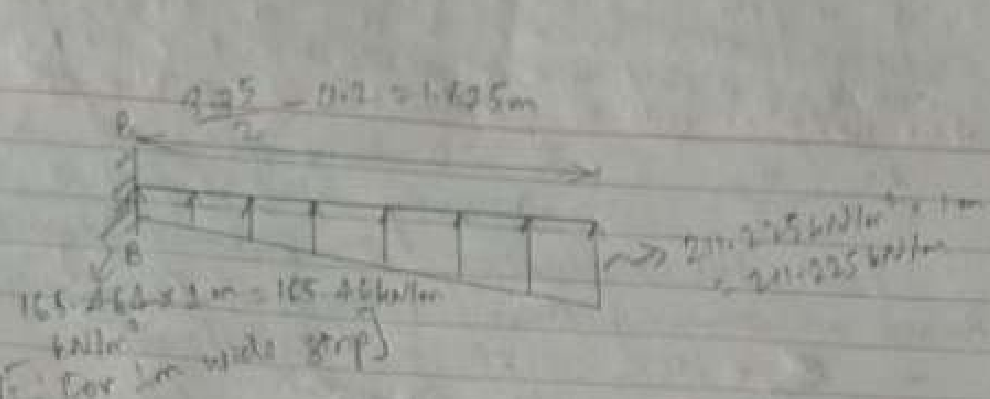
(a) Based on BM :-

$$d = \sqrt{\frac{M_u}{Q\,b}}$$

Find 'd' for M_u at faces of the column. Take the largest. From observation, M_{uy} at right face of column will be greater than M_{uy} at left face b/w of the p_u distribution along the length.

doesn't mean at y-y, just tells about the direction of the moment i.e. about y-y dirxn.

So, M_{uy} at right face :-



Avg. UDL $= \dfrac{165.46 + 211.23}{2} = 188.35\ kN/m$

$\therefore M_{uy} = (188.35 \times 1.675) * \bar{x} \;\; \dfrac{1.675}{2} = 274.78\, kNm$ $\rightarrow 0.871m$

$d = \sqrt{\dfrac{274.78 \times 10^6}{3.457 \times 1000}} = 281.94\ mm \sim 282\,mm$

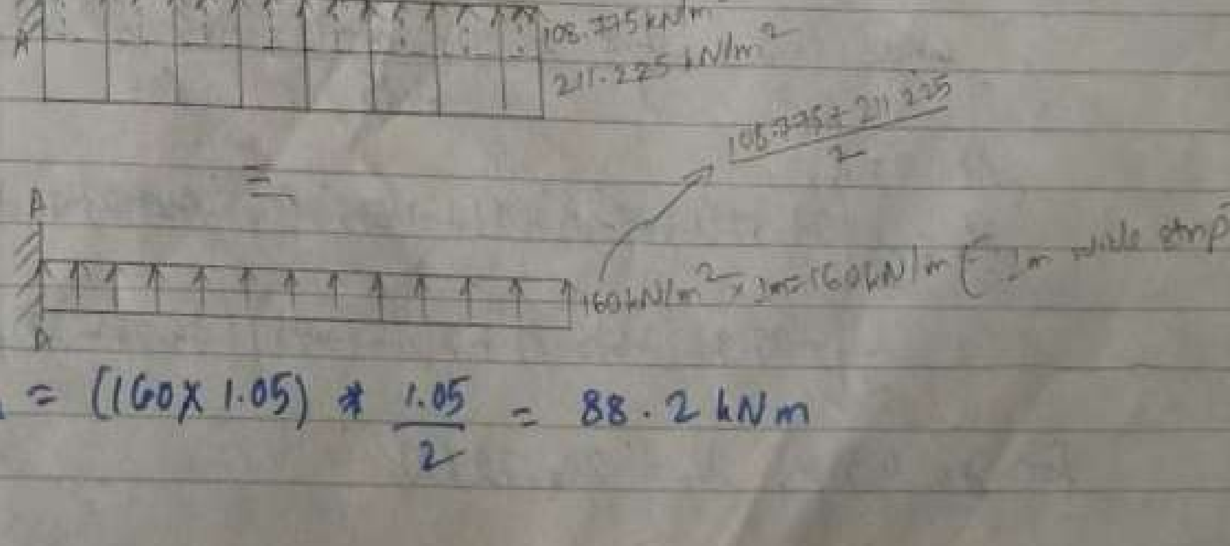
[$\because Q = 0.36 f_{ck} \dfrac{x_{u,l}}{d}\left(1 - 0.416\dfrac{x_{u,l}}{d}\right)$

for Fe415, $\dfrac{x_{u,l}}{d} = 0.48 \;\therefore Q = 3.457$

For 1m width, $b = 1000\,mm$]

From symmetry, M_{ux} at both the horz. faces of the column are equal. So, calc. BM at any one, say upper face:

fig. (i)



$\therefore M_{ux} = (160 \times 1.05) * \dfrac{1.05}{2} = 88.2\ kNm$

Unlike in previous problem, where we calculated 'd' from BM, one-way & two-way shear conditions & chose the largest, here, we directly use a multiple of 'd' obtd. from BM consideration bcoz otherwise, finding 'd' from shear conditions will be complicated in this problem due to non-uniform soil

$d = \sqrt{\dfrac{M_u}{Qb}}$. Q & b being the same & M_u being smaller, obviously, the 'd' for M_{un} will be smaller.

$\therefore$ From BM consideration, $d = 282\,mm$. Adopt $d = 2\times 282 = 564 \approx 570\,mm$

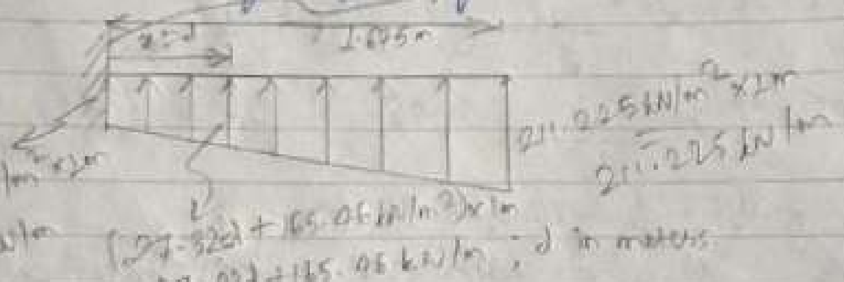
(b) Based on one-way shear: [Check if $\tau_{uv} \leq k\tau_{uc}$]

$k\tau_{uc} = 1\times 0.29 = 0.29\ N/mm^2$

$\tau_{uv} = \dfrac{V_u}{b_0 d}$; $b_0 = 1000\,mm$

Crit. sections on footing for one-way shear are at dist. 'd' outward from the column faces. Due to similar reasons as those for BM consideration, calculating V_u at dist. 'd' right of the right column face and at 'd' above the upper column face will be enough.

For V_u at 'd' right of the right face of column :-



$$V_u(x) = \frac{1}{2}\left[S(l-x)^2 + 2(x-l)(a+Sx)\right] + \frac{1}{3}\left[-S(l-x)^2 + 2(x-l)(b-a-Sx)\right]$$

where $S = \dfrac{b-a}{l} = \dfrac{211.225 - 165.464}{1.675} = 27.32$

At $x = d$,

$$V_u = \frac{1}{2}\left[27.32(1.675-d)^2 + 2(d-1.675)(165.46+27.32d)\right] + \frac{1}{3}\left[-27.32(1.675-d)^2 + 2(d-1.675)(45.76-27.32d)\right]$$

For $d = 0.57\,m$, $V_u = 216.72\,kN$

pressure p_u & thus a UVL loading rather than the simpler UDL.
And why multiply d obtd from BM condition? Also 'Two way a SR UR [illegible] for which d must > d_{bal} and it is d_{bal} that we get from $\sqrt{\frac{M_u}{Q\,b}}$ (ii)
BM doesn't govern footing thickness as much as shear does, so $d = d_{bal} \times (2 \text{ to } 3)$

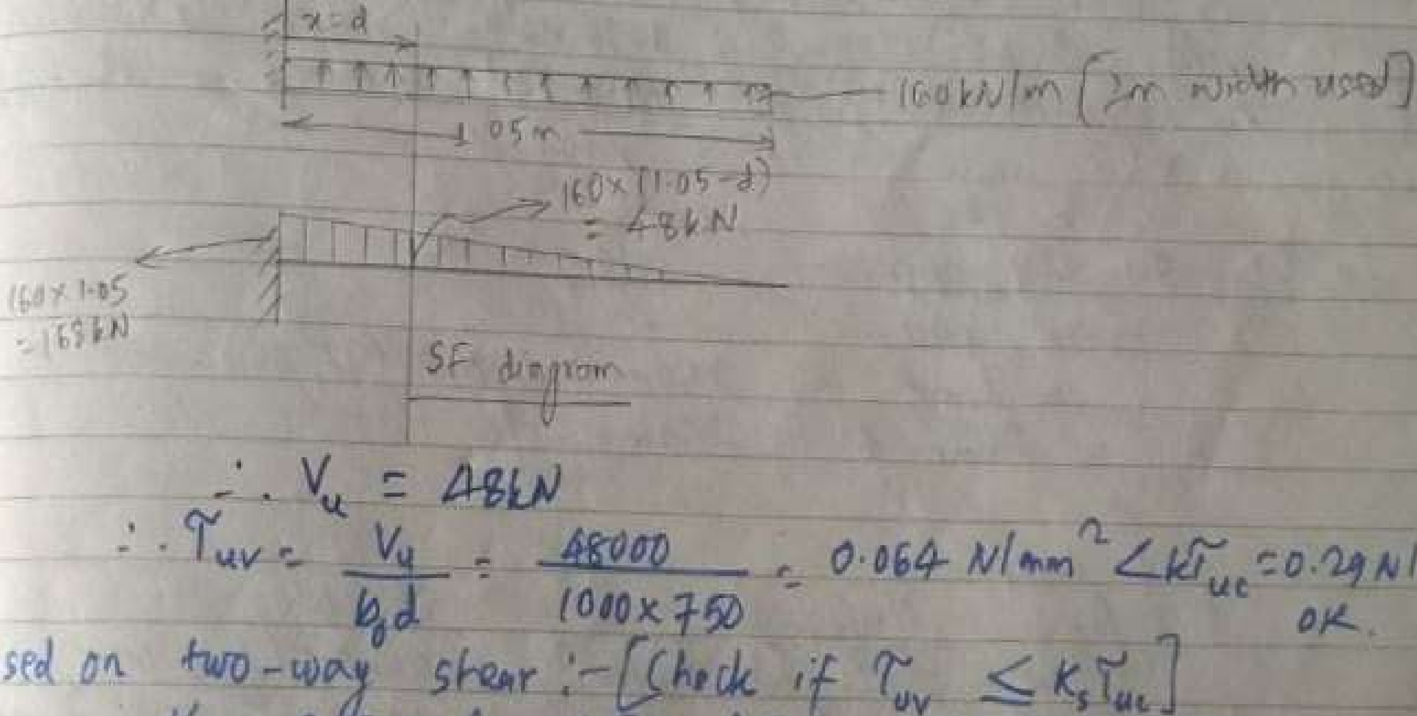
$$\therefore \tau_{uv} = \frac{216.72 \times 10^3\ N}{1000\,mm \times 570\,mm} = 0.38\ N/mm^2$$

Since $\tau_{uv} > k\tau_{uc}$, inc. d to 750 mm

(then, $\cancel{\tau_{uv}}$ $V_u = 183.7\ kN$

$\Rightarrow \tau_{uv} = \frac{183.7 \times 10^3}{1000 \times 750} = 0.245\ N/mm^2$
$< k\tau_{uc} = 0.29\ N/mm^2$
OK.

For V_u at 'd' above the upper face of column :-
loading fig. = fig. (i)



$$\therefore V_u = 48\,kN$$

$$\therefore \tau_{uv} = \frac{V_u}{b_0 d} = \frac{48000}{1000 \times 750} = 0.064\ N/mm^2 < k\tau_{uc} = 0.29\ N/mm^2$$
OK.

(c) Based on two-way shear :- [Check if $\tau_{uv} \le K_s \tau_{uc}$]

$K_s = 0.5 + \frac{B}{L} = 0.5 + \frac{2.5}{3.75} = 1.167 \not> 1 \therefore K_s = 1$

$\tau_{uc} = 0.25\sqrt{f_{ck}} = 0.25\sqrt{25} = 1.25\ N/mm^2$

$$\tau_{uv} = \frac{p_u\left[LB - (l+d)(b+d)\right]}{2\left[(l+d) + (b+d)\right] d} = \frac{160\left[3.75 \times 2.5 - (0.4 + 0.75)^2\right]}{2\left[2(0.4 + 0.75)\right] \times 0.75} = 0.3[illegible]\ N/[illegible]$$

As $\underset{(0.373)}{\tau_{uv}} < \underset{(1.25)}{K_s \tau_{uc}}$, safe.

$\therefore$ Adopt $D = d + \phi/2 + cc = 750 + \frac{20}{2} + 70 = 830\,mm$; $d = 750\,mm$, $\phi = 20\,mm$, $cc = 70\,mm$.

reinf for $M_{uy,max}$

(iv) Reinforcement bars :- As $M_{uy,max} > M_{ux,max}$, place A_{st_1} at bottom.

In x-x dirxn :

$$A_{st_1} = \frac{M_{uy,max}}{0.87f_y(d-0.416x_u)} = \frac{274.78\times10^6}{0.87\times415(750-0.416x_u)}$$

$$x_u = \frac{0.87f_yA_{st_1}}{0.36f_{ck}b} = \frac{0.87\times415A_{st_1}}{0.36\times25\times1000} = 0.0401A_{st_1}$$

cause 1m width was used to calc. M_{uy}

$$\Rightarrow A_{st_1} = \frac{761058.025}{750-0.01668A_{st_1}}$$

$$\Rightarrow 0.01668A_{st_1}^2 - 750A_{st_1} + 761058.025 = 0$$

$$\Rightarrow A_{st_1} = 43925.29\,mm^2,\ 1038.74\,mm^2 > A_{st}^{min} = 0.12\%\ (bD) = 0.12\%\times1000\times830 = 996\,mm^2\ OK$$

$$\Rightarrow n = \frac{43925.29}{\pi10^2},\ \frac{1038.74}{\pi10^2}$$

$$= 140,\ 4$$

$$\Rightarrow S = \frac{1000\,mm}{140},\ \frac{1000\,mm}{4}$$

$$= 7\,mm,\ 250mm < Min.(3d, 450) = 450\,mm\ OK$$

$\therefore A_{st_1} = 20mm\ \phi$ @ 250mm c/c

In y-y dirxn :

$$A_{st_2} = \frac{M_{ux,max}}{0.87f_y(d-0.416x_u)} = \frac{88.2\times10^6}{0.87\times415(d-0.416x_u)}$$

Since A_{st_1} is placed at bottom, 'd' for $A_{st_2} = 750-\phi = 750-20 = 730\,mm$

$$x_u = \frac{0.87f_yA_{st_2}}{0.36f_{ck}b} = \frac{0.87\times415A_{st_2}}{0.36\times25\times1000} = 0.0401A_{st_2}$$

cos 1m width ($\leftrightarrow$) was used to calc. M_{ux}

$$\Rightarrow A_{st_2} = \frac{244287.49}{730-0.01668A_{st_2}}$$

$$\Rightarrow A_{st_2} = 43427.75\,mm^2,\ 337.24\,mm^2 < A_{st}^{min} = 0.12\%(bD) = 996\,mm^2$$

1m

NOTE: A_{st_2} in the expr for A_t represents total area of steel in the corresponding direction, not just for 1m width.

$\therefore$ Provide $A_{st_2} = A_{st}^{min} = 996\ mm^2$ per 1m width

In the central zone of width $= B = 2.5m$, provide :

$$A_t = \frac{2A_{st_2}}{\frac{L}{B}+1} = \frac{2\times(996\times 3.75)}{\frac{3.75}{2.5}+1} = 2988\ mm^2 > A_{st}^{min} = 0.12\%(bD) = 2490\ mm^2$$

2500

$\therefore$ Provide $A_t = 2988\ mm^2$ → area reqd. in width 2.5m (central zone)

i) $n = \frac{2488}{\pi 10^2} = \cancel{12}\ 10$

ii) $S = \frac{2500}{10} = 250\ mm < Min.(3d, 450)$ OK.

Say 220 mm

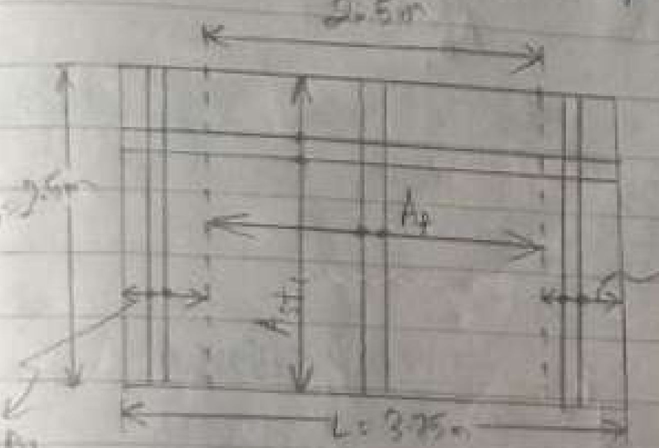
$\therefore A_t = 20mm\,\phi$ @ 220 mm c/c

In the end zones, provide $A_{st_2} - A_t = 747 < A_{st}^{min} = 0.12\%(bD) = 1245\ mm^2$

996×3.75−2988   3750−2500

$A_{st_2} - A_t = 1245\ mm^2 \Rightarrow n = \frac{1245}{\pi 10^2} = 4 \Rightarrow S = \frac{3750-2500}{4} = 312.5\ mm.$

ie $20mm\,\phi$ @ 300 mm



$A_{st_1} = 20mm\,\phi$ @ 250 mm c/c

$A_t = 20mm\,\phi$ @ 220 mm c/c

$A_{st_2} - A_t = 20mm\,\phi$ @ 300 mm c/c

(v) Check bearing stress :-

no need of avg. stress due to M_u wh it is in there due to P_u, just P_u/A

Bearing stress at the base of column $= \frac{P_u}{A_{col.}}\left(=\frac{1500\times10^3}{400\times400}\right) + \cancel{\frac{M_u}{I_{yy}}e}$

$$= 9.375 + \cancel{\frac{300\times10^6}{\frac{400\times400^3}{12}}\times 200}$$

$= \cancel{37.5\ N/mm^2}$

Allowable bearing pressure $= 0.45 f_{ck}\sqrt{\frac{A_1}{A_2}}$

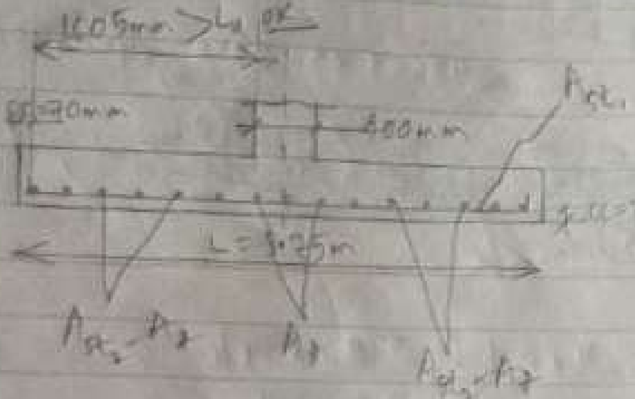
$$A_1 = (l+4d)(b+4d) = (400+4\times 750)^2$$
$$A_2 = bl = 400^2$$
$$\therefore \sqrt{\frac{A_1}{A_2}} = 8.5 \not> 2 \quad \therefore \text{Take } \sqrt{\frac{A_1}{A_2}} = 2$$

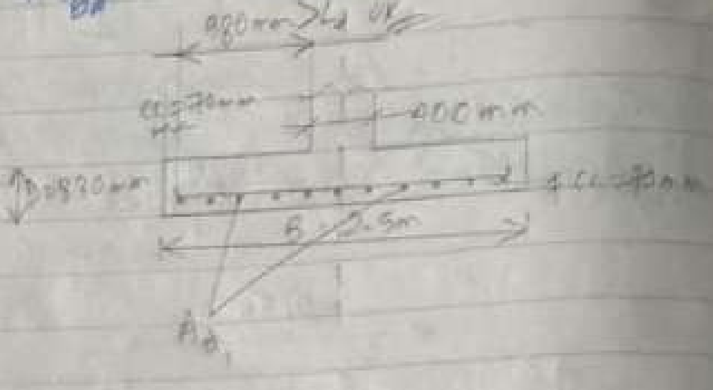
Allowable bearing pressure $= 0.45 \times 25 \times 2 = 22.5\ N/mm^2$

Since bearing pressure < Allowable bearing pressure, OK.

(vi) Check development length:-

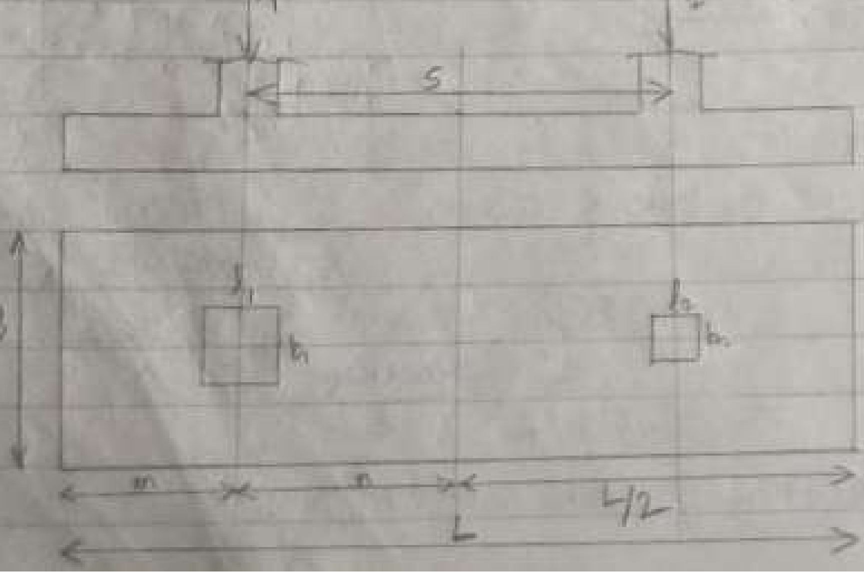
$$\text{Reqd. dev. length } (L_d) = \frac{0.87 f_y \phi}{4 \tau_{bd}} = \frac{0.87 \times 415 \times 20}{4 \times 2.24} = 806\ mm$$





Class #25

(*) Design combined footing:-



$P_1 = 1500 kN$

$P_2 = 1000 kN$

$b_1 = l_1 = 800 mm$

$b_2 = l_2 = 600 mm$

$S = 4m$

$m = 1.72m$

$q_{allow} = 200 kN/m^2$

M20, Fe415

Combined footing is not designed for 1m wide strip. Whole dimensions are used.

Sol^n:-

(I) Find the size of combined footing :

Area of footing (A) $= \dfrac{1.1(P_1+P_2)}{q_{allow}} = 13.75 m^2$

For length of footing, imposing the condition that the resultant of P_1+P_2 acts at the mid-point of L :

Taking moment about line of action of P_1 :

Length of footing so fixed will produce axial loading, i.e., uniform soil pressure

$P_2 * S = (P_1+P_2) * n$

$\Rightarrow n = \dfrac{P_2 S}{P_1+P_2} = \dfrac{1000 \times 4}{2500} = 1.6m$

$\therefore$ Length of footing $(L) = 2(m+n)$

$= 2(1.72+1.6)$

$= 6.64m$

$\therefore$ Breadth of footing $(B) = \dfrac{A}{L} = \dfrac{13.75}{6.64} = 2.07m \sim 2.2m$.

(II) Determine soil upwards pressure:

In longer dir^n:

$p_u = \dfrac{1.5(P_1+P_2)}{L} = \dfrac{1.5 \times 2500}{6.64} = 564.76 kN/m$

In shorter dir^n:

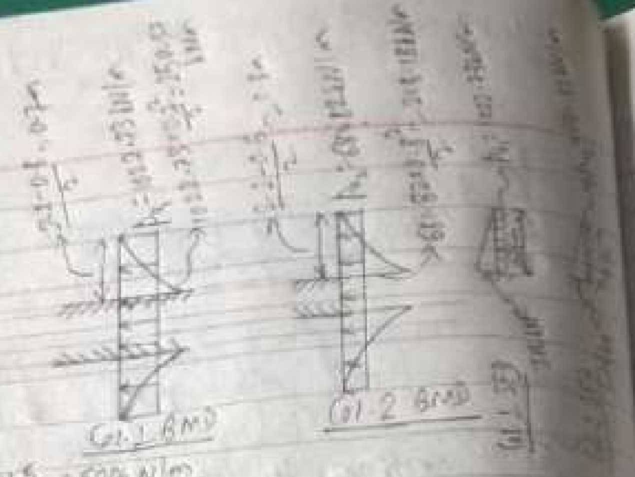
For column 1:

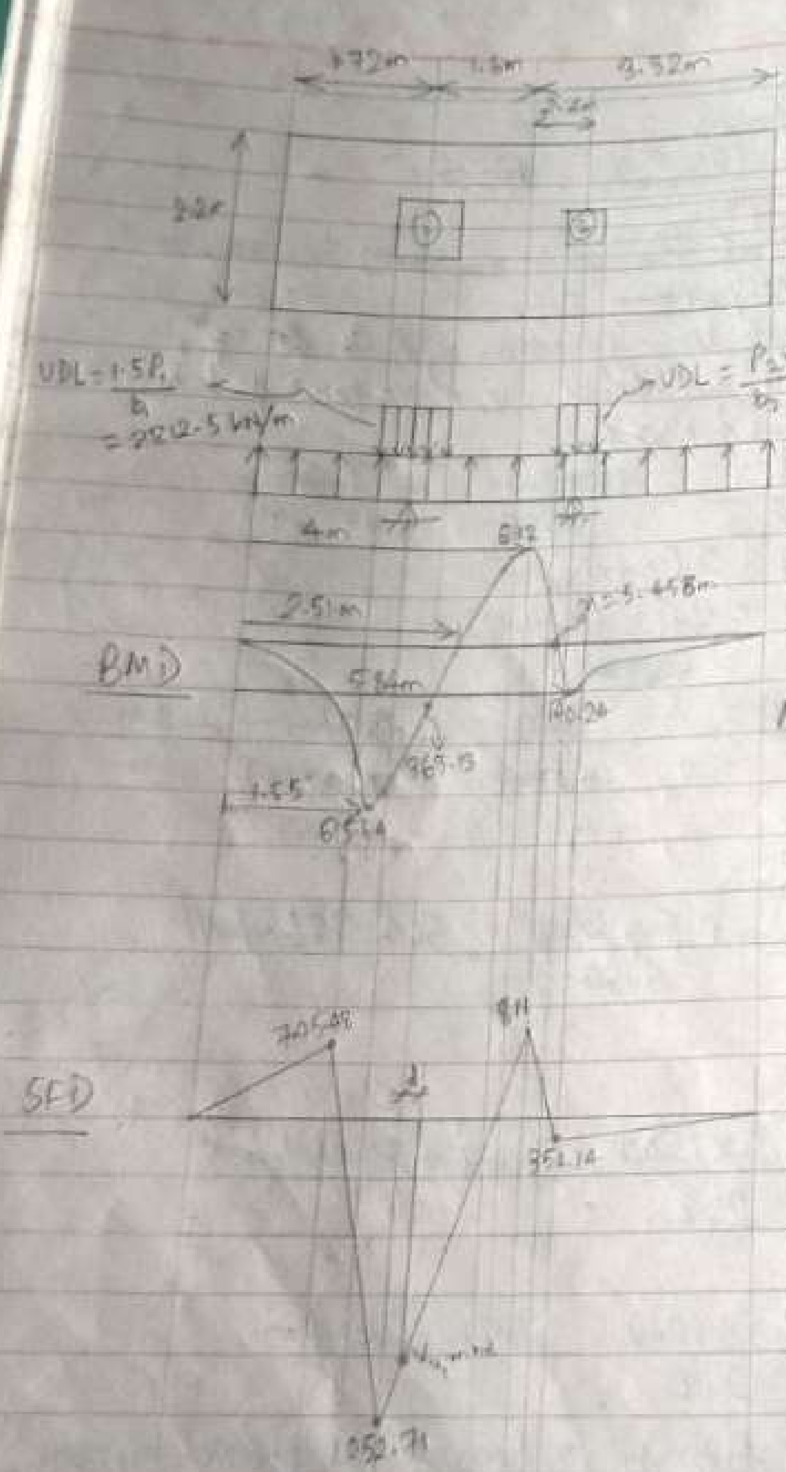
$p_u = \dfrac{1.5 P_1}{B} = \dfrac{1.5 \times 1500}{2.2} = 1022.73 kN/m$

For col. 2:

$p_u = \dfrac{1.5 P_2}{B} = \dfrac{1.5 \times 1000}{2.2} = 681.82 kN/m$

(III) Find responses (BM & SF) of footing in longer & shorter dir^{n}s:





$$M_x = \frac{564.76x^2}{2} - \frac{2812.5(x-1.32)^2}{2} \; ; x <$$

$$M_x = \frac{564.76x^2}{2} - 2812.5\times0.8\times(x-1.72) - \frac{2500\times(x-5.42)^2}{2} \; ; x \in [5.12, 6.5]$$

$$V_x = \frac{dM_x}{dx}$$

(IV) Design footing for max responses :-

(1) Determine depth of footing :

(a) Depth reqd. to resist max. BM :-

$$d > d_{BM} = \sqrt{\frac{M_{u,max}}{Qb}} = \sqrt{\frac{615.64 \times 10^6}{Q * 2200}}$$

not 1000mm coz. M_u was calculated due to whole 2.2m width, not just due to 1m wide strip

$$Q = 0.36 f_{ck} \frac{x_{u,l}}{d}\left(1 - 0.416\frac{x_{u,l}}{d}\right)$$

$$= 0.36 \times 20 \times 0.48 (1 - 0.416 \times 0.48)$$

$$= 2.766$$

$\therefore d_{BM} = 318.072$ mm

Adopt $d = 820$ mm

(b) Check for one-way shear :

is $\tau_{uv} \leq k\tau_{uc}$?

As D will obv. be > 300 mm, $k = 1$ [Cl. 40.2.1.1]

$$\% p = \frac{A_{st}^{min}}{bd} = \frac{0.12\% \; bD}{bd} \simeq 0.12\%$$

for M20, $\tau_{uc} = 0.28$ N/mm² [Table 19]

$$\tau_{uv} = \frac{V_{u,max}}{b_0 d}$$

in combined footing, b_0 is never taken 1m coz V_u is calculated using UDL obtd. by using whole width, not just 1m wide strip.

$V_{u,max}$ = Max. SF out of ~~all~~ SF's at all the crit. sections (ie 'd' outward from column) = $V_{u,max}$ in longer dirn (in combined footing, this is always the case)

From SFD diag., largest SF = 1052.71 kN. At d = 820mm right of it,

$$V_u = V_{u,max} = \frac{4 - (1.72 + 0.8/2) - d}{4 - (1.72 + 0.8/2)} * 1052.71 = 593.55 \text{ kN}$$

$b_0 = 2200$ mm

$$\therefore \tau_{uv} = \frac{V_{u,max}}{b_0 d} = \frac{593.55 \times 10^3}{2200 \times 820} = 0.329 \text{ N/mm}^2 > k\tau_{uc} = 0.28 \text{ N/mm}^2$$

$\therefore$ Adopt $d = 900\,mm$

Then,

$$V_{u,max} = \frac{1.88 - d}{1.88} \times 1052.7 = 548.75\ kN$$

$$\therefore \tau_{uv} = \frac{V_{u,max}}{b_o d} = \frac{548.75 \times 10^3}{2200 \times 900} = 0.277\ N/mm^2 < k\tau_{uc} = 0.28\ N/mm^2$$

OK.

(c) Check for two-way shear: $\tau_{uv} \le k_s \tau_{uc}$

For column 1 :-

$$V_u = \cancel{1.5}P_{u1} - p_u (l_1 + d)(b_1 + d)$$ [See Sinha @ 11.65]

$$p_u = \frac{1.5 (P_1 + P_2)}{LB} = \frac{1.5 \times 2500}{6.64 \times 2.2} = 256.71\ kN/m^2$$

~~$V_u = 1.5 \times$~~ $P_{u1} = 1.5 \times P_1 = 1.5 \times 1500 = 2250\ kN$

$$\therefore V_u = 2250 - 256.71 (0.8 + 0.9)^2 = 1508.11\ kN$$

$$\tau_{uv} = \frac{V_u}{b_o d} = \frac{1508.11 \times 10^3}{b_o \times 900}$$

$$b_o = 2(0.8 + d + 0.8 + d) = 6.8\ m$$

$$\therefore \tau_{uv} = \frac{1508.11 \times 10^3}{6800 \times 900} = 0.246\ N/mm^2$$

$$k_s = 0.5 + \frac{B}{L} = 0.5 + \frac{2.2}{6.64} = 0.831$$

$$\tau_{uc} = 0.25\sqrt{f_{ck}} = 0.25\sqrt{20} = 1.118$$

$$\therefore \tau_{uv} = 0.246\ N/mm^2 < k_s \tau_{uc} = 0.929\ N/mm^2$$ OK.

So, adopt $d = 900\,mm$, $\phi = 20\,mm$, $cc = 70\,mm$

$$\therefore D = d + \frac{\phi}{2} + cc = 980\,mm$$

(ii) Reinforcement bars:-

(a) Reinf. bars in longer dirxn:- [SR URS]

$$A_{st,}\ \text{for}\ (M_u = 615.64\ kNm) = \frac{M_u}{0.87 f_y (d - 0.416 x_u)} = \frac{615.64 \times 10^6}{0.87 \times 415 (900 - 0.416 x_u)}$$

2200 mm as M_u has been calc. using whole width

$$x_u = \frac{0.87 f_y A_{st_1}}{0.36 f_{ck} b} = \frac{0.87 \times 415}{0.36 \times 20 \times 2200} A_{st_1} = 0.0228 A_{st_1}$$

$$\therefore A_{st_1} = \frac{1705.14 \times 1000}{900 - 0.00948 A_{st_1}}$$

$$\Rightarrow 0.00948 A_{st_1}^2 - 900 A_{st_1} + 1705.14 \times 10^3 = 0$$

$$\Rightarrow A_{st_1} = \cancel{94994.81\ mm^2},\ 93002.71\ mm^2,\ \cancel{1944}\ 1934\ mm^2$$

$$\cancel{\Rightarrow n =}\ A_{st_1} < A_{st}^{min} = 0.12\%\ bD)$$

$$= \frac{0.12}{100} \times 2200 \times 980 = 2587.2\ mm^2$$

$$\therefore \text{Take } A_{st_1} = A_{st}^{min} = 2587.2\ mm^2$$

$$\Rightarrow n = \frac{2587.2}{\pi 10^2} = 9$$

$$\Rightarrow s = \frac{2200}{9} = 244 \sim 220\ mm < Min(3d, 450)\ OK.$$

$\therefore A_{st_1} = 20$ mm ϕ @ 220 mm c/c at bottom

$$A_{st_2} \text{ for } (M_u = 612\ kNm) = \frac{612 \times 10^6}{0.87 \times 415 (900 - 0.416 \times x_u)}$$

$$x_u = \frac{0.87 \times 415}{0.36 \times 20 \times 2200} A_{st_2}$$

$$\therefore A_{st_2} \approx 2587\ mm^2$$

$$\therefore s \approx 220\ mm$$

$\therefore A_{st_2} = 20$ mm ϕ @ 220 mm c/c at top

$$A_{st_3} \text{ for } (M_u = 140.24\ kNm) = \frac{140.24 \times 10^6}{0.87 \times 415 (900 - 0.416 \times 0.0228 A_{st_3})}$$

$$\Rightarrow 0.00948 A_{st_3}^2 - 900 A_{st_3} + 388422.66 = 0$$

$$\Rightarrow A_{st_3} = 94503.15\ mm^2,\ 433.56\ mm^2 < A_{st}^{min} = 2587.2\ mm^2$$

$$\therefore \text{Take } A_{st_3} = A_{st}^{min}$$

$\therefore A_{st_3} = 20$ mm ϕ @ 220 mm c/c at bottom

is generally taken as $l_1+1.5d$, with the column lying in the middle of this range.
on footing [illegible] are used, but 'SR' instead. The longitudinal extent or influence of this [illegible]
$b = b_1 + 1.5d$ are while calculating $M_u = 250.57$ kNm, for UDL in shorter dirn, not the total design [illegible]

(6) Reinf. bars in shorter dirn :- [SR ORS]

$$A_{st_4} \text{ for } (M_u = 250.57\,kNm) = \frac{250.57 \times 10^6}{0.87 \times 415 (900 - 0.416 x_u)}$$

$$x_u = \frac{0.87 \times 415 A_{st_4}}{0.36 \times 20 \times \cancel{600} (l_1 + 1.5d)}$$

$$= \frac{50.146 A_{st_4}}{800 + 1.5 \times 900}$$

$$= 0.023 A_{st_4}$$

$$\therefore A_{st_4} = \frac{694003.6}{900 - 0.00957 A_{st_4}}$$

$$\Rightarrow 0.00957 A_{st_4}^2 - 900 A_{st_4} + 694003.6 = 0$$

$$\Rightarrow A_{st_4} = 93266.34\,mm^2,\ 777.54\,mm^2$$

$$A_{st_4} < A_{st}^{min} = 0.12\%\, bD$$

$$= \frac{0.12}{100} \times (l_1 + 1.5d) \times 980$$

$$= 2528.4\,mm^2$$

$$\therefore \text{Adopt } A_{st_4} = A_{st}^{min} = 2528.4\,mm^2$$

$$\Rightarrow n = \frac{2528.4}{\pi 10^2} = 9$$

$$\Rightarrow S = \frac{b}{9} = \frac{l_1 + 1.5d}{9} = \frac{2150}{9} = 238 \sim 220\,mm$$

$$A_{st_5} \text{ for } (M_u = 218.18\,kNm) = \frac{218.18 \times 10^6}{0.87 \times 415 (900 - 0.416 x_u)}$$

$$x_u = \frac{0.87 \times 415 A_{st_5}}{0.36 \times 20 \times (l_2 + 1.5d)} = \frac{50.146 A_{st_5}}{600 + 1.5 \times 900} = 0.0257 A_{st_5}$$

$$\therefore A_{st_5} = \frac{604293.03}{900 - 0.0107 A_{st_5}}$$

$$\Rightarrow 0.0107 A_{st_5}^2 - 900 A_{st_5} + 604293.03 = 0$$

$$\Rightarrow A_{st_5} = 83435.27\,mm^2,\ 676.88\,mm^2 < A_{st}^{min} = 0.12\%\, bD$$

$$= \frac{0.12}{100} \times (l_2 + 1.5d) \times 980$$

$$= 2293.2\,mm^2$$

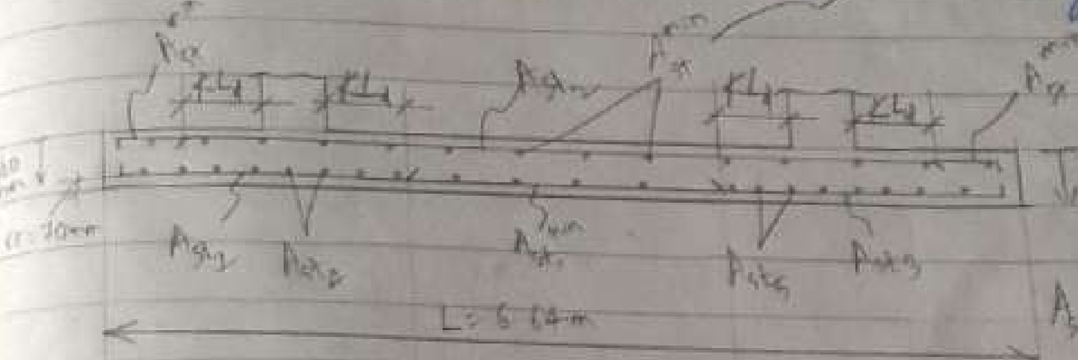
$\therefore$ Adopt $A_{st_5} = 2293.2\,mm^2$

∴) $n = \frac{2293.2}{\pi 10^2} = 8$

∴) $S = \frac{b}{8} = \frac{l_2 + 1.5d}{8} = \frac{1950}{8} = 243.75$

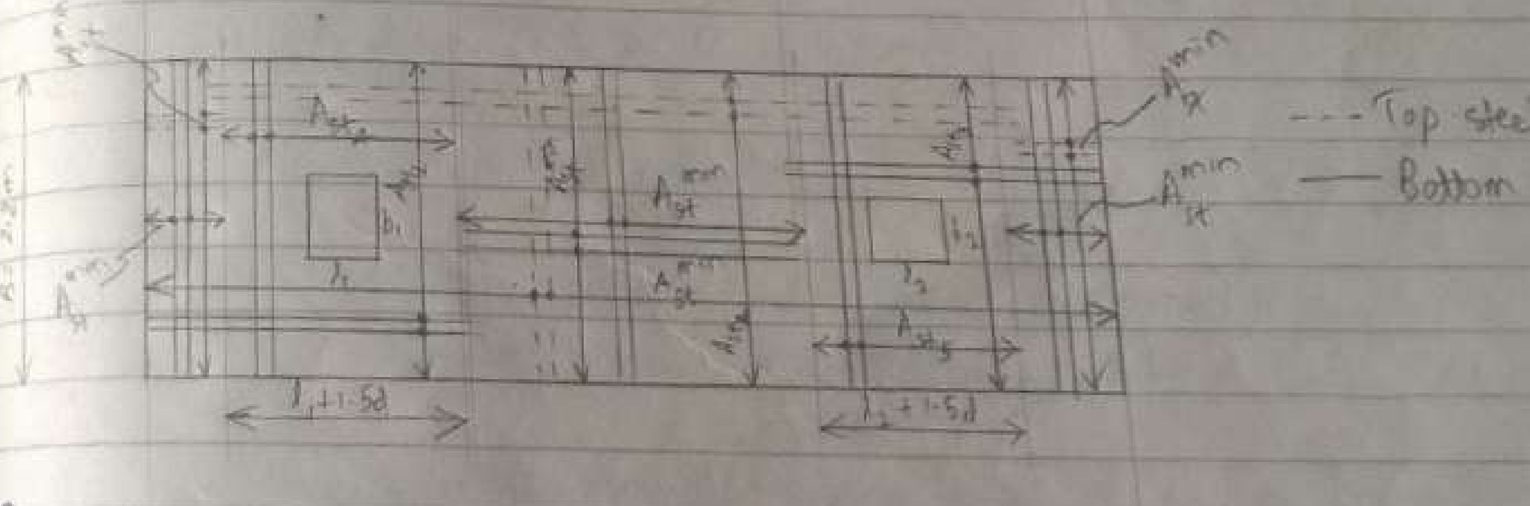
Say 220 mm

For possible upside tension in transverse (shorter dirn) bending.



$A_{st_1} = A_{st_2} = A_{st_3} = A_{st_4} = A_{st_5} = A_{st}^{min}$

$= 20\,mm\,\phi$ @ 220 mm c/c



(v) Check dev. length from face of col. in shorter dirn:

Reqd. dev. length $L_d = \frac{0.87 f_y \phi}{4 \tau_{bd}}$

For M20 & deformed bars, $\tau_{bd} = 1.6 \times 1.2 = 1.92\,N/mm^2$

$\therefore L_d = \frac{0.87 \times 415 \times 20}{4 \times 1.92} = 940.23\,mm$

$$= \frac{0.12}{100} \times (l_2 + 1.5d) \times 980$$

$$= 2293.2\,mm^2$$

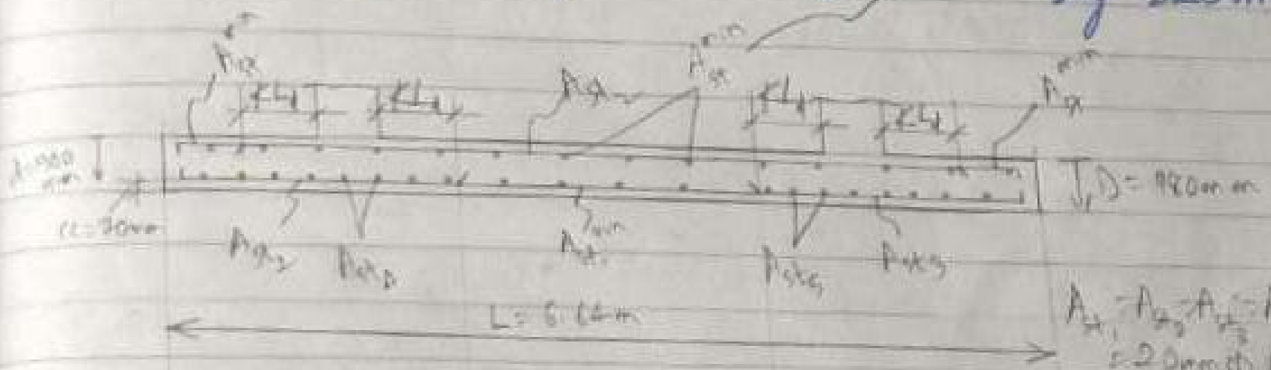
$\therefore$ Adopt $A_{st_5} = 2293.2\,mm^2$

2) $n = \frac{2293.2}{\pi\, 10^2} = 8$

3) $S = \frac{b}{8} = \frac{l_2 + 1.5d}{8} = \frac{1950}{8} = 243.75$

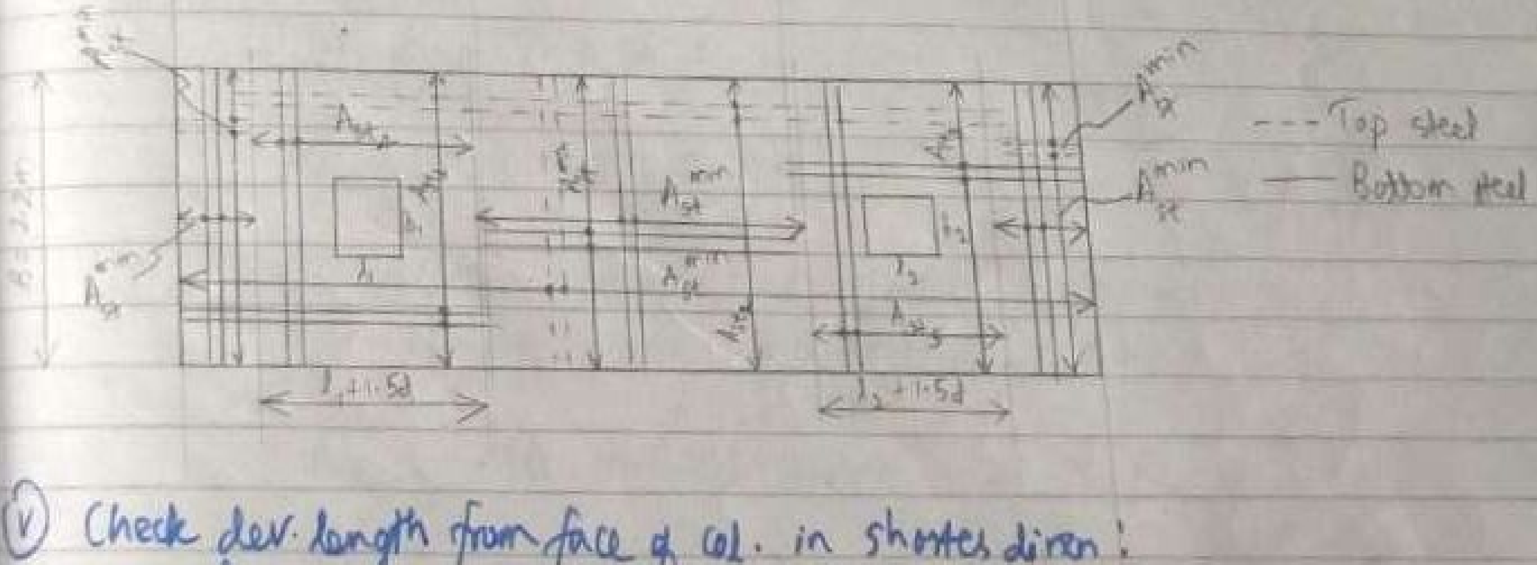
Say 220 mm

For possible upside tension in transverse (shorter dirn) bending.



$A_{st_1} = A_{st_2} = A_{st_3} = A_{st_4} = A_{st_5} = A_{st}^{min}$

$= 20\,mm\, \phi$ @ 220 mm c/c



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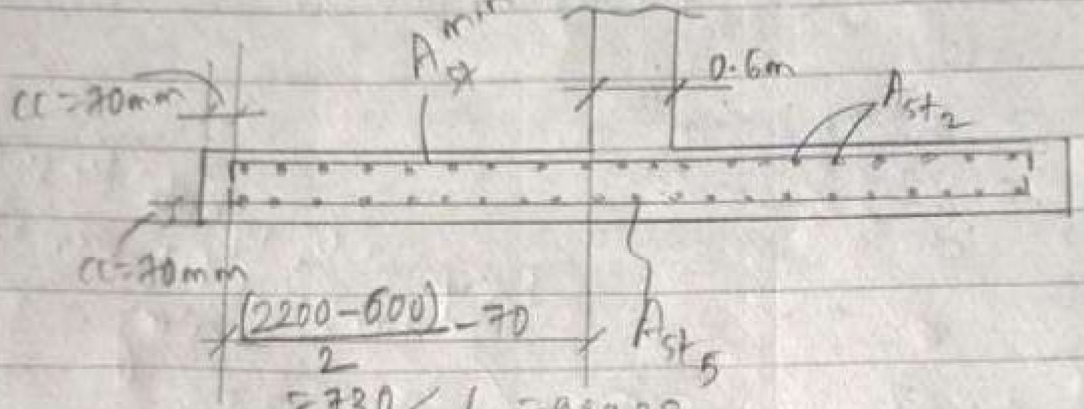
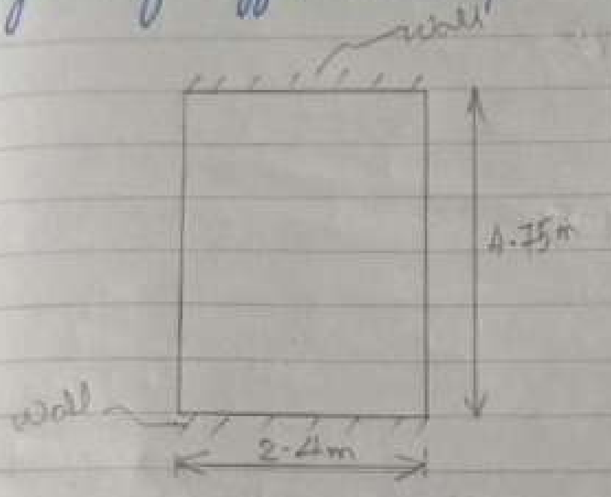


$= 730 < L_d = 940.23$

So, ?

Class # 26

(1) Design dog-legged stair for the following data :-



Floor height = 3.52m
Floor finish = 0.6 kN/m^2
Wall thickness = 300mm
Live load = 3 kN/m^2
M20, Fe415

Plan of staircase

Soln:—

1. Geometrical Design :-

Width of stair = $\frac{2.4}{2}$ = 1.2m

No. of rises = $\frac{3.52 \times 10^3}{\text{Rises}}$

let Rises = 160mm and Tread = 250mm

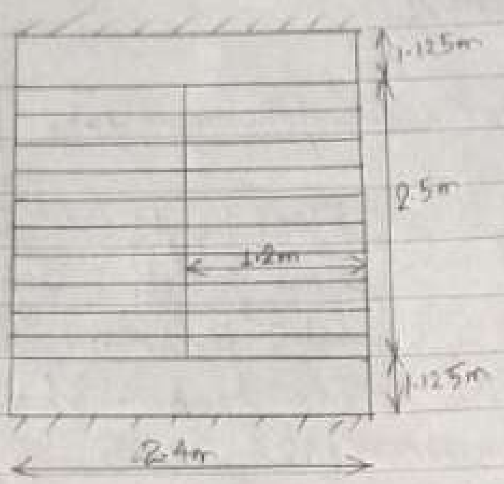
$\therefore$ No. of rises = $\frac{3.52 \times 10^3}{160}$ = 22

No. of rises in each flight = $\frac{22}{2}$ = 11

No. of tread in each flight = 11 − 1 = 10

Horizontal length of going = Tread × 10 = 2500 mm = 2.5m

Width of landing = $\frac{4.75 - 2.5}{2}$ = 1.125m



2. Structural design :-

(i) Find depth of waist slab :-

Take $d = \frac{\text{span(eff)}}{20} = \frac{\text{going} + 1.125/2 + 1.125/2}{20}$

$= \frac{2.5 + 1.125}{20}$

$= 181.25$ mm

Adopt $D = 250$ mm

Using cc: 20 mm & $\phi = 16$mm,

$D = d + \frac{\phi}{2} + cc$

$\therefore d = 250 - \frac{16}{2} - 20 = 222$mm